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# Scale invariant non linear sigma model at finite temperatures

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**Abstract.** The recently proposed renormalization group improved optimized perturbation theory is employed to evaluate the pressure of the two dimensional non linear sigma model at finite temperatures. We explicitly show how this powerful resummation method can turn the lowest (one loop) perturbative contribution to the pressure, which is not RG invariant, into a non perturbative quantity exhibiting scale invariance .

## 1. Introduction

The development of reliable approximation techniques to solve the equations of motion describing a physical system has always been considered a problem of fundamental importance in all branches of physics since most realistic situations are described by non linear interactions. In practice this means that an exact solution may not be easily attainable and theoretical predictions require that the evaluation of a physical quantity be performed within an appropriate approximation framework. Sometimes, when a problem cannot be exactly solved one can attempt to separate it into two parts with one of them representing an exactly solvable case. Then, if the underlying dynamics allows, one can treat the other piece as a "perturbation" and recur to the well established perturbation theory (PT) to approximately solve the problem by successively evaluating the terms which appear in a series written in powers of a (small enough) parameter which characterizes the perturbative interaction. Being relatively easy to implement, PT has been successfully employed to treat various widely different problems. However, the same approximation does not have the same capability in the case of quantum chromodynamics (QCD), which is the fundamental theory of strong interactions, since in this case the interaction parameter $(4\pi\alpha_s)$ is not small at the energy scales relevant for nuclear physics. Today, the development of powerful computers offers the possibility to solve these non perturbative problems by employing the numerical methods of the so-called lattice field theory (LFT), which has been very successful in the description of the QCD phase transitions at finite temperatures and near vanishing baryonic densities [1]. However, the numerical sign problem, which arises at finite chemical potentials, has not yet been completely solved, preventing that the method be

successfully used to describe the complete QCD phase diagram. A possible analytical alternative is to employ a variational approximation where the result of a related solvable case is rewritten in terms of a variational parameter. This strategy is adopted in the optimized perturbation theory (OPT) [2, 3] where, for scalar theories, a harmonic term $(1 - \delta)m^2$ is added to the potential energy density while the anharmonic coupling, $\lambda$, is rescaled as $\lambda \to \delta\lambda$ with $\delta$ being formally treated as a (small) interaction parameter. Having evaluated a physical quantity to the desired order in $\delta$ one sets $\delta = 1$, and fixing the mass parameter, $m$, in a variational fashion. The OPT has been successfully used in physical situations relevant for hadronic [4] and condensed matter physics [5, 6, 7], while a similar technique, known as screened perturbation theory (SPT) [8], has been extended in order to cope with gauge theories. The resulting approximation, which provides a resummation of hard thermal loop perturbation theory (HTLpt) [9], has already been applied to QCD. Unfortunately, all these variational methods predict that scale invariance is not observed by thermodynamical physical observables such as the pressure, as many applications to scalar theories show [8, 10]. In the case of QCD the results obtained in Ref. [9] show a good agreement between HTLpt and LFT simulations at the "central" arbitrary renormalization scale value, $M = 2\pi T$, but the agreement is lost when the scale is varied even by a moderate amount. An alternative, combining OPT with renormalization group (RG) invariance, has been recently proposed [11]. This technique, known as renormalization group optimized perturbation theory (RGOPT), was originally used within the framework of the Gross-Neveu model and QCD at vanishing temperatures. More recently, the RGOPT has been extended to the finite temperature domain in an application to the scalar $\lambda\phi^4$ theory producing encouraging results [12]. Here, we illustrate how the method works by evaluating the pressure of the non linear sigma model (NLSM) in 1+1 dimensions which, despite of being a simple model, shares some common features with QCD such as asymptotic freedom, trace anomaly and the generation of a mass gap. At finite temperatures this model has been first analyzed in Ref. [13] and more recently in Refs. [14, 15]. For our present purpose we only need to compute the lowest order (one loop) contribution to the pressure. A more complete discussion (up to the two-loop level) will be given elsewhere [16]. The work is organized as follows. In the next section we recall the two dimensional NLSM, presenting the one loop perturbative result in Sec. 3, while the RGOPT is applied in Sec. 4 and some numerical results illustrated in Sec. 5. Our conclusions are given in Sec. 6.

## 2. The NLSM in 1+1-dimensions
The two-dimensional NLSM Lagrangian density, for $\pi_i$ fields with $N - 1$ components, can be written as [17]

$$\mathcal{L}_0 = \frac{1}{2}(\partial\pi_i)^2 + \frac{g_0(\pi_i\partial\pi_i)^2}{2(1 - g_0\pi_i^2)} - \frac{m_0^2}{g_0}\left[\left(1 - g_0\pi_i^2\right)^{1/2} - 1\right], \tag{1}$$

where the last parcel represents a constant and $m_0$ represents a mass parameter whose physical role will be specified later. Expanding he above theory to zeroth-order yields

$$\mathcal{L}_0 = \frac{1}{2}\left[(\partial\pi_i)^2 + m_0^2\pi_i^2\right] + \mathcal{E}_0 + \mathcal{O}(g_0), \tag{2}$$

where we have introduced a field independent term, $\mathcal{E}_0$, which corresponds to an infinite renormalization of the "zero-point" energy [3].

Within the imaginary time formalism the propagator is $1/(\omega_n^2 + \mathbf{p}^2 + m^2)$ where $\omega_n = 2\pi nT$ represent the bosonic Matsubara frequencies ($n = 0, \pm 1, \pm 2 \cdots$) and $T$ is the temperature. In this work, the divergent integrals are regularized using dimensional regularization (within the minimal subtraction scheme $\overline{\text{MS}}$) which, at finite temperature, and $d = 2 - \epsilon$ dimensions can be easily implemented by using [18]

$$\int \frac{d^2 p}{(2\pi)^2} \to T \sum\!\!\!\!\!\!\int_{\mathbf{p}} \equiv T \left( \frac{e^{\gamma_E} M^2}{4\pi} \right)^{\epsilon/2} \sum_{n=-\infty}^{+\infty} \int \frac{d^{1-\epsilon}\mathbf{p}}{(2\pi)}, \tag{3}$$

where $\gamma_E$ is the Euler-Mascheroni constant and $M$ is the $\overline{\mathrm{MS}}$ arbitrary regularization energy scale. Before performing the finite temperature evaluations we recall that in two-dimensions no spontaneous symmetry breaking of the global $O(N)$ symmetry can take place (at any coupling value) [19].

## 3. Perturbative pressure and scale invariance

If one considers the (one-loop) zeroth-order contribution to the pressure only the free gas type of term contributes yielding [13]

$$P = -\frac{(N-1)}{2} I_0(T) + \mathcal{E}_0, \tag{4}$$

where

$$I_0(T) = T \sum\!\!\!\!\!\!\int_{\mathbf{p}} \ln \left( \omega_n^2 + \omega_{\mathbf{p}}^2 \right), \tag{5}$$

with the dispersion $\omega_{\mathbf{p}}^2 = \mathbf{p}^2 + m^2$.

Then, performing the sum over the Matsubara's frequencies within the $\overline{\mathrm{MS}}$ scheme one obtains

$$I_0(m,T) = \frac{m_0^2}{2\pi} \left\{ \frac{1}{\epsilon} + \left[ \frac{1}{2} - \ln \left( \frac{m}{M} \right) \right] \right\} + T^2 \frac{2}{\pi} J_0(m/T). \tag{6}$$

In the above expression the thermal integral, $J_0(T)$, reads

$$J_0(m/T) = \int_0^\infty dz \ln \left( 1 - e^{-\omega_z} \right), \tag{7}$$

where we have defined the dimensionless quantity $\omega_z^2 = z^2 + y^2$ with $z = |\mathbf{p}|/T$ and $y = m/T$. Then, to this lowest perturbative order the renormalized pressure can be obtained by setting $m_0 = m$, $g_0 = g$, where $m$ is renormalized mass and $g$ is the renormalized coupling. The only divergence can be eliminated by the zero point subtraction term, $\mathcal{E}_0 = (N-1)m^2/(4\pi\epsilon)$, so that the finite pressure is simply

$$P = -\frac{(N-1)}{2} I_0^{\mathrm{r}}(T), \tag{8}$$

where

$$I_0^{\mathrm{r}}(m,T) = \frac{m^2}{2\pi} \left[ \frac{1}{2} - \ln \left( \frac{m}{M} \right) \right] + T^2 \frac{2}{\pi} J_0((\frac{m}{T}). \tag{9}$$

Next, let us consider the complete renormalization group (RG) operator defined by

$$M \frac{d}{dM} \equiv M \frac{\partial}{\partial M} + \beta \frac{\partial}{\partial g} - \frac{n}{2} \zeta + \gamma_m m \frac{\partial}{\partial m}. \tag{10}$$

Since the pressure is represented by a zero point Green's function ($n = 0$) we only need to consider the $\beta$ and $\gamma_m$ functions at the one-loop level [20] $\beta = -b_0 g^2$ and $\gamma_m = -\gamma_0 g$ where we have defined the RG coefficients in our normalization: $b_0 = (N-2)/(2\pi)$ and $\gamma_0 = (N-3)/(8\pi)$.

Let us now apply the RG operator to the zeroth-order pressure. Since $\beta$ and $\gamma_m$ are at least of order-$g$ and $J_0(T)$ is scale independent, one gets

$$M\frac{dP}{dM} = -(N-1)\frac{m^2}{4\pi} , \qquad (11)$$

which explicitly shows that the one loop pressure is not scale-invariant. Following Refs. [11, 12] one can fix this problem by adding a finite field independent term $m^2/g\sum s_k g^k$, where the coefficient $s_k$ is fixed so that the pressure becomes RG invariant as we now demonstrate. Employing this RGOPT prescription one can then write the zeroth-order pressure as

$$P = -\frac{(N-1)}{2}I_0^{\rm r}(T) + \frac{m^2}{g}s_0 . \qquad (12)$$

Then, the RG equation, $MdP/dM = 0$, leads to

$$-(N-1)\frac{m^2}{4\pi} + \left(\beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m}\right)\frac{m^2 s_0}{g} = 0 , \qquad (13)$$

fixing $s_0 = 1$. In the next section we show how this RG invariant perturbative pressure can be interpolated in order to generate non perturbative results.

## 4. RG improved optimized perturbation theory

The next step is to perform the replacements [11, 12] $m \to (1-\delta)^a m$ and $g \to \delta g$, into the renormalized and RG invariant perturbative pressure that was obtained in the previous section and then fix the RGOPT exponent, $a$. It is important to note that within the standard OPT this parameter is fixed in an *ad hoc* way (for example, $a = 1/2$ in the case of scalar theories [2, 3, 10]). Note also that in the case of massless theory, such as the case studied here, $m$ can also be interpreted as an infra red regulator. To fix the arbitrary mass parameter the RGOPT adopts the same variational criterion used within the standard OPT. Namely [21],

$$\frac{\partial P^{\rm RGOPT}}{\partial m}\Big|_{m=\bar{m}} = 0 , \qquad (14)$$

which implies that the RG operator is *reduced* to

$$\left(M\frac{\partial}{\partial M} + \beta\frac{\partial}{\partial g}\right)P^{\rm RGOPT} = 0 . \qquad (15)$$

Performing the replacements, re-expanding to the zeroth order and using $s_0 = 1$, one can finally write

$$P^{\rm RGOPT} = -\frac{(N-1)}{2}I_0^{\rm r}(T) + \frac{m^2}{g}(1-2a). \qquad (16)$$

Next, to fix the RGOPT exponent $a$ one requires that the pressure given by Eq. (16) satisfies the *reduced* RG relation, Eq. (15), obtaining

$$a \equiv \frac{\gamma_0}{b_0} = \frac{(N-3)}{4(N-2)}, \qquad (17)$$

which is also the value found in the context of the scalar $\lambda\phi^4$ theory [12]. Using $a = \gamma_0/b_0$ and the definitions for $b_0$ and $\gamma_0$ one can finally write

$$P^{\rm RGOPT} = -\frac{(N-1)}{2}I_0^{\rm r}(T) + (N-1)\frac{m^2}{(4\pi)g(M)b_0}. \qquad (18)$$

where, at the one loop level, the running coupling is given by

$$g(M) = g(M_0) \left[ 1 + g(M_0) b_0 \ln \left( \frac{M}{M_0} \right) \right]^{-1} . \tag{19}$$

Before presenting some numerical results let us quickly illustrate how the RGOPT is able to produce non perturbative results already at this lowest order. Applying the optimization criterion, Eq. (14), to the RGOPT pressure, Eq. (18), yields a non trivial gap equation for $\bar{m}$ which, at $T = 0$, leads to following the result for $\bar{m}$,

$$\bar{m}(0) = M \exp \left( -\frac{1}{g(M) b_0} \right) . \tag{20}$$

Moreover, as usual [22] the large-$N$ result can be exactly reproduced when the $N \to \infty$ limit is appropriately taken within our formalism.

## 5. Numerical results

In order to make contact with the usual SPT/HTLpt results [8, 9] let us first define a temperature dependent scale, $M = 2\pi T \alpha$, where the case $\alpha = 1$ determines the "central" value. To obtain the pressure at high temperatures one can consider the high-$T$ expansion for $J_0(T)$ [18]

$$J_0(m/T) = -\frac{\pi^2}{6} + \frac{\pi m}{2T} + \left( \frac{m}{2T} \right)^2 \left[ \ln \left( \frac{m e^{\gamma_E}}{4\pi T} \right) - \frac{1}{2} \right] + \mathcal{O}(m^4/T^4) , \tag{21}$$

Then, using the Stefan-Boltzmann result for the NLSM [14], $P_{SB} = (N-1)\pi T^2/6$, one obtains

$$\frac{P^{\mathrm{RGOPT}}(\alpha)}{P_{SB}} = 1 - 6\hat{m}(\alpha) - 6\hat{m}^2(\alpha) \left[ L_T(\alpha) - \frac{1}{b_0 g(\alpha, T)} \right] + O(\hat{m}^4) , \tag{22}$$

where, again in accordance with Refs. [8, 9], we have defined $\hat{m} = m/(2\pi T)$, $L_T(\alpha) = \ln(\hat{M}(\alpha) e^{\gamma_E}/2)$, and $\hat{M}(\alpha) = M/(2\pi T) = \alpha$. Setting $M_0 = 2\pi T_0$, where $T_0$ is a reference temperature one easily obtains $g(\alpha, T)$ from Eq. (19). Next, the optimized mass can be easily obtained by applying Eq. (14) to Eq. (22) which gives

$$\hat{m}(\alpha) = \frac{1}{2} \left( \frac{1}{b_0 g(\alpha, T)} - L_T(\alpha) \right)^{-1} . \tag{23}$$

Note that using the previous optimized mass gap solution, Eq. (23), within Eq. (22), the latter takes a much simpler expression (in the high-$T$ limit here considered):

$$\frac{P^{\mathrm{RGOPT}}(\alpha)}{P_{SB}} = 1 - 3\hat{m}(\alpha) + O(\hat{m}^4) . \tag{24}$$

In order to compare the RGOPT and ordinary PT results, one can obtain the latter directly from Eq. (8) in the massless limit, which after straightforward algebra gives

$$\frac{P^{\mathrm{PT}}(\alpha)}{P_{SB}} = 1 - \frac{3}{2} \gamma_0 \, g(\alpha, T). \tag{25}$$

On the other hand, the purely perturbative thermal "screening" mass at one-loop order can be obtained starting from the self-energy [17]:

$$\Gamma^{(2)}(p) = p^2(1 + g_0 I_1) + m_0^2 \left[ 1 + \frac{(N-1)}{2} g_0 I_1 \right] + \mathcal{O}(g^2) , \tag{26}$$

where $I_1$ is the basic (Euclidean) one-loop integral, in $\overline{\text{MS}}$ renormalization scheme,

$$I_1 \equiv T \sum_{\mathbf{p}} \!\!\!\!\!\!\! \int \frac{1}{\omega_n^2 + \mathbf{p}^2 + m^2} = \frac{1}{2\pi}\left(\frac{1}{\epsilon} - \ln\frac{m}{M} - 2J_1(m/T)\right), \tag{27}$$

with the thermal part $J_1(m/T)$ having the high-T expansion (compare Eq. (21))

$$J_1(m/T) \equiv -\frac{T^2}{m}\frac{\partial J_0(\frac{m}{T})}{\partial m} \simeq -\frac{\pi}{2}\frac{T}{m} - \frac{1}{2}\ln\left(\frac{m\,e^{\gamma_E}}{4\pi T}\right) + \mathcal{O}(m^2/T^2). \tag{28}$$

Taking thus the pole mass $p^2 \equiv -m_D^2$ in Eq. (26) after mass renormalization, gives, in the massless limit $m \to 0$ relevant for the pure thermal one-loop mass:

$$m_D(\alpha) = \frac{N-3}{8}\,g(\alpha,T)\,T \equiv \pi\,\gamma_0\,g(\alpha,T)\,T, \tag{29}$$

which may be compared with the RGOPT nonperturbative one-loop result (23).
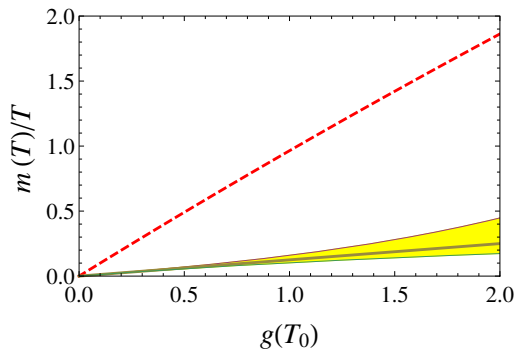


**Figure 1.** The thermal mass $m(T)/T$ for fixed $T = T_0$ as a function of the reference coupling, $g(T_0)$ for $\alpha = 0.5, 1$, and 2. Dashed line (RGOPT), upper continuous line (PT at $\alpha = 0.5$), middle continuous line (PT at $\alpha = 1$) and lower continuous line (PT at $\alpha = 2$).
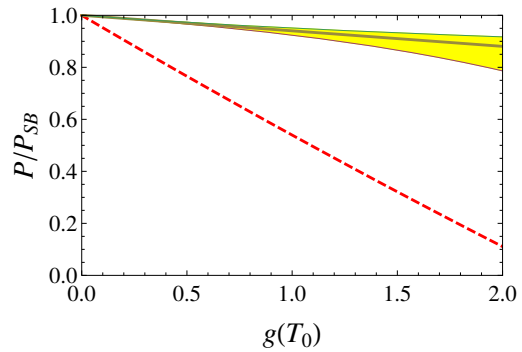
**Figure 2.** Normalized pressure $P/P_{SB}$ as a function of $g(T_0)$ at $T = T_0$ for $\alpha = 0.5, 1$, and 2. Dashed line (RGOPT), upper continuous line (PT at $\alpha = 2$), middle continuos line (PT at $\alpha = 1$) and lower continuous line (PT at $\alpha = 0.5$).

Choosing $N = 4$, $\alpha = 0.5, 1$, and 2 we can now investigate different thermal quantities. Fig. 1 shows the PT and RGOPT (thermal) masses $m(T)/T$ for fixed $T = T_0$ as a function of the reference coupling, $g(T_0)$. The RGOPT mass displays exact scale invariance, in contrast to the PT mass (29). Next, Fig. 2 shows $P/P_{SB}$ as a function of $g(T_0)$, also illustrating the exact scale invariance of the RGOPT pressure, and exhibiting a behavior similar to the one found for the $\lambda\phi^4$ model [12]. In particular, one should not be surprised by the different behavior (slope) as function of the coupling, with respect to PT one-loop results, of the RGOPT mass and pressure in Figs. 1 and 2 respectively. First, in contrast with the physical one-loop Debye (pole) mass, the optimized mass (23) is only an intermediate unphysical quantity in the pressure optimization procedure. Second, while comparing the different pressures as function of the running coupling Eq. (19) is sensible as concerns the scale dependence, at this stage the coupling value is not a physical quantity, being not fixed at some well-defined input scale and therefore essentially arbitrary in both PT and RGOPT cases. However, the overall physical consistency of our RGOPT

pressure result can be checked by noting that, when the *arbitrary* mass in Eq. (24) is replaced with the *physical* thermal mass $m_D$ Eq. (29), one consistently recovers the standard perturbative PT pressure as function of the coupling Eq. (25) (see Ref. [12] for a detailed discussion of similar results for the scalar $\phi^4$ model).

Finally Fig. 3 shows $P/P_{SB}$ as a function of $T/T_0$ for the fixed reference coupling, $g(T_0) = 1$, clearly displaying again the exact scale invariance of the nonperturbative RGOPT result. Although it is not obvious from the figure, note that the RGOPT pressure tends consistently towards the Stefan-Boltzmann limit for $T/T_0 \to \infty$, while the PT pressure reaches this limit more rapidly. This difference for a given reference coupling, and in particular the fact that the RGOPT pressure is substantially smaller than PT for moderate and small $T/T_0$, indicates that the RGOPT captures (resums) more essentially nonperturbative content.
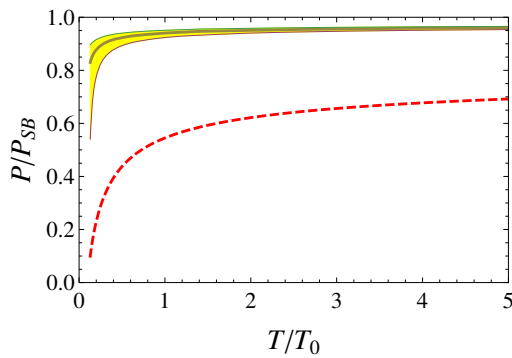


**Figure 3.** $P/P_{SB}$ as a function of $T/T_0$ for the fixed reference coupling, $g(T_0) = 1$, $\alpha = 0.5, 1$, and 2. Dashed line (RGOPT), upper continuous line (PT at $\alpha = 0.5$), middle continuous line (PT at $\alpha = 1$) and lower continuous line (PT at $\alpha = 2$)

## 6. Conclusions

Using a simple model which shares common features with QCD, such as asymptotic freedom, we have briefly described how the RGOPT technique can generate scale invariant non perturbative results from the lowest order contribution to the pressure. As we have shown, within this framework one starts by requiring that the ordinary "free gas" type of contribution be RG invariant, which implies that an extra (field independent) term should be added to the action. Next, the Lagrangian density is deformed by the interpolating term, $(1-\delta)^a m$, and the arbitrary parameters, $a$ and $m$, fixed by combining the RG equations with an optimization variational criterion. Circumventing the severe scale dependence issues observed within related variational methods (such as OPT, SPT, and HTLpt) the RGOPT method described here stands as a new potential alternative to LFT concerning the description of compressed strongly interacting matter.

**References**
[1] Aoki Y, Endrodi G, Fodor Z, Katz S D and Szabo K K 2006 *Nature* **443** 675
    Aoki Y, Fodor Z, Katz S D and Szabo K K 2006 *Phys Lett* B **643** 46
[2] Okopińska A 1987 *Phys Rev* B **35** 1835
    Duncan A and Moshe M 1988 *Phys Lett* B **215** 352
[3] Chiku S and Hatsuda T 1998 *Phys Rev* D **58** 076001
[4] Kneur J L, Pinto M B and Ramos R O 2010 *Phys Rev* C **81** 065205
    Ferroni L, Koch V and Pinto M B 2010 *Phys Rev* C **82** 055205

    Restrepo T E, Macias J C, Pinto M B and Ferrari G N 2015 *Phys Rev* D **91** 065017
[5]  Kleinert H 2003 *Mod Phys Lett* B **17** 1011
    Kastening B 2003 *Phys Rev* A **68** 061601
    Kneur J L, Neveu A and Pinto M B 2004 *Phys Rev* A **69** 053624
[6]  Caldas H, Kneur J L, Pinto M B and Ramos R O 2008 *Phys Rev* B **77** 205109
[7]  Kneur J L, Pinto M B and Ramos R O 2013 *Phys Rev* D **88** 045005
[8]  Karsch F, Patkos A and Petreczky P 1997 *Phys Lett* B **401** 69
    Andersen J O, Braaten E and Strickland M 2001 *Phys Rev* D **63** 105008
    Andersen J O and Strickland M 2005 Ann Phys **317** 281
[9]  Andersen J O, Braaten E and Strickland M 1999 *Phys Rev Lett* **83** 2139
    Andersen J O, Leganger L E, Strickland M and Su N 2011 *JHEP* **1108** 05
    Haque N, Bandyopadhyay A, Andersen J O, Mustafa M G, Strickland M and Su N 2014 *JHEP* **1405** 027
[10] Farias R L S, Krein G and Ramos R O 2008 *Phys Rev* D **78** 065046
[11] Kneur J L and Neveu A 2010 *Phys Rev* D **81** 125012
    Kneur J L and Neveu A 2012 *Phys Rev* D **85**, 014005
    Kneur J L and Neveu A 2013 *Phys Rev* D **88** 074025
    Kneur J L and Neveu A 2015 *Phys Rev* D **92** 074027
[12] Kneur J L and Pinto M B 2015 *Phys Rev* D **92** 116008
    Kneur J L and Pinto M B 2016 *Phys Rev Lett* **116** 031601
[13] Dine M and Fischler W 1981 *Phys Lett* B **105** 207
[14] Seel E, Smith D, Lottini S and Giacosa F 2013 *JHEP* **1307** 010
[15] Andersen J O, Boer D and Warringa H J 2004 *Phys Rev* D **69** 076006
[16] Ferrari G N, Kneur J L, Pinto M B and Ramos R O *in preparation*
[17] Zinn-Justin J 2002 *Quantum field theory and critical phenomena* (Oxford: Clarendon Press)
[18] Kapusta J I 1985 *Finite-Temperature Field Theory* (Cambridge: University Press)
[19] Mermin N D and Wagner H 1966 *Phys Rev Lett* **17** 1133
[20] Hikami S and Brézin E 1978 *J Phys* A **11** 1148
[21] Stevenson P M 1981 *Phys Rev* D **23** 2916
[22] Gandhi S K, Jones H F and Pinto M B 1991 *Nucl Phys* B **359** 429