COSMOLOGICAL NEUTRINOS AND RYDBERG ATOMS

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Abstract

The event rate in the level excitation of Rydberg atoms by the cosmological background of very low energy light neutrinos is estimated. The result is encouraging and deserves further theoretical and experimental study.

One of the clear predictions of the standard big bang model of the expanding Universe is the existence of a cosmological background of light neutrinos ($m_\nu < 1$ MeV) with density N_ν and temperature T_ν such that, per neutrino species (see, for instances [1]):

$$N_\nu = \frac{3}{11} N_\gamma \tag{1}$$

and

$$T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma, \tag{2}$$

where N_γ and T_γ are the density and temperature of the cosmological photon background. As is well known the cosmological background of photons was discovered in 1965[2], and N_γ and T_γ are now well determined: $N_\gamma \simeq 400$ cm^{-3} and $T_\gamma \simeq 2.73$ K. From (1) and (2), for the three species of light neutrinos, we expect

$$N_\nu \simeq 327 \text{ cm}^{-3}, \quad T_\nu \simeq 1.9 \text{ K}. \tag{3}$$

The existence of the neutrino background was never positively demonstrated, and relations (1) and (2) and the estimates (3) were never tested.

The main reason why the background of neutrinos is so difficult to detect is related to the effective weakness of the weak interaction (mediated by the heavy bosons $W^\pm$ and Z^0) at low energy. While in electron - electron scattering the cross section at a given momentum transfer q goes as

$$\sigma_{ee} \sim \frac{\alpha^2}{q^2}, \tag{4}$$

where α is the electroweak coupling constant, in neutrino - electron scattering, with neutrinos of energy E_ν, it behaves as

$$\sigma_{\nu e} \sim \frac{\alpha^2 E_\nu^2}{M^4}, \tag{5}$$

where M is the vector boson mass.

With a neutrino background of temperature $T = 1.9$ K, typical energies and energy transfers are of the order of 10^{-4} eV. With these values for $|q|$ and E_ν in (4) and (5) we obtain

$$\frac{\sigma_{ee}}{\sigma_{\nu e}} \sim 10^{60}. \tag{6}$$

Such a large number is indeed very discouraging. However, at energies $\geq$ 8 MeV, neutrino - electron elastic scattering can already be used to detect neutrinos in the Kamiokande detector[3].

Previous attempts at resolving the enormous difficulty to measure neutrino - electron elastic scattering at very low energies were based on the idea of coherent near-forward elastic scattering[4]. The event rate, instead of being proportional to N, the number density of electrons or nucleons (of the order of 10^{23}, say), becomes proportional to N^2. However, the most optimistic proposed measurements, based on reflection as well as refraction from bulk matter, were shown not to be correct since the equilibrium of forces from all directions (reinforced by the presumable equality of ν and $\bar{\nu}$ fluxes) makes such an effect quite negligible[5].

Our suggestion here for the detection of the cosmological neutrino background is based on the excitation of Rydberg atoms by the 1.9 K neutrino bath. The energies involved are in the range of 10^{-4} - 10^{-5} eV, of the order of the transition energies in Rydberg atoms[6].

In fact, let us consider as reference a hydrogen-like atom and a transition $n \to n' = n + 1$, where n is an effective quantum number with

$$\Delta E_n \simeq R\left(\frac{1}{n^2} - \frac{1}{n'^2}\right), \tag{7}$$

R being the Rydberg constant, $R = 13.6$ eV. For transitions with $\Delta E_n < 10^{-4}$ eV one needs, from (7), $n > 60$. This is a typical situation for an excited Rydberg atom.

The physics of the interaction of very low energy neutrinos with atoms is in itself a quite interesting subject. While for energies above the binding energy the physics of neutrinos, apart from the value of the effective coupling constant, is similar to the physics of electrons, for energies below the binding energy the physics of neutrinos is much simpler, as the neutrino never affects the overall stability of the atom. Techniques for relatively high energies, like the Born aproximation, can be extended to low energies, in the case of neutrinos, as the atomic field is never strongly disturbed and the atom maintains its structure.

In the study of inelastic interactions of neutrinos with atoms, physics is governed by three very different scales: the scale set up by the mass of the Z^0, characterizing the range of the weak interaction ($\frac{1}{M} \sim 10^{-11}$ eV^{-1}); the scale connected with the size of the Rydberg atom $R_n \sim n^2 a_0$, where a_0 is the Bohr radius (for $n \sim 100$, $R_n \sim 1$ eV^{-1} and $\Delta R_n \sim 10^{-2}$ eV^{-1}); the scale related to the ΔE_n transitions, ($\frac{1}{\Delta E_n} \sim 10^5$ eV^{-1}).

Let us now consider inelastic collisions of neutrinos in a framework equivalent to the Born aproximation (see, for instance[7]). As a neutrino passes close to an atom, the atom feels the potential

$$V = \alpha_Z \frac{\exp(-M|\vec{R}' - \vec{R}|)}{|\vec{R}' - \vec{R}|}, \tag{8}$$

where $\vec{R}'$ represents the position of the external electron, relative to the centre of mass of the atom, $\vec{R}$ the position of the neutrino, and α_Z is the electroweak coupling of the Z^0 to the electron. For a $n \to n'$ transition, with Z^0 absorption by the external electron, we have the transition matrix element

$$V_{nn'} = \alpha_Z \int \psi^*_{n'}(\vec{R}') \frac{\exp(-M|\vec{R}' - \vec{R}|)}{|\vec{R}' - \vec{R}|} \psi_n(\vec{R}')\ d\vec{R}', \tag{9}$$

where ψ_n and $\psi_{n'}$ are stationary wave functions. We assume that the neutrino in this process behaves as an undisturbed plane wave with $v \simeq c$. The resulting transition probability amplitude, at a given impact parameter b, is

$$a_{nn'}(b^2) = -i \int_{-\infty}^{\infty} V_{nn'}(b^2, z) \exp[i(E_{n'} - E_n)z]\, dz, \tag{10}$$

where $b^2 + z^2 = R^2$ (see Fig. 1).

As the wavelength of the neutrino is very large, larger than the atomic radius, $E_\nu^{-1} >> R_n$, 10^4 eV^{-1} in comparison with 1 eV^{-1}, the neutrino can not be treated as a point particle and the impact parameter is not a good quantum number. The usual impact parameter sumation is not applicable here. In fact the region $0 \leq b \leq R_n$ corresponds essentially to s-wave ($l = 0$) scattering. We thus define an average transition probability amplitude as

$$\overline{a_{nn'}(b^2)} = \frac{1}{R_n^2} \int_0^{R_n} a_{nn'}(b^2)\, db^2, \tag{11}$$

and write the, s-wave scattering transition cross section as

$$\sigma_{nn'} = \frac{\pi}{E_\nu^2} |\overline{a_{nn'}(b^2)}|^2. \tag{12}$$

The calculation of (9) depends naturally on the radial and angular behaviour of the wave functions ψ_n and $\psi_{n'}$, and the transitions may be of different types (electric and magnetic). Here

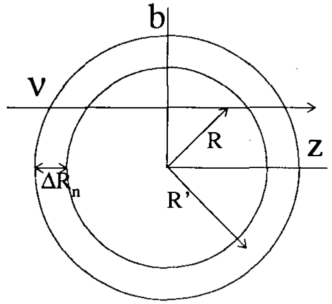

Figure 1: Interaction of a neutrino ν with an atom. The neutrino moves along z direction and has, at a given instant the position $\vec{R}$ relative to the centre of the mass of the atom. The position of the electron at the level n is given by $\vec{R'}$. The impact parameter is b. The wave functions ψ_n and $\psi_{n'}$ are supposed to be significantly different from zero only within the ΔR_n shell.

we are only interested in order-of-magnitude estimates and simply use the property of the radial Rydberg functions being peaked at the classical radius R_n, neglecting angular dependence.

We perform first the angular integration in (9), treating ψ_n and $\psi_{n'}$ as functions only of $R' \equiv |\vec{R'}|$, to obtain

$$V_{nn'} = \alpha_Z \pi \int_0^\infty \frac{R'}{RM} [\exp(-M|R-R'|) - \exp(-M|R+R'|)] \psi^*_{n'}(R')\psi_n(R')dR'. \tag{13}$$

We now make use of the fact that the interaction is of a short-range nature: only the contributions with $R \simeq R'$ survive. In the large M limit we then have

$$M \exp(-M|x|) \longrightarrow \delta(x) \ , \ M \to \infty \tag{14}$$

and

$$V_{nn'}(R) \simeq \frac{\alpha_Z \pi}{M^2} \psi^*_{n'}(R)\psi_n(R). \tag{15}$$

Next we estimate the integral (10). The Rydberg wave functions, $\psi_n(R)$ and $\psi_{n'}(R)$ are significantly diffrent from zero only in the region ΔR_n around R_n. On the other hand we note that

$$\int_0^\infty \psi^*_{n'}(R)\psi_n(R)dR = \frac{1}{4\pi}\left\langle\frac{1}{R^2}\right\rangle_{nn'} \simeq \frac{1}{4\pi}\frac{1}{R_n^2}. \tag{16}$$

As the wave functions are concentrated around $R_n \simeq 1$ eV^{-1} and the energy difference ΔE_n is of the order of 10^{-5} eV, the relevant region of the integration in (10) is $|z| \leq R_n$, and the argument of the exponential is essentially zero $|\Delta E_n \, z| \leq 10^{-5}$. We thus have

$$a_{nn'}(b^2) \simeq -\frac{i2\alpha_Z\pi}{M^2}\int_0^\infty \psi^*_{n'}(R)\psi_n(R)dz. \tag{17}$$

Having in mind (16), we approximate $\psi^*_{n'}(R)\psi_n(R)$ in the overlap region $[R_n, R_n + \Delta R_n]$ by its mean value:

$$\psi^*_{n'}(R)\psi_n(R) = \frac{1}{4\pi R_n^2}\frac{1}{\Delta R_n} \qquad R_n \leq R \leq R_n + \Delta R_n \tag{18}$$

$$= 0 \qquad \text{otherwise.} \tag{19}$$

We thus obtain
if $b \leq R_n$

$$|a_{nn'}(b^2)| = \frac{\alpha_Z}{2M^2} \frac{1}{R_n^2 \Delta R_n} [((R_n + \Delta R_n)^2 - b^2)^{1/2} - (R_n^2 - b^2)^{1/2}]; \quad (20)$$

if $R_n < b \leq R_n + \Delta R_n$

$$|a_{nn'}(b^2)| = \frac{\alpha_Z}{2M^2} \frac{1}{R_n^2 \Delta R_n} ((R_n + \Delta R_n)^2 - b^2)^{1/2}; \quad (21)$$

if $b > R_n + \Delta R_n$

$$|a_{nn'}(b^2)| = 0. \quad (22)$$

The integration (10) gives

$$|\overline{a_{nn'}(b^2)}|^2 = \frac{\alpha_Z}{2M^2} \frac{1}{R_n^4 \Delta R_n} \frac{2}{3} [\, 3R_n^2 \Delta R_n + 3R_n \Delta R_n^2 + \Delta R_n^3]. \quad (23)$$

Finally, we obtain for the transition cross section (12), neglecting terms in $\Delta R_n / R_n$

$$\sigma_{nn'} = \frac{1}{\pi} \left(\frac{M_W}{M_Z}\right)^2 \frac{G_F^2}{R_n^4} \frac{1}{E_\nu^2}, \quad (24)$$

where we have introduced the Fermi coupling constant G_F.

In order to estimate the event rate, one needs to multiply (24) by the neutrino flux ρv and the number of atoms present. For the neutrino flux, from eq.(3), and taking $v \simeq c$, we obtain

$$\rho v \simeq 10^{17} \text{ m}^{-2}\text{s}^{-1} \quad (25)$$

With 10^{23} atoms in a cubic metre of low density material, we obtain from (24) and (25), for the number of events

$$N_{events} \simeq 10^{-12} \text{ m}^{-3}\text{s}^{-1}, \quad (26)$$

This is a very small number. However two remarks should be taken into account:

1) The stimulated transitions from the level n are not just to a single level n'. One has to sum over the states n' in the range $10^{-5} \leq E'_n - E_n < 10^{-4}$ eV . As $E'_n - E_n \simeq \frac{2(n'-n)}{n} E_n$ we obtain $n' - n \simeq 10$. Including the degenerate l, m states the number of final states in the 10^{-5} - 10^{-4} eV region becomes $\sim 10^3$.

2) The neutrino wavelength is very large, of the order of 10^4 eV^{-1}, when compared with the Rydberg atom size, $R_n \sim 1$ eV^{-1}. This means that the neutrino, with a given phase, sees coherently in the z direction of the order of 10^3 atoms. This introduces a correction to (26) of the order of $\sim 10^3$.

Including in (26) the corrections 1) and 2) above, one obtains

$$N_{events} \simeq 10^{-6}\text{m}^{-3}\text{s}^{-1} \simeq 30 \text{ m}^{-3}\text{yr}^{-1}. \quad (27)$$

This is a satisfactory result but, of course, it should not be taken at face value as some of our aproximations, for instance (18), may be too crude. In fact, detailed calculations, requiring the knowledge of the atomic wave functions and the excitation spectrum[8)], are needed in order to obtain more reliable estimates of the event rate. On the other hand, the neutrino background energy spectrum should be handle, in a precise way when estimating the number of events in a given energy range.

We can imagine several difficulties when trying to put our suggestion in practice. Rydberg atoms are not stable and so they have to be continuously excited, by laser pumping techniques, from the ground state to the level n. This probably requires working with atomic beams. The $n \to n'$ excitation by atomic collisions may force the use of low atomic densities, and thus decrease the event rate, to eliminate such competing process. The main source of background is due to $n \to n'$ excitation by photon radiation. These processes have been studied and the temperature dependence of the effect is well under control[6]. This requires the experiment to be carried out at very low temperature, of the order of mK, to drastically reduce the photon density. A discussion of these difficulties appears in[9], where Rydberg atoms were proposed to study galactic axions.

In conclusion, we believe that the possibility of detecting cosmological background neutrinos by exciting Rydberg atoms deserves further study, both theoretical and experimental.

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