Primordial gravitational waves in bouncing universe

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Both inflationary and ekpyrotic scenarios can account for the origin of the large scale structure of the universe. It is often said that detecting primordial gravitational waves is the key to distinguish both scenarios. We show that this is not true if the gauge kinetic function is present in the ekpyrotic scenario. In fact, primordial gravitational waves sourced by the gauge field can be produced in an ekpyrotic universe. We also study scalar fluctuations sourced by the gauge field and show that it is negligible compared to primordial gravitational waves. This comes from the fact that the fast roll condition holds in ekpyrotic models.

1. Introduction

Inflation has succeeded in solving several issues in big bang cosmology and explaining the temperature anisotropy of the cosmic microwave background radiation (CMB) and the large scale structure of the universe. However, it is known that bouncing universe models[1] such as the ekpyrotic scenario[2] based on superstring theory[3] can do the same job[4].[a] Therefore, it is important to clarify which scenario is actually realized in the early stage of the universe.

In the ekpyrotic scenario, the primordial fluctuations are produced in a slowly contracting (ekpyrotic) phase. The spectrum of the scalar and tensor vacuum fluctuations becomes blue-tilted in the phase. We therefore need an additional scalar field to explain the temperature anisotropy of the CMB[6]. Moreover, in the ekpyrotic scenario, the amplitude of primordial gravitational waves[7] is quite small and practically unobservable[8]. Hence, it is often said that, if we could detect the primordial gravitational waves, we would be able to disprove the ekpyrotic scenario. However, if there could exist another mechanism for producing gravitational waves in the ekpyrotic scenario, the story would be completely different. Indeed, we show that there exists a mechanism for producing abundant gravitational waves in the ekpyrotic phase.

The key is the presence of magnetic fields in the early universe. Observationally, there are several evidences for magnetic fields to exist on various cosmological scales[9]. Although the origin of primordial magnetic fields is unknown, the presence of magnetic fields on extra galactic scales[10] implies that the seed of magnetic fields must be produced in the early universe. Notably, there are attempts to make primordial magnetic fields with the gauge kinetic function in an inflationary universe[11] or in a bouncing universe[12].

This report is a review of our previous paper[13]. We first show that scale invariant magnetic fields can be produced in the ekpyrotic phase in the presence of the gauge kinetic function. Next, we show that the magnetic fields can be a source of abundant

[a]The pre-big bang scenario is also a kind of the models[5]. Our conclusion could apply to it too.

gravitational waves (such mechanism works also in inflation[14]). It turns out that the gravitational wave spectrum is nearly scale invariant (slightly blue) at the end of the ekpyrotic phase. Hence, it is difficult to discriminate between inflation and the ekpyrotic scenario by merely detecting primordial gravitational waves. We also study scalar fluctuations induced by the magnetic fields and show that the tensor to scalar ratio should be more than unity, which implies that scalar fluctuations in the CMB should be dominated by quantum fluctuations produced by an additional scalar field as is often assumed in the ekpyrotic scenario.

2. Ekpyrotic phase

The ekpyrotic scenario can be described by a four-dimensional effective theory with a scalar field ϕ moving in an effective potential $V(\phi)$ specified below. The action reads

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)\right] \ , \tag{1}$$

where M_{pl} represents the reduced Planck mass, g is the determinant of the metric $g_{\mu\nu}$, and R is the Ricci scalar. The scalar field represents the separation l between two branes $l \sim e^{\phi}$. The contracting universe ($\dot{\phi} < 0$) is connected to the expanding universe ($\dot{\phi} > 0$) through a bounce (a collision of two branes). The scalar and tensor vacuum fluctuations are produced in the contracting phase where the scalar field rolls down a negative steep potential

$$V(\phi) \simeq V_0 e^{\lambda\frac{\phi}{M_{pl}}} \ , \tag{2}$$

where V_0 is a negative constant. Note that λ is also negative and satisfies the fast roll condition $|\lambda| \gg 1$ to keep isotropy of the universe. Thus, we can take an isotropic metric ansatz in this phase as

$$ds^2 = a(\tau)\left[-d\tau^2 + dx^2 + dy^2 + dz^2\right] \ , \tag{3}$$

where we used a conformal time τ. It is straightforward to derive scaling solutions from Eqs. (1)–(3):

$$a(\tau) = a_{end}\left(\frac{-\tau}{-\tau_{end}}\right)^{\frac{2}{\lambda^2-2}} , \quad \frac{\phi(\tau)}{M_{pl}} = \phi_0 - \frac{2\lambda}{\lambda^2-2}\ln(-M_{pl}\tau) \ , \tag{4}$$

where τ_{end} (< 0) and a_{end} represent the moment and the scale factor at the end of the ekpyrotic phase, respectively. The obtained vacuum scalar and tensor power spectrums are blue-tilted, so that we need an additional scalar field to explain the CMB observation[6]. Then, the ekpyrotic scenario predicts the nearly scale invariant scalar power spectrum and the blue-tilted tensor power spectrum. The situation is different from inflation where both spectra are nearly scale invariant.

3. Scale Invariant Magnetic Fields

Let us consider a gauge field coupled with ϕ in the contracting phase as

$$-\frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu} \ , \tag{5}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the gauge field and $f(\phi)$ represents the gauge kinetic function. Now, we take the gauge kinetic function as exponential type functional form which is ubiquitous in models obtained from dimensional reduction

$$f(\phi) = f_0 e^{\rho \frac{\phi}{M_{pl}}} \ , \tag{6}$$

where it has been set to be unity at the end of the ekpyrotic phase, and then there is no strong coupling problem. As is discussed in Ref. 13, one can obtain a expression of Fourier coefficient of magnetic fields defined by $\vec{B}(\tau, \boldsymbol{x}) \equiv \frac{f}{a^2}\left(\nabla \times \vec{A}(\tau, \boldsymbol{x})\right)$ on the background (4) as

$$\mathcal{B}_k(\tau) = \frac{3\sqrt{2}}{8}(\lambda^2 - 2)^2 k^{-3/2}\left(\frac{-\tau}{-\tau_{end}}\right)^{-\frac{2\lambda^2}{\lambda^2-2}} H_{end}^2 \ . \tag{7}$$

Here, we have eliminated ρ by requiring the scale invariance of magnetic fields at the end of the ekpyrotic phase. τ_{end} and H_{end} are the conformal time and the Hubble parameter at the end of the ekpyrotic phase, respectively. For example, if we set $H_{end} = 10^{-5} M_{pl}$ and $\lambda = -17$, where back reaction from electromagnetic fields is negligible, the amplitude of the magnetic field at the end of the ekpyrotic phase is about 10^{49} G. Thus, the cosmological magnetic fields observed at present can be produced in the ekpyrotic scenario[9]. Remarkably, such magnetic fields can also induce abundant primordial gravitational waves.

4. Gravitational waves from Magnetic Fields

As is shown in Refs. 13, 14, one can get the tensor sector of the action (1) in the presence of the gauge field (5) as

$$S_{GW} = \int d\tau d^3x \left[\frac{M_{pl}^2}{8} a^2 \left(h'_{ij}h'^{ij} - \partial_k h_{ij}\partial_k h^{ij}\right) + \frac{1}{2}a^4\left(E_iE_j + B_iB_j\right)h^{ij} \right] \ , \tag{8}$$

where h_{ij} is the transverse traceless tensor, namely, gravitational waves. We see that electric and magnetic fields work as a source of gravitational waves. From above action with scale invariant magnetic fields (7), one can estimate the power spectrum of gravitational waves:

$$P_s(k) \simeq \frac{27}{16\pi^4}\lambda^8\left(\frac{H_{end}}{M_{pl}}\right)^4 \ln\left[\frac{k}{k_{in}}\right] \ . \tag{9}$$

There is a factor $\left(\frac{H_{end}}{M_{pl}}\right)^4$ in the spectrum (9) because of the nonlinear contribution of the magnetic fields (7). One can see that sourced gravitational waves have a nearly scale invariant spectrum. This conclusion is different from the well-known blue-tilted spectrum in the ekpyrotic scenario[8]. Most importantly, there appears a factor λ^8 in $P_s(k)$. For example, if we set $H_{end} = 10^{-5}M_{pl}$ and $\lambda = -17$ to produce the observed magnetic field, the amplitude of the power spectrum is about 10^{-11}. This is comparable with the gravitational waves in the inflationary universe $\sim \left(\frac{H_{end}}{\pi M_{pl}}\right)^2$. Therefore, we can not discriminate between inflation and the ekpyrotic scenario just by detecting primordial gravitational waves.

5. Scalar Fluctuations from Magnetic Fields

The power spectrum of scalar fluctuations induced by magnetic fields can be calculated as same as tensor fluctuations. The result is

$$\mathcal{P}_s(k) \simeq \frac{243}{1024\pi^4}\lambda^8\left(\frac{H_{end}}{M_{pl}}\right)^4 \ln\left[\frac{k}{k_{in}}\right] \ . \tag{10}$$

From Eqs. (9) and (10), the tensor to scalar ratio r_{source} is given by

$$r_{source} \simeq 7 \ . \tag{11}$$

Since the tensor to scalar ratio becomes larger than unity, we can say that the scalar fluctuations sourced by the scale invariant magnetic field are negligible in the ekpyrotic scenario. Therefore, the ekpyrotic model with a gauge field is compatible with the CMB data.

6. Conclusion

We studied the role of the gauge kinetic function in the ekpyrotic scenario and showed that abundant gravitational waves sourced by the gauge field can be produced. As a demonstration, we first showed that scale invariant magnetic fields can be produced in the ekpyrotic phase. It turned out that the magnetic fields induce nearly scale invariant gravitational waves (slightly blue) and the amplitude could be comparable with that of the inflationary universe. It turned out that it is difficult to disprove the ekpyrotic scenario by detecting primordial gravitational waves. In order to distinguish both scenarios, it is necessary to look at the details of the spectrum such as the tilt of the spectrum or the non-gaussianity[15]. Observing the distinction of higher order scalar perturbations is also important[16]. We should mention that the idea of finding an ekpyrotic model with observable gravitational waves on CMB scales using sourced fluctuations was put forward for the first time in Ref. 17 by investigating a different model with explicit parity violation. Our model has no explicit parity violation. Moreover, we also showed that the scalar fluctuations induced by the magnetic field are smaller than the sourced gravitational waves. Generally, as far as the fast roll condition is satisfied, the tensor to scalar ratio becomes more than unity in any ekpyrotic models with the gauge kinetic function. Therefore, our scenario would be compatible with the CMB data provided that nearly scale invariant scalar fluctuations are produced in a standard way with an additional scalar field[6].

It should be noted that we must check the non-gaussianity of the primordial scalar fluctuations in the present model[18]. Moreover, we should consider a bounce process from contracting to expanding to connect the spectrum at the end of the ekpyrotic phase with observables. We have not looked into this issue in this paper since the mechanism is model dependent and the detailed analysis is beyond the scope of this paper[1]. However, actually, although we fixed the parameters such as ρ, λ, H_{end} for simplicity in this paper, we can tune these parameters in our scenario so that our conclusion becomes valid for any ekpyrotic bouncing models. Therefore, our conclusion is robust.

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