SOME METHODS OF BETATRON SPECTRUM MEASUREMENTS

A.I.Dzergatch, G.F.Senatorov, V.A.Uvarov

Radiotechnical Institute, Academy of Sciences, USSR

While investigating betatron beam motion in synchrotrons the main attention is usually payed to the measurement of the beam center-of-mass betatron frequency oscillations, but it is desirable to have information on the particle betatron frequency distribution inside the beam i.e. spectrum.

Betatron spectrum measurement may be carried out by investigating the beam center-of-mass response on the excitation field.

First consider the oscillations excited by $F(v)=F_m \exp(jq_0\omega_0 t)$ signal, (ω_0 -rotation frequency). The betatron oscillations are usually excited by a field, its length being much shorter than betatron oscillation wavelength. Therefore the particle transversal coordinate variations during the flight through this field may be disregarded. Suppose also that the excitation field keeps constant during the time of flight.

The signal V(t) induced across the pick-up electrodes by the beam in which particles have different betatron oscillation frequencies is proportional to the beam center-of-mass displacement $\bar{x}(\vartheta)$ from its stable position:

$$\bar{x}(\vartheta) = \int_{Q_1}^{Q_2} x(\vartheta, Q) P(Q)\, dQ \Big/ \int_{Q_1}^{Q_2} P(Q)\, dQ, \qquad (I)$$

where $x(\vartheta,Q)$ is "smoothed" solution for betatron oscillations equation with equal frequencies $-Q\omega_0$; $P(Q)$ -is betatron spectrum, $\vartheta=\omega_0 t$

Assuming $Q \gg 2\Delta Q=(Q_2-Q_1)$ we obtain:

$$\bar{x}(\vartheta) = \frac{2F_0}{\Delta Q\sqrt{\pi}}(A^2+B^2)^{1/2}\cos(q_0\vartheta+\psi), \qquad (2)$$

where $\psi=\mathrm{arc\,tg}(A/B)$, $F_0=F_m/Q_0$, $Q_0=(Q_1+Q_2)/2$

A and B, for example, for rectangular spectrum being uniform at the intervals Q_1 and Q_2, after N turns equal to:

$$A=\sum_{K=1}^{N} G_K \frac{\sin 2\pi K q_0}{K}; \quad B=\sum_{K=1}^{N} G_K \frac{\cos 2\pi K q_0}{K} \qquad (2a)$$

and for the spectrum $\cos^2[\pi(Q-Q_0)/(Q_2-Q)]$

$$A=\sum_{K=1}^{N} G_K \frac{\sin 2\pi K q_0}{K[(2\pi K\Delta Q)^2-1]}; \quad B=\sum_{K=1}^{N} G_K \frac{\omega 2\pi K q_0}{K[(2\pi K\Delta Q)^2-1]} \qquad (2b)$$

$$G_K = \sin 2\pi K Q_0 \cdot \sin 2\pi K \Delta Q$$

If M is an integer nearest to Q and Q_0, whereas L is that to q_0 and $\hat{Q}=|Q-M|$, $\hat{Q}_0=|Q_0-M|$, $\hat{q}_0=|q_0-L|$ we obtain the values A, B and ψ (with accuracy to π) independent of M and L, and other conditions being the same the oscillations amplitude will be equal at any excitation frequency $q_0\omega_0$ if q_0 differs from an integer by the same value of $\hat{q}$.*

Consider the case of small $N \ll 1/2\pi\Delta Q$ when $\sin 2\pi K\Delta Q \approx 2\pi K\Delta Q$ and $(2\Delta QK)^2 \ll I$.

As $x(\vartheta)$ decreases rapidly with increase of $|\hat{q}_0-\hat{Q}_0|$ we consider only the values of frequency detuning of the order ΔQ then with actual $\hat{Q}$ and $\hat{q}_0$; therefore after several turns

$$\bar{x}(\vartheta)=(N+1)F_0 \exp\{j[q_0\vartheta+N\pi(\hat{q}_0-\hat{Q}_0)]\} \qquad (3)$$

Both the oscillation amplitude and its initial growth rate are independent on ΔQ_1 and $|\hat{q}_0-\hat{Q}_0|$ (Fig.I).

The size of this linear region for a given nonlinearity is inversely proportional to ΔQ. The phase difference between the exciting field and the excited oscillations with small $(\hat{q}_0-\hat{Q}_0) \ll \frac{1}{2\pi N}$ varies lineary with detuning (Fig.2).

The steady-state amplitude of the excited oscillations ($N \gg 1/2\pi\Delta Q$) with constant exciting frequency $\hat{q}_0=\hat{Q}_0$ is inversely proportional to ΔQ.

*All the above cited is true for the accelerator with one sufficiently short bunch. For an accelerator with bunches uniformly distributed inside the ring the excitation is optimal with where = 0, ±I, ±2 ...

When the exciting frequency q_0 differs from Q_0, the maximum steady-state amplitude of excited oscillations for rectangular spectrum occurs at $\hat{q}_0=\hat{Q}_1$, or $\hat{Q}_2$ where $|\bar{x}| \to \infty$. The steady state phase varies monotonously from $\pi/2$ to $-\pi/2$ within frequency range of $Q_1 Q_2$ passing zero at $\hat{q}_0 \approx \hat{Q}_0$.

For the spectrum $\cos^2\pi(Q-Q_0)(Q_2-Q_1)$ the oscillation amplitude is finite at all the values of $\hat{q}_0$. Maximum amplitude with small ΔQ (which usually occurs) coincides with Q_0. The oscillation phase in practice varies linearly with $\hat{q}_0$ from $\pi/2$ to $-\pi/2$, $\hat{q}$ varying from Q_1 to Q_2 (Fig.I,2).

For the more complicated cases several maximum steady-state amplitudes at the exciting frequency different from $\hat{Q}_0$ occur at the interruption points of the first derivative $P(Q)$.

For measuring ΔQ and $\hat{Q}$ the resonant excitation technique of betatron oscillations has been used at ITEP 7 Gev Accelerator.

The signal with the accelerating field frequency passes to a pulse frequency divider through a gate which is being opened for the excitation period of betatron oscillations (Fig.3). Changing the dividing factor from I6 to 72 any value of $\hat{q}_0$ from 0.097 to 0.44 by steps of 0.006 to 0.05 could be obtained.

The frequency divider output signal passes to a pulse generator producing the pulses with the amplitudes from 0 to 30 kv and duration from 0.8 to 10 μsec at deflector electrode.

For observing the coherent betatron oscillations pick-up electrodes (4) and a bandpass-regulated L.F.filter rejecting all the frequency component but $\omega_0 q_0$ were used.

While determining $\hat{Q}_0$ the dividing factor was being adjusted till the filter output had the maximum value.

Fig.4 shows the oscillogram of the filter output at the proton energy of I Gev. The oscillogram in Fig.4a corresponds to the exciting field frequency ($\hat{q}_0=7/24$) giving maximum oscillation amplitude (~Imm). The oscillogram in Fig.4b is given to compare the same filter output with $\hat{q}_0$=7/22. With $\hat{q}_0$=7/22 and $\hat{q}_0$=7/26 the coherent signal between the electrodes is practically absent.

To reveal the fine structure $P(Q)$ at the end of acceleration cycle the constant exciting frequency has been used, its value could be varied from one acceleration cycle to another.

Fig.6 shows the amplitude and phase dependences of the filter output on the exciting field frequency after I00 turns of the beam. There is also a curve calculated from (3b) for N=I00 and ΔQ = 0.0I2.

According to these methods it is possible to measure $\hat{Q}_0$ and ΔQ during 3-4 cycles with high precision, but the measurement of the spectrum form takes much more time.

There is also a possibility of not so precise but more quick measurement of betatron frequencies spectra. This method propose to excite coherent betatron oscillations by a single pulse of the exciting field. These oscillations decaying because of betatron frequency spread are measured with the help of some devices sensible to the beam center-of-mass transversal coordinates.

Taking into account, as it was before, the "smoothed" betatron oscillations equation and assuming the duration of the excitment small as compared with $T_0=2\pi/\omega_0$, one can show that the signal being measured is proportional to

$$h_s(t)=\sum_{K=0}^{\infty}\left\{\int_{Q_1}^{Q_2}\frac{P(Q)}{Q}\sin Q\omega_0(T_0+KT)\,\delta[t-(T+KT_0)]\,\delta Q\right\} \qquad (4)$$

where T is the time of particle flight from the exciting field to the nearest electrode ($T \ll T_0$). That is the signal looks like a series of short pulses originating at the moments $t_K=KT_0$ with amplitudes

$$A_K=\int_{-\infty}^{+\infty}\frac{P(Q)}{Q}\sin 2\pi K Q\, dQ \qquad (5)$$

(the integration boundaries may be spread from $-\infty$ to ∞ since the spectrum $P(Q)$ is limited).

Similarly the second measuring device displaced by an angle $\beta=\pi/2Q_0$ where Q_0 is the expected mean value of Q, will give pulses with amplitudes:

$$B_K \approx \int_{-\infty}^{+\infty} \frac{P(Q)}{Q} \cos 2\pi K Q \, dQ \qquad (6)$$

Further having the numbers A_K and B_K one can find $P(Q)$ using the sine- and cosine-Fourier transformations.

In order that the set of numbers A_K and B_K give the possibility of determining the function $P(Q)$ it is necessary according to the well-known rule of information theory that the azimuthal interval between the pairs of measuring devices be less than $\Delta\vartheta = 1/2\Delta Q$

In practice we have usually $\Delta Q \ll 0.08$ which corresponds to $\Delta\vartheta > 2\pi$ that is one pair of measuring devices is quite enough. Under the real conditions it is expedient to excite the oscillation by a rectangular pulse of duration T_0. In this case with uniform particle distribution by azimuth the signal amplitudes coincide with the above calculated. If azimuthal particle density is not constant then the signals are modulated accordingly. This modulation may be removed, for instance, by means of a lowpass filter with bandpass

So this method gives the possibility of obtaining the betatron frequencies spectrum during the time of the order of tens or hundreds of microseconds.

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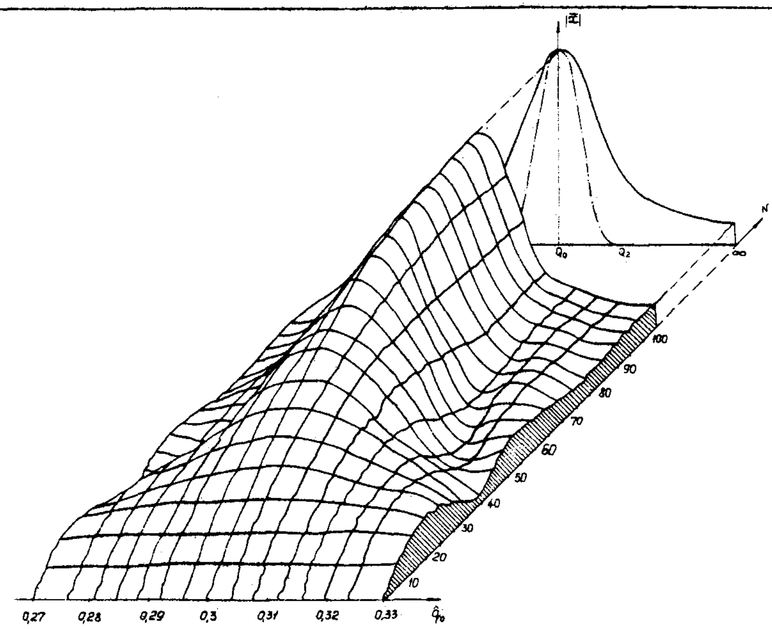


Fig 1 Excited oscillation amplitude vs number of turns for the $\cos^2 \frac{\pi(Q-Q_0)}{2\Delta Q}$ distribution ($\Delta Q = 0.01$, $Q_0 = 0.3$) and with the exciting frequency as a parameter.

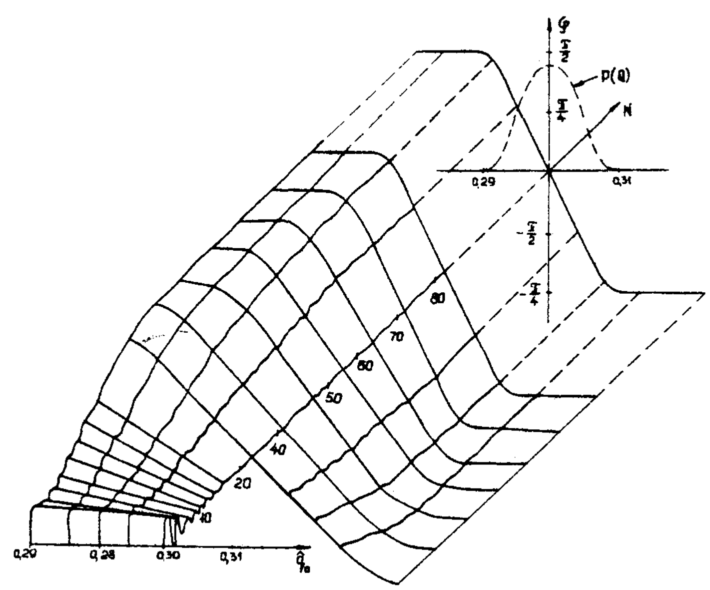


Fig 2 Excited oscillation phase vs. number of turns for the $\cos^2 \frac{\pi(Q-Q_0)}{2\Delta Q}$ distribution ($\Delta Q = 0.01$, $Q_0 = 0.3$) and with the exciting frequency as a parametr.

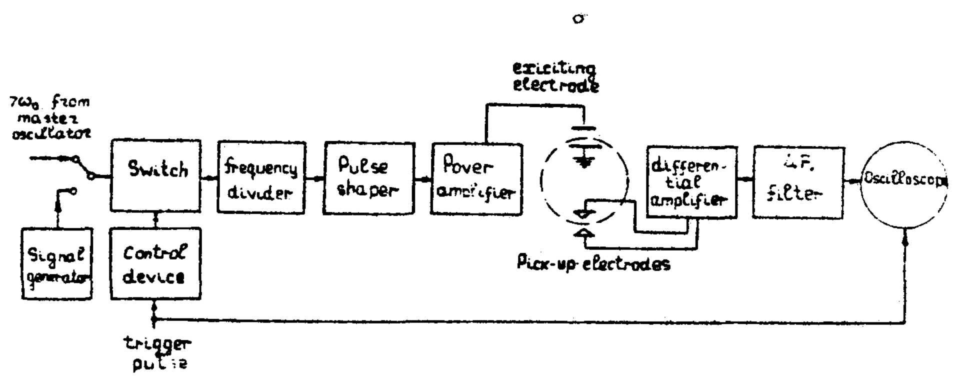
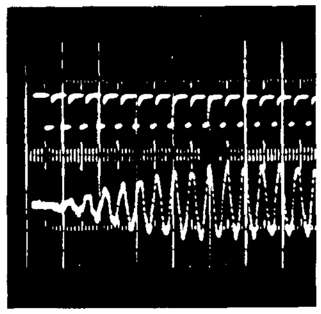


Fig. 3 Block diagram of exciting and monitoring betatron oscillation devices



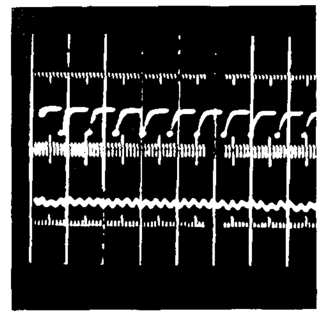


Fig. 4. The LF filter output (lower trace) and exiting signal (upper trace) Proton energy 1 GeV.
a) $\hat{q}_o = 7/24$ b) $q_o = 7/22$

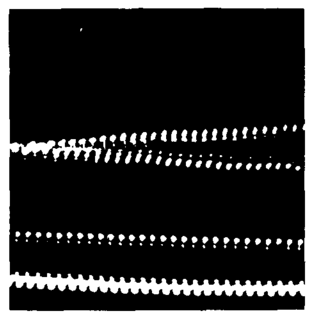


Fig. 5 The LF filter output (upper trace) Proton energy ~ 6 GeV.

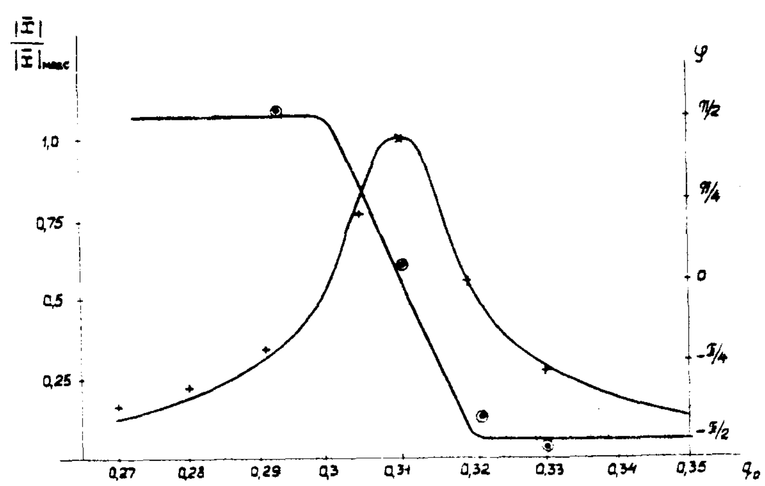


Fig 6 Relative amplitude and phase of bunch center-of-mass
— calculated from (2) for $\Delta Q = 0.012$ and $\hat{Q}_o = 0.31$
\+ measured amplitude
⊙ measured phase