



Will LAGEOS and LARES 2 succeed in accurately measuring frame-dragging?

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Abstract The current LAGEOS–LARES 2 experiment aims to accurately measure the general relativistic Lense–Thirring effect in the gravitomagnetic field of the spinning Earth generated by the latter’s angular momentum $\mathbf{J}$. The key quantity to a priori analytically assess the overall systematic uncertainty is the ratio $\mathcal{R}^{J_2}$ of the sum of the classical precessions of the satellites’ nodes Ω induced by the Earth’s oblateness J_2 to the sum of their post-Newtonian counterparts. *In principle*, if the sum of the inclinations I of both satellites were *exactly* 180° , the semimajor axes a and the eccentricities e being *identical*, $\mathcal{R}^{J_2}$ would *exactly* vanish. Actually, it is *not* so by a large amount because of the departures of the *real* satellites’ orbital configurations from the *ideal* ones. Thus, J_2 impacts not only directly through its own uncertainty, but also *indirectly* through the errors in all the other physical and orbital parameters entering $\mathcal{R}^{J_2}$. The consequences of this fact are examined in greater details than done so far in the literature. The Van Patten and Everitt’s proposal in 1976 of looking at the sum of the node precessions of two counter-orbiting spacecraft in (low-altitude) circular polar orbits is revamped rebranding it POLAR Relativity Satellites (POLARES). Regardless the specific type of satellite and tracking technologies that may be eventually adopted, it *might* be *conceptually* superior to the LAGEOS–LARES 2 one from the point of view of the orbital characteristics since, given the *same* semimajor axes and eccentricities of the existing laser-ranged cousins, its $\mathcal{R}^{J_2}$ is *less* sensitive to the impact of the deviations from its *ideal* orbital configuration.

1 Introduction

To its first post-Newtonian (1pN) order [56], the General Theory of Relativity (GTR) predicts, among other things, that

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any mass-energy currents of a source that moves in some way contribute its overall gravitational field with an own, peculiar term encoded in the off-diagonal elements g_{0i} , $i = 1, 2, 3$ of the spacetime metric tensor $g_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$. In the case of an isolated rigidly rotating body, such an additional component of its gravitational field, dubbed as gravitomagnetic [41, 42, 61, 72–74], is proportional to its spin angular momentum $\mathbf{J}$. The previous denomination has nothing to do with the magnetic field induced by electric currents, being simply due to the formal resemblance of the linearized Einstein’s field equations in the weak-field and slow-motion limit with those of the Maxwellian electromagnetism. Among the variety of gravitomagnetic phenomena [18, 62, 65, 66], there is also the so-called Lense–Thirring (LT) effect [35, 43] pertaining the orbit a test particle in geodesic motion around its spinning primary. According to recent historical studies [53–55], it would be more appropriate to call it the Einstein–Thirring–Lense effect. Be that as it may, it consists of generally small cumulative changes of the orientation of the orbital plane in space and of the orbit in the orbital plane itself, being its shape and size left unaltered [31].

By restricting to our solar system, proposals to measure the LT orbital precessions of natural and artificial bodies revolving about, e.g., the Sun and Jupiter were put forth over the past years; for an overview, see [33], and references therein. To date, some rare, still inconclusive tests have been performed with Mercury in the solar field [47–49] and with the spacecraft Juno [3] around Jupiter [17, 23].

The situation looks quite different with regards to the Earth in various respects. The idea of using artificial satellites to measure the LT effect in the terrestrial gravitomagnetic field dates back to the late 1950s [2, 24, 25]. Actual attempts to detect the gravitomagnetic orbital precessions have been underway since the mid-1990s [9] by monitoring the motion of some passive geodetic satellites [50] tracked with the Satellite Laser Ranging (SLR) technique [15], as

was first proposed to be done with LAGEOS [14] by Cugusi and Proverbio [16]. For a comprehensive overview, see, e.g., [58], and references therein. In this respect, among such spacecraft, a prominent role is currently played by LAGEOS (dubbed L hereafter), in orbit since 1976, and its cousin LARES 2 (LR 2) [46], launched on July 13, 2022. They are dense metallic spheres entirely covered by retroreflectors [38] bouncing back the laser pulses routinely sent to them from several ground-based SLR stations belonging to the International Laser Ranging Service (ILRS) [51]. The shape and composition of these satellites greatly reduce the impact of many non-gravitational disturbing accelerations [44], thus making them the man-made objects that come closest to the concept of test particles in pure geodesic motion.

To date, the only undisputed test of a gravitomagnetic effect is the one carried out with the Gravity Probe B (GP-B) mission [19,20] in the circumterrestrial space. It measured the Pugh–Schiff precessions of the spins [57,67] of four gyroscopes carried onboard a drag-free spacecraft with a $\simeq 19\%$ accuracy [21,22].

A major source of systematic bias in all the attempts to measure the LT effect with terrestrial satellites is represented by the disturbances induced by the multipolar expansion of the Newtonian component of the Earth's gravity potential accounting for its departure from spherical symmetry [26,75]. Indeed, the resulting classical orbital shifts [5,28] have the *same* temporal behaviour of the LT ones, together to the fact that they exhibit *much larger* nominal magnitudes. Among them, the largest by far are those due to the first even zonal harmonic coefficient J_2 of degree $\ell = 2$ and order $m = 0$ of the geopotential accounting for the Earth's oblateness.

In order to gain useful insight into the relative sizes of all such competing features of motion, it is convenient to reason in terms of the standard Keplerian orbital elements [34]. In some Earth-centered asymptotic inertial (ECI) reference frame $\mathcal{K}$ an axis of which is *exactly* aligned with the terrestrial angular momentum, the longitude of the ascending node Ω of the test particle is displaced by both the gravitomagnetic and the classical quadrupolar gravitational fields according to [5,35]

$$\dot{\Omega}_{\text{LT}} = \frac{2GJ}{c^2 a^3 (1 - e^2)^{3/2}}, \quad (1)$$

$$\dot{\Omega}_{J_2} = -\frac{3}{2} n_K \left(\frac{R}{p} \right)^2 \cos I, \quad (2)$$

where c is the speed of light in vacuum, G is the Newtonian constant of gravitation, a is the semimajor axis, e is the eccentricity, $p := a(1 - e^2)$ is the semilatus rectum, I is, in this case, the inclination of the orbital plane to the equatorial plane of the central body whose mass and equatorial

radius are M and R , respectively. Furthermore, $\mu := GM$ is its standard gravitational parameter, and $n_K := \sqrt{\mu/a^3}$ is the Keplerian mean motion of the test particle. The longitude of the ascending node Ω is an angle reckoned in the adopted reference plane Π of $\mathcal{K}$ from the x direction to the unit vector

$$\hat{l} = \{\cos \Omega, \sin \Omega, 0\} \quad (3)$$

of the line of nodes, which is the intersection of the orbital plane with Π itself. The versor $\hat{l}$ is directed toward the ascending node Ω , which is the point where the orbiter crosses Π from below. In principle, the argument of pericentre ω , which is an angle counted in the orbital plane from $\hat{l}$ to the line of apsides oriented toward the point of closest approach, also undergoes, among other things, a secular LT precession [35]. Nonetheless, it has been a long time since, after some initial attempts [9–11] involving also the perigee of LAGEOS 2 [27], it was decided not to use such an orbital element anymore. Indeed, it is heavily perturbed by a host of non-gravitational accelerations [36,37,39].

An remarkable feature of Eqs. (1)–(2) is that, if on the one hand, the LT node precession does *not* depend on I at all, on the other hand, the classical rate gains on *opposite* sign if I is switched by 180° . Thus, if there were two satellites the sum of whose inclinations is *exactly* 180° , all the other orbital parameters being *identical*, the *sum* of their nodes would allow, *in principle*, to add up the LT rates while the nominally much larger competing precessions due to J_2 would *exactly* cancel out. The same property would automatically extend also to *all* the other disturbing Newtonian rates induced the *even zonal* harmonics J_ℓ , $\ell = 4, 6, \dots$ of higher degree since it turns out that they are *all* proportional to Eq. (2) through certain functions of I which are left unaffected by the replacement $I \rightarrow 180^\circ - I$ [28].

This is just the line followed by Ciufolini and coworkers who, in a series of recent papers [12,13], claimed to be able to perform in the near future a LT test accurate to $\simeq 0.2\%$ with LAGEOS and LARES 2 whose inclinations are $I^L \simeq 110^\circ$ and $I^{\text{LR}2} \simeq 70^\circ$, while their semimajor axes and eccentricities are *almost* identical, as per Table 1.

Figure 1 shows the orbital geometry of LAGEOS and LARES 2.

Relying upon [7], Ciufolini and coworkers look at the *sum* of the node precessions of LAGEOS and LARES 2 in order to extract a sufficiently clean LT signal. In this respect, the key quantity to *analytically* assess the systematic bias due to J_2 on the LT signal is the ratio

$$\mathcal{R}^{J_2} := \frac{\dot{\Omega}_{J_2}^L + \dot{\Omega}_{J_2}^{\text{LR}2}}{\dot{\Omega}_{\text{LT}}^L + \dot{\Omega}_{\text{LT}}^{\text{LR}2}} \quad (4)$$

of the sum of the classical precessions of LAGEOS (L) and LARES 2 (LR 2) to the sum of the LT precessions of the same satellites. In an *ideal* case, characterized by *identical*

Table 1 Relevant orbital parameters of LAGEOS and LARES 2 [12, Tab. 1]. They are mean values over 127 days

	Semimajor axis a (km)	Eccentricity e	Inclination I (°)
LAGEOS	12, 270.020705	0.00403	109.8469
LARES 2	12, 266.1359395	0.00027	70.1615

semimajor axes and eccentricities and by *exactly* supplementary inclinations, $\mathcal{R}^{J_2}$ would vanish. In fact, as pointed out in [30], even its *nominal* value is *by far not* zero because of the departures of the *actual* satellites' orbital configurations from the previously mentioned *ideal* scenario. This important fact opens the door to potentially relevant systematic errors induced by the uncertainties in *all* the relevant physical and orbital parameters entering the analytical expression of $\mathcal{R}^{J_2}$. In other words, the Earth's oblateness has an impact not only *directly* through its own uncertainty, but also *indirectly* through the errors in the other parameters leaking into the largely nonvanishing expression of Eq. (4).

Aim of the present work is to examine such an important issue in greater detail than has been done so far in the literature [13, 30], and to find possible alternatives by reexamining an earlier proposal put forth in 1976 by Van Patten and Everitt [76, 77].

The paper is organized as follows. Section 2 treats the consequences of the precession of the Earth's spin axis on $\mathcal{R}^{J_2}$. In particular, Sect. 2.1 deals with the impact of the uncertainties in those parameters which act as scaling factors on the recalculated expression of the ratio of the summed classical to relativistic node precessions: G (Sect. 2.1.1) and J (Sect. 2.1.2). The effect of the secular variation of J_2 is treated in Sect. 2.2. A quick assessment of the bias due to J_4 is the subject of Sect. 3. The scenario envisaged by Van Patten and Everitt is revisited in Sect. 4. In it, the features of counter-orbiting test particles along identical orbits, however inclined, are reviewed in Sect. 4.1 showing its equivalence with the LAGEOS–LARES 2 system, while the case of polar orbits is examined in Sect. 4.2. In Sect. 5, the criticisms by Ciufolini et al. [13] are addressed. In particular, the issue of the correct error propagation is the subject of Sect. 5.1, while Sect. 5.2 is dedicated to the impact of the uncertainties in the orbital elements: the semimajor axis (Sect. 5.2.1) and the inclination (Sect. 5.2.2). Section 6 summarizes the findings and offers conclusions.

2 The direct and indirect consequences of the precession of the Earth's axis on $\mathcal{R}^{J_2}$

The ECI used in satellite geodesy is the Geocentric Celestial Reference System (GCRS) [52]. It is essentially characterized, among other things, by the Earth's Mean Equator and Mean Equinox (MEME) at 12:00 Terrestrial Time on 1

January 2000 (J2000.0), being also dubbed as J2000 system [71]. Its x axis is aligned with the mean vernal equinox. Its z axis is aligned with the Earth's rotation axis (or equivalently, the celestial North Pole) as it was at *that* time. The y axis is rotated by 90° East about the celestial equator [71]. More precisely, as per the Recommendation 2 of the IAU 2006 Resolution B.2 by the International Astronomical Union (IAU) [52], the *orientation* of GCRS coincides *by default* with that of the International Celestial Reference System (ICRS). The principal plane Π of the latter and its origin are chosen to be as *close as possible* to the Earth's mean equator and equinox at J2000.0; there is a fixed offset, known as frame bias, of about 23 milliarcseconds (mas) between the two systems [6]. See Sect. 5.2.2 for how this impacts the realistic accuracy in knowing the satellites' inclinations.

LARES 2 was launched 21.53 years *after* the reference epoch J2000.0. Thus, data analyses aimed to detect the LT effect with LAGEOS and LARES 2 will be necessarily carried out starting at least from *that* date onwards. Furthermore, they can generally last for decades. Actually, it *does matter* since, in the meantime, the Earth's spin axis $\hat{k}$ *has changed* mainly due to several physical processes, the most relevant of which is the precession of the equinoxes having a period of approximately 26,000 years [70]. In fact, the LT and classical orbital precessions *generally* depend on the orientation of $\hat{k}$ with respect to the inertial frame adopted. They are equal to [31]

$$\dot{\Omega}_{\text{LT}} = \frac{2GJ \csc I}{c^2 a^3 (1 - e^2)^{3/2}} \hat{k} \cdot \hat{m}, \quad (5)$$

$$\dot{\Omega}_{J_2} = -\frac{3}{2} n_K \left(\frac{R}{p} \right)^2 \csc I (\hat{k} \cdot \hat{m}) (\hat{k} \cdot \hat{h}), \quad (6)$$

where

$$\hat{m} = \{-\cos I \sin \Omega, \cos I \cos \Omega, \sin I\}, \quad (7)$$

$$\hat{h} = \{\sin I \sin \Omega, -\sin I \cos \Omega, \cos I\}. \quad (8)$$

The unit vector $\hat{h}$ is directed along the satellite's orbital angular momentum, while $\hat{m}$ lies in the orbital plane in such a way that $\hat{l} \times \hat{m} = \hat{h}$ holds. It should be recalled that (1)–(2) hold *only* at J2000.0 when $\hat{k}_{\text{J2000.0}} = \{0, 0, 1\}$. At this point, it may be the case to recall that, for an arbitrary orientation of $\hat{k}$ in space, also the inclination I undergo long-term shifts

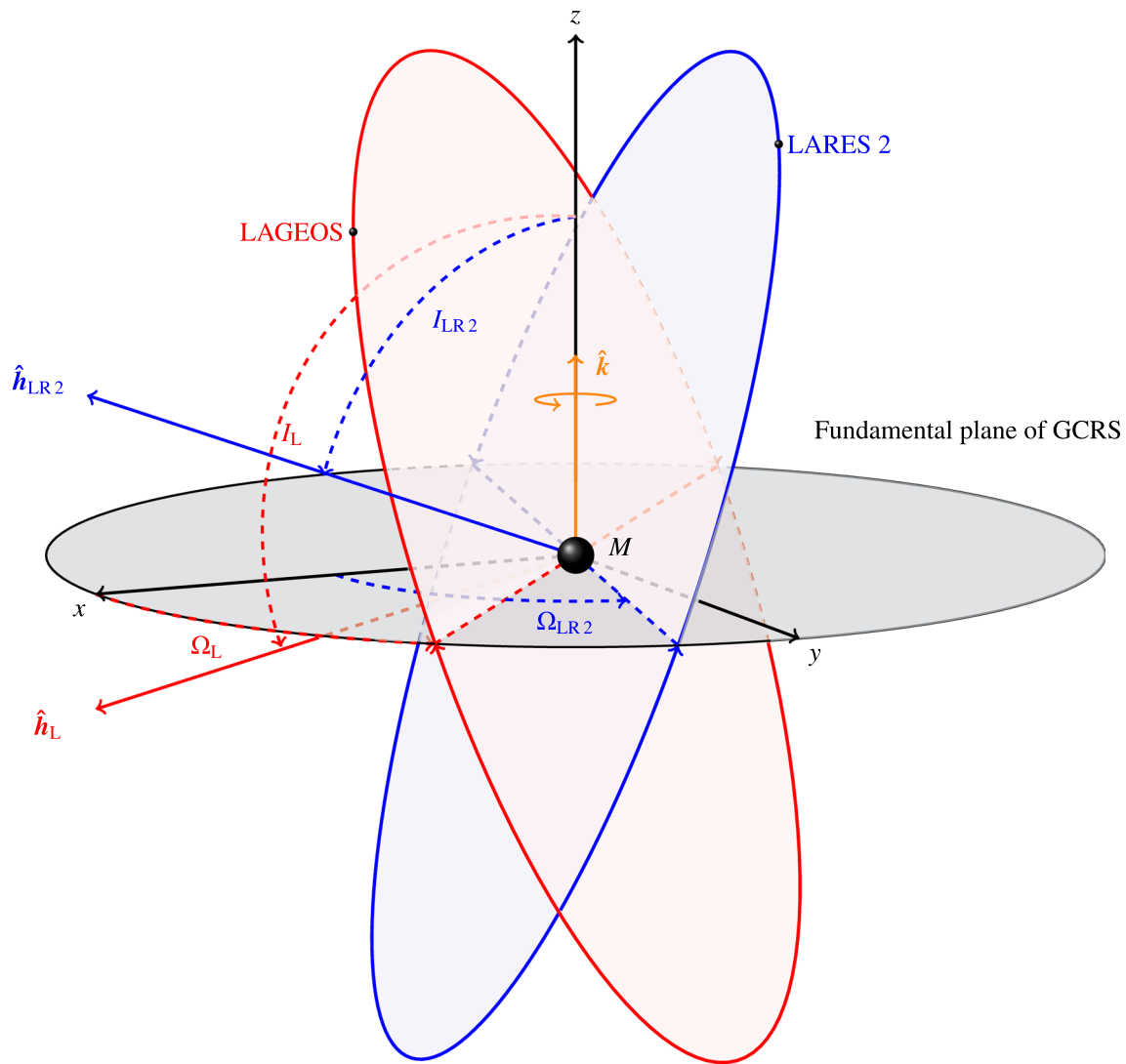


Fig. 1 Orbital configurations of LAGEOS (in red) and LARES 2 (in blue) at the epoch T_1 of the launch of LARES 2 with respect to the Geocentric Celestial Reference System (GCRS) whose fundamental plane is shown shaded in grey. The Earth's spin axis $\hat{k}$, in orange, is depicted according to Eq. (11). The ratio of the semimajor axis a of LARES 2 to that of LAGEOS, to which an arbitrary reference value has been

assigned just for illustrative purpose, is as in Table 1 from which the eccentricities e and inclinations I are retrieved as well. The longitudes of the ascending node Ω at T_1 are as in (15)–(16). The unit vectors $\hat{h}$, given by Eq. (8), are directed towards the orbital angular momenta of the satellites. The sizes of the Earth and of the satellites' orbits are *not* in scale

given by

$$\dot{I}_{LT} = \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}} \hat{k} \cdot \hat{I}, \quad (9)$$

$$\dot{I}_{J_2} = -\frac{3}{2} n_K \left(\frac{R}{p} \right)^2 (\hat{k} \cdot \hat{I}) (\hat{k} \cdot \hat{h}), \quad (10)$$

Thus, one may wonder to what extent the expression of $\mathcal{R}^{J_2}$ used by Iorio [30] and Ciufolini et al. [13], *calculated with Eqs. (1)–(2)*, is adequate. The answer is negative, as it will be shown in the following.

The first step is using Eqs. (5)–(6) to calculate an expression for $\mathcal{R}^{J_2}$ valid for the *epoch of the launch of LARES 2*, denoted in the following as T_1 . To this aim, one has first to obtain the Earth's spin axis $\hat{k}_{T_1}$ just at T_1 . By taking into account only the effect of the precession for simplicity, this task can be accomplished by means of the standard formulas providing the orientation of the mean equator and equinox of the generic epoch T with respect to the MEME [45, p. 176]. It turns out

$$\hat{k}_{T_1} = \{-0.00209215, -5.04 \times 10^{-6}, 0.99999781\}. \quad (11)$$

By parameterizing the Earth's spin axis in terms of the right ascension (R.A.) α and declination (decl.) δ of the Earth's North Pole of rotation as

$$\hat{k} = \{\cos \alpha \cos \delta, \sin \alpha \cos \delta, \sin \delta\}, \quad (12)$$

Equation (11) corresponds to

$$\alpha_{T_1} = 0.13815807^\circ, \quad (13)$$

$$\delta_{T_1} = 89.88012829^\circ. \quad (14)$$

The next step is obtaining the values of the longitudes of the ascending node of LAGEOS and LARES 2 at T_1 , not provided in [12, Tab. 1]. This can be approximately done by retrieving their values at any epoch by means of, say, the WEB interface provided at <https://www.n2yo.com/>, and propagating them backward in time to T_1 by means of Eq. (2). Thus, one finally has

$$\Omega^L \simeq 49.55^\circ, \quad (15)$$

$$\Omega^{LR2} \simeq 76.15^\circ. \quad (16)$$

By calculating Eqs. (5)–(6) with Eq. (11), Eqs. (15)–(16) and the values of the other orbital parameters listed in Table 1, one obtains for Eq. (4)

$$\mathcal{R}^{J_2} \Big|_{T_1} \simeq 59\,161.9. \quad (17)$$

The figure of Eq. (17) is even about 12 times larger than that calculated in [30] by means of Eqs. (1)–OdolJ2. A nominal value of $\mathcal{R}^{J_2}$ as large as that provided by Eq. (17) is potentially a serious issue. Indeed, even relatively small errors in some of the physical and orbital parameters entering Eq. (4) may propagate inducing too large a systematic bias to meet the accuracy goal stated by Ciufolini et al. [12, 13]. In this respect, the results by Iorio [30] about the consequences of the uncertainties in a , e and I retain their overall validity, even if a reassessment of the latter ones may be needed; see Sects. 5.2.1 to 5.2.2.

Furthermore, it must be remarked that, actually, $\mathcal{R}^{J_2}$ is time-dependent due, among other things, to the slow precessional motion of $\hat{k}$. The precession transformation between arbitrary epochs can be worked out as detailed in [45, pp. 176–177]. By applying the resulting calculation to the Earth's spin axis between T_1 and a generic epoch $T_2 > T_1$ allows to plot the nominal value of $\mathcal{R}^{J_2}$, calculated with Eqs. (5)–(6), against T_2 assumed as independent variable; see Fig. 2.

2.1 The impact of some physical parameters acting as scaling factors

Here, the impact of the Newtonian constant of gravitation G and the Earth's angular momentum J is evaluated separately.

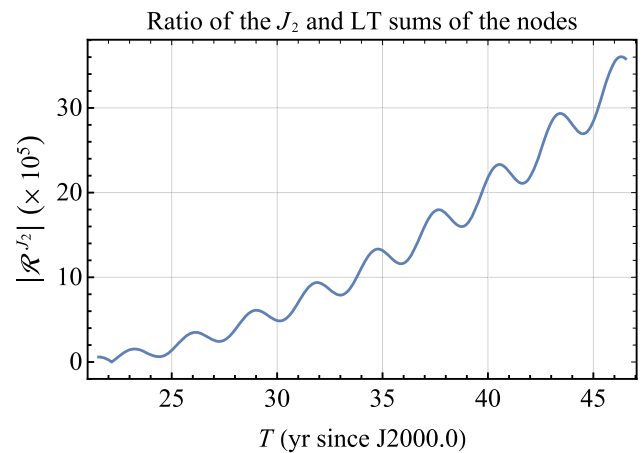


Fig. 2 Plot of the absolute value of Eq. (4), calculated with the general expressions for the LT and classical node rates of Eqs. (5)–(6), as a function of the epoch T over a hypothetical data analysis time span 25 year long starting from the LARES 2 launch date. The precession of the Earth's spin axis $\hat{k}$, worked out as explained in [45, pp. 176–177], was included along with the temporal displacements of I and Ω as per Eqs. (10) and (6). Also the secular variation of the Earth's oblateness was implemented as per Eq. (22)

Since both act as scaling factors for the LT node precessions, they could be removed, at least in principle, by using a suitable linear combination of the nodes of LAGEOS, LARES 2 and some other probes like LAGEOS 2. Such an approach, proposed for the first time by Shapiro [68] in a different scenario, was extended years later also to some SLR targets of the LAGEOS family by Ciufolini [8]. In particular, the possibility of linearly combining the nodes of L, L 2 and a satellite which was intended to have the same orbital parameters of the current LR 2, was explicitly considered by Iorio et al. [32]. In fact, Ciufolini et al. [12, 13] have so far only considered the sum of the nodes of LAGEOS and LARES 2, giving no signs of wanting to consider any other alternative. Furthermore, also the aforementioned approach would not be free from drawbacks. Indeed, since the coefficients weighing the residuals of the orbital elements entering the linear combinations are theoretically calculated from some mean values of the satellites' orbital parameters, they would be affected by the uncertainties in the latter ones, thus not allowing for a perfect cancellation of the scaling parameters whose bias one wish to remove [29].

2.1.1 The impact of the uncertainty in the Newtonian constant of gravitation

Let the impact of the uncertainty in G , overlooked by either Iorio [30] and Ciufolini et al. [13], be considered.

On the one hand, G enters Eq. (6) only through μ , which is one of the parameters that are accurately estimated in data reductions of satellites' SLR observations. On the other hand,

the product GJ entering Eq. (5) does *not* (yet?) fall within such a category; thus, the impact of our imperfect knowledge of G on Eq. (4) must be evaluated *separately*.

According to the 2022 CODATA recommended values, provided by the National Institute of Standards and Technology (NIST) and retrievable on the at <https://physics.nist.gov/cgi-bin/cuu/Value?bg>, its relative uncertainty is currently

$$\frac{\sigma_G}{G} = 2.2 \times 10^{-5}. \quad (18)$$

By scaling Eq. (17) by (18), an error in $\mathcal{R}^{J_2}$ as large as

$$\sigma_{\mathcal{R}^{J_2}} \leq \left| \frac{\partial \mathcal{R}^{J_2}}{\partial G} \right|_{\mu=\text{const}}^{T_1} \sigma_G = \left| \mathcal{R}^{J_2} \right|_{T_1} \frac{\sigma_G}{G} = 1.3, \quad (19)$$

corresponding to a $\simeq 130\%$ systematic bias due to J_2 in the expected LT signature, occurs.

Interestingly, even with the J2000.0 expression of $\mathcal{R}^{J_2}$, calculated with Eqs. (1)–(2) and used in [13,30], Eq. (18) would still yield a $\simeq 11\%$ oblateness-driven systematic uncertainty in the sum of the LT node precessions.

2.1.2 The impact of the uncertainty in the Earth's angular momentum

By citing relevant works, Ciufolini et al. [13, Tab. 2] *convincingly* demonstrated that the realistic relative error in the Earth's angular momentum is actually *smaller* than guessed by Iorio [30, Sect. 4], being of the order of

$$\frac{\sigma_J}{J} \simeq 10^{-6}. \quad (20)$$

If, on the one hand, Eq. (20) is small enough not to pose problems to the J2000.0 expression of $\mathcal{R}^{J_2}$ used in [13,30], on the other hand it is not entirely so for Eq. (17). Indeed, by rescaling it by Eq. (20) one has

$$\sigma_{\mathcal{R}^{J_2}} \leq \left| \frac{\partial \mathcal{R}^{J_2}}{\partial J} \right| \sigma_J = \left| \mathcal{R}^{J_2} \right|_{T_1} \frac{\sigma_J}{J} = 0.059, \quad (21)$$

corresponding to a $\simeq 6\%$ bias.

2.2 The impact of the uncertainty in the secular variation of the Earth's oblateness

The first even zonal harmonic J_2 of the geopotential is brought in the numerator of Eq. (4) by Eq. (6) as a multiplicative parameter common to the node rates of both LAGEOS and LARES 2.

Indeed, a variety of physical processes induce certain time-dependent variations of the Earth's oblateness which can be expressed as

$$J_2(T) = \bar{J}_2 + \dot{J}_2 \left(\frac{T - T_{\text{rf}}}{P} \right) + J_2^c \cos \left[\frac{2\pi}{P} (T - T_{\text{rf}}) \right]$$

$$+ J_2^s \sin \left[\frac{2\pi}{P} (T - T_{\text{rf}}) \right], \quad (22)$$

where $\bar{J}_2$ is meant here as the coefficient of degree $\ell = 2$ and order $m = 0$ of the unconstrained static field, $\dot{J}_2$ is the amplitude of the linear trend, in yr^{-1} , J_2^c and J_2^s are the amplitudes of the harmonic annual variations, T is an arbitrary epoch, T_{rf} is some reference epoch depending on the Earth's gravity field solution adopted, $P = 365.25$ d is the duration of the year, in days. As an example, for the model ITSG-Grace2018, obtained from 162 months of GRACE.¹ data collected from April 2002 to June 2017 and retrievable at <http://doi.org/10.5880/ICGEM.2018.003>, the reference epoch T_{rf} is June 1, 2010.

Should only the static component of J_2 affect Eq. (4), as seemingly assumed by [13], no problems would arise, at least at first sight. Indeed, the present-day level of the *formal* relative uncertainty in $\bar{J}_2$ is at the

$$\frac{\sigma_{\bar{J}_2}}{\bar{J}_2} \simeq 10^{-9} \quad (23)$$

level, as for ITSG-Grace2018. However, Ciufolini et al. [13] conservatively considered the *calibrated* relative uncertainty in the static part of J_2 released by the model GGM05S [60]

$$\frac{\sigma_{\bar{J}_2}}{\bar{J}_2} \simeq 2.4 \times 10^{-7}. \quad (24)$$

By rescaling Eq. (17) by Eq. (24), one gets a systematic bias on the summed LT node precessions of nearly 1.4%, which is nearly one order of magnitude larger than the overall accuracy level claimed by Ciufolini et al. [13].

In fact, also the mismodeling in the *other* components of Eq. (22) has, in principle, to be taken into account. Limiting just to the secular trend of J_2 , it yields a correction ΔJ_2 to the static value of the Earth's oblateness which, at the epoch T_1 of the LARES 2 launch, is

$$\Delta J_2(T_1) \simeq 6.3 \times 10^{-10}. \quad (25)$$

The relative uncertainty in Eq. (25), retrievable from the published error in $\dot{J}_2$ according to, e.g., ITSG-Grace2018, turns out to be as large as

$$\frac{\sigma_{\Delta J_2}}{\Delta J_2} \Big|_{T_1} = \left| \frac{\sigma_{\dot{J}_2}}{\dot{J}_2} \right| \simeq 1.4 \times 10^{-2}. \quad (26)$$

It corresponds to an error in $\mathcal{R}^{J_2}$, calculated by replacing $J_2 \rightarrow \Delta J_2(T_1)$ in Eq. (6) entering Eq. (4), of the order of

$$\sigma_{\mathcal{R}^{J_2}} \leq \left| \frac{\partial \mathcal{R}^{J_2}}{\partial \Delta J_2} \right|_{T_1} \sigma_{\Delta J_2} \simeq 5 \times 10^{-4}. \quad (27)$$

¹ Values of the low-degree even zonals from Earth's gravity solutions obtained only from SLR satellites like, e.g., IGG_UPWr_SLR retrievable at <https://doi.org/10.1016/j.rse.2024.113994>, would not be suitable since they would be a priori imprinted just by the LT effect, not explicitly solved-for in them.

However, due to the non-trivial overall time dependence of $\mathcal{R}^{J_2}$ introduced by the precessional motion of $\hat{\mathbf{k}}$ and by the temporal evolution of I and Ω , it would be advisable to fully account also for $\dot{J}_2$. Its *nominal* impact on $\mathcal{R}^{J_2}$ was accounted for in producing Fig. 2.

3 The impact of the other Earth's gravity field multipoles

In principle, also the other even zonal harmonics J_ℓ , $\ell = 4, 6, \dots$ of higher degree should be taken into account.

A quick evaluation about the nominal impact of $J_4 \simeq 10^{-7}$ of the sum of the nodes can be easily inferred simply by rescaling Eq. (4) by

$$\frac{J_4}{J_2} \left(\frac{R}{a} \right)^2 \simeq 4 \times 10^{-4}. \quad (28)$$

Thus, from Eq. (17) it can be guessed

$$\mathcal{R}^{J_4} \simeq 23.9. \quad (29)$$

An application of the scaling factor of Eq. (28) to the time series of Fig. 2 clearly shows that a $\simeq 10^3$ bias due to the imperfectly cancelled node precessions due to J_4 would indirectly occur over the years because of the Earth's spin axis precession.

4 Revisiting the van Patten–Everitt proposal for two counter-orbiting polar satellites

In 1976, Van Patten and Everitt [76, 77] suggested to measure the LT effect by using a pair of low-altitude counter-orbiting drag-free Earth's satellites A and B in circular polar motion. In addition to tracking data from existing ground stations, satellite-to-satellite Doppler ranging data should have been taken near the poles.

Apart from *obvious* differences in terms of *cost* and *technologies* to be employed, such a proposal is *conceptually* equivalent to that put forth by Ciufolini [7] *ten* years later (see Sect. 4.1) and, within certain limits, *even better* from the point of view of the overall accuracy because of the unavoidable departures from the *ideal* orbital configuration (see Sect. 4.2).

4.1 Counter-revolving satellites along identical, arbitrarily inclined orbits

In fact, the orbits of A and B may not necessarily pass through $\mathbf{J}$ in order to yield a *conceptually* equivalent scenario to that envisaged by Ciufolini [7]. This is proven as follows.

By assuming *ideally* identical semimajor axes and eccentricities, a satellite B is counter-revolving with respect to

another satellite A if the conditions

$$I_B = 180^\circ - I_A, \quad (30)$$

$$\Omega_B = \Omega_A + 180^\circ \quad (31)$$

exactly hold. Indeed, from Eqs. (3) and (7)–(8), along with the trigonometric identities

$$\sin(180^\circ - \beta) = \sin \beta, \quad (32)$$

$$\cos(180^\circ - \beta) = -\cos \beta, \quad (33)$$

$$\sin(180^\circ + \beta) = -\sin \beta, \quad (34)$$

$$\cos(180^\circ + \beta) = -\cos \beta, \quad (35)$$

it turns out that

$$\hat{\mathbf{l}}_B = -\hat{\mathbf{l}}_A, \quad (36)$$

$$\hat{\mathbf{m}}_B = \hat{\mathbf{m}}_A, \quad (37)$$

$$\hat{\mathbf{h}}_B = -\hat{\mathbf{h}}_A. \quad (38)$$

Equation (38) implies that the sense of motion of B along its orbit is just *opposite* to that of A, while Eqs. (36)–(37), together with Eqs. (30)–(31), guarantee that the orbital planes of A and B *coincide*. Finally, Eqs. (36)–(38) yield

$$\hat{\mathbf{l}}_B \times \hat{\mathbf{m}}_B = \hat{\mathbf{m}}_A \times \hat{\mathbf{l}}_A = -\hat{\mathbf{h}}_A = \hat{\mathbf{h}}_B. \quad (39)$$

It should be noted that, so far, *no* assumptions on the mutual orientation of $\hat{\mathbf{k}}$ and $\hat{\mathbf{h}}_{A/B}$, were made at all; in other words, the condition of passage through the primary's poles was *not* adopted. From Eqs. (5)–(6), (32) and (36)–(38) it straightforwardly turns out that

$$\dot{\Omega}_{\text{LT}}^B = \dot{\Omega}_{\text{LT}}^A, \quad (40)$$

$$\dot{\Omega}_{J_2}^B = -\dot{\Omega}_{J_2}^A, \quad (41)$$

which hold for an *arbitrary* orientation of $\hat{\mathbf{k}}$ in space.

This proves that the orbital configurations by Ciufolini [7] and by Van Patten and Everitt [76, 77] are *conceptually* equivalent, even *regardless* of the *inclination* of the orbital planes.

4.2 The polar orbital configuration

For a generic orientation of $\hat{\mathbf{k}}$ in space, parameterized in terms of RA and decl. as per (12), the condition that the orbital plane contains the primary's spin axis is *ideally* fulfilled if

$$I = 90^\circ, \quad (42)$$

$$\Omega = \alpha \quad (43)$$

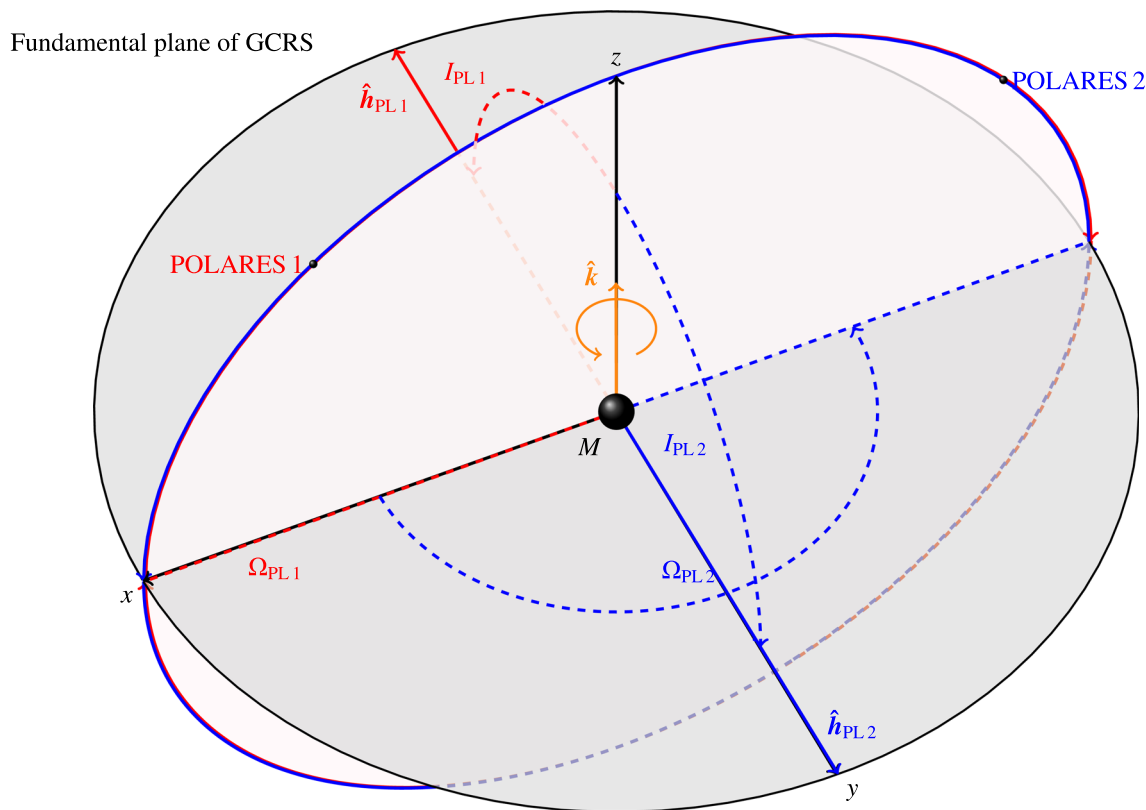


Fig. 3 Orbital configurations of POLARES 1 (in red) and POLARES 2 (in blue) at the epoch T_1 of the launch of LARES 2 with respect to the Geocentric Celestial Reference System (GCRS) whose fundamental plane is shown shaded in grey. The Earth's spin axis $\hat{k}$, in orange, is depicted according to Eq. (11). The ratio of the semimajor axis a of POLARES 2 to that of POLARES 1, to which an arbitrary reference value has been assigned just for illustrative purpose, is as in Table 1

from which the eccentricities e are retrieved as well. The inclinations I and the longitudes of the ascending node Ω are given by Eqs. (42)–(43) up to an offset of $\delta\gamma = \pm 1$ arcsecond for both orbital elements. The unit vectors $\hat{h}$, given by Eq. (8), are directed towards the orbital angular momenta of the satellites. The sizes of the Earth and of the satellites' orbits are *not* in scale. The view is from above the fundamental plane of GCRS

exactly hold. Indeed, from Eqs. (12) and (8), it turns out that Eqs. (42)–(43) yield just

$$\hat{h} \cdot \hat{k} = 0. \quad (44)$$

Figure 3 depicts such a scenario, which may be branded as POLAR Relativity Satellites (POLARES).

For the sake of definiteness, the *same* values as LAGEOS and LARES 2 were used for the semimajor axes and the eccentricities of the counter-revolving satellites POLARES 1 and 2, and the Earth's spin axis orientation was chosen as given by Eq. (11). Furthermore, departures $\pm\delta\gamma$ with

$$\delta\gamma = 1 \text{ arcsecond} = 0.00027^\circ \quad (45)$$

from the *ideal* conditions of Eqs. (30)–(31) and (42)–(43) were adopted.

A peculiar advantage of a polar orbit configuration is that, *in principle*, it allows to cancel out all the classical long-term rates of change of the node induced by the even zonal harmonics, as per Eqs. (6) and (44); see also [69].

Figures 4, 5 and 6 show that Eq. (4), calculated with Eqs. (5)–(6), is *less* sensitive to departures from the *ideal* configuration established by Eqs. (30)–(31) and (42)–(43) than in the case of LAGEOS and LARES 2.

It turns out that, within the range given by Eq. (45), $\mathcal{R}^{J_2}$ is mainly sensitive to the conditions of Eqs. (30) and (42) on the orbital inclinations. Indeed, its maximum *nominal* value can reach the level of about 100, which, however, is nearly *600 times smaller* than Eq. (17) for LAGEOS and LARES 2. Instead, the conditions on the nodes of Eqs. (31) and (43) can be somewhat relaxed, at least from the point of view of the reduction of the systematic bias due to J_2 .

Thus, the earlier scenario envisaged by Van Patten and Everitt [76,77] is worth of being reconsidered. In view of the recent advances in accurate modeling the non-gravitational perturbations and in the SLR technique, it may be implemented also with common geodetic satellites should the measurement of the LT effect be (*one of*) its *main* goal. Such

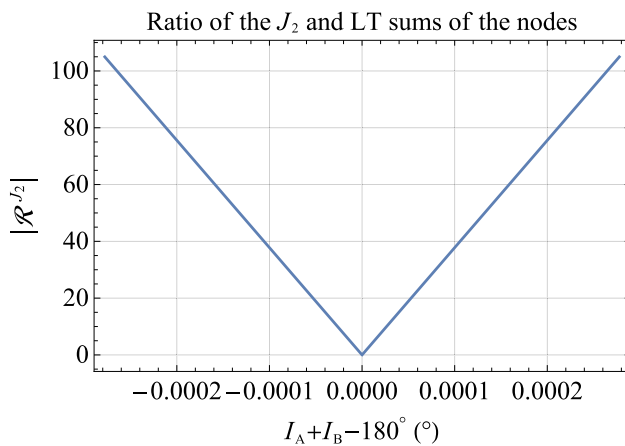


Fig. 4 Plot of the absolute value of Eq. (4), calculated with Eqs. (11), (31) and (42)–(43) in Eqs. (5)–(6), as a function of the departure from the *ideal* condition of Eq. (30) within a range 2 arcseconds wide. The values for the semimajor axes and the eccentricities listed in Table 1 were used

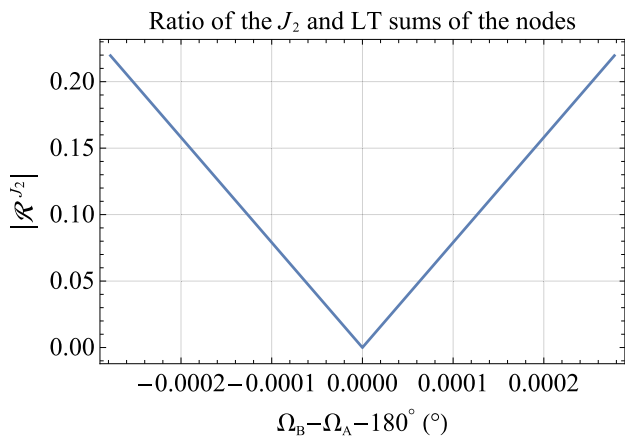


Fig. 5 Plot of the absolute value of Eq. (4), calculated with Eqs. (11), (30) and (42)–(43) in Eqs. (5)–(6), as a function of the departure from the *ideal* condition of Eq. (31) within a range 2 arcseconds wide. The values for the semimajor axes and the eccentricities listed in Table 1 were used

a possibility would certainly deserve further investigations relying upon the past ones [4, 63, 64, 78].

5 Some comments on the criticisms raised by Ciufolini et al.

Ciufolini et al. [13] raised certain criticisms to [30] which essentially boil down to the following.

5.1 The issue of the error propagation

Ciufolini et al. [13] repeatedly accused Iorio [30] of ignoring even the most basic notions of error propagation in reaching his conclusions. Such an allegation would be based only (or

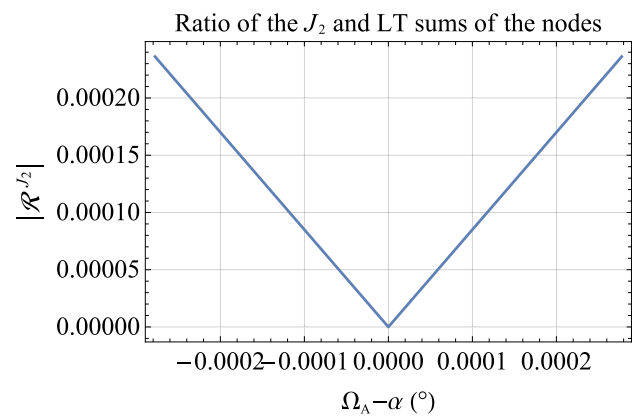


Fig. 6 Plot of the absolute value of Eq. (4), calculated with Eqs. (11), (30)–coO and (42) in Eqs. (5)–(6), as a function of the departure from the *ideal* condition of Eq. (43) within a range 2 arcseconds wide. The values for the semimajor axes and the eccentricities listed in Table 1 were used

mainly?) on the fact that Iorio [30] did not include (the *static* part of) J_2 in the set of the parameters affected by errors to be propagated in $\mathcal{R}^{J_2}$. Actually, it was just a matter of (sound) choice since, as correctly shown by Ciufolini et al. [13] themselves, the error of (the *static* part of) J_2 impacts the overall uncertainty in the *J2000.0* expression of $\mathcal{R}^{J_2}$ adopted by Iorio [30] and Ciufolini et al. [13] to a negligible level.

On the contrary, it is precisely Ciufolini et al. [13] who seem to ignore how to properly propagate errors, judging by what they have done with the uncertainties on a and I . Indeed, it is well known that if $f(q_1, q_2, \dots, q_N)$ is an explicit function of N parameters q_i , $i = 1, 2, \dots, N$ affected by experimental or observational errors σ_{q_i} , $i = 1, 2, \dots, N$, an upper bound of the total uncertainty in f can be calculated as

$$\sigma_f \leq \sum_{i=1}^N \left| \frac{\partial f}{\partial q_i} \right| \sigma_{q_i}. \quad (46)$$

If q_i , $i = 1, 2, \dots, N$ are assumed to be mutually uncorrelated, then one can write

$$\sigma_f = \sqrt{\sum_{i=1}^N \left(\frac{\partial f}{\partial q_i} \right)^2 \sigma_{q_i}^2}. \quad (47)$$

It is precisely what Iorio [30] correctly did with $f \equiv \mathcal{R}^{J_2}$, assumed as function of the Newtonian constant of gravitation, of the Earth's standard gravitational parameter, equatorial radius, quadrupole mass moment and angular momentum, and of the semimajor axes, eccentricities and inclinations of LAGEOS and LARES 2 in assessing the impact of their uncertainties on $\mathcal{R}^{J_2}$ itself. Instead, Ciufolini et al. [13] incorrectly considered just the *numerator* of $\mathcal{R}^{J_2}$, made of the sum of the classical node precessions, as a function of the aforementioned orbital elements, by keeping the *denomina-*

tor, made of the sum of the LT node rates, *fixed*. Instead, inexplicably, when it came to propagating the error on J , which enters the *denominator* of $\mathcal{R}^{J_2}$, Ciufolini et al. [13] treated the latter as a *variable* quantity dependent on J itself by taking the derivative of the denominator *only*. All this, together with the *ad hoc* choice of the magnitudes of the errors in the orbital parameters, for the sole purpose of obtaining values more favorable to their preconceived assumptions. Suffice it to say that Ciufolini et al. [13], after having criticized with dubious arguments the values adopted by Iorio [30] as representative of the experimental uncertainties in a , e and I , in the end decided that $\sigma_{a_{LR2}} = 0.1$ mm by Iorio [30] was fine, while the figures by σ_I by Iorio [30] himself were to be rejected; See the discussion about such a guess for σ_a of the LAGEOS-type satellites in Sect. 5.2.1. Even from the strongly biased point of view of Ciufolini et al. [13] themselves, such an unjustified “cherry-picking” attitude, as well as being contradictory and incorrect, is also counterproductive. Indeed, Iorio [30] showed that the bias on $\mathcal{R}^{J_2}$ due to $\sigma_e \simeq 10^{-5}$, tacitly accepted by Ciufolini et al. [13] since they did not criticize them, if *correctly* calculated, amounts to $\simeq 120\%$ of the LT signature.

5.2 The realistic assessment of the uncertainties in the Keplerian orbital parameters

About the correct evaluation of the uncertainties affecting the Keplerian orbital elements and their impact on the total error budget, their exceedingly small errors invoked by Ciufolini et al. [13] are likely the mere *formal*, *statistical* ones sorting out of the least-square estimation procedure of the satellites’ data reduction. As such they are, by no means, representative of the *physically realistic* uncertainties which have to be assessed from other pieces of information. Below, a pair of examples will be given.

5.2.1 The semimajor axis

In the unperturbed Keplerian motion, the semimajor axis a is calculated as

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu} \right)^{-1}, \quad (48)$$

where r and v are the geocentric distance and speed of the satellite, respectively. The relative error in a due to the uncertainty in μ , averaged over one orbital period, turns out to be equal to the relative error in μ itself [29] which, for the Earth, amounts to

$$\frac{\sigma_\mu}{\mu} = 1 \times 10^{-9}. \quad (49)$$

Such a figure is based on the online version of [52, Chap. 1] which quotes [59]. Thus, the *realistic* uncertainty in the semi-

major axis of a LAGEOS-type satellite *cannot* reasonably be smaller than

$$\sigma_a \gtrsim \frac{\sigma_\mu}{\mu} a = 0.01 \text{ m} = 1 \text{ cm}, \quad (50)$$

which is *even* 1 – 2 orders of magnitude *larger* than the figures proposed in [30] and, for some reasons, accepted by Ciufolini et al. [13].

By reassessing the impact of the uncertainties in a on $\mathcal{R}^{J_2}$ with Eq. (50) yields

$$\sigma_{\mathcal{R}^{J_2}} \leq \left| \frac{\partial \mathcal{R}^{J_2}}{\partial a_L} \right| \sigma_{a_L} + \left| \frac{\partial \mathcal{R}^{J_2}}{\partial a_{LR2}} \right| \sigma_{a_{LR2}} \simeq 1.1. \quad (51)$$

5.2.2 The inclination

The fact that the an accuracy as high as 0.01 mas in the inclinations of LAGEOS and LARES 2 claimed by Ciufolini et al. [13] should be deemed as *physically unrealistic* can be proven as follows.

The inclination I the orbital plane of any satellite is reckoned from the principal plane of the ECI adopted, or, equivalently, from the latter’s pole. Thus, the *realistic* accuracy in I *cannot* certainly be *better* than that of the reference directions themselves with respect to which it is defined. The orientation of the ICRS, which coincides by default with that of GCRS, slightly differs from the MEME by a fixed offset expressed in terms of the three angles ξ_0 , η_0 and $d\alpha_0$; ξ_0 and η_0 are the celestial pole offsets at J2000 and $d\alpha_0$ is the offset in right ascension of the J2000 mean equatorial frame with respect to the GCRS [6]. As far as the pole is concerned, according to [52, Sect. 2.1.1], the discrepancy between different determinations of ξ_0 and η_0 is of the order of $\simeq 0.5$ mas and $\simeq 1.8$ mas, respectively; such figures can be reasonably assumed as representative of the uncertainty in the GCRS equator, at least due to the frame bias. Furthermore, the direction of the ICRS pole is maintained fixed relative to the quasars within ± 20 microarcseconds (μas) corresponding to 0.02 mas [1, 40, 52]. This shows that the *physically meaningful* uncertainty in knowing the inclination I of *any* satellite cannot be better of

$$\sigma_I \simeq 0.5 - 2 \text{ mas} \quad (52)$$

By reassessing the impact of the uncertainties in I on $\mathcal{R}^{J_2}$ with Eq. (52) yields

$$\sigma_{\mathcal{R}^{J_2}} \leq \left| \frac{\partial \mathcal{R}^{J_2}}{\partial I_L} \right| \sigma_{I_L} + \left| \frac{\partial \mathcal{R}^{J_2}}{\partial I_{LR2}} \right| \sigma_{I_{LR2}} \simeq 1.1. \quad (53)$$

5.3 The other criticisms by Ciufolini et al.

All the rest of the criticisms by Ciufolini et al. [13] don’t make the case, completely bypassing the *real* problem of the

estimation of the LT effect in the data reduction raised in [30, Sect. 5].

Suffice it to say that Ciufolini et al. [13], in reply to [30, Sect. 5] who described in general terms the standard orbit determination procedure in satellite geodesy, astrodynamics and astronomy, wrote that “of course, in standard space geodesy there is not such a thing as “simultaneously” estimating the coefficients of the Earth gravitational field with “the propagation of electromagnetic waves” and “the behaviour of measuring devices””. Actually, in, e.g., [71, p. 1], one can just read: “For the satellite orbit determination problem the minimal set of parameters will be the position and velocity vectors at some given epoch. In subsequent discussions, this minimal set will be expanded to include dynamic and measurement model parameters, which may be needed to improve the prediction accuracy.” Furthermore, in [71, p. 3] it is written: “[...] the orbit determination procedure may be used to obtain better estimates of the location of tracking stations, adjust the station clocks, calibrate radar biases, obtain a better estimate of geophysical constants, and so on.”

However, this is *not* the important point, but rather the fact that, *after almost 30 years* since the first attempts [9], neither Ciufolini and (past and present) coworkers nor anyone else have ever so far estimated a dedicated solve-for parameter of the LT effect in the least-square procedure of data reduction along with all the other parameters routinely estimated, whatever they may be, in satellite geodesy studies. This would be the *only* significant breakdown in the long history of LT tests with SLR. By repeating such estimations with different data sets and background reference models, and inspecting the covariance and correlation among *all* the estimated parameters, *including also a LT one* would be the *only* correct and unambiguous way to proceed, as, on the other hand, already done by other teams in the Solar and Jovian scenarios [17, 23, 47, 48]. Ciufolini et al. [13] did not offer *any* answer at all to this important point. They limit themselves to cite just a few decades-old works whose authors have used times series of satellites’ orbital elements to determine some non-gravitational physical effects, thus ignoring the “evolution over the past four decades” [71, p. 1] of the satellite orbit determination.

6 Summary and conclusions

Recently, Ciufolini and coworkers firmly reaffirmed their belief about the possibility of successfully performing a $\simeq 0.2\%$ test of the post-Newtonian LT effect in the field of the Earth with the passive geodetic satellites LAGEOS and LARES 2 tracked with the Satellite Laser Ranging technique.

Should the terrestrial gravitomagnetic field be explicitly modeled and estimated along with other parameters in dedicated satellites’ data reductions, it would be possible to assess

the overall systematic uncertainty in the *standard way*, common to *satellite geodesy, astronomy and astrodynamics*, by inspecting the *correlations* among the *estimated* LT parameter(s) and all the other determined ones contained in the covariance matrix of the fit. It is worthwhile noticing that this *is* just the way other teams of researchers recently performed their own LT tests with Mercury in the field of the Sun and with the probe Juno around Jupiter. Such an approach should be repeated by *varying* the *data sets* of LAGEOS and LARES 2 themselves and the *background reference models* adopted such as, e.g., different Earth’s gravity field solutions produced by several institutions worldwide with data collected by dedicated spacecraft (GRACE, GOCE, GRACE-FO, other geodetic satellites) during different time spans. Given that, for unknown reasons, nearly *30 years after* the first tests this has *not* yet been done nor does it seem that it will be done in future tests, it is therefore more necessary than ever to resort to a “*offline*”, *a priori* evaluation of the error budget based on *analytical* methods and pieces of information collected in a variety of means.

The ratio of the sum of the Newtonian oblateness-driven node precessions of LAGEOS and LARES 2 to the sum of their LT counterparts fits well the scope. It can be viewed as a function of the orbital and physical parameters of the satellites and the Earth, respectively, all affected by observational uncertainties of various nature. *In principle*, such a ratio would *vanish* if the orbital sizes and shapes of both satellites were *identical* and the sum of their inclinations were *precisely* 180° .

Recently, it was pointed out by the present author that the *actual* mean values of the such orbital parameters of LAGEOS and LARES 2, averaged over a hundred days, do *not* allow to meet this stringent goal, its resulting non-zero *nominal* value being equal to almost 5000. This unfortunate circumstance implies that the Earth’s quadrupole moment sensibly impacts the proposed LT test also *indirectly* through the errors in the physical and orbital parameters entering the aforementioned ratio.

Actually, previous estimates of such an important systematic bias were based *incorrectly* on formulas for the well known standard classical and relativistic node precessions which hold *only* in an inertial reference system *exactly* aligned with the Earth’s spin axis. On the one hand, the former one is the Geocentric Celestial Reference System (GCRS), whose *fixed* orientation in space *nearly* coincides with the Earth’s mean equator and equinox at the epoch *J2000.0*. GCRS is just the inertial reference system used in data analyses of Earth’s satellites. On the other hand, LARES 2 was launched about 22 *years after* that epoch. Then, *general* formulas for the node precessions of interest, valid for an *arbitrary* orientation of the *precessing* Earth’s spin axis, should be used. By repeating the above analytical calculation for the epoch of the launch of LARES 2 by *consistently propa-*

gating the errors in *all* the parameters entering the ratio of the summed classical to relativistic precessions shows that its *nominal* value is *even larger* than its J2000.0 counterpart, amounting now to about 59,000. Thus, the *indirect* impact of the inaccurately known physical and orbital parameters entering it is *even larger* than in the J2000.0 case recently investigated in the literature. The uncertainties in parameters such as J_2 itself and the Earth's angular momentum J which did not have a remarkable impact in the J2000.0 case, now also play a non-negligible role-to the percent order-in pushing the *realistically* achievable accuracy away from the final 0.2% accuracy goal sought for the LT test with LAGEOS and LARES 2. Among them, the impact of the secular rate of change of the Earth's quadrupole mass moment should deserve a careful evaluation.

The *exceedingly small* errors in the orbital parameters of LAGEOS and LARES 2 claimed by Ciufolini and coworkers are *unlikely* representative of any *realistic, physically meaningful* uncertainty in them, being just the *mere statistical* errors of the fitting procedure performed in the data reduction. The present-day relative uncertainty in the Earth's standard gravitational parameter entering the calculation of the semimajor axis a does *not* allow to *realistically* know it with an accuracy *better* than about 1 cm. The satellite's orbital inclination I , being, by definition, reckoned from the GCRS pole axis, can only be known with an accuracy *necessarily* limited by that of the orientation of GCRS itself, being the latter of the order of $\simeq 0.5$ –2 *milliarcseconds*.

It has been shown that, as far as the *orbital configuration* is concerned, the earlier proposal by Van Patten and Everitt of two counter-orbiting drag-free satellites in *identical* polar orbits is *conceptually equivalent* to the LAGEOS–LARES 2 one in the sense that *both* imply that, *in principle*, the sum of the LT node precessions add up while the classical ones due to J_2 cancel out, irrespectively of their altitudes and eccentricities provided that they are *ideally* equal for both the hypothesized spacecraft. Notably, this feature does *not* necessarily hold *only* for polar orbits.

Furthermore, it turns out that a pair of counter-revolving satellites in *nearly* equal orbits both passing *almost* exactly through the Earth's poles would represent a scenario *less sensitive* to the impact of the unavoidable departures from the *idealized* one than LAGEOS and LARES 2. Indeed, by assuming for the sake of definiteness the *same* orbital sizes and shapes of the latter ones, the *nominal* ratio of the summed classical to relativistic node precessions would reach a maximum of *just 100* for deviations of the inclinations from their *ideal* values of up to 1 *arcseconds*.

The Van Patten–Everitt proposal, revamped and rebranded POLAR RELativity Satellites (POLARES), may be imple-

mented even with passive geodetic satellites tracked with the SLR technique only.

Data Availability Statement This manuscript has no associated data. [Author's comment: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.]

Code Availability Statement This manuscript has no associated code/software. [Author's comment: Code/Software sharing not applicable to this article as no code/software was generated or analysed during the current study.]

Declarations

Conflict of interest I declare no conflicts of interest.

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