FRW UNIVERSE WITH VARIABLE G AND Λ TERM IN $f(R,T)$ GRAVITY

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In this paper, we have studied the FRW metric for variable G and Λ in $f(R,T)$ gravity with the modified Chaplygin gas equation of state *i.e.* $p = A\rho - \frac{B}{\rho^n}$. We have used the hybrid exponential law (HEL) for scale factor to obtain the solution of the field equations. Here also we have discussed some physical behaviour of the model.

Key words: FRW Metric, $f(R,T)$ gravity, Variable G and Λ, Hybrid Exponential Law (HEL), Modified Chaplygin gas.

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1. INTRODUCTION

The observation of supernovae type Ia and Wilkinson Microwave Anisotropy Probe (WMAP) ([1]-[2]) provides the evidence mass density of our universe is low $(\Omega \sim 0.3)$ ([3] and see references there in). The literatures of modern cosmology suggests that a point of universe is filled up with dark energy. The dark energy has been addressed by various slow rolling scalar field, one of the prominent candidates of them is cosmological constant.

Cosmological constant Λ and gravitational constant G are the two important parameters of Einstein's field equation. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein's field equation. In a evolving universe, it appears to look at this constant as a function of time. There are significant observational evidence that the expansion of the universe is undergoing a late time acceleration. In other words in the context of general relativity some kind of dark energy varies slowly with time and space, which dominates the current composition of the cosmos. The origin and nature of such field poses a completely open question. Riess *et al.* [4] have presented an analysis of 156SNe including a few at $z > 1.3$ from the Hubble Space Telescope (HST) "GOOD ACS" Treasury survey. Type Ia supernove observation [5], leads to a conclusion that expansion of the universe is accelerating. Observations strongly favour a small and positive value of the effective cosmological constant at the present epoch.

Dirac [6] was first suggested that gravitational constant is not independent of time and this has been studied by several authors [7]-[10]. Numerous modifications of general relativity to allow for a variable G based on different arguments have been proposed [11]. The Large Number Hypothesis (LNH) proposed by Dirac leads to a cosmology when G varies with time. Variation of G has many interesting consequences in astrophysics. Canuto and Narlikar [12] have shown that G-varying cosmology is consistent with whatsoever cosmological observations available at present. In modern cosmology study of G and Λ play a important role because it may responsible for the acceleration of the universe. Among all the possiable alternatives the simplest and most theoretically appealing possibility for dark energy is the energy density stored on the vaccum state of all existing fields in the universe *i.e.* $\rho_v = \frac{\Lambda}{8\pi G}$, where Λ is the cosmological constant. However a constant Λ can not explain the huge difference between the cosmological constant inferred from observation and the vaccum energy density resulting from quantum field theories. In an attempt to solve this problem, variable Λ was introduced such that Λ was large in the early universe and then decayed with evolution [13]. The Λ term has also been interpreted in terms of Higg's scalar field [14] as well as function of temperature and related it to the process of broken symmetries [15]. The cosmological constant Λ as a function of time has been extensively discussed by several authors in various variable G theories in different contexts. A number of authors, *e.g.*, Kalligas *et al.* [16], Arbab [17], Abdussattar and Vishwakarma [18] proposed linking of variations of G and Λ within the framework of general relativity. This approach is appealing as it leaves the form of Einstein equations formally unchanged by allowing a variation of G to be accompanied by change in Λ. Pradhan and Yadav [19] investigated bulk viscous anisotropic cosmological models with variable G and Λ. Pradhan *et al.* [20] derived FRW universe with verying G and Λ. Since Bianchi type I spaces are subsequent generalization of zero curvature FRW models, Singh *et al.* [21] obtained some Bianchi type I models with variable G and Λ. Singh *et al.* [22] obtained early viscous universe with variable G and Λ. Bianchi type I models in the presence of a perfect fluid with time varying G and Λ in general relativity has been discussed by Singh and Tiwari [23]. Cosmological models with variable G and Λ in space-times of higher dimensions has been explained by Singh and Kotambkar [24]. Singh and Kale [25] dealt with Bianchi type I, Kantowski-Sachs and Bianchi type III anisotropic models of the universe filled with a bulk viscous cosmic fluid in the presence of variable G and Λ. Bali and Tinker [26] investigated Bianchi type III bulk viscous barotropic fluid cosmological model with variable G and Λ which leads to inflationary phase of the universe. Verma and Shriram [27] obtained Bianchi type III bulk viscous barotropic fluid cosmological model with variable G and Λ in simple and systematic way. Dark energy models with time-dependent G has been investigated by Ray *et al.* [28]. Mukhopadhyay *et al.* [29] studied higher dimensional dark energy with time variable G and Λ. Recently, two-fluid anisotropic cosmological model with Variable G and Λ has been studied by Samanta [30]. Tiwari *et al.* [31] studied the Polytropic bulk viscous cosmological model with variable G and Λ in general relativity and higher order corrections of the extended Chaplygin gas cosmology with varying G and Λ has been studied by Khurshudyan *et al.* [32]

The idea of an accelerated expansion of the universe and led to the search for a new type of matter which violates the strong energy condition *i.e.*, $\rho + 3p < 0$. The matter considered to be responsible for such a condition to be satisfied at a certain stages of evolution of the universe is referred to as dark energy. There are several different candidates for dark energy. The type of dark energy represented by a scalar field is often called quintessence. The simplest candidate for dark energy is the cosmological constant Λ. In particular, one can try another type of dark energy, the so-called Chaplygin gas which obeys an equation of state like $p = \frac{-B}{\rho}$ $(B > 0)$ [33]-[34], where p and ρ are the pressure and energy density respectively. Subsequently, the above equation was generalized to the form $p = \frac{-B}{\rho^n}$, $0 \leq n \leq 1$ (see [35]-[37]). There are some works on modified Chaplygin gas obeying the equation of state as follows (see [38] -[40]):

$$p = A\rho - \frac{B}{\rho^n}, \text{ for } A > 0. \tag{1}$$

The aforesaid survey of literature clearly indicates that there has been interest in studying variable G and Λ in different theory with different space-time geometry. Motivated by the above observations we have studied the homogeneous and isotropic FRW metric with variable G and Λ in $f(R,T)$ modified gravity. Modify gravity is obtain by modifying the geometrical part of Einstein-Hilbert action of general relativity. Modify gravity is of great importance because it can successfully explain the rotation curve of galaxies and the motion of galaxy clusters in the universe. There are various modify gravity namely $f(R)$, $f(G)$, $f(R,G)$ and $f(R,T)$ theory of gravity.

In this paper we deal with $f(R,T)$ gravity, which has recently developed by Herko *et al.* [41] where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the stress energy tensor. They have obtained the gravitational field equations in the metric formalism, as well as the equations of motion for test particles, which follow from the covariant divergence of the stress energy tensor. The $f(R,T)$ gravity model depends on a source term, representing the variation of the matter stress energy tensor with respect to the metric. General expression of source term is obtained as a function of the matter Lagrangian L_m and for specific choice of L_m would generates a specific set of field equations. Depending upon the choice of $f(R,T)$, we have different particular models. In literature Harko *et al.* [41] (see also for details on $f(R,T)$ modify gravity), we have found three class of $f(R,T)$ as follow

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{2}$$

In this paper we have concentrated on the first class as $f(R,T) = R + 2f(T)$. Recently several authors [42]-[45] have studied the $f(R,T)$ modify gravity in different contexts. For finding the solution of the field equations in $f(R,T)$ modify gravity, we have used the following ansatz for the scale factor of the universe [46] :

$$a(t) = Ct^{\alpha}e^{\beta t} \tag{3}$$

where $C > 0$, $\alpha \geq 0$, and $\beta \geq 0$ are constant. In literature, this generalized form of the scale factor is called Hybrid Expansion Law (HEL). HEL is a mixture of power-law and exponential-law cosmologies. Power-law and exponential-law is obtain as a special cases of HEL, when $\alpha = 0$ and $\beta = 0$ in (3) respectively. The assumption $\alpha > 0$ and $\beta > 0$ leads to a new cosmology.

2. BASIC FIELD EQUATIONS

In this section we have consider the homogeneous and isotropic space-time given by Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{4}$$

where $k(=0,\pm 1)$ is the curvature parameter.
The energy momentum tensor for a perfect fluid is given by

$$T_i^j = (p+\rho)u_i u^j - p\delta_i^j \tag{5}$$

Here u^i is the flow vector satisfying $g_{ij}u^i u^j = 1$. Where ρ is the energy density and p is the isotropic pressure and we take $c = 1$. Using equation (5) we have obtain

$$T_0^0 = \rho, \quad T_1^1 = T_2^2 = T_3^3 = -p \tag{6}$$

The field equations for $f(R,T)$ gravity model with variable G and Λ is given as

$$\begin{aligned} f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) - \Lambda(t)g_{ij} \\ = [8\pi - f_T(R,T)]G(t)T_{ij} - f_T(R,T)\Theta_{ij} \end{aligned} \tag{7}$$

where

$$\begin{aligned} T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{ij}}L_m \quad ,\Theta_{ij} = -2G(t)T_{ij} - pg_{ij}, \\ f_R(R,T) = \frac{\partial f(R,T)}{\partial R} \text{ and } f_T(R,T) = \frac{\partial f(R,T)}{\partial T}. \end{aligned} \tag{8}$$

Here $f(R,T)$ is an arbitrary function of Ricci scalar (R) and trace (T) of the stress energy momentum tensor T_{ij}. L_m is the matter Lagrangian density.
Now we take the function $f(R,T)$ as (Harko *et al.* [41])

$$f(R,T) = R + 2f(T) \tag{9}$$

Using (5), (9) in (7) the field equations takes the form

$$G_{ij} - \Lambda(t)g_{ij} = \left[8\pi + 2f'(T)\right] G(t)T_{ij} + [2pf'(T) + f(T)]g_{ij} \tag{10}$$

where the overhead prime indicates differentiation with respect to the argument. We consider

$$f(T) = \lambda T \tag{11}$$

where λ is a constant(Harko *et al.* [41]).
Using equation (11), we obtain the field equations as

$$G_{ij} - \Lambda(t)g_{ij} = [8\pi + 2\lambda]\, G(t)T_{ij} + (\rho - p)\lambda g_{ij} \tag{12}$$

For the metric (4), the field equations (10) take the form

$$3\left(\frac{\dot{a}}{a}\right)^2 + \frac{3k}{a^2} - \Lambda(t) = [(8\pi + 2\lambda)G(t) + \lambda]\, \rho - p\lambda \tag{13}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \Lambda(t) = -\left[(8\pi + 2\lambda)G(t) + \lambda\right] p + \lambda\rho \tag{14}$$

3. SOLUTION OF THE FIELD EQUATIONS

The field equations contain two equation and five unknown namely: p, ρ, G, Λ, a. Therefore for finding the exact solution we need three more plausible conditions. For which we have used equation (1), (3) and with out loss of generality we considered the ρ in the form

$$\rho = \frac{1}{a^\gamma}, \tag{15}$$

where $\gamma > 0$ is a constant.
Using (1), (3) and (15) in (13), we obtain

$$\begin{aligned} \Lambda(t) + \frac{8\pi + 2\lambda}{C^\gamma t^{\alpha\gamma} e^{\gamma\beta t}} G(t) &= 3\left(\frac{\alpha}{t} + \beta\right)^2 \\ &+ \frac{3k}{C^2 t^{2\alpha} e^{2\beta t}} + \frac{\lambda\left[A - BC^{(1-n)\gamma} t^{(1-n)\alpha\gamma} e^{(1-n)\gamma\beta t} - 1\right]}{C^\gamma t^{\alpha\gamma} e^{\gamma\beta t}} \end{aligned} \tag{16}$$

Substituting (1), (3) and (15) in (14), we get

$$\begin{aligned}-\Lambda(t)+&\frac{(8\pi+2\lambda)(A-BC^{(1-n)\gamma}t^{(1-n)\alpha\gamma}e^{(1-n)\gamma\beta t})}{C^{\gamma}t^{\alpha\gamma}e^{\gamma\beta t}}G(t)=\\ &=\frac{\lambda\left[1-A+BC^{(1-n)\gamma}t^{(1-n)\alpha\gamma}e^{(1-n)\gamma\beta t}\right]}{C^{\gamma}t^{\alpha\gamma}e^{\gamma\beta t}}\\ &-\left[\frac{3\alpha^2-2\alpha}{t^2}+\frac{6\alpha\beta}{t}+3\beta^2+\frac{k}{C^2t^{2\alpha}e^{2\beta t}}\right].\end{aligned} \tag{17}$$

Adding (16) and (17) we obtain

$$G(t)=\frac{C^{\gamma}t^{\alpha\gamma}e^{\gamma\beta t}\left[\frac{2\alpha}{t^2}+\frac{2k}{C^2t^{2\alpha}e^{2\beta t}}\right]}{(8\pi+2\lambda)(1+A-BC^{(1-n)\gamma}t^{(1-n)\alpha\gamma}e^{(1-n)\gamma\beta t})} \tag{18}$$

With the help of (18), from (16) we obtain

$$\begin{aligned}\Lambda(t)=3\left(\frac{\alpha}{t}+\beta\right)^2+&\frac{3k}{C^2t^{2\alpha}e^{2\beta t}}\\ &-\frac{\lambda}{C^{\gamma}t^{\alpha\gamma}e^{\gamma\beta t}}\left[1-A+BC^{(1-n)\gamma}t^{(1-n)\gamma\alpha}e^{(1-n)\gamma\beta t}\right]\\ &-\frac{1}{1+A-BC^{(1-n)\gamma}t^{(1-n)\gamma\alpha}e^{(1-n)\gamma\beta t}}\left[\frac{2\alpha}{t^2}+\frac{2k}{C^2t^{2\alpha}e^{2\beta t}}\right].\end{aligned} \tag{19}$$

The deceleration parameter of the model is

$$q=-\frac{a\ddot{a}}{(\dot{a})^2}=\frac{\alpha}{(\alpha+\beta t)^2}-1 \tag{20}$$

From this we observe that the HEL universe evolves with a variable deceleration parameter, and a transition from deceleration to acceleration takes place at

$$t=\frac{\sqrt{\alpha}-\alpha}{\beta} \tag{21}$$

which restrict α in the range $0<\alpha<1$.

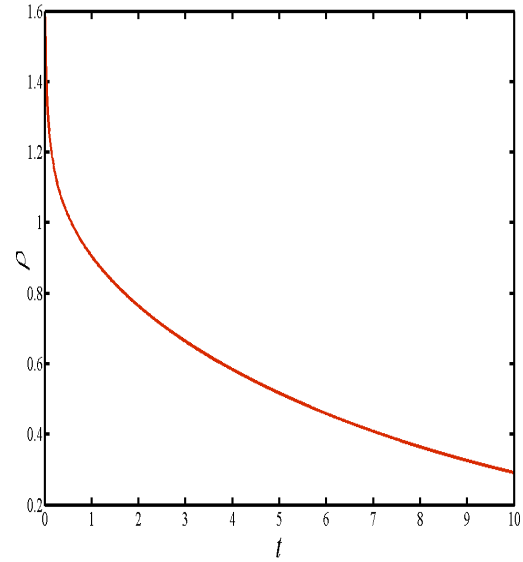

Fig. 1 – Density ρ *versus* time t for $\alpha = 0.2$, $\beta = 0.2$, $\gamma = 0.5$ and $C = 1$.

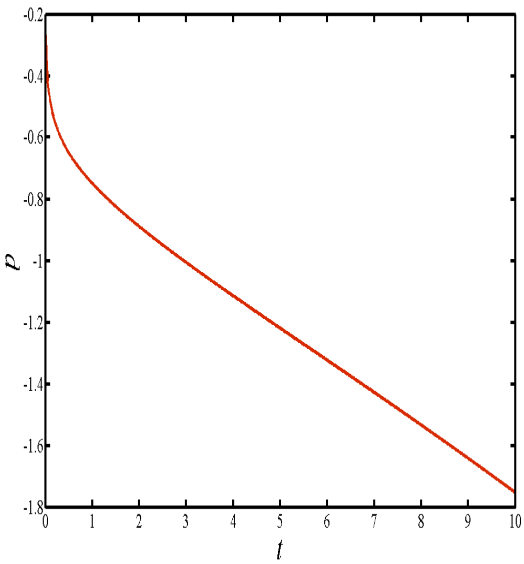

Fig. 2 – Pressure P *versus* time t for $\alpha = 0.2$, $\beta = 0.2$, $\gamma = 0.5$, $A = \frac{1}{3}$, $B = 1$, $n = 0.5$ and $C = 1$.

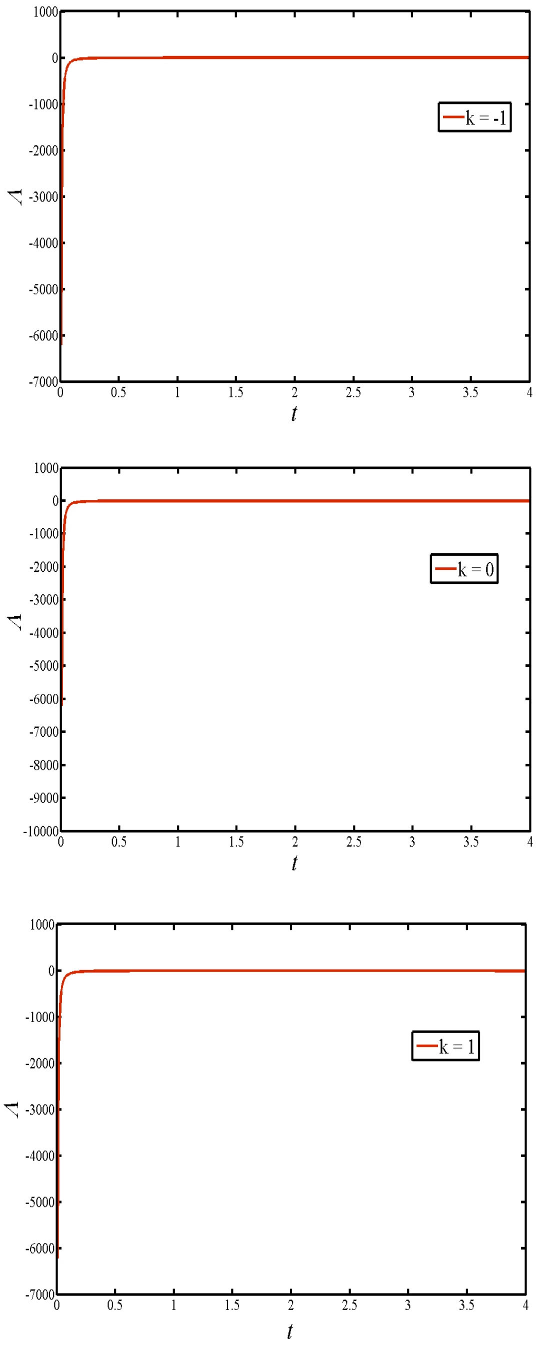

Fig. 3 – $\Lambda(t)$ *versus* time t with $\alpha=\beta=0.2$, $C=1$, $\gamma=n=0.5$, $A=\frac{1}{3}$ and $B=1$ for $k=0,\pm1$.

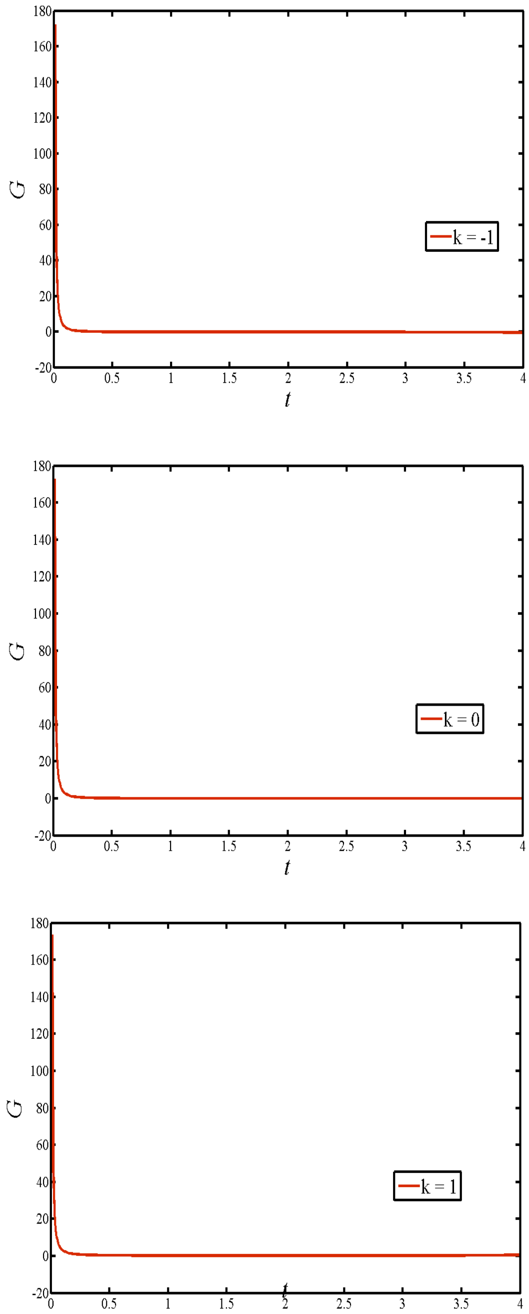

Fig. 4 – Gravitational constant $G(t)$ *versus* time t with $\alpha = \beta = 0.2$, $C = 1$, $\gamma = n = 0.5$, $A = \frac{1}{3}$ and $B = 1$ for $k = 0, \pm 1$.

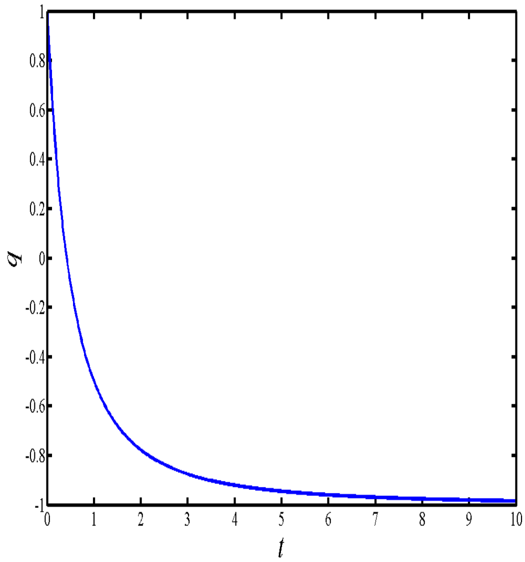

Fig. 5 – Deceleration parameter q *versus* time t for $\alpha = \beta = 0.5$.

4. CONCLUDING REMARKS

We have studied the FRW universe with variable G and Λ filled with perfect fluid in $f(R,T)$ modify gravity. We have assumed the HEL for the average scale factor, which yields power-law and exponential-law in its special cases. We find that the HEL universe exhibits a transition from deceleration to acceleration, which is an essential feature of the dynamics of the evolution of the universe. From Figure 1 it is noticed that, the energy density ρ decreases with increase in time *i.e.* $\rho \to 0$ as $t \to \infty$. In Figure 2 it is observed that, the pressure decreases from positive to negative with the evolution of time. Figure 3, we have plotted the cosmological constant $\Lambda(t)$ against time. It is noticed that $\Lambda(t)$ approaches towards zero from negative values with the evolution of time. In other world we can say $\Lambda < 0$. It is also noticed that cosmological constant $\Lambda(t)$ show similar behaviour for $k = -1$,$k = 0$ and $k = 1$.

Gravitational constant $G(t)$, which also decreases with increase in time and it approaches to zero *i.e.* $G(t) \to 0$ as $t \to \infty$ (see Figure 4). The behaviour of $G(t)$ is similar for $k = 0, \pm 1$. Finally, Figure 5 shows the behaviour of deceleration parameter against time. It represents that with the evolution of time, deceleration parameter approaches towards -1. Three different ranges for deceleration parameter: -0.715 ± 0.045, $-0.658^{+0.061}_{-0.057}$, $-0.461^{+0.031}_{-0.033}$ has been discussed by Xu *et al.*[47]. Here the deceleration parameter agreed with the result of Xu *et al.* [47].

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