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REVIEW ON SCATTERING-AMPLITUDE ANALYSIS *

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Abstract: Attempts to determine πN scattering amplitudes at around 6 GeV/c have been made since results of several scattering measurements with polarized-proton targets became available. We discuss new πp charge-exchange polarization data by comparing with various theoretical models and review πN amplitude analyses.

Résumé : Des tentatives d'extraction des amplitudes de diffusion π N à 6 GeV/c ont été faites en utilisant les résultats d'expériences de diffusion sur des cibles de protons polarisés. Nous discutons les nouvelles données de l'échange de charge π p en passant en revue ces analyses en amplitudes ainsi que divers modèles théoriques.



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Since preliminary results of the measurements of R and A parameters[1] and of πp charge-exchange polarization[2] became available, a long-waiting crucial task to deduce πN scattering amplitudes has been attempted by several authors. In this talk, I would like, first, to compare results of new πp charge-exchange polarization data[3] obtained from ANL ZGS with various theoretical predictions, second, to review πN-amplitude analyses with an emphasis on the latest analyses[4] using the new data,[3, 5] and, finally, to discuss measurements needed to improve scattering-amplitude uncertainty.

New Data and Theoretical Predictions in πp Charge-Exchange Polarization

There are numerous theoretical papers discussing πp charge-exchange polarization. Here I compare some of them with the new data.[3] With the exception of the prediction by Barger and Phillips,[6] which was made before any data in the region $|t| > 0.3$ became available, most of the predictions have perhaps been biased by the preliminary data.[2] Some of the models are actually fitted to the data whenever a preliminary stage was completed, and others stick with the consistency of their fundamental principles.

Figure 1(a) shows predictions by i) Ref.6 (solid) based on a pole model and finite-energy sum rules, ii) Ref. 7 (dotted) using a new two-pole model, and iii) Ref. 8 (dashed) using exchange degeneracy, SU 3 symmetry, and absorption effects. Figure 1(b) shows predictions by i) Ref. 9 (solid) using ρ Regge pole plus a background term, ii) Ref. 10 (dotted) using fixed-t dispersion relation and a dual-absorption model, (a similar prediction was made by using the complex Regge-pole model[11]) and iii) Ref. 12 (dashed) using a dual-absorption model. Figure 1(c) shows predictions by i) Ref. 13 (solid), ii) Ref. 14 (dotted), applying a Reggeized absorption model, and iii) Ref. 15 (dashed) using a new absorption model.

There is also one prediction[16] that shows a tremendous energy dependence, but if one takes 5 GeV/c of Ref. 3 and 8 GeV/c of Ref. 2, there hardly exists such an energy dependence.

Review on πN Amplitude Analysis

Ringland and Roy[17] attempted to determine the nonflip scattering amplitudes (I = 1) in πN charge-exchange scattering using the preliminary data of Ref. 2 and results of phase-shift analyses. Their conclusion is somewhat dependent on their theoretical assumptions.

The first direct analysis to determine both I = 0 and I = 1 amplitudes was performed by Halzen and Michael[18] up to $|t| = 0.625$ and then followed by Cozzika et al.,[19] who extended the analysis up to $|t| = 1.0$. Both of these papers defined amplitudes relative to H^{o}_{++}. Giffon fitted the experimental data to a certain formula and their results are in agreement with those of other authors,[18, 19] but in disagreement with the Regge model and the FESR.[20] Using methods of analytic data analysis, Kelly[21] extracted the amplitudes in the angular range $0 < |t| < 1.5$ and attempted to determine the absolute phase.

Although there is no major discrepancy among the above-mentioned papers, we observe some significant difference in determining errors in amplitude values. Here I would like to discuss the most recent analyses with an improved method of analysis using the recent data of πp charge-exchange polarization and $\pi^{\pm}p$ differential-cross-section data.

It is well known that in principle one can determine seven amplitudes algebraically from seven measurements. However, in reality, we cannot determine amplitudes uniquely without making proper assumptions. The reliability of such assumptions naturally depends upon the accuracy of the experimental data. First, I discuss individual t-by-t analysis, and then t-dependent analysis. These two different analyses compensate each other, and their advantages and disadvantages are summarized as:

Method	Advantages	Disadvantages
t-by-t analysis	Mostly algebraic calculations and limited amount of the data fitting.	i) Eight solutions exist at each $\|t\|$ value. Therefore assume a "shortest-path" approach to determine the smoothest solution passing through each of the eight solutions at individual t. ii) Errors are not realistically calculated. iii) All the data available are not used.
t-dependent analysis	i) All the data available are used for fitting, particularly using the method of an accelerated convergence expansion. ii) Errors can be estimated realistically.	Although the t-by-t analysis gives a good clue to a proper formula to be fitted, some bias exists and a large number of parameters are involved. Some risk is involved about the uniqueness of the final solution.

We have performed two different analyses as follows:

(1) Amplitude Analysis 1 t-by-t analysis)

First we determined the seven amplitudes algebraically from seven measurements [$d\sigma\ (\pi^+p \rightarrow \pi^+p)/dt$, $d\sigma\ (\pi^-p \rightarrow \pi^-p)/dt$, $d\sigma\ (\pi^-p \rightarrow \pi^o n)/dt$, $P(\pi^+p \rightarrow \pi^+p)$, $P(\pi^-p \rightarrow \pi^-p)$, $P(\pi^-p \rightarrow \pi^o p)$, and $R(\pi^-p \rightarrow \pi^-p)$, denoted by σ^+, σ^-, σ^o, P^+, P^-, P^o, and R^-, respectively] , obtaining the eight solutions at each momentum transfer. These solutions were used as the starting point in a gradient search, including the additional measurements[1] of R $(\pi^+p \rightarrow \pi^+p)$ and A $(\pi^-p \rightarrow \pi^-p)$, denoted by R^+ and A^- respectively.

A "shortest-path" approach was used to determine the smoothest solution passing through each of the eight solutions at each individual t. At a given momentum transfer t_2, the distances of each solution from each of the eight at the previous momentum transfer t_1 are calculated from differences in the corresponding amplitudes at t_2 and t_1 with an appropriate metric. We define the distance between solutions as follows: Let $H_1, \ldots, H_7$ be real or imaginary parts of various helicity amplitudes; $M_{ij} = \partial^2 \chi^2 / \partial H_1 \partial H_j$ is the error matrix evaluated at a solution. Then the distance is defined to be

$$d_{12} = \sum_{i=j}^{7} \sum_{j=1}^{7} [H_i(t_2) - H_i(t_1)]\, M_{ij}\, [H_j(t_2) - H_j(t_1)] .$$

In analogy to "shortest-path" constructions in phase-shift analysis, we find solutions with smoothest dependence on momentum transfer by minimizing the total distance between neighboring solutions.

The new charge-exchange polarization data[3] at 5 GeV/c and the recent σ^+ and σ^- data[5] at 6 GeV/c were used together with those data of σ^o, P^+, P^-, and R^- used in Ref. 18 and R^+ and A^- as mentioned earlier. The "shortest-path" solutions covering the $|t|$ range from 0 to 0.625 are given in the plots of Fig. 2. The errors are taken from the diagonal elements of the error matrix and are to be used for relative comparison only. Our convention for helicity amplitudes H_{++} and H_{+-} (following Ref. 18) is such that cross section $d\sigma/dt = |H_{++}|^2 + |H_{+-}|$, in mb/ $(GeV/c)^2$.

(2) Amplitude Analysis II (t-dependent analysis)

We have done the t-dependent searches to enforce continuity by fitting the data to a particular analytic form. Our analysis differs from Ref. 2 in that we determine amplitudes up to an overall t-dependent phase. Our view is that, since the measured quantities at a given energy are unchanged if every helicity amplitude is multiplied by this phase, it is impossible to determine this phase from the data.

The right- and left-hand cuts are mapped (we call the mapping function ω) onto the edge of a unifocal elipse in the $\cos\theta_{c.m.}$ plane with $\cos\theta_{c.m.} = \pm 1$ as fixed points. We write the helicity amplitudes as products of diffractive terms and sixth-order polynomials in ω:

$$H_j = g_j \exp\{-a_j[(4m_\pi^2 - t)^{1/2} - 2m_\pi]\} \times \left[1 + \sum_{n=1}^{6} b_n^j(\omega - 1)^n\right],$$

where g_j, a_j, and b_n^j are the varied parameters and m_π is the pion mass.

The amplitudes at $P_{lab} = 6.0$ GeV/c were obtained by using all the data points (σ^+ and σ^- from Ref. 5, σ^o from the CERN compilation,[22] P^+ and P^- from Borghini et al.,[23] P^o from Ref. 4 and Drobnis et al.,[24] and R^+, R^-, and A^- from Ref. 1).

A variable metric method [25] incorporating an analytic gradient with respect to search parameters was used to obtain fits to the data.

Figure 3 shows the results at 6.0 GeV/c . (Note that new results are slightly different from those in Ref. 4) The error bands shown in this figure with respect to the best-fit values were obtained by shifting each data point in random manner about its measured value, weighting this shift by a Gaussian function of width given by the experimental error, and repeating the fitting procedure for each set of such random shifts. The fits obtained are found to be unique within the stated errors.

We note that criticism was made by Fox and Griss[26] for ACE phase-shift analyses. However, for this work we limit ourselves to fitting new data over the limited t-range where there are complete measurements. Therefore the use of the ACE method should be considered proper.

Scattering-amplitude Uncertainty versus Experimental Errors

Earlier Fox[27] examined the effect of amplitude determination with respect to experimental errors by using a method similar to our analysis I. Here we study the effect by using analysis II. We either raise or reduce the present experimental error and determine the amplitude uncertainty accordingly. Our conclusion differs from those in Ref. 27. This difference may be caused by the fact that we used all the available data. Although the details of this investigation will be described elsewhere, we outline some of the results, which are $|t|$-dependent, by averaging the entire $|t|$ region is:

<table>
<tr><th rowspan="2">Assumed experimental error in R^+ and R^-</th><th colspan="6">Scattering-amplitude uncertainty</th></tr>
<tr><th>RE H^0_{+-}</th><th>IM H^0_{+-}</th><th>RE H^1_{++}</th><th>IM H^1_{++}</th><th>RE H^1_{+-}</th><th>IM H^1_{+-}</th></tr>
<tr><td>Half the present error</td><td rowspan="2">No change</td><td>Half the present error</td><td rowspan="2" colspan="2">No appreciable change</td><td rowspan="2" colspan="2">No appreciable change</td></tr>
<tr><td>Twice the present error</td><td>Three times the present error</td></tr>
</table>

Assumed experimental error in P^0	Scattering-amplitude uncertainty			
	RE H^1_{++}	IM H^1_{++}	RE H^1_{+-}	IM H^1_{+-}
Half the present error	0.8 times the present error	Half the present error		One third the present error
Twice the present error	1.2 times the present error	Twice the present error	No change	1.5 times the present error

It is also clear that a determination of R and A parameters beyond $|t| = 0.6$ is badly needed. In fact, our results of H^1_{+-} amplitude differ from those in Refs. 18 to 21 particularly at around $|t| = 0.6$. Although such difference was mainly due to the new data[3], the continuity at $|t| = 0.6$ is not well established with presently available data.

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Fig. 1 πp charge-exchange polarization P(t) and theoretical predictions by : (a) Ref. 6 (solid),

Ref. 7 (dotted), and Ref. 8 (dashed); (b) Ref. 9 (solid), Ref. 10 (dotted), and Ref. 12 (dashed); (c) Ref. 13 (solid), Ref. 14 (dotted), and Ref. 15 (dashed).

Fig. 2 πN Amplitudes at 6.0 GeV/c. The filled and hollow symbols represent real and imaginary parts of the amplitude. Data points are at $-t = 0.0,\ 0.125,\ 0.250,\ 0.375,\ 0.500$ and $0.625\ (\text{GeV/c})^2$.

Fig. 3 πN amplitudes at 6.0 GeV/c as determined from t-dependent fit shown by dark lines. The error bands are represented by shaded regions.

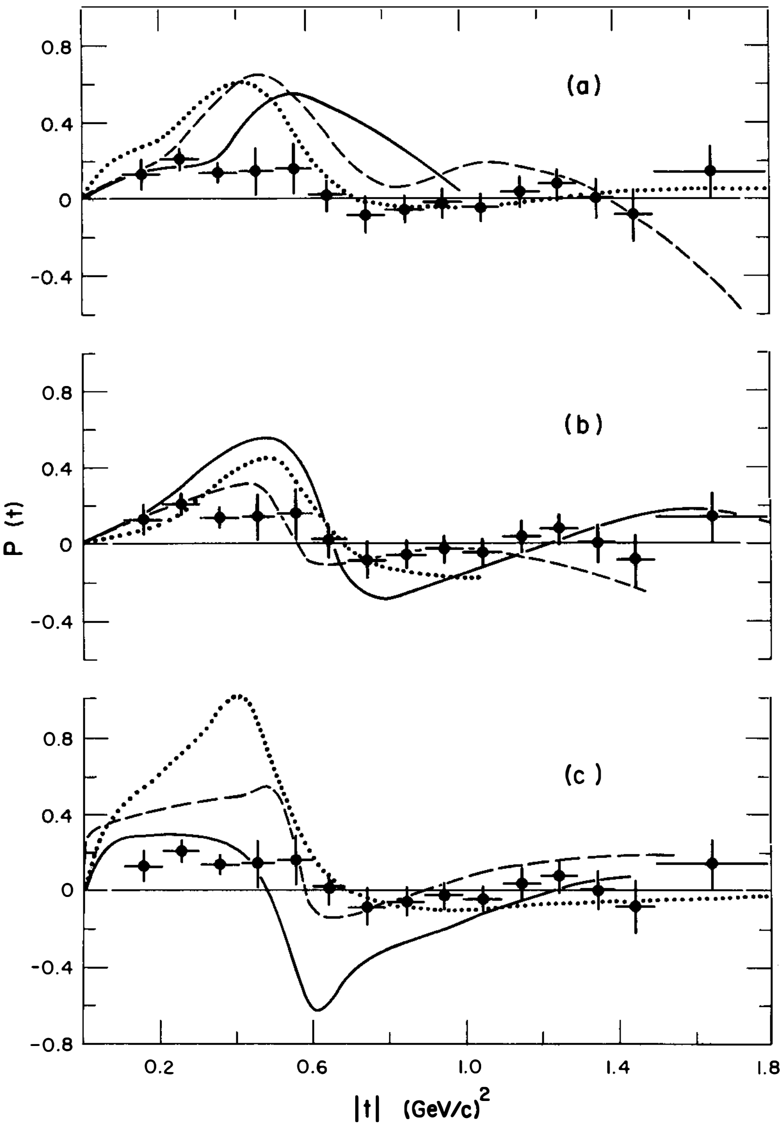
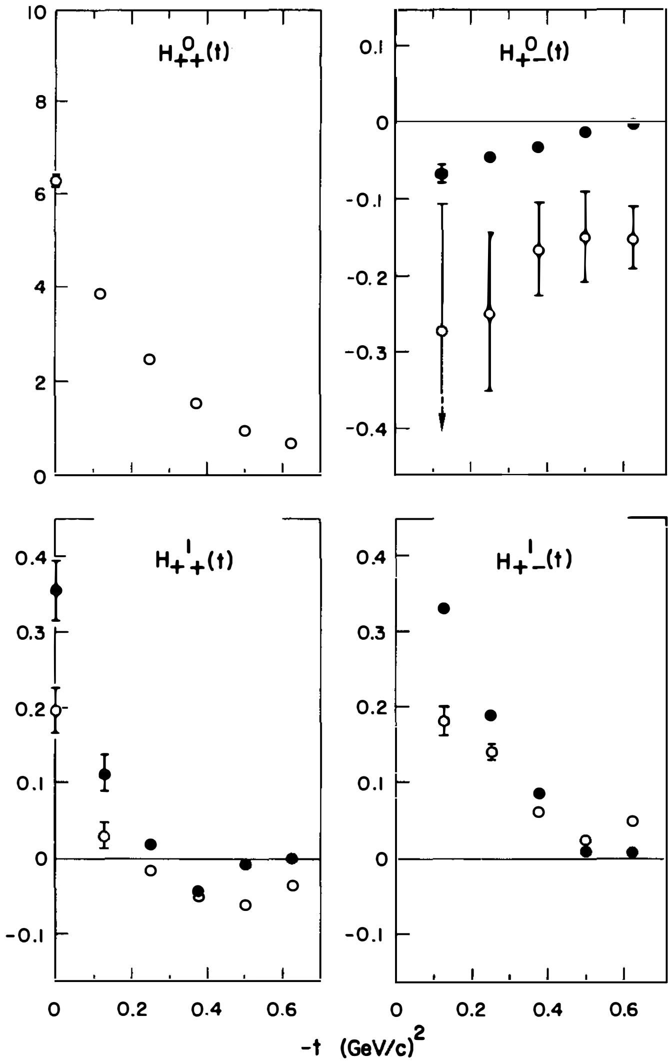
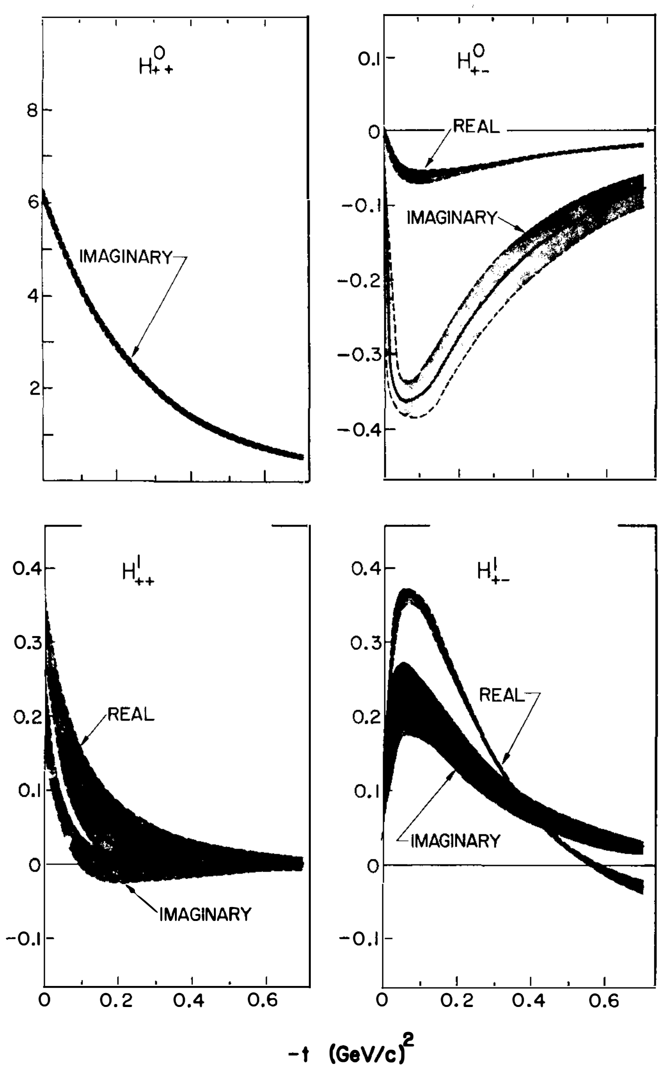


Fig. 1



$H^{0}_{++}(t)$
$H^{0}_{+-}(t)$
$H^{1}_{++}(t)$
$H^{1}_{+-}(t)$
10
8
6
4
2
0
0.1
0
-0.1
-0.2
-0.3
-0.4
0.4
0.3
0.2
0.1
0
-0.1
0
0.2
0.4
0.6
-t (GeV/c)^2

Fig. 2



H^0_{++}
IMAGINARY
H^0_{+-}
REAL
IMAGINARY
H^1_{++}
REAL
IMAGINARY
H^1_{+-}
REAL
IMAGINARY
-t (GeV/c)^2

Fig. 3