The study of b-s anomaly decays in covariant quark model

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The work is devoted to the study of b-s anomaly decays. We compute the relevant form factors in the framework of the covariant quark model with infrared confinement in the full kinematical momentum transfer region. The calculated form factors are used to evaluate branching fractions and polarization observables in the $B \to K\pi + \mu^+\mu^-$ decay caused by the presence of the intermediate scalar K_0^* and in the cascade decay $B \to \phi(\to K^+K^-)\ell^+\ell^-$. We compare the obtained results with available experimental data and the results from other theoretical approaches.

1 Introduction

The main aim of particle physics is to determine the fundamental principles that govern matter, energy, space and time. Standard Model (SM) in particle physics provides a thoroughly tested basis for describing the matter particles (quarks and leptons), together with bosons of strong and electroweak interactions (gluons,photons, W and Z bosons). However, historical data suggests that Standard model is not complete and that it is just a low-energy limit of a more fundamental theory. One has to admit very impressive progress achieved in the last decade in the electroweak and quantum chromodynamics perturbative calculations. This became possible due to the development of new techniques and computer codes for multi- loop and multi-leg calculations. Today the accuracy of theoretical calculations competes with that of experimental data and further progress is on the way in both the cases. Worth noting that Standard model has been extremely successful in explaining the results of experiments for particle physics. The outstanding success of SM in the description of almost all experimental data in particle physics is manifested in the electroweak pool for different observables. Nevertheless, in recent years observed discrepancies in B-meson rare decays with the predictions of SM. Flavour-changing neutral currents have been prominent tools in high-energy physics in the search for new degrees of freedom, due to their quantum sensitivity to energies much higher than the external particles involved and can be instrumental in order to determine where to look for new physics. During the last decade a lot of observables, including the branching ratios, CP and the angular asymmetry in inclusive and exclusive decay modes of B-meson were measured by B-factories and at LHC experiments. These data allow to explore the spiral structure in the interactions with the flavour-changing and a possible existence of new sources of CP violation.

The transition $b \to s\ell^+\ell^-$ mediated by Flavor-Changing Neutral Current (FCNC) is one of the key point in the SM which allows one to look for the possible manifestation of New

Physics. The physical processes induced by this transition are currently studied in great details at the LHC. The most popular and well–analyzed among them are the rare B-meson decays $B \to K^*(\to K\pi)\mu^+\mu^-$ and $B_s \to \phi(\to K^+K^-)\mu^+\mu^-$. The decay $\Lambda_b \to \Lambda(\to p\pi)\,\ell^+\ell^-$ can be considered to be a welcome complement to the above decay channels. The LHCb Collaboration [1] reported a measurement of form-factor independent angular observables in the decay $B \to K^*\mu^+\mu^-$. One observable was found to be in disagreement with the SM on the level of 3.7 σ. The improved measurements of the isospin asymmetries and branching fractions for $B \to K\mu^+\mu^-$ and $B \to K^*\mu^+\mu^-$ decays were reported in [2]. The isospin asymmetries were consistent with the SM, whereas some branching fractions were found to be slightly lower than the theoretical predictions. An angular analysis and a measurement of the differential branching fraction of the decay $B^0_s \to \phi\mu^+\mu^-$ were presented in [3]. The results of the angular analysis are consistent with the SM. However, the differential branching fraction in one bin was found to be more than 3 σ below the SM predictions. The observed discrepancies (sometimes called "$b \to s\ell\ell$") have generated a plenty of theoretical studies [4]-[15] involving the various scenarios of NP and analysis of the uncertainties from hadronic contributions.

2 The $b - s$ transition form factors in the covariant quark model

We define dimensionless form factors for scalar mesons by

$$
\begin{aligned}
< H_2(p_2)\,|\,\bar{s}\,O^\mu\,b\,|H_1(p_1)> &= F_+(q^2)\,P^\mu + F_-(q^2)\,q^\mu\,, \\
< H_2(p_2)\,|\,\bar{s}\,i\sigma^{\mu\nu}q_\nu(1+\gamma^5)\,b\,|\,H_1(p_1)> &= -\frac{1}{m_1+m_2}\,\big(P_\mu\,q^2 - q_\mu\,Pq\big)\;F_T(q^2)\,,
\end{aligned}
\tag{1}
$$

and for vector mesons as:

$$
\begin{aligned}
&\langle\phi(p_2,\epsilon_2)\,|\,\bar{s}\,O^{\,\mu}\,b\,|\,B_s(p_1)\rangle \; = \\
= \;& N_c\,g_{B_s}\,g_\phi\!\int\!\frac{d^4k}{(2\pi)^4 i}\,\widetilde{\Phi}_{B_s}\Big(-(k+w_{13}p_1)^2\Big)\,\widetilde{\Phi}_\phi\Big(-(k+w_{23}p_2)^2\Big) \\
\times \;& \mathrm{tr}\Big[O^{\,\mu}\,S_b(k+p_1)\,\gamma^5\,S_s(k)\;\not\epsilon_2^{\,\dagger}\,S_s(k+p_2)\Big] \\
= \;& \frac{\epsilon^{\,\dagger}_\nu}{m_1+m_2}\Big(-g^{\mu\nu}\,P\cdot q\,A_0(q^2) + P^{\,\mu}\,P^{\,\nu}\,A_+(q^2) + q^{\,\mu}\,P^{\,\nu}\,A_-(q^2) \\
& + i\,\varepsilon^{\mu\nu\alpha\beta}\,P_\alpha\,q_\beta\,V(q^2)\Big),
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
&\langle\phi(p_2,\epsilon_2)\,|\,\bar{s}\,(\sigma^{\,\mu\nu}q_\nu(1+\gamma^5))\,b\,|\,B_s(p_1)\rangle \; = \\
= \;& N_c\,g_{B_s}\,g_\phi\!\int\!\frac{d^4k}{(2\pi)^4 i}\,\widetilde{\Phi}_{B_s}\Big(-(k+w_{13}p_1)^2\Big)\,\widetilde{\Phi}_\phi\Big(-(k+w_{23}p_2)^2\Big) \\
\times \;& \mathrm{tr}\Big[(\sigma^{\,\mu\nu}q_\nu(1+\gamma^5))\,S_b(k+p_1)\,\gamma^5\,S_s(k)\;\not\epsilon_2^{\,\dagger}\,S_s(k+p_2)\Big] \\
= \;& \epsilon^{\,\dagger}_\nu\Big(-(g^{\mu\nu}-q^{\,\mu}q^{\,\nu}/q^2)\,P\cdot q\,a_0(q^2) + (P^{\,\mu}\,P^{\,\nu} - q^{\,\mu}\,P^{\,\nu}\,P\cdot q/q^2)\,a_+(q^2) \\
& + i\,\varepsilon^{\mu\nu\alpha\beta}\,P_\alpha\,q_\beta\,g(q^2)\Big).
\end{aligned}
\tag{3}
$$

where $P = p_1 + p_2$ and $q = p_1 - p_2$, $\epsilon_2^\dagger \cdot p_2 = 0$, $p_1^2 = m_1^2 \equiv m_{B_s}^2$, $p_2^2 = m_2^2 \equiv m_\phi^2$ and the weak matrix $O^\mu = \gamma^\mu(1-\gamma^5)$. Since there are three quarks involved in these processes, we introduce the notation with two subscripts $w_{ij} = m_{q_j}/(m_{q_i} + m_{q_j})$ $(i, j = 1, 2, 3)$ so that $w_{ij} + w_{ji} = 1$. The form factors are calculated in the full kinematical region of momentum transfer squared.

The results of our numerical calculations are with high accuracy approximated by the parametrization

$$F(q^2) = \frac{F(0)}{1 - as + bs^2}\,, \qquad s = \frac{q^2}{m_1^2}\,, \tag{4}$$

the relative error is less than 1%. In our work we evaluate the b-s transition form factors assuming that mesons are ordinary two-quark states.

3 The $B \to K^*(\to K\pi)\mu^+\mu^-$ decay

Recently, it has been paid much attention to the rare flavor-changing neutral current decay $B \to K^*(\to K\pi)\mu^+\mu^-$. One of the reason was the first measurement of form-factor independent angular observables performed by LHCb-collaborationn [1, 16]. It has been claimed that there is a 3.7σ deviation from the Standard Model (SM) prediction for one of the angular observables. Much effort has been spent to explain this deviation by invoking the effects of new physics (NP) (see Refs. [17]-[23]). The main emphasis of the above mentioned papers was focusing on the search of the physical observables which have low sensitivity to the form factors. In addition to the NP effects, the uncertainties related to the presence of the intermediate scalar resonance K_0^* decaying also into $K\pi$ are intensively discussed in the literature Refs. [24]-[28]. In many papers, the Breit-Wigner form for the $K\pi$-mass spectra is used. However, this assumption can not be justified for the broad scalar resonances like the $K_0^*(800)$ meson. The improvement of the description was done in the paper [29] by invoking the chiral perturabation theory for $K\pi$-interaction. This issue was also generalized to $B_s \to K\pi\ell\bar{\nu}$ in Ref. [30]. As well-known, short-distance physics is under control in the description of the rare B-decays whereas the effects of long-distance physics described by the hadronic form factors lead to large uncertainties since it involves nonperturbative QCD.

Some remarks should be done before the calculations. The internal structure of the light scalar mesons is not yet well established, see for review [31] and [32]. Since they have large decay widths it is difficult to distinguish them from background. There are interpretations of these objects as four-quarks states and/or gluballs. Here, we describe the scalar mesons as two-quark states and evaluate the B-S form factors within our approach but when we use the calculated form factors in the matrix element of the cascade decay $B \to K_0^*(\to K\pi)\ell^+\ell^-$ we take into account the line shape of the K_0^* which reflect the broad width of this resonance. We will use the notation from PDG'14 [33] for the scalar mesons below 1 GeV.

We are going to explore the influence of the intermediate scalar K_0^* meson on the angular decay distribution of the cascade decay $B \to K\pi + \mu^+\mu^-$. Therefore, we give the maximum values of the form factors in Table 3 and branching ratios in Table 3 obtained for *size parameter* $\Lambda_S = 1.5$ GeV. The results for the e-mode are almost identical to those of the μ-mode and will not be shown separately. We compare the obtained results with those from other approaches. One can see that our values for branching ratios are less almost twice compare with those from other approaches.

Let us briefly discuss the impact of scalar resonance K_0^* on $B \to K^*(\to K\pi)\ell^+\ell^-$ decay. As well known, the narrow $K^*(892)$ vector resonance is described by a Breit-Wigner parametriza-

Table 1: The values of the form factors at $q^2 = 0$ in the covariant quark model ($\Lambda_S = 1.5$ GeV) and other approaches

$B - S$	$F(0)$	Our work [34]	[35]	[36]	[37]	[38]	[39]
$B_d^0 - a_0^+(980)$	$F_+(0)$	0.192	0.58	0.56			
	$F_T(0)$	0.182	0.78				
$B_s^0 - K_0^{*+}(800)$	$F_+(0)$	0.274	0.44	0.53			
	$F_T(0)$	0.292	0.60				
$B_s^0 - f_0(980)$	$F_+(0)$	0.254	0.45	0.44	0.19	0.35	0.12
	$F_T(0)$	0.285	0.60	0.58	0.23	0.40	-0.08
$B_d^0 - K_0^{*0}(800)$	$F_+(0)$	0.306	0.50	0.46			
	$F_T(0)$	0.306	0.67	0.58			
$B_d^0 - f_0(500)$	$F_+(0)$	0.210					
	$F_T(0)$	0.203					

Table 2: The branching fractions for the semileptonic and rare B-decays into light scalar mesons and lepton pairs

Decay modes	Branching fractions			
	Our work [34] ($\Lambda_S = 1.5$ GeV)	[40]	[37]	[38]
$B_d^0 \to a_0^+(980)\mu^-\bar{\nu}_\mu$	0.52×10^{-4}	$(2.74 \pm 0.40) \times 10^{-4}$		1.84×10^{-4}
$B_d^0 \to a_0^+(980)\tau^-\bar{\nu}_\tau$	0.11×10^{-4}	$(1.31 \pm 0.23) \times 10^{-4}$		1.01×10^{-4}
$B_s^0 \to K_0^{*+}(800)\mu^-\bar{\nu}_\mu$	1.23×10^{-4}	$(2.06 \pm 0.31) \times 10^{-4}$		1.42×10^{-4}
$B_s^0 \to K_0^{*+}(800)\tau^-\bar{\nu}_\tau$	0.25×10^{-4}	$(1.07 \pm 0.19) \times 10^{-4}$		0.88×10^{-4}
$B_d^0 \to K_0^{*0}(800)\mu^+\mu^-$	3.47×10^{-7}	$(7.31 \pm 1.21) \times 10^{-7}$		
$B_d^0 \to K_0^{*0}(800)\tau^+\tau^-$	0.61×10^{-7}	$(1.33 \pm 0.36) \times 10^{-7}$		
$B_s^0 \to f_0(980)\mu^+\mu^-$	2.45×10^{-7}	$(5.14 \pm 0.78) \times 10^{-7}$	0.95×10^{-7}	5.21×10^{-7}
$B_s^0 \to f_0(980)\tau^+\tau^-$	0.42×10^{-7}	$(0.74 \pm 0.17) \times 10^{-7}$	1.1×10^{-7}	0.38×10^{-7}
$B_d^0 \to K_0^{*0}(800)\bar{\nu}\nu$	2.53×10^{-6}	$(6.30 \pm 0.97) \times 10^{-6}$		
$B_s^0 \to f_0(980)\bar{\nu}\nu$	1.79×10^{-6}	$(4.39 \pm 0.63) \times 10^{-6}$	0.87×10^{-6}	

tion and the given cascade B-decay can be calculated by using the narrow width approximation. But it is not true in the case of the broad scalar $K_0^*(800)$ meson. There are several parametrizations of the $K-\pi$ line shapes in the literature, see, for instance, their discussion in Ref. [29]. We will use for the time being the parametrization accepted in Ref. [30] which integrated value in the K^*-resonance region is equal to

$$\int_{(m_{K^*}-\delta_m)^2}^{(m_{K^*}+\delta_m)^2} dm_{K\pi}^2 |L_S(m_{K\pi}^2)|^2 = 0.17, \quad \text{where} \quad \delta_m = 100\,\text{MeV}. \tag{5}$$

Then we scale the calculated value for the differential decay rate $d\Gamma(B \to K_0^*(800)\mu^+\mu^-)$ by this factor and compare with those for $B \to K(892)\mu^+\mu^-)$ decay. We display the behavior of the ratio

$$R(q^2) = \frac{2/3\, d\Gamma(B \to K^*(892)\mu^+\mu^-)}{2/3\, d\Gamma(B \to K^*(892)\mu^+\mu^-) + 0.17 d\Gamma(B \to K_0^*(800)\mu^+\mu^-)} \tag{6}$$

in Fig.1 which may be compared with the finding of Ref. [25]. The integrated ratio (numerator and denominator are integrated separately in the full kinematical region of q^2) gives the size of the S-wave pollution to the branching ratio of the $B \to K^*\ell^+\ell^-$ decay about 6%.

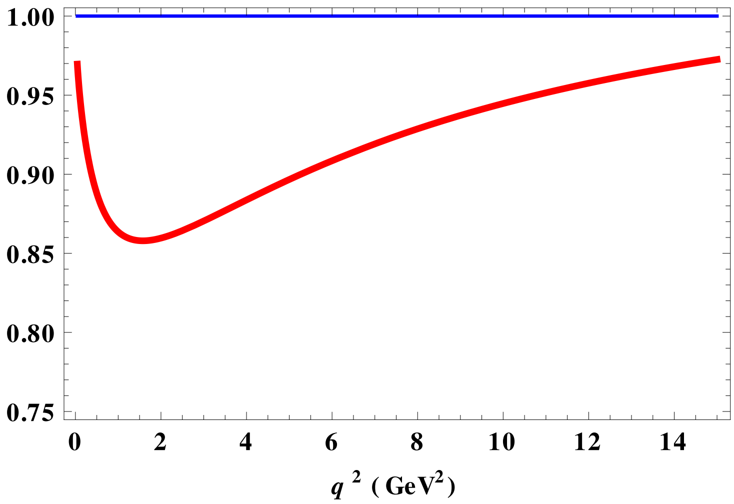

Figure 1: The ratio of the differential decay rate $d\Gamma(B \to K^*(892)(\to K^0\pi^+)\mu^+\mu^-))$ to the full differential decay rate $d\Gamma(B \to K^*(892)(\to K^+\pi^-)\mu^+\mu^-) + d\Gamma(B \to K_0^*(\to K^+\pi^-)\mu^+\mu^-)$.

4 The $B_s^0 \to \phi(\to K^+K^-)\ell^+\ell^-$ decay

This decay channel was first discovered and studied by CDF collaboration [41, 42], later been studied by LHCB collaboration [22, 43]. Despite the fact that the angular distributions are in good agreement with the SM expectations, branching ratio of decay had a 3.1σ disagreement with the prediction of the SM [12, 22].

Table 3: The form factors at maximum recoil $q^2 = 0$.

	$V^c(0)$	$A_0^c(0)$	$A_1^c(0)$	$T_1^c(0)$	$T_3^c(0)$
Our work [45]	0.31 ± 0.03	0.28 ± 0.03	0.27 ± 0.03	0.27 ± 0.03	0.18 ± 0.02
Ref. [46]	0.32		0.29	0.28	
Ref. [47]	$0.434{\pm}0.035$	0.474 ± 0.037	$0.311{\pm}0.029$	$0.349{\pm}0.033$	0.175 ± 0.018
Ref. [48]	0.406 ± 0.020	0.322 ± 0.016	0.320 ± 0.016	0.275 ± 0.014	0.133 ± 0.006
Ref. [49]	0.43	0.38	0.30	0.35	0.25
Ref. [50]	0.25 ± 0.05	0.30 ± 0.05	0.19 ± 0.04		
Ref. [51]	0.44	0.42	0.34	0.38	0.26
Ref. [52]	0.26 ± 0.07	0.31 ± 0.07	$0.18^{+0.06}_{-0.05}$	$0.23^{+0.06}_{-0.05}$	0.19 ± 0.05
Ref. [53]	0.329	0.279	0.232	0.276	0.170
Ref. [54]	0.339 ± 0.017		0.271 ± 0.014	0.299 ± 0.016	0.191 ± 0.010

The $B_s^0 \to \phi\ell^+\ell^-$ decay is similar to the $B \to K^*\ell^+\ell^-$ decay. The B_s meson production is suppressed compared to the B^0 meson by the relation $f_s/f_d \simeq 1/4$, but the narrow resonance ϕ provides a clean set of data with low background. The main difference between $B_s^0 \to \phi\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays is that the final state do not contain information about the initial state of the meson, whether it was B_s or $\bar{B}_s$. Experimentally procurable angular observables in the $B_s^0 \to \phi\ell^+\ell^-$ decay are CP-averaged $F_l, S_{3,4,7}$ and CP-asymmetries $A_{5,6,8,9}$[3, 44].

We display in Table 3 the form factors obtained in our model and compare them with those from other approaches.

The rare decay $b \to s\ell^+\ell^-$ is described in terms of the effective Hamiltonian [55]:

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\lambda_t \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu), \tag{7}$$

where $C_i(\mu)$ and $\mathcal{O}_i(\mu)$ are the Wilson coefficients and local operators, respectively. $\lambda_t = |V_{tb}V_{ts}^*|$ is the product of CKM matrix elements. Note that we drop small corrections proportional to $\lambda_u = |V_{ub}V_{us}^*|$. By using the effective Hamiltonian defined by Eq. (7) one can write the matrix element of the exclusive transition $B_s \to \phi\ell^+\ell^-$ as

$$\begin{aligned} \mathcal{M} &= \frac{G_F}{\sqrt{2}} \cdot \frac{\alpha\lambda_t}{\pi} \cdot \Big\{ C_9^{\rm eff} < \phi \,|\, \bar{s}\, \gamma^\mu\, P_L\, b \,|\, B_s > \big(\bar{\ell}\gamma_\mu \ell\big) \\ &- \frac{2\bar{m}_b}{q^2} C_7^{\rm eff} < \phi \,|\, \bar{s}\, i\sigma^{\mu\nu} q_\nu\, P_R\, b \,|\, B_s > \big(\bar{\ell}\gamma_\mu \ell\big) \\ &+ C_{10} < \phi \,|\, \bar{s}\, \gamma^\mu P_L\, b \,|\, B_s > \big(\bar{\ell}\gamma_\mu\gamma_5 \ell\big) \Big\}, \end{aligned} \tag{8}$$

where $C_7^{\rm eff} = C_7 - C_5/3 - C_6$. One has to note that matrix element in Eq.(8) contains both a free quark decay amplitude coming from the operators $\mathcal{O}_7$, $\mathcal{O}_9$ and $\mathcal{O}_{10}$ (gluon magnetic penquin $\mathcal{O}_8$ does not contribute) and, in addition, certain long-distance effects from the matrix elements of four-quark operators $\mathcal{O}_i$ $(i = 1, \ldots, 6)$ which usually are absorbed into a redefinition of the short-distance Wilson-coefficients. The Wilson coefficient $C_9^{\rm eff}$ effectively takes into account, first, the contributions from the four-quark operators $\mathcal{O}_i$ $(i = 1, ..., 6)$ and, second, the nonperturbative effects coming from the $c\bar{c}$-resonance contributions which are as usual parametrized by the Breit-Wigner ansatz [56]:

$$
\begin{aligned}
C_9^{\rm eff} &= C_9 + C_0 \left\{ h(\hat{m}_c, s) + \frac{3\pi}{\alpha^2}\,\kappa \sum_{V_i=\psi(1s),\psi(2s)} \frac{\Gamma(V_i \to l^+l^-)\, m_{V_i}}{{m_{V_i}}^2 - q^2 - i m_{V_i}\Gamma_{V_i}} \right\} \\
&- \frac{1}{2} h(1,s)\,(4C_3 + 4C_4 + 3C_5 + C_6) \\
&- \frac{1}{2} h(0,s)\,(C_3 + 3C_4) + \frac{2}{9}\,(3C_3 + C_4 + 3C_5 + C_6)\ , \qquad (9)
\end{aligned}
$$

where $C_0 \equiv 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$. Here the charm-loop function is written as

$$
\begin{aligned}
h(\hat{m}_c, s) &= -\frac{8}{9}\ln\frac{\bar{m}_b}{\mu} - \frac{8}{9}\ln \hat{m}_c + \frac{8}{27} + \frac{4}{9}x \\
&- \frac{2}{9}(2+x)|1-x|^{1/2} \begin{cases} \left(\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi\right), & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} < 1, \\ 2\arctan\frac{1}{\sqrt{x-1}}, & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} > 1, \end{cases} \\
h(0,s) &= \frac{8}{27} - \frac{8}{9}\ln\frac{\bar{m}_b}{\mu} - \frac{4}{9}\ln s + \frac{4}{9}i\pi,
\end{aligned}
$$

where $\hat{m}_c = \bar{m}_c/m_1$, $s = q^2/m_1^2$ and $\kappa = 1/C_0$. In what follows we drop the charm resonance contributions by putting $\kappa = 0$. We will use the value of $\mu = \bar{m}_{b\,\rm pole}$ for the renormalization scale. Besides the charm-loop perturbative contribution, two loop contributions have been calculated in [57, 58]. They effectively modify the Wilson coefficients as

$$
\begin{aligned}
C_7^{\rm eff} &\to C_7^{\rm eff} - \frac{\alpha_S}{4\pi}\left(C_1 F_1^{(7)} + C_2 F_2^{(7)}\right), \\
C_9^{\rm eff} &\to C_9^{\rm eff} - \frac{\alpha_S}{4\pi}\left(C_1 F_1^{(9)} + C_2 F_2^{(9)}\right) \qquad (10)
\end{aligned}
$$

where the two-loop form factors $F_{1,2}^{(7,9)}$ are available in Ref. [58]. A global analysis of $b \to s\ell\ell$ anomalies has been performed in Ref. [4] with the NNLL corrections included. It was shown that they amount up to 15%. The discussion of the non-local $c\bar{c}$ contributions maybe also found in Ref. [11].

We are aiming to compare our results for the branching fractions and angular observables with the experimental data recently reported by the LHCb Collaboration [3] and the results of global analyses performed in Ref. [4]. A set of so-called optimized observables P_i have been constructed (see [18] and references therein) by taking appropriate ratios of the form factors in such a way to minimize the hadronic uncertainties. It seems however more difficult to give them a clean physical interpretation, as it was the case for A_{FB} and F_L.

The optimized observables have not been given explicitly in [3]. Their numerical values were obtained in [4] by converting the results for the CP averages $S_{3,4,7}$ into the optimized observables.

Finally, we present our results for the binned observables in Tables 4-7. Here, we take into account the NNLL corrections for the Wilson coefficients which have been calculated in [57, 58]. The NNLL corrections contribute up to 20% in the region of small transferred momentum squared $q^2 \leq 6$ GeV^2 but their influence in the region of large q^2 is really negligible. Using this optics one can address the 3.3σ deviation seen by [3] for branching fraction in the $1-6$ GeV range. In the covariant quark model this discrepancy is much reduced. The remaining deviation (1.4σ) shrinks is even further if the two-loop corrections for the Wilson coefficients are taken into account, down to 1.1σ. With such error reduction one cannot claim a discrepancy with the SM any longer.

Overall one observes a good description of the data by the covariant quark model and the agreement becomes even better if the two-loop corrections are taken into account. The biggest discrepancy of 2.5σ observed for F_L in the lowest bin $0.1 \leq q^2 \leq 2$ GeV is reduced to 1.7σ when these corrections are taken into account.

The remaining deviations do not exceed 2.0σ and only two of them are greater than 1.5σ if the two loops corrections are neglected (branching fraction and S_4, both for $15 \leq q^2 \leq 17$ GeV). When they are taken into account most measurements lie within one standard deviation, the only two exceeding 1.5σ are S_4 for $15 \leq q^2 \leq 17$ and the above mentioned measurement of F_L.

Table 4: Binned observables for $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$.

$10^7\mathcal{B}(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	[4]	Expt. [3]
$[0.1, 2]$	0.99 ± 0.2	0.86 ± 0.17	1.81 ± 0.36	1.11 ± 0.16
$[2, 5]$	0.90 ± 0.18	0.95 ± 0.19	1.88 ± 0.31	0.77 ± 0.14
$[5, 8]$	$--$	1.25 ± 0.25	2.25 ± 0.41	0.96 ± 0.15
$[11, 12.5]$	0.84 ± 0.17	0.88 ± 0.18	$--$	0.71 ± 0.12
$[15, 17]$	1.15 ± 0.23	1.19 ± 0.24	$--$	0.90 ± 0.13
$[17, 19]$	0.75 ± 0.15	0.77 ± 0.15	$--$	0.75 ± 0.13
$[1., 6.]$	1.56 ± 0.31	1.64 ± 0.33	$--$	1.29 ± 0.19
$[15, 19]$	1.89 ± 0.28	1.95 ± 0.29	2.20 ± 0.16	1.62 ± 0.20

Table 5: Binned observables for $F_L(B_s \to \phi\mu^+\mu^-)$.

$F_L(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	[4]	Expt. [3]
$[0.1, 2]$	0.37 ± 0.07	0.46 ± 0.09	0.46 ± 0.09	0.20 ± 0.09
$[2, 5]$	0.72 ± 0.14	0.74 ± 0.15	0.79 ± 0.03	0.68 ± 0.15
$[5, 8]$	$--$	0.57 ± 0.11	0.65 ± 0.05	0.54 ± 0.10
$[11, 12.5]$	0.40 ± 0.08	0.40 ± 0.08	$--$	0.29 ± 0.11
$[15, 17]$	0.34 ± 0.07	0.34 ± 0.07	$--$	0.23 ± 0.09
$[17, 19]$	0.33 ± 0.06	0.33 ± 0.06	$--$	0.4 ± 0.14
$[1, 6]$	0.69 ± 0.14	0.71 ± 0.14	$--$	0.63 ± 0.09
$[15, 19]$	0.34 ± 0.07	0.34 ± 0.07	0.36 ± 0.02	0.29 ± 0.07

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Table 6: Binned observables for $S_3(B_s \to \phi\mu^+\mu^-)$.

$S_3(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	[4]	Expt. [3]
$[0.1, 2]$	0.0031 ± 0.0006	0.0023 ± 0.0005	0.02 ± 0.02	-0.05 ± 0.13
$[2, 5]$	-0.035 ± 0.007	-0.039 ± 0.008	-0.01 ± 0.01	-0.06 ± 0.21
$[5, 8]$	$--$	-0.082 ± 0.016	-0.03 ± 0.02	-0.10 ± 0.25
$[11, 12.5]$	-0.15 ± 0.03	-0.15 ± 0.03	$--$	-0.19 ± 0.21
$[15, 17]$	-0.23 ± 0.05	-0.23 ± 0.05	$--$	-0.06 ± 0.18
$[17, 19]$	-0.29 ± 0.06	-0.29 ± 0.06	$--$	-0.07 ± 0.25
$[1, 6]$	-0.034 ± 0.007	-0.039 ± 0.008	$--$	-0.02 ± 0.13
$[15, 19]$	-0.25 ± 0.05	-0.25 ± 0.05	-0.22 ± 0.01	-0.09 ± 0.12

Table 7: Binned observables for $S_4(B_s \to \phi\mu^+\mu^-)$.

$S_4(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	[4]	Expt. [3]
$[0.1, 2]$	-0.038 ± 0.008	-0.031 ± 0.006	-0.06 ± 0.03	-0.27 ± 0.23
$[2, 5]$	0.19 ± 0.04	0.21 ± 0.04	0.16 ± 0.03	0.47 ± 0.37
$[5, 8]$	$--$	0.28 ± 0.06	0.25 ± 0.02	0.10 ± 0.17
$[11, 12.5]$	0.30 ± 0.06	0.30 ± 0.06	$--$	0.47 ± 0.25
$[15, 17]$	0.31 ± 0.06	0.31 ± 0.06	$--$	0.03 ± 0.15
$[17, 19]$	0.32 ± 0.06	0.32 ± 0.06	$--$	0.39 ± 0.3
$[1, 6]$	0.17 ± 0.03	0.19 ± 0.04	$--$	0.19 ± 0.14
$[15, 19]$	0.31 ± 0.06	0.31 ± 0.06	0.31 ± 0.00	0.14 ± 0.11

Table 8: Binned observables for $S_7(B_s \to \phi\mu^+\mu^-)$.

$S_7(B_s \to \phi\mu^+\mu^-)$	2 loop	1 loop	[4]	Expt. [3]
$[0.1, 2]$	0.0065 ± 0.0013	0	0.03 ± 0.01	0.04 ± 0.12
$[2, 5]$	0.0065 ± 0.0013	0	0.02 ± 0.01	-0.03 ± 0.21
$[5, 8]$	$--$	0	0.01 ± 0.00	0.04 ± 0.18
$[11, 12.5]$	0.0021 ± 0.0004	0	$--$	0.00 ± 0.16
$[15, 17]$	0.00087 ± 0.0002	0	$--$	0.12 ± 0.15
$[17, 19]$	0.00034 ± 0.00007	0	$--$	0.20 ± 0.26
$[1, 6]$	0.0065 ± 0.0013	0	$--$	-0.03 ± 0.14
$[15, 19]$	0.00066 ± 0.00013	0	0.00 ± 0.03	0.13 ± 0.11

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