Microscopic - Macroscopic Description of Ground State Nuclear Masses

A. Bhagwat[1],* M. Gupta[2], and Y. K. Gambhir[2,3]
[1] *UM-DAE Centre for Excellence in Basic Sciences, Mumbai 400 098, INDIA*
[2] *Manipal University, Manipal 576104, INDIA and*
[3] *Dept. of Physics, IIT Bombay, Powai, Mumbai 400076, INDIA*

Introduction

The study of ground state nuclear masses has been a subject of interest for a number of years. A variety of different models have been reported in the literature so far [1–6] that describe the ground state masses of the nuclei spanning the entire periodic table with varying degrees of precision. The idea behind the present investigation is to develop a mic - mac model for ground state nuclear masses. Nuclei with mass numbers larger than or equal to 50 have been selected for the investigation. These are 1919 in number (including even - even, even - odd, odd - even as well as odd - odd).

Results and Discussion

The present calculations proceed in two steps. **Step 1:** Liquid Drop Model: For a system of N neutrons and Z protons, the liquid drop part of the total binding energy is assumed to be [6, 7]:

$$\begin{aligned} E_{LDM} &= a_v \left[1 + \frac{4k_v}{A^2} T_z \left(T_z + 1\right)\right] A \\ &+ a_s \left[1 + \frac{4k_s}{A^2} T_z \left(T_z + 1\right)\right] A^{2/3} \\ &+ \frac{3e^2 Z^2}{5 r_o A^{1/3}} - C_4 \frac{Z^2}{A} + E_p \end{aligned} \quad (1)$$

where, $A = N + Z$; e^2 is square of electronic charge; T_z is third component of isospin, defined by $\left|\frac{N-Z}{2}\right|$; a_v, a_s, k_v, k_s, r_o and C_4 are parameters of the model. E_p is the pairing energy, and following Möller - Nix [4], it is defined by:

$$\begin{aligned} E_p &= \bar{\Delta}_p \ , Z \text{ odd and } N \text{ even} \\ &= \bar{\Delta}_n \ , N \text{ odd and } Z \text{ even} \\ &= \bar{\Delta}_p + \bar{\Delta}_n + \bar{\Delta}_{np} \ , Z \text{ odd and } N \text{ odd} \\ &= 0 \ , N \text{ even and } Z \text{ even} \end{aligned}$$

with,

$$\begin{aligned} \bar{\Delta}_p &= \frac{d_p}{Z^{1/3}} \\ \bar{\Delta}_n &= \frac{d_n}{N^{1/3}} \\ \bar{\Delta}_{np} &= \frac{d_{np}}{A^{2/3}} \end{aligned} \quad (2)$$

In these equations, d_p, d_n and d_{np} are parameters.

The experimental binding energies have been adopted from Audi's 2003 evaluation [8], and the binding energies have been corrected for binding energy of electrons. Chi square fit to determine the free parameter yields *rms* of 2.47 MeV. The values of parameters obtained are: a_v = -15.5810; k_v = -1.8199; a_s = 18.4576; k_s = -2.3075; r_o = 1.1975; C_4 = 1.7891; d_n = 5.1857; d_p = 5.0536; d_{np} = -8.7129.

Step 2: The fluctuations: Treatment of fluctuations is the most difficult part of the exercise. It is clear that the fluctuations include shell energy as well as microscopic pairing energy, and both of these depend on shell structure. There could in principle be a number of ways to parameterise the fluctuations, but the simplest way to do that is through a discrete Fourier transform inspired ansatz for the fluctuations. Assume for the sake of sim-

*Electronic address: ameeya@cbs.ac.in

plicity that the fluctuations depend on number of neutrons outside a 'core' and number of protons outside the suitable 'core'. Thus, we here have the case of a two dimensional discrete Fourier transform, with variables given by:

$$x_1 \;=\; \left|\frac{N-N_c}{N_c}\right| \;\;,\;\; x_2 \;=\; \left|\frac{Z-Z_c}{Z_c}\right| \quad (3)$$

The values of N_c are assumed to be 2, 8, 20, 50, 82, 126 and 172 whereas the values of Z_c are assumed to be 2, 8, 20, 50, 82, 126. The 'core' for a given neutron number is obtained by looking at its proximity to adjacent magic numbers. For example, for $N = 16$, N_c is assumed to be 20.

The fluctuations, defined by:

$$\delta E \;=\; E_{Expt.} - E_{LDM} \quad (4)$$

are parameterised as:

$$\delta E \;=\; \sum_{n=1}^{N}\sum_{m=1}^{M} a_{nm}\cos\left(2\pi\frac{nx_1}{N}+2\pi\frac{mx_1}{M}\right) + \sum_{n=1}^{N}\sum_{m=1}^{M} b_{nm}\sin\left(2\pi\frac{nx_1}{N}+2\pi\frac{mx_1}{M}\right) \quad (5)$$

with a_{nm} and b_{nm} being free parameters of the model. In practice, N and M are taken to be equal, hence, for a given N, this model has $2N^2$ parameters. The value of N is assumed to be 3 here. Chi square minimisation for these parameters yields *rms* deviation of 1.15 MeV, which is a tremendous improvement over the LDM binding energies. The resulting difference between the experimental and the corresponding experimental binding energies are plotted in Fig. (1) (denoted by 'LDM+FT'). It is very clearly seen that the strong shell effects are well taken care of, the resulting difference is much smoother. This difference is required to be parameterised further. Work along these lines is in progress.

Recently, Bhagwat [10] has been able to reduce the *rms* deviation to merely 266 keV, which is the smallest reported in the literature so far, using a mic - mac model inspired by the semi - classical trace formulas [9].

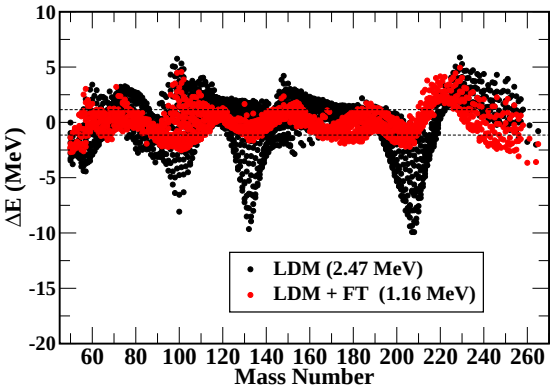

FIG. 1: Difference between experimental and calculated binding energies.

Acknowledgments

Partial financial support by the Department of Science and Technology (DST), Govt. of India to two of us (AB and YKG) (Project No. SR/S2/HEP-034/2009) is gratefully acknowledged.

References

[1] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **A601**, 141 (1996).
[2] L. Satpathy and R. C. Nayak, At. Data Nucl. Data. Tab. **39**, 241 (1988).
[3] J. Duflo, A. P. Zuker, Rev. Mod. Phys. **75**, 1021 (1995).
[4] P. Möller, J. R. Nix, W. D. Myers and W. J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[5] S. Goriely, N. Chamel and J. M. Pearson, Phys. Rev. Lett. **102**, 152503 (2009).
[6] A. Bhagwat, X. Viñas, M. Centelles, P. Schuck and R. Wyss, Phys. Rev. C **86**, 044316 (2012) and Phys. Rev. C **81**, 044321 (2010).
[7] K. Pomorski and J. Dudek, Phys. Rev. C **67**, 044316 (2003).
[8] G. Audi, A. H. Wapstra and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[9] M. Brack and R. K. Bhaduri, *Semi-classical Physics* (Addison - Wesley Publishing Co., 1997).
[10] A. Bhagwat, to be published in Phys. Rev. C and private communication.