Institute of Physics Publishing
doi:10.1088/1742-6596/37/1/016
Journal of Physics: Conference Series **37** (2006) 90–95
IX Mexican Workshop on Particles and Fields

How to solve the Schwinger-Dyson equations once and for all gauges

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Abstract.
Study of the Schwinger-Dyson equation (SDE) for the fermion propagator to obtain dynamically generated chirally asymmetric solution in any covariant gauge is a complicated numerical exercise specially if one employs a sophisticated form of the fermion-boson interaction complying with the key features of a gauge field theory. However, constraints of gauge invariance can help construct such a solution without having the need to solve the SDE for every value of the gauge parameter. Starting from the Landau gauge where the computational complications are still manageable, we apply the Landau-Khalatnikov-Fradkin tansformation (LKFT) on the dynamically generated solution and find approximate analytical results for arbitrary value of the covariant gauge parameter. We also compare our results with exact numerical solutions.

1. Introduction

Schwinger-Dyson Equations [1] (SDE) of quantum field theories are an ideal framework to study non-perturbative phenomena such as dynamical chiral symmetry breaking (DCSB). We take up this question in a simple but non-trivial gauge theory: quantum electrodynamics in 3 dimensions (QED3). Owing to its simplicity as compared with its four-dimensional counterpart and quantum chromodynamics (QCD), the quenched version of QED3 is particularly neat to unfold the intricacies of the SDE for the fermion propagator (FP). It provides us with an excellent laboratory to study one of its unwelcome features, namely, the lack of gauge invariance of the associated physical observables which hampers a fully reliable predictive power of the said equations in the non-perturbative regime of interactions.

A natural path to try to achieve gauge invariance in non-perturbative studies of SDE is to construct a vertex, which, in the weak coupling regime would match onto their perturbative counterparts. For the study of DCSB, it implies constructing a very complicated form for the three-point fermion-boson vertex if one wants to obtain correct gauge covariance of the FP and the vertex at the same time [1]. This has recently been achieved in QED3 [4] at the one loop level. The vertex ansatz proposed there guarantees that the resulting FP satisfies its Landau-Khalatnikov-Fradkin transformation [5] (LKFT) to two orders and the vertex itself to the first order in their respective perturbative expansions in quenched QED3.

[1] This amounts to a formidable numerical excersice for arbitrarily large values of the gauge parameter [2, 3].

LKFT of the Greens functions describe the specific manner in which these functions transform under a variation of gauge. As LKFT are non perturbative in nature, we expect them not only to be satisfied at every order in perturbation theory but also in phenomena which are realized only non perturbatively, such as DCSB. One of the reasons why these transformations have played a less significant and practical role in nonperturbative studies is the fact that momentum space calculations are very tedious, as reflected in [6].

The goal of this work is to show how the LKFT of a dynamically generated solution to the SDE for the FP can be helpful to study the gauge dependence of the said Greens function in a straightforward manner for arbitrarily large values of the gauge parameter. The idea is rather simple: solve the SDE for the FP in Landau gauge where the numerical exercise involved is still manageable; approximate its key features analytically; take its LKFT and study DCSB. This work is organized as follows: In Sect. 2, we recall the solution of the dynamically generated FP in the Landau gauge rainbow approximation and we approximate it with a simple analytic parametric form; in Sect. 3, we perform the LKFT of the dynamically generated FP to get the same in an arbitrary covariant gauge. We also discuss various limiting cases of our results. We display a graphic interpretation of the results in Sect. 4 and present our conclusions in Sect. 5.

2. DCSB in Rainbow Approximation

In the rainbow approximation, it is a common practice to write the SDE for the FP as :

$$\frac{1}{F(p;\xi)} = 1 - \frac{\alpha\xi}{\pi p^2}\int_0^\infty dk k^2 K_F(k;\xi)\left[1 - \frac{k^2+p^2}{2kp}\ln\left|\frac{k+p}{k-p}\right|\right] , \tag{1}$$

$$\frac{\mathcal{M}(p;\xi)}{F(p;\xi)} = \frac{\alpha(\xi+2)}{\pi p}\int_0^\infty dk k K_M(k;\xi)\ln\left|\frac{k+p}{k-p}\right| , \tag{2}$$

with

$$K_F(p;\xi) = \frac{F(p;\xi)}{p^2+\mathcal{M}^2(p;\xi)} = \frac{K_M(p;\xi)}{\mathcal{M}(p;\xi)} . \tag{3}$$

Owing to the fact that in the Landau gauge, $F(p;0)=1$, it has long served as a favourite gauge where one only has to solve the equation :

$$\mathcal{M}(p;0) = \frac{2\alpha}{\pi p}\int_0^\infty dk k K_M(k;0)\ln\left|\frac{k+p}{k-p}\right| . \tag{4}$$

The corresponding numerical solution has the following key features : It behaves like a constant for low-p and falls as $1/p^2$ for large momentum [3]. We can therefore approximate this behavior with analytical continuous functions and perform the LKFT exercise to see wheater those key features remain intact or get modified. We do so next.

2.1. Analytical Approximations

We can approximate the numerical solution by the following function :

$$\mathcal{M}(p;0) = \frac{M_0\, m_0^2}{p^2+m_0^2} . \tag{5}$$

With this approximation, we find that the LKFT excercise is still not trivial to perform. Therefore, we need a further simplification. We can re-write the approximation to the mass function as follows :

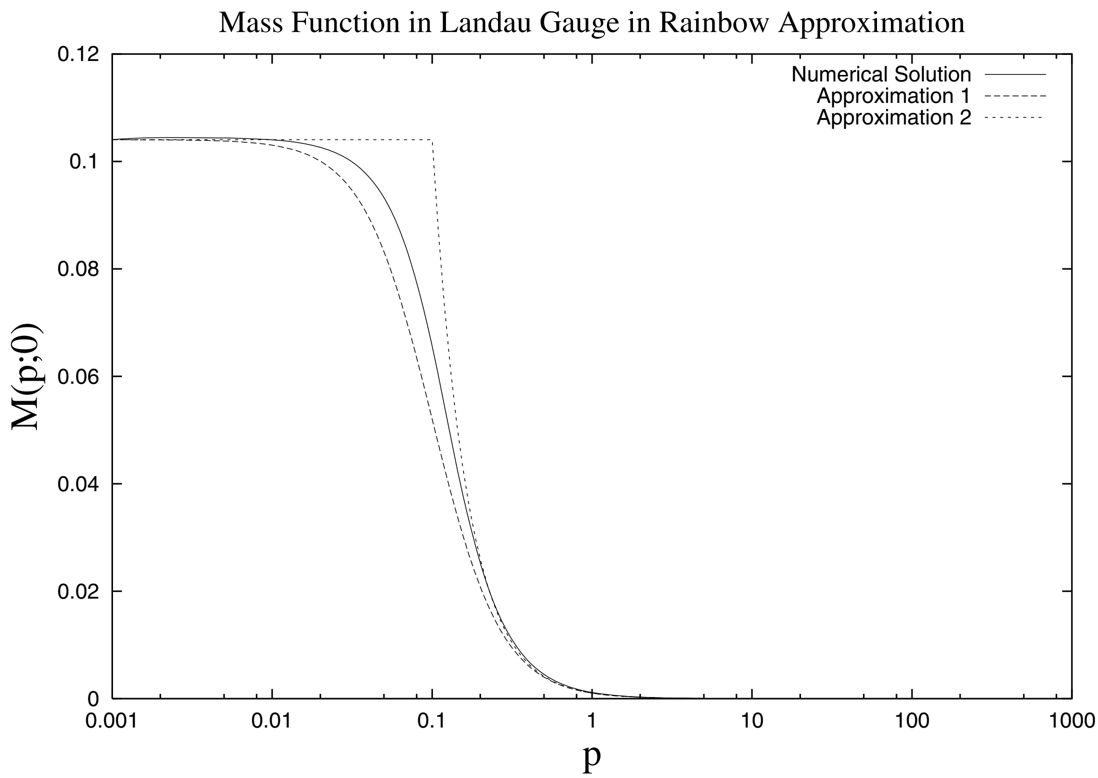

Figure 1. The mass function in the Landau gauge for the bare vertex. Approximations proposed in Eq. (5) and Eq. (6) are also shown.

$$\mathcal{M}(p;0) = M_0 \left[\theta(m_0 - p) + \frac{m_0^2}{p^2} \theta(p - m_0) \right] \; . \tag{6}$$

As shown in Fig. (1), Eqs. (5,6) provide a good approximation. We are now in a position to proceed, and will do so in the next section.

3. LKFT for the FP in Rainbow Approximation

The LKFT exercise for the FP in rainbow approximation can be performed ([7]) to analyse the dynamically generated behaviour of the FP in the asymptotic limits of momenta, i.e., when $p >> m_0$ and $m_0 >> p$.

3.1. Large-p Behaviour of FP

In the large-p limit, the mass function and the wavefunction renormalization have been found to have the following form :

$$\mathcal{M}(p;\xi) = \frac{C_3(\xi)}{p^2} + \mathcal{O}\left(\frac{1}{p^3}\right) \; , \tag{7}$$

$$F(p;\xi) = 1 + \mathcal{O}\left(\frac{1}{p}\right) \; , \tag{8}$$

where

$$C_3(\xi) = \left[m_0^2 M_0 + \frac{4am_0(m_0^2+3M_0^2)}{3M_0\pi} \right] . \tag{9}$$

We proceed now to analyze the behavior of F and $\mathcal{M}$.

3.2. Low-p Behaviour of FP

An analogous analysis for the low-p regime yields :

$$\mathcal{M}(p;\xi) = \frac{C_1(\xi)}{C_2(\xi)} + \mathcal{O}(p^2) , \tag{10}$$

$$F(p;\xi) = -\frac{C_1^2(\xi)}{C_2(\xi)} - C_2(\xi)p^2 + \mathcal{O}(p^4) , \tag{11}$$

where

$$\begin{aligned} C_1(\xi) = & \left[\frac{m_0 M_0}{a^4}\left(3m_0 - \frac{4a}{\pi}\right) + \frac{2am_0}{\pi(a^2+m_0^2)}\left(\frac{1}{M_0} - \frac{m_0^2 M_0}{a^4}\right) \right. \\ & \left. -\frac{2}{\pi}\left(\frac{1}{M_0} + \frac{3m_0^2 M_0}{a^4}\right) \arctan\frac{m_0}{a} \right] \end{aligned} \tag{12}$$

and

$$C_2(\xi) = \frac{2}{3M_0^2\pi}\left[\frac{am_0(3a^2+5m_0^2-2M_0^2)}{(a^2+m_0^2)^2} - 3\arctan\frac{m_0}{a} \right] . \tag{13}$$

Expectedly, $\mathcal{M}$ is flat for small values of p, and it falls off as $1/p^2$ for its large values. On the other hand, F is also a constant for small values of p. For the large values, it approaches 1. Thus the p-dependence of the dynamically generated FP for the small and large values of the momentum continues to have the same qualitative features in an arbitrary covariant gauge as the ones in the Landau gauge and in its neighbourhood, [3].

4. Results in Plots

From the corresponding expressions in the low and large momentum regimes for F and $\mathcal{M}$, we perform the following parametrisation for the FP in arbitrary gauge :

$$\mathcal{M}(p;\xi) = M_\xi \left[\theta_1 + \frac{m_\xi^2}{p^2}\theta_2 \right] \tag{14}$$

$$F(p;\xi) = F_\xi \theta_1 + \theta_2 , \tag{15}$$

where $\theta_1 = \theta(m_\xi - p)$, $\theta_2 = \theta(p - m_\xi)$, and

$$M_\xi = \frac{C_1(\xi)}{C_2(\xi)}, \qquad M_\xi m_\xi^2 = C_3(\xi), \qquad F_\xi = -\frac{C_1^2(\xi)}{C_2(\xi)} . \tag{16}$$

- In Fig. (2), we have plotted $F(p;\xi)$ in several gauges. Comparing them with the ones obtained by solving SDE with the bare vertex ansatz, one sees that the difference is not enormous, reassuring the correctness of the method employed.
- In Fig. (3), we have plotted the $\mathcal{M}(p;\xi)$ in several gauges in order to compare the results of directly solving SDE against the ones obtained by employing the LKFT. We find reasonably good agreement.

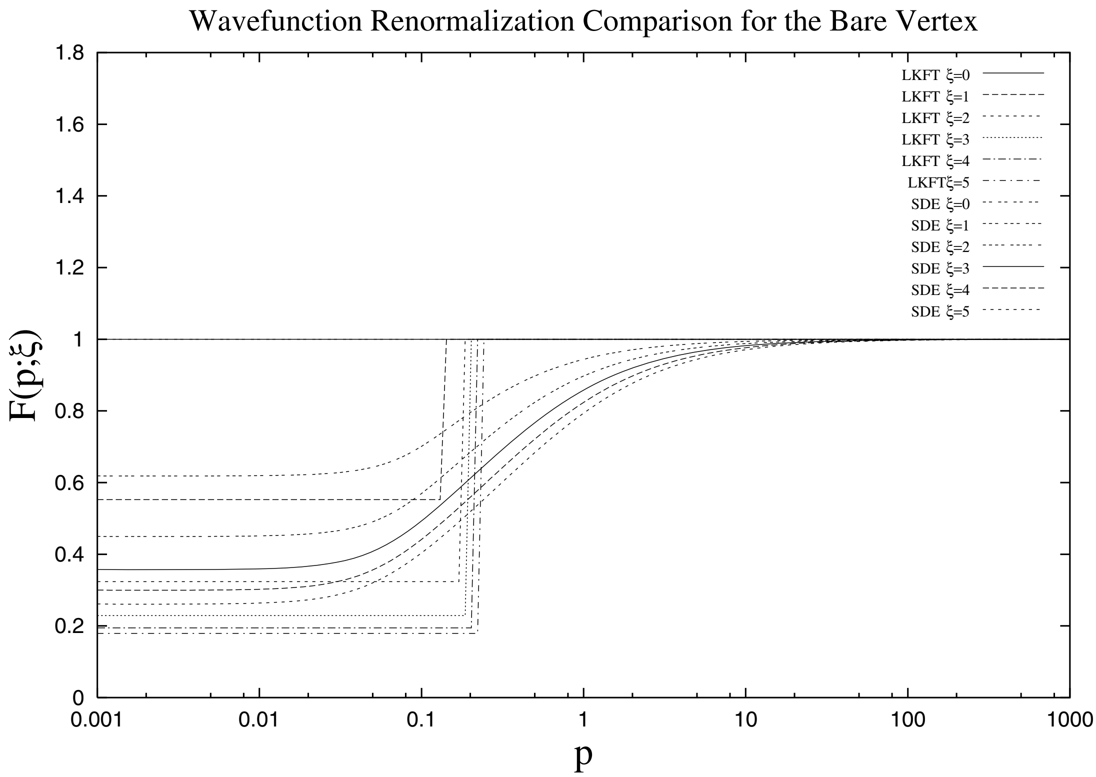

Figure 2. $F(p;\xi)$ for the bare vertex employing LKFT. For a comparison, we also plot the results obtained by directly solving SDE.

5. Conclusions

Knowledge of the FP in an arbitrary covariant gauge is extremely useful. If we are dealing with SDE studies, where it is hard to obtain solutions for arbitrarly large values of the gauge parameter, LKFT comes to rescue. We only need to solve the SDE for the FP in Landau gauge and its LKFT is found to have the key features of a dynamically generated FP for an arbitrarly large value of the gauge parameter. Hopefully our reasonings can be taken to more complicated scenarios, like the inclusion of an ansatz for the full vertex [7], the unquenched version of QED3 and yet, more compliated theories, like QCD.

Acknowledgments

AB wishes to thank CIC and CONACyT for their support under grants 4.10 and 32395-E respectively. AR whishes to thank the Alvarez-Buylla grant.

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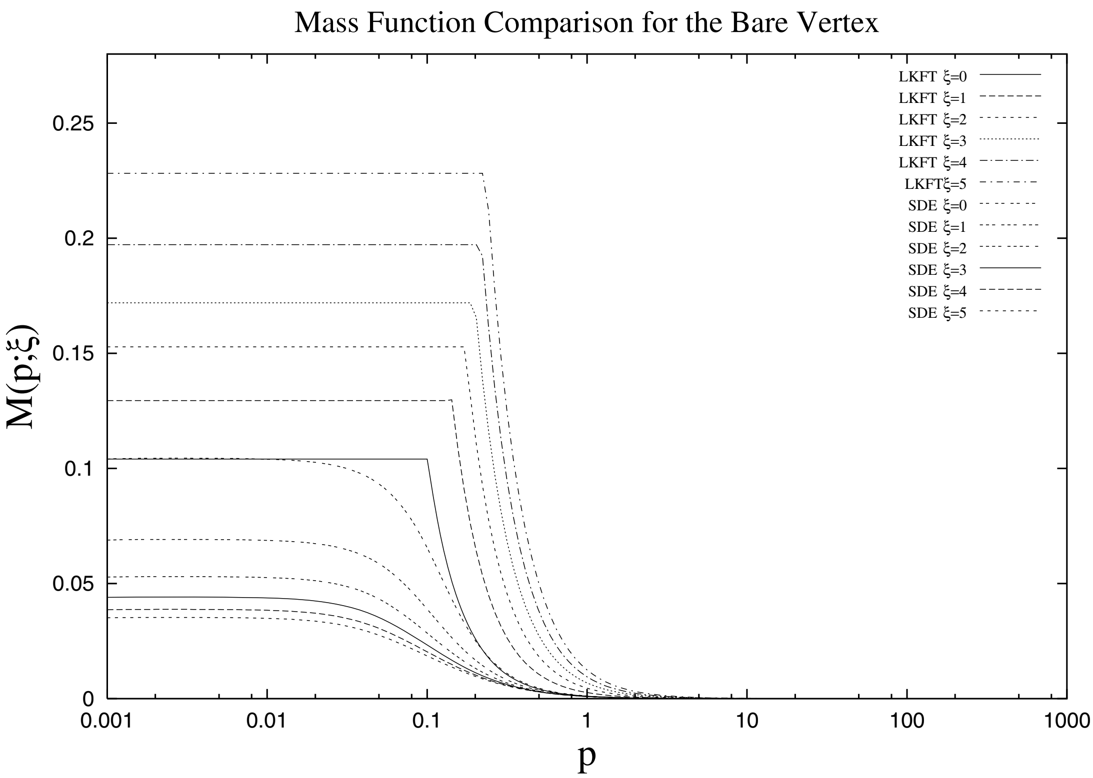

Figure 3. $\mathcal{M}(p;\xi)$ for the bare vertex employing LKFT. For a comparison, we also plot the results obtained by directly solving SDE.

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