

CP violation in beauty and charm at LHCb

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Abstract. Precision measurements of CP violating observables in the decays of b and c hadrons are powerful probes to search for physics beyond the Standard Model. The most recent results on CP violation in the decay, mixing and interference of both b and c hadrons obtained by the LHCb Collaboration with Run I and years 2015–2016 of Run II are presented, including the first observation of CP violation in the charm system. In particular world best constraints and world first measurements are provided for CKM elements, unitarity angles and charm parameters. Prospects for future sensitivities are also discussed.

1. Introduction

Although the Standard Model (SM) has been very successful describing particles and their interactions, we already know it must be an effective theory whose validity range stops at some higher energy scale. One of the most intriguing unexplained phenomena is the large matter– antimatter asymmetry—which requires a sizeable Charge–Parity (CP) asymmetry [1]—that leads to current Universe, a matter dominated one. From the SM point of view there is only one source of CP violation in the quark sector, the complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix, which is orders of magnitude smaller than the needed to match the nature asymmetry. The CKM is a 3×3 matrix completely defined by three real rotation angles and a complex phase which are not predicted by the SM but need to be measured.

Many theories predict the existence of, yet unobserved, heavy particles, referred to as Beyond the Standard Model (BSM) physics. These can appear by means of higher order contributions to a given process, in the so called box and penguin diagrams. These new particles would introduce modifications of observables such as the CP violating parameters. Therefore precision measurements in the beauty and charm system are one of the best probes to test the SM and search for BSM contributions. These are sensitive to mass scales reaching up to 100 TeV.

In the literature [2], three types of CP violation are often distinguished from asymmetries between amplitudes, \mathcal{A} , detailed below.

CP violation in decay. When the amplitudes of the processes A_f and $\bar{A}_{\bar{f}}$ (the conjugate process) are different, then there is CP violation given that

$$\left| \frac{A_f}{\bar{A}_{\bar{f}}} \right| \neq 1 \quad \Rightarrow \quad \mathcal{A}_D = \frac{\bar{\Gamma} - \Gamma}{\bar{\Gamma} + \Gamma} = \frac{|\bar{A}_{\bar{f}}/A_f|^2 - 1}{|\bar{A}_{\bar{f}}/A_f|^2 + 1}.$$

This is the only source of CP violation in charged mesons, since these are not affected by the mixing [3].



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CP violation in the mixing. Arises when the parameters that control the mixing, p and q , are not the same

$$\left| \frac{q}{p} \right| \neq 1 \quad \Rightarrow \quad \mathcal{A}_M = \frac{d\bar{\Gamma}/dt - d\Gamma/dt}{d\bar{\Gamma}/dt + d\Gamma/dt} = \frac{1 - |q/p|^4}{1 + |q/p|^4},$$

a kind of CP violation that has been seen and measured in semi-electronic decays. Notice that this asymmetry of time-dependent decay rates is, in fact, non-time-dependent. If this CP violation is manifested, it would indicate that in these processes the mixture is biased to one of the mesons.

CP violation in interference. CP violation can occur between the decay of the meson and its particle-antiparticle oscillation, in the entangled state that occurs before the decay. If both mesons share the same final state, $f = \bar{f}$, then they can decay to the final state having or not previously oscillated. It is possible to observe CP violation, through $\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$ parameter, even in the absence of direct and indirect CP violation, $|\lambda_f| = 1$, i.e.,

$$\Im(\lambda_f) \neq 0 \quad \Rightarrow \quad \mathcal{A}_\lambda = \frac{d\Gamma(\bar{M} \rightarrow f)/dt - d\Gamma(M \rightarrow f)/dt}{d\Gamma(\bar{M} \rightarrow f)/dt + d\Gamma(M \rightarrow f)/dt} \propto \Im(\lambda_f) \sin(\Delta m t).$$

Hereinafter a selection of the most recent results of LHCb in CP violation are presented, covering mainly searches of CP asymmetries in beauty and charm decay channels.

2. Beauty physics

2.1. Direct CPV

This section is centred in direct CP violation in B meson decays, the only one manifestation in B^\pm mesons. Tree-body charmless decays allow to study strong phases in short distance processes as well as in long distance ones with hadron-hadron interactions in the final state as $KK \leftrightarrow \pi\pi$ exchange and interference between intermediate states. Regarding this there are two main results coming from LHCb: $B^\pm \rightarrow \pi^\pm K^+ K^-$ already published, and $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ in preparation. Further, an updated measurement of $B^+ \rightarrow J/\psi \rho^+$ is presented.

2.1.1. $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays [4] This study uses 3 fb^{-1} of Run 1 data. Separated Dalitz plot analysis are performed for B^+ (with about 2000 events) and B^- (1600 events). The Dalitz Plot amplitude is parametrised with seven components, using isobar model, the most important ones are the non-resonant and $\rho^0(1450)$, representing about 30% each. The rescattering amplitude, produces a negative CP asymmetry, $\mathcal{A} = (-66 \pm 4 \pm 2)\%$ which is the largest CPV effect ever coming from a single amplitude

2.1.2. $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decays [5] This analysis uses 3 fb^{-1} of Run 1 data, that translates into about 20000 signal decays. Three different models are used to handle the complicated S-wave parametrisation: isobar, where each contribution has a clear physical meaning; K-matrix, which is unitary by construction; and quasi-model-independent approach, that fits regions of the Dalitz Plot directly from data. All three are in broad agreement, but isobar model is the one which worst fits to data. There are lots of resonances in $\pi^+ \pi^-$ pairs, making this one of the most difficult analysis performed at LHCb. Three different kinds of CP asymmetries are observed: huge asymmetry in S-P interference around the $\rho^0(770)$ pole with over 25σ statistical significance, being the first observation of CPV in a quasi-two-body interference; large asymmetry in $f_2(1270)$ tensor, at over 10σ , the first evidence of CP violation involving a tensor; and asymmetry in S-wave at low $\pi^+ \pi^-$ mass with over 10σ statistical significance where there is a flip of sign in m_{KK} threshold. Finally, no asymmetry is observed in $\rho - \omega$ mixing.

2.1.3. $B^+ \rightarrow J/\psi \rho^+$ decay [6] The analysis of this decay channel measures its branching-fraction, relative to $B^+ \rightarrow J/\psi K^+$ because of its well known branching-fraction and statistics, and the direct CP asymmetry, using 3 fb^{-1} of Run 1 data. The branching-fraction was measured to be $\mathcal{B}(B^+ \rightarrow J/\psi \rho^+) = (3.81^{+0.25}_{-0.24} \pm 0.35) \times 10^{-5}$ and the direct CP asymmetry $\mathcal{A}_{\text{CP}}(B^+ \rightarrow J/\psi \rho^+) = -0.045^{+0.056}_{-0.057} \pm 0.008$, that is compatible with $B^0 \rightarrow J/\psi \rho$ as expected from isospin symmetry [7]. Both are the most precise measurements to date, and compatible with the previous BaBar result [8]. This asymmetry value can be used to place penguin constraints in measurements of ϕ_s in $B_s^0 \rightarrow J/\psi \phi$.

2.2. Measurement of the CKM phase γ [9, 10]

The least well determined angle of the the unitary triangle is γ , which is the only one determined from tree-level only processes. This implies that these measurements are not expected to be strongly affected from BSM contributions.

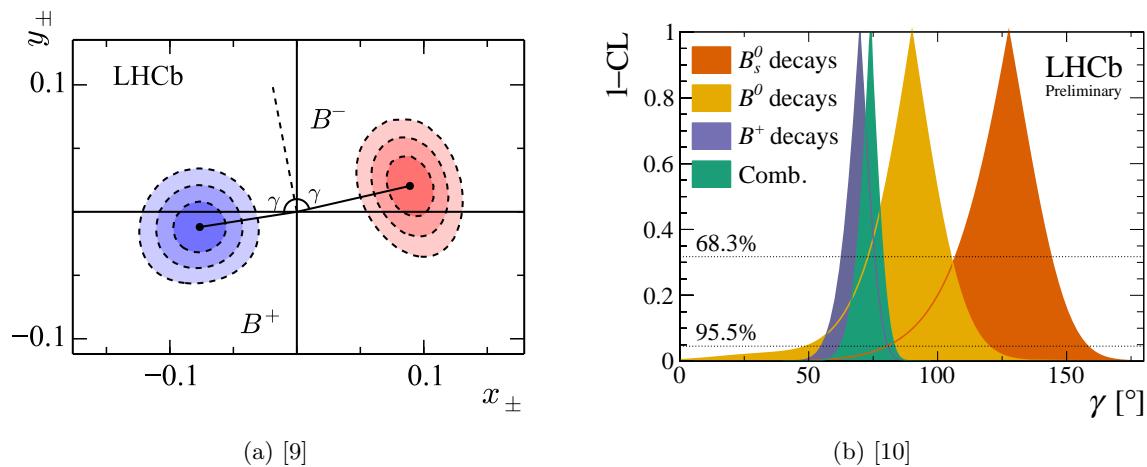


Figure 1: (a) Measurement of γ in B^{\pm} decay channels. (b) Confidence level curves for the combination of the γ measurements and the contributions for the individual beauty meson.

Experimentally this angle is measured exploiting the interference between $b \rightarrow c$ and $b \rightarrow u$ transitions, by means of the CP asymmetry in $B^{\pm} \rightarrow D K^{\pm}$ and $B_q^0 \rightarrow D_q^+ K^-$ decays. The first approach [9] requires an excellent $k - \pi$ separation as well as the difficulty of triggering purely hadronic final states, resulting in an experimentally challenging measurements. The second approach [10], actually two approaches, involves looking at the time-dependent CP asymmetry of these decays. These three approaches are combined by LHCb to achieve the most precise single-experiment measurement to date of γ . The LHCb measurement is $(74.5^{+5.0}_{-5.8})^\circ$ [11] and the HFLAV combination is $(73.5^{+4.2}_{-5.1})^\circ$ both consistent within $\sim 2\sigma$ with the UTfit derivation $(65.8 \pm 2.2)^\circ$, which does not include the tree-level measurements (it is the result from applying all of the other constraints).

The measurement of this angle is a non trivial test on the KM theory of CPV single-phase hypothesis and the possibility of contributions of new physics in tree-level diagrams. Finally, there exist some small internal tensions between $B_{s,d}^0$ and B^{\pm} measurements of γ as can be seen in Figure 1.

2.3. Measurement of the electroweak phase ϕ_s

There are several channels analysed in order to measure this phase, which arises from the interference between mixing and decay in B_s^0 channels. Its expression is $\phi_s = \phi_M - 2\phi_D = \arg \lambda$,

measuring alongside $\Delta\Gamma_s$ and Γ_s , in time-dependent analyses. New Physics (NP) can appear in ϕ_M , the mixing phase, both in tree and penguin diagrams, and also in ϕ_D , the decay phase, in penguin diagrams. Two penguin examples were selected, $B_s^0 \rightarrow \phi\phi$ and $B_{s,d}^0 \rightarrow K^{*0}\bar{K}^{*0}$; and also two tree level examples that proceed through a $b \rightarrow c\bar{c}s$ transition, $B_s^0 \rightarrow J/\psi K^+K^-$ and $B_s^0 \rightarrow J/\psi\pi^+\pi^-$, where $\phi_s^{c\bar{c}s}$ is expected to be $\phi_s \approx -2\beta_s$. The β_s angle is very well predicted, coming from experimental measurements and it is derived from collaborations as CKMfitter within SM, to be $-2\beta_s = 0.0368^{+0.0010}_{-0.0007}$, [12]. Any deviation from this value, small or big, would imply NP, thus there are lots of efforts to measure it; direct CP violation can be observed too, by means of $|\lambda| \neq 1$.

Table 1: Latest results from $B_s^0 \rightarrow J/\psi K^+K^-$ decay channel. Table 2: Latest results from $B_s^0 \rightarrow J/\psi\pi^+\pi^-$ decay channel.

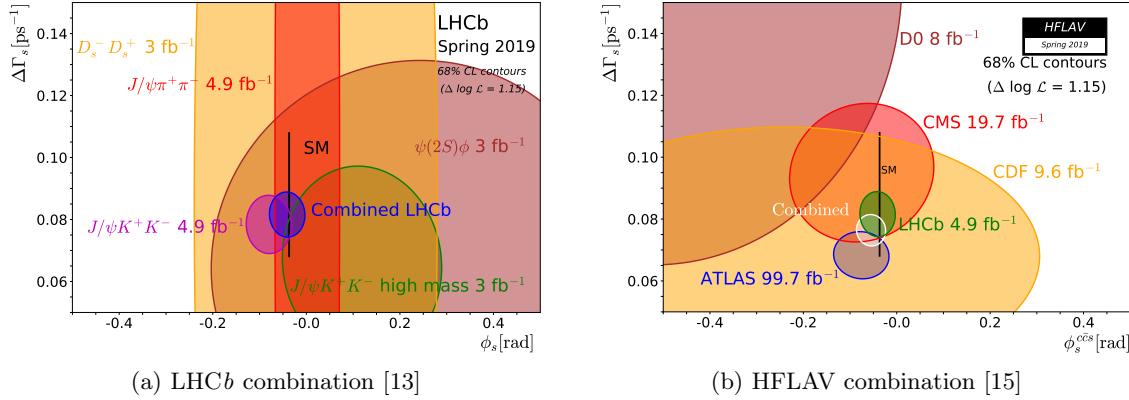
Parameter	Value
ϕ_s	$-83 \pm 41 \pm 6$ mrad
$ \lambda $	$1.012 \pm 0.016 \pm 0.006$
$\Gamma_s - \Gamma_d$	$-4.1 \pm 2.4 \pm 1.5$ fs $^{-1}$
$\Delta\Gamma_s$	$77 \pm 8 \pm 3$ fs $^{-1}$

Parameter	Value
ϕ_s	$-57 \pm 60 \pm 11$ mrad
$ \lambda $	$1.01^{+0.08}_{-0.06} \pm 0.03$
$\Gamma_H - \Gamma_d$	$-50 \pm 4 \pm 4$ fs $^{-1}$

2.3.1. $B_s^0 \rightarrow J/\psi K^+K^-$ decay [13] This is the golden plated channel to measure ϕ_s and was analysed using 1.9fb^{-1} of Run2 data representing about 117000 events. The measurement is centered at $m(K^+K^-) \in [990, 1050]\text{ MeV}/c^2$ mass window, dominated by the $\phi(1020)$ resonance including a small S-wave contribution. The average decay-time resolution is $\sigma_{\text{eff}} = 45.5\text{ fs}^{-1}$. The decay-time and angular efficiencies are estimated with simulation and matched to data. There is a 30% higher tagging power when comparing with Run 1. To fit this data a four dimensional unbinned maximum likelihood fit is performed to the three helicity angles and the time. The fit is performed in six bins of K^+K^- pair mass, to take into account S and P-wave interference after integrating in the dikaon mass, which does not vanish. The obtained results are shown in Table 1, where ϕ_s is consistent with a non-zero CP violation predicted within the SM and with no CP violation in the interference of B_s^0 meson mixing and decay, while $|\lambda|$ is compatible with zero direct CP violation.

2.3.2. $B_s^0 \rightarrow J/\psi\pi^+\pi^-$ decay [14] This channel is also used to measure ϕ_s , and its analysis is similar to the one described above. In this case $f_0(980)$ is the main resonance mixed with other S and D wave resonances in the 1.9 (Run 2) fb^{-1} of data (about 33000 events) used. The average decay-time resolution is $\sigma_{\text{eff}} = 41.5\text{ fs}^{-1}$. Decay-time and angular efficiencies are estimated with simulation and matched to data. In order to fit the data an unbinned maximum likelihood fit is performed by means of a five-dimensional amplitude analysis taking into account the three helicity angles, the time and the dipion mass. The obtained parameters are shown in Table 2.

2.3.3. HFLAV combination of $\phi_s^{c\bar{c}s}$ [15] HFLAV collaboration makes a combination of all the results given by different experiments. They are in agreement with previous measurements and the SM prediction. There is a reduction on the experimental uncertainty of about 30 % from the average before Moriond 2019 as represented in Figure 2, mainly thanks to ATLAS and LHCb latest results, and ϕ_s is 2.5σ different from zero. LHCb and ATLAS have a large tension, about 4σ , in Γ_s . The LHCb value of ϕ_s is -41 ± 25 mrad and the HFLAV one is -55 ± 21 mrad.

Figure 2: Two-dimensional likelihood scans of ϕ_s and $\Delta\Gamma_s$.

2.3.4. $B_s^0 \rightarrow \phi\phi$ decay [16] Using 3 (Run 1) + 2 (Run 2) fb⁻¹ of data (about 8500 events) a four-dimensional fit is performed to both helicity angles and time. The parameters Γ_s , $\Delta\Gamma_s$ and Δm_s are fixed to known values. The mass spectrum is mainly composed of P-wave $\phi(1020)$ and S-wave $f_0(980)$ contributions. Detector efficiency and decay-time resolution are determined with simulation whilst decay-time efficiency is determined with data. There is an improvement on experimental uncertainty of 25% on $\phi_s^{s\bar{s}s}$, and 40 % on $|A_0|^2$ from previous analysis, which is in agreement with QCD predictions as shown in Table 3.

Table 3: Latest results from $B_s^0 \rightarrow \phi\phi$ decay channel.

Parameter	Value
$\phi_s^{s\bar{s}s}$	$-73 \pm 115 \pm 27$ mrad
$ \lambda $	$0.99 \pm 0.05 \pm 0.01$
$ A_0 ^2$	$0.381 \pm 0.007 \pm 0.012$

Table 4: Results from $B_s^0 \rightarrow K^{0*}\bar{K}^{0*}$ time-dependent analysis [17].

Parameter	Value
$\phi_s^{d\bar{d}s}$	$-100 \pm 130 \pm 140$ mrad
$ \lambda $	$1.035 \pm 0.034 \pm 0.089$

2.3.5. $B_{s,d}^0 \rightarrow K^{*0}\bar{K}^{*0}$ decays [18] The first LHCb study of $B_d^0 \rightarrow K^{*0}\bar{K}^{*0}$ decay is an untagged and time-integrated analysis assuming $\Delta\Gamma \approx 0$ and negligible CP violation in the mixing and the decay. The data is composed of 3 (Run 1) fb⁻¹ that is fitted against a 5-dimension model: helicity angles and the two $m(K\pi)$ masses. Since $B_{s,d}^0 \rightarrow K^{*0}\bar{K}^{*0}$ are U-spin partners, B_d can be used to control the penguin pollution. The same analysis is performed to $B_s^0 \rightarrow K^{*0}\bar{K}^{*0}$ and it is compatible with the previous time-dependent analysis [18] that gave the results listed in Table 4. It was found that $|A_0|^2(B_d^0) \gg |A_0|^2(B_s^0)$, since these values are so different whilst not expecting so, this translates into a thus unexpected value of the B_s^0/B_d^0 branching-fraction ratio $R_{sd} = \frac{\mathcal{B}(B_s^0)}{\mathcal{B}(B_d^0)} \frac{|A_0|^2(B_s^0)}{|A_0|^2(B_d^0)} \frac{1-y^2}{1+y \cos \phi_s}$ which is measured to be $R_{sd}^{\text{exp}} = 3.43 \pm 0.38$, far away from agreement with the theoretical prediction [19] $R_{sd}^{\text{theo}} = 16.4 \pm 5.2$.

3. Charm physics

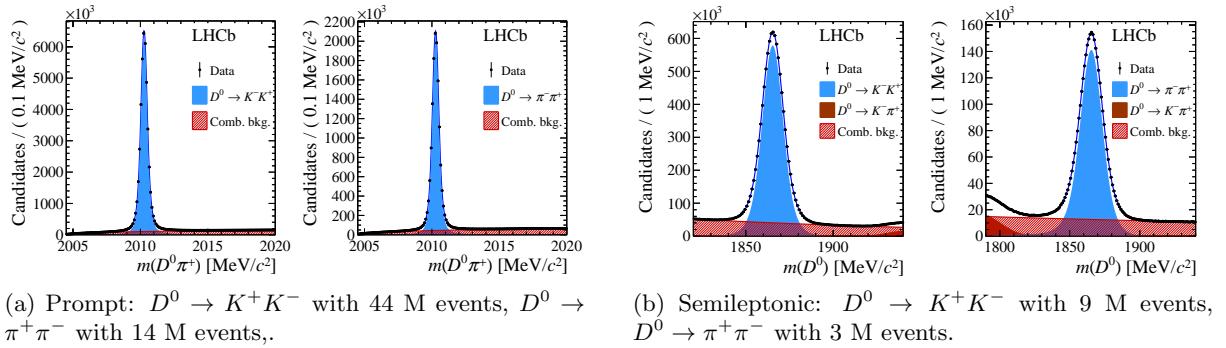
After the first experimental observations of the slow mixing rate of the D^0 flavour oscillations, the level of attention on charm physics increased significantly, getting entirely complementary to the B and K mesons, for mixing and CP violation. Here the direct and indirect CP violation search and the best measurements of charm mixing parameters are presented.

3.1. Direct CP violation in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays [20]

By means of 5.9 fb^{-1} (almost full Run 2) of data using both prompt $m(D^0\pi_{\text{soft}}^+)$ and semileptonic $m(D^0)$, LHCb has found for the very first time CP violation in charm as it is shown in Figure 3. The raw asymmetry in Cabibbo suppressed $D^0 \rightarrow h^+h^-$ decays is $\mathcal{A}(D \rightarrow h^+h^-) = \mathcal{A}_{\text{CP}} + \mathcal{A}_{\text{detector}} + \mathcal{A}_{\text{production}}$ that includes both physics and detector effects. To get rid of them it is computed $\Delta\mathcal{A}_{\text{CP}} = \mathcal{A}(K^+K^-) - \mathcal{A}(\pi^+\pi^-) = \mathcal{A}_{\text{CP}}(K^+K^-) - \mathcal{A}_{\text{CP}}(\pi^+\pi^-)$. This difference of asymmetries is primarily sensitive to direct CPV. Using the latest results of $\frac{\Delta\langle t \rangle}{\tau(D^0)} = \frac{\langle t \rangle_{KK} - \langle t \rangle_{\pi\pi}}{\tau(D^0)} = 0.115 \pm 0.002$ and $\mathcal{A}_{\Gamma} \approx -a_{\text{CP}}^{\text{ind}} = (-2.8 \pm 2.8) \times 10^{-4}$ it is possible to calculate $a_{\text{CP}}^{\text{dir}}$, the direct CP asymmetry,

$$a_{\text{CP}}^{\text{dir}} = \Delta\mathcal{A}_{\text{CP}} + \frac{\Delta\langle t \rangle}{\tau(D^0)} \mathcal{A}_{\Gamma} = (-15.7 \pm 2.9) \times 10^{-4}.$$

This means that CP violation was observed at 5.4σ statistical significance, consistent with the SM expectations. Due to its large uncertainties because of strong interactions, the value ranges between $10^{-4} - 10^{-3}$.



(a) Prompt: $D^0 \rightarrow K^+K^-$ with 44 M events, $D^0 \rightarrow \pi^+\pi^-$ with 14 M events.,.

(b) Semileptonic: $D^0 \rightarrow K^+K^-$ with 9 M events, $D^0 \rightarrow \pi^+\pi^-$ with 3 M events.

Figure 3: Raw asymmetries from fits to the $m(D^0\pi^+)$ or $m(D^0)$ distributions.

3.2. Indirect CP violation search in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decay channels [21]

A new measurement of \mathcal{A}_{Γ} with 1.9 fb^{-1} was presented with the asymmetries in the mixing and in the interference are measured to be $\mathcal{A}_{\Gamma}(K^+K^-) = (1.3 \pm 3.5 \pm 0.7) \times 10^{-4}$ for kaons and $\mathcal{A}_{\Gamma}(\pi^+\pi^-) = (11.3 \pm 6.9 \pm 0.8) \times 10^{-4}$ for pions. Neglecting decay phases, it is possible to get a combined value

$$\mathcal{A}_{\Gamma}(K^+K^- + \pi^+\pi^-) = (0.9 \pm 2.1 \pm 0.7) \times 10^{-4}$$

that means that there is no evidence for CPV in mixing or interference so far, the result is statistically limited yet.

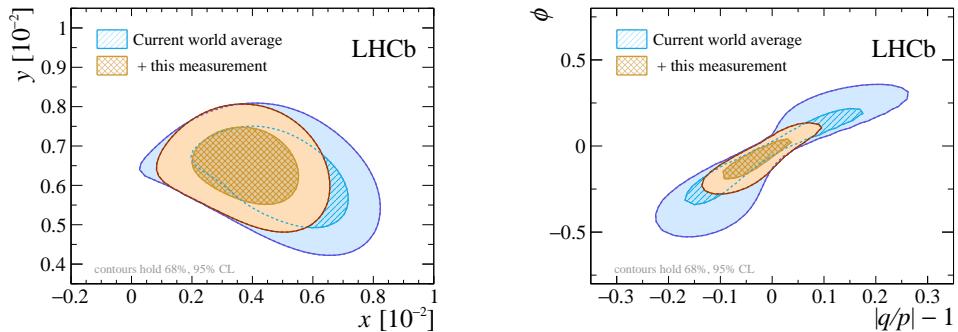


Figure 4: Mixing parameters evolution with this new $LHCb$ measurement.

3.3. Charm mixing parameters [22, 23]

$LHCb$ provides now the most precise single-experiment measurements to date of the charm mixing parameters that are shown in Figure 4. To achieve these results a time-dependent Dalitz plot analysis is made, using the bin-flip method. Data is binned in Dalitz plot coordinates where the binning scheme is chosen to have approximately constant strong-phase differences. Then the yield ratio, $R_{bj}^\pm(x_{CP}, y_{CP}, \Delta x, \Delta y)$, is measured between $[-b, +b]$ bins as function of decay time. Two analysis are performed: one from 2018 that measures y_{CP} and another from 2019 for x_{CP} . Both analysis are compatible with symmetry hypothesis regarding the search of CP violation in $D^0 - \bar{D}^0$ mixing. With 3 fb^{-1} of data, about 2.3 million events, and using both prompt and semileptonic tagging, it is measured [22]

$$x_{CP} = 0.27 \pm 0.16(\text{stat}) \pm 0.04(\text{syst})\% \\ \Delta x = -0.053 \pm 0.070(\text{stat}) \pm 0.022(\text{syst})\%$$

that is also the first evidence of $x > 0$ when combining with previous measurements. On the other hand, using 3 fb^{-1} of data coming from semimuonic B decays it is measured [23]

$$y_{CP} = 0.57 \pm 0.13(\text{stat}) \pm 0.09(\text{syst}).$$

4. Conclusions

To summarise, the following $LHCb$ highlights were presented:

- Time-dependent analyses are compatible with the SM and produce the strongest constraints in the different ϕ_s .
- Very large direct CPV manifestations in DP regions of charmless 3-body decays, possibly due to strong phases originated in rescattering.
- First observation of CPV involving a tensor.
- First observation of CP violation in charm decays that corresponds to direct CP violation found by means of ΔA_{CP} . There is no evidence for indirect one yet.
- Most precise determination of the mixing parameters x_{CP} and y_{CP} from a single experiment. Also first evidence of $x > 0$, contributing to the emerging evidence for a positive nonzero mass difference between the neutral charm-meson eigenstates.
- All results are statistically limited: large room for improvements with next runs of $LHCb$.

Concerning beauty physics, $LHCb$ is currently dominating the CKM angle $\gamma = (74.0 \pm 5.0)^\circ$, and the precision on γ will improve after Upgrade II, being 0.35° its expected uncertainty. The expected precision on $\phi_s^{c\bar{c}s}$ after Upgrade II will be ~ 3 mrad from all modes combined. We are now set for precision studies on CPV in charm and future measurements with HL-LHC will lead to significant improvements in precision [24].

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