Formulation of relativistic dissipative hydrodynamics of spin-1/2 particles from kinetic theory

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Heavy-ion collisions (HICs) have improved our understanding of the properties of strongly interacting hot QCD matter. The matter produced in these extreme events has shown fluid-like properties with minimum viscosity [1] and consequently, the theory of relativistic hydrodynamics has played a significant role in describing many of its properties with unprecedented accuracy [2]. The recent experimental observation of global spin-polarization of hadrons in non-central HICs [3] gave rise to the necessity of formulating a theory of relativistic ideal spin-hydrodynamics with equilibrated spin degrees of freedom. The development of the ideal spin-hydrodynamics required the introduction of a chemical potential-like rank-2 tensor related to spin, called spin-polarization tensor. It was also shown that under global equilibrium, this spin-polarization tensor should become the thermal vorticity. The assumption of global equilibrium of spin was further strengthened by its success in explaining the global spin-polarization phenomenon in Λ hyperons. However, the same assumption of equilibration led to contradictory results in the case of longitudinal spin-polarization [4], indicating the requirement of inclusion of non-trivial physics such as dissipation. Keeping this in mind, we developed a theory of relativistic dissipative spin-hydrodynamics in the absence of any external force and later extended it to include the effects of the magnetic field.

To formulate the dissipative spin hydrodynamics, in this work [5, 6] we started with a semi-classical expansion of the Wigner function of spin-1/2 particles and then constructed the kinetic theory within the relaxation time approximation (RTA). This allowed us to set up the first-order equations of relativistic dissipative hydrodynamics to describe a spin-polarizable relativistic fluid. The conserved currents of the system, such as the particle four-flow and energy-momentum tensor, were found to have no corrections and are identical to those obtained in the case of unpolarized fluids. We further determined the behavior of the spin current and found that it depends on the gradients of multiple hydrodynamic variables. This led to the first identification of several novel transport coefficients related to spin transport in a spin-polarizable relativistic fluid. We also found the evolution of the spin-polarization tensor to depend on multiple hydrodynamic gradients as well.

We then incorporated the effect of magnetic field, which may be produced during the non-central HICs due to charged spectators receding with relativistic velocities. In this work [7], we assumed the constituent particles of the fluid to be magnetizable and spin-polarizable. Consequently, the Boltzmann equation was modified to include Lorentz and Mathisson force terms. Similar to Bargmann-Michel-Telegdi equations, a term with a derivative of the internal angular momentum appeared in the transport equation, which was dropped later for the sake of simplification. By taking appropriate moments of the Boltzmann equation, we obtained the correct hydrodynamic equations. Interestingly, all dissipative currents were found to depend on multiple hydrodynamic gradients, some of which implied coupling between the magnetic field and spin-polarization tensor. We further evaluated the magnetization tensor of the fluid and found it to be proportional to the spin-polarization tensor, which in the case of a rigid rotor be-

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comes the fluid vorticity. This established the suspected existence of magneto-mechanical effects like Einstein-de Hass and Barnett effects in a spin-polarizable and magnetizable fluid. The evolution of spin-polarization tensor established its connection with vorticity in a magnetizable fluid, which was missed previously [6]. While the conservation laws for energy-momentum and angular momentum tensors of the fluid were satisfied in the absence of a magnetic field, the same is no longer true in the presence of a magnetic field. Their respective divergences are equal to the force and torque exerted by the external current. However, the violation of these conservation laws for the fluid was of no concern since the total system (including the source of the external magnetic field) still satisfies them. Thus we successfully constructed the theories of spin-hydrodynamics and spin-magnetohydrodynamics from kinetic theory by extending RTA to incorporate the spin degrees of freedom.

Noting that the equilibration mechanism of the system may significantly influence the origin of longitudinal spin-polarization, we then focused on two aspects - (i) modification of RTA for a fluid of multiple species with different relaxation timescales and (ii) incorporation of an equation of state of the hot QCD matter. For the first case, we considered a fluid consisting of three different species, which can interact with each other. Such interactions need not have the same timescales. To accommodate these different timescales for different processes, we modified the RTA, proposed by Anderson and Witting. The conservation laws of this work [8] restricted the number of independent relaxation timescales to two. We found that while the total transport coefficients remain unaffected, the transport properties of individual species were modified, which may be significant for the thermalization process of individual species including their spin degrees of freedom. This may be crucial for observables related to the polarization of hadrons in HICs. As for the second case, we constructed a theory of relativistic hydrodynamics for a set of quasiparticles [9, 10]. Their distribution functions, as well as the transport equations, were modified to incorporate the equation of state of the hot QCD matter that was obtained from Lattice QCD. A significant deviation of the transport properties was noted as the system moved toward the transition temperature. Consequently, the thermalization and isotropization processes were modified. The evolution of the system turned out to be more realistic with the second-order dissipative corrections for specific initial conditions. Such modifications may be necessary to describe the evolution of spin degrees of freedom and thereby the spin-polarization phenomena.

This thesis work has shown the importance of dissipative effects for the description of polarizable fluids and established a framework that can be used for realistic simulation to explain some unresolved puzzles in HICs.

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