II. 2 HADRON PRODUCTION IN e^+e^- COLLISIONS—ONE- AND TWO-PHOTON PROCESSES*

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Hadron production in e^+e^- collisions plays a fundamental role in particle physics since one studies the synthesis of hadronic matter from pure electromagnetic energy. In fact, as has been emphasized recently, states of both even and odd charge conjugation are involved, since higher order electromagnetic production processes are important in the energy range of the Frascati, SLAC, CEA, and DESY storage rings. In particular, the two-photon processes of Fig. 1(d) and 1(e) lead to logarithmically increasing cross sections, which, although order α^4, eventually dominate a decreasing e^+e^- annihilation cross section at high energy. In this short talk, I will briefly review the various components of hadron production in e^+e^- collisions and report on some of the relevant contributed papers to this conference, including two contributions which evaluate the possibility of detecting neutral weak currents in purely leptonic amplitudes.

One-Photon Annihilation: $e^+e^- \to \gamma^* \to$ hadrons (C = -)

The cross section of most current and critical interest is of course the order α^2 one-photon annihilation cross section [Fig. 1(a)]. The exciting implication for hadronic physics if the annihilation cross section scales (i.e., $s\sigma(e^+e^- \to \text{hadrons}) \to C$ at large s), the significance of the value of C for the light-cone algebra and constituent models of the hadrons, and the question of the continuation of scaling laws from deep inelastic electron-proton scattering are discussed in Professor Drell's review.[1] Predictions for specific exclusive channels are of interest and have been the focus of an abundant literature.[2] In a contribution to this conference, G. Kramer and T. Walsh[3] develop a systematic treatment of quasi-two-body production, using a helicity amplitude formalism, and they catalog the implications of polarization-correlation measurements. They also present a vector dominance model for various two-body resonance cross sections: cross sections near threshold in the range $1 < \sigma < 100$ nb for processes like

$$e^+e^- \to \pi^0\omega,\ \pi A_2,\ \pi A_1,\ \epsilon\rho,\ \rho^+\rho^-$$

are possible depending on sizes of the various form factors. Other estimates and discussions of quasi-two-body processes are given in Ref. 2.

Two-Photon Annihilation: $e^+e^- \to \gamma^* + \gamma^* \to$ hadrons (C = +)

There has been very little discussion of this interesting amplitude [Fig. 1(b)] in the literature.[4] Constituent (parton) models predict the cross section to fall as s^{-1} or $\log s/s$; i.e., the same as the corresponding lepton-production cross sections. The calculation of the C = + hadronic amplitudes for specific channels involves the same uncertainties as the Cottingham formula for mass differences, the nucleon polarization correction to the hyperfine splitting of hydrogen, and the two-photon correction to electron-proton elastic scattering. The interference of this amplitude with the one-photon annihilation amplitude causes an asymmetry of charged hadron production relative to the incident electron direction and contributes a cross section of order α^3.

Hadronic Photon Emission: $e^+e^- \to \gamma^* \to$ hadrons (C = +) + γ

This interesting process, which permits the study of two photon (one real plus one time-like) couplings of various C = + hadronic systems, has been discussed in considerable detail by Einhorn and Creutz.[5] Unlike the previously discussed processes, the hadronic system is not produced at rest in the e^+e^- CM system.

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The cross section is of order α^3, decreases with s, and leads to asymmetries with the incident electron direction due to the interference with the amplitude in which the real photon is emitted from an incident lepton line.

Hard Photon Emission: $e^+e^- \to \gamma + \gamma^* \to \gamma$ + hadrons (C = -)

This ordinary radiative correction process—where the incident lepton bremstrahlungs a photon of large momentum [Fig. 1(c)]—can play an insidious role in multiparticle hadron production experiments in which the energy of the final hadrons is not determined. The bremsstrahlung allows the production process to occur at a much lower energy—at the expense of one power of α. In fact, if the actual annihilation cross section has asymptotic behavior $\sigma(s) \to \alpha^2 S^{-N}$, then the order α^3 radiatively-corrected cross section only falls as s^{-1}, for any N > 1. If N = 1, then

$$\sigma(e^+e^- \to \gamma + \gamma^* \to \gamma + \text{hadrons}) \sim \frac{\alpha^3}{s} \log s/s_{min}$$

where s_{min} is a constant determined by the threshold dependence of σ_0. The total momentum of the produced hadronic system ("Fireball") moves dominantly along the beam direction, leading to a forward-backward cross section. Discussion of the effects of this type of radiative correction, especially for ρ^0 production, is given in A. Litke's thesis.[6] Other calculations of radiative corrections are given in Ref. 7. Clearly, much more work is needed in this important area.

The Double Fireball Process: $e^+e^- \to \gamma^* + \gamma^* \to$ hadrons (C = -1) + hadrons (C = -1)

In an interesting contribution to this Conference, Cheng and Wu[8] emphasize that the "double fireball" process see Fig. 1(d) could be a dominant cross section for specific hadronic channels at high energies at fixed momentum transfer (small production angles). The form of the cross section (which follows from $j = \frac{1}{2}$ lepton exchange in the t channel) for the production of two fireballs, each of mass of order m, is

$$\frac{d\sigma}{d\Omega_{cm}} \sim \frac{\alpha^4 |t|}{\left(|t| + m^2\right)^2} \quad , \quad \sigma \sim \frac{\alpha^4}{s}$$

where $|t| \sim s\theta^2$ is the momentum transfer. The sharp peaking at $\theta \sim 0$ (zero production angle relative to the beam direction) is of the same origin as that of the single fireball radiative correction. Using the above form, one sees that the two-fireball process becomes comparable with an isotropic cross section of order α^2/s if $\sqrt{s} > m/\alpha$, $\theta^2 \sim m^2/s^2 \sim \alpha$; i.e., very small production angles. Thus the total one-photon annihilation cross section will generally overwhelm the two-fireball cross section at present energies and normal angles. On the other hand, for the case of a specific channel suppressed in one-photon annihilation, especially forward-backward $\rho^0 + \rho^0$, the two-fireball process can be the dominant production mechanism at high energies and fixed momentum transfer ($\theta \to 0$). Cheng and Wu[8] also demonstrate that final state interactions are negligible between the two fireballs at high s.

The Two-Photon Process: $ee \to ee\gamma^*\gamma^* \to ee$ hadrons (C = +)
$ee \to ee\gamma \to ee$ hadrons (C = -)

In contrast to the e^+e^- annihilation cross sections, the cross section for the two-photon process[9-16] (in which the leptons survive in the final state—see Fig. 1(e) and 1(f)) is logarithmically increasing. For $s \gg s_{min} \gg 4m_e^2$, the cross sections are of order

$$\sigma \sim \frac{\alpha^4}{s_{min}} \log^2 \frac{s}{m_e^2} \log \frac{s}{s_{min}} \qquad (C = +)$$

$$\sim \frac{\alpha^4}{s_{min}} \log \frac{s}{m_e^2} \log \frac{s}{s_{min}} \qquad (C = -)$$

In fact, the total cross section for $e^+e^- \to e^+e^-\mu^+\mu^-$ is larger than the annihilation cross section $e^-e^+ \to \mu^+\mu^-$

for $E > 1$ GeV ($s > 4$ GeV^2). The incident leptons are scattered predominantly in the forward direction, with approximately half of the leptons falling within a forward cone of opening angle $\sqrt{m_e/E}$. A simple practical estimate for the C = + cross section is obtained from the equivalent photon approximation

$$\sigma_{ee\to eeX} = \int_0^E \frac{d\omega_1}{\omega_1} N(\omega_1) \int_0^E \frac{d\omega_2}{\omega_2} N(\omega_2)\, \sigma_{\gamma\gamma\to X}(s = 4\omega_1\omega_2)$$

corresponding to the annihilation of two virtual bremsstrahlung beams each along their respective incident lepton direction. $\sigma_{\gamma\gamma\to X}$ is the cross section for real photon annihilation at $s = 4\omega_1\omega_2$.

The simple Weisacker-Williams form

$$N_{WW}(\omega) = \frac{2\alpha}{\pi}\left[\frac{E^2 + (E-\omega)^2}{2E^2}\right] \log E/m_e$$

does give the correct total cross section behavior for $m_e/E \to 0$, but often is inaccurate to ± 30% in applications. However, forms for $N(\omega)$ have been presented[12,14] which are, in fact, very accurate for most applications and which, in fact, reduce to the exact answer if the leptons in the final state are confined to a small forward angle ($\theta_e^{max} \ll 1$).

The two-photon process is, of course, separable from the various annihilation processes by tagging (either one or both of the final leptons), or, most dramatically, by the use of e^-e^- collisions, as will be possible at DESY. We thus have the exciting potential to measure the (crossed Compton) processes $\gamma + \gamma \to$ hadrons for both real and virtual (spacelike) photons. Over the past two years, many comprehensive calculations of various two-photon processes have been exactly calculated and extensive work has been done on development reliable approximation methods.[12-18] A detailed review and further references are given in my Cornell talk.[16] Some typical two-photon cross sections taken from the work of Ref. 14 are shown in Fig. 2 and 3.

In a detailed contributed paper to this Conference, C. J. Brown and D. M. Lyth[18] give general formulas for ee → ee hadrons based on a helicity amplitude representation of the virtual $\gamma + \gamma \to$ hadrons cross section. They also derive an equivalent photon approximation for N(E) identical to that of Ref. 14 . Their numerical checks on the equivalent photon approximation are particularly instructive: for $ee \to ee\pi^+\pi^-$ (Born amplitudes) at E = 2 GeV, the approximate form is essentially indistinguishable from the exact calculation if both leptons are detected within $\theta^e_{max} = 0.1$ (which gives, in fact, 90% of the total cross section); there is a +5% error at $\theta^e_{max} = 0.35$ (where $|q^2| \lesssim 0.1$ GeV^2), and a +10% error in the case of the total cross section. The equivalent photon approximation used in these works (closely related to the work of Dalitz and Yennie) is much more accurate than the simple "Weisacker-Williams" approximation which retains only the leading $\log s/m_e^2$ factor and gives errors of 30% or more. Brown and Lyth[18] also verify that the kinematic approximation of ignoring the correlation angle ϕ_e between the lepton-scattering planes in these calculations is generally justified. On the other hand, measurements of the lepton coplanarity angle can provide parity information on the production of the hadron state. In some kinematical situations, e.g., noncoplanar pion pairs, the equivalent photon approximation is useless, and exact calculations are required.

One of the simplest two-photon processes, but probably of the most critical current interest, is the process $\gamma + \gamma \to \pi^+ + \pi^-$ for (almost) real photons. Measurements of this process provide a determination of $\pi\pi$ phase shifts (via a Watson theorem in the elastic region) exclusive of hadronic-target complications, as well as a check of the low-energy and soft-pion theorems for the crossed ($s \leftrightarrow t$) Compton amplitude. More specifically, the $\sigma(\epsilon)$ and all C = +, ℓ-even, positive-parity resonances are accessible from γ-γ annihilation. The magnitude of the resonance couplings to two photons is very much model dependent. In a contribution to this Conference, Lyth[19] argues from dispersion theory and specific assumptions on the asymptotic behavior of the $\gamma\gamma \to \pi\pi$ amplitude, that the σ coupling will be weak, and Born approximation should be reliable at small $\pi\pi$ invariant mass. Chanowitz and Ellis[20] reach a similar conclusion on the basis of assumptions on the dimensionality of current and a low-energy theorem. On the other hand, Goble and Rosner[21] (using current algebra arguments), Brodsky, Kinoshita, and Terazawa[14] and Sarker[22] have given models in which the σ plays a dominant role in the $J = 0^+ \gamma\gamma \longrightarrow \pi\pi$ channel. Other predictions have been given by F. Yndurian[23], B. Schremp-Otto et al.,[24] G. Scheirholz and K. Sundermeyer.[25] The analytic procedure for extracting the $\gamma\gamma \longrightarrow \pi\pi$ amplitudes from the $ee \longrightarrow ee\pi\pi$ measurements is discussed by C. Carlson and Wu-Ki Tung.[26] Estimates for other hadronic channels are reviewed in my Cornell talk.[16]

There is also the potential for measuring extremely interesting virtual photon-photon annihilation cross sections. These include processes such as (a) $\gamma^* + \gamma \longrightarrow X$, "deep-inelastic scattering on a photon target" (where one lepton is detected forward, the other at large angles) discussed by Brodsky, Kinoshita, and Terazawa,[27] Walsh,[28] and Fujikawa[29] (electromagnetic contributions); (b) $\gamma^* + \gamma^* \longrightarrow \pi^+\pi^-$ – the connection to the pion mass difference discussed by T. M. Yan[30]; (c) $\gamma^* + \gamma^* \longrightarrow X$ (scaling, Regge limits) discussed by Kunszt and Ter-Antonyan[31]; Walsh and Zerwas,[32] Kingsley,[33] Carlson,[34] and Terazawa.[35] I would also like to emphasize the importance of measurements of $\gamma^*(q^2) + \gamma \longrightarrow \pi^+\pi^-$. As shown by Close, Gunion, and myself,[36] the local behavior of the electromagnetic current evident in the SLAC-MIT measurements of scaling implies the existence of a component of this amplitude which is independent of q^2 at fixed $\pi\pi$ invariant mass. Similarly, the consequences of the Adler-Bell anomaly contribution in the virtual $\gamma\gamma \longrightarrow \pi^0$ amplitude can be studied.

The two-photon processes are clearly of great theoretical interest and will broadly extend the physics capabilities of the high-luminosity storage rings. The first results on hadronic production from this channel are eagerly awaited.

Weak Interactions

The possible detection of neutral weak currents by electron-positron annihilation is discussed in contributions to this Conference by V. K. Cung, A. H. Mann, and E. A. Paschos,[37] and by G. V. Grigoryan and V. A. Khoze.[38] The asymmetry of the spin-averaged cross section for $e^+e^- \longrightarrow \mu^-\mu^+$ due to the interference of a postulated W^0 (Z^0) annihilation amplitude with the one-photon annihilation amplitude is given by Cung et al. as

$$A = \frac{d\sigma(\theta,\phi) - d\sigma(\pi-\theta,\phi)}{d\sigma(\theta,\phi) + d\sigma(\pi-\theta,\phi)} = \frac{\sqrt{2}\, G_0 M_W^2}{4\pi\alpha} \; \frac{S}{S - M_W^2} \; \frac{2\cos\theta}{2 - \sin^2\theta\left(1 + |P_+||P_-|\cos 2\phi\right)}$$

(The muon charge must be detected.) The asymmetry is enhanced if the circulating e^+ and e^- beams are transversely polarized, which is expected to happen theoretically $\left(\vec{P}_+ = -\vec{P}_-, |P_\pm| < 0.927\right)$ due to the effects of synchrotron radiation.[39] Taking the maximum polarization, and $G_0 = G_F$, one obtains an asymmetry A ($s = 64\ \text{GeV}^2$, $\phi = 0$, $\theta = 65°$) = -2.0%. Already, however, neutrino and meson decay measurements indicate limits on the neutral current below $G_0 = G_F$. (See the reviews of B. W. Lee and D. H. Perkins, this Conference.) Thus, measurements of weak-electromagnetic interference will inevitably require very high energy and high-luminosity colliding-beam facilities.

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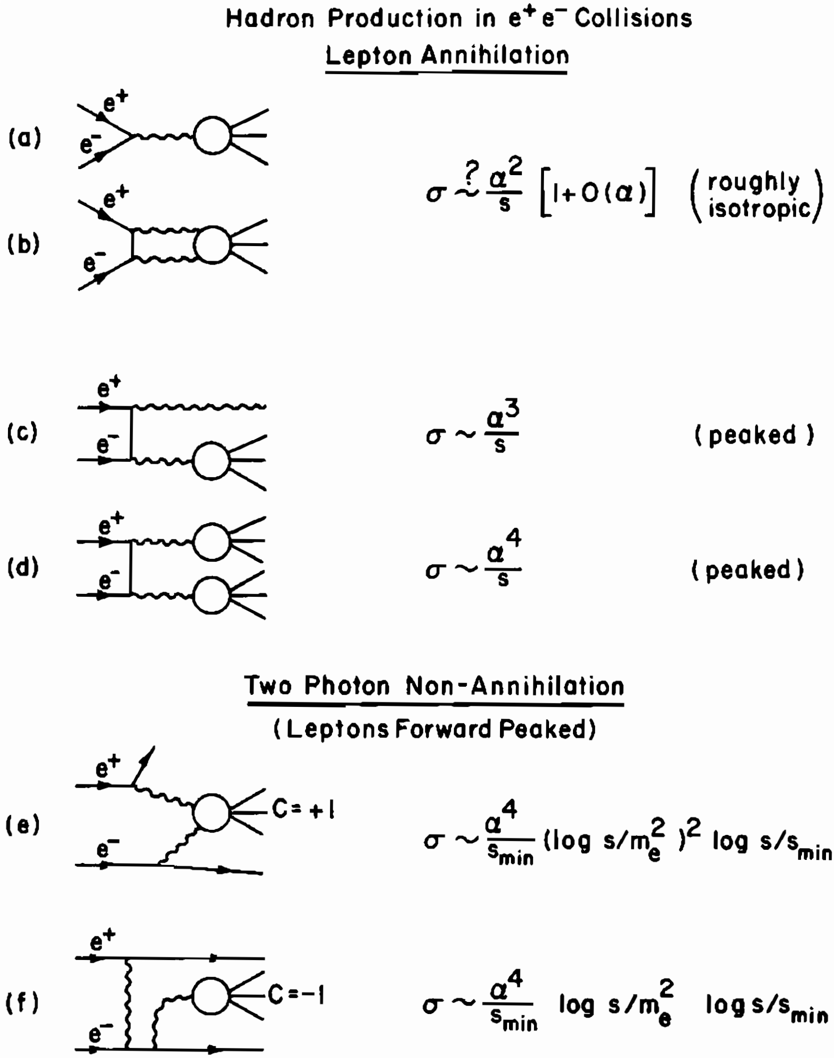


Fig. 1. The various components to hadron production in e^+e^- collisions.

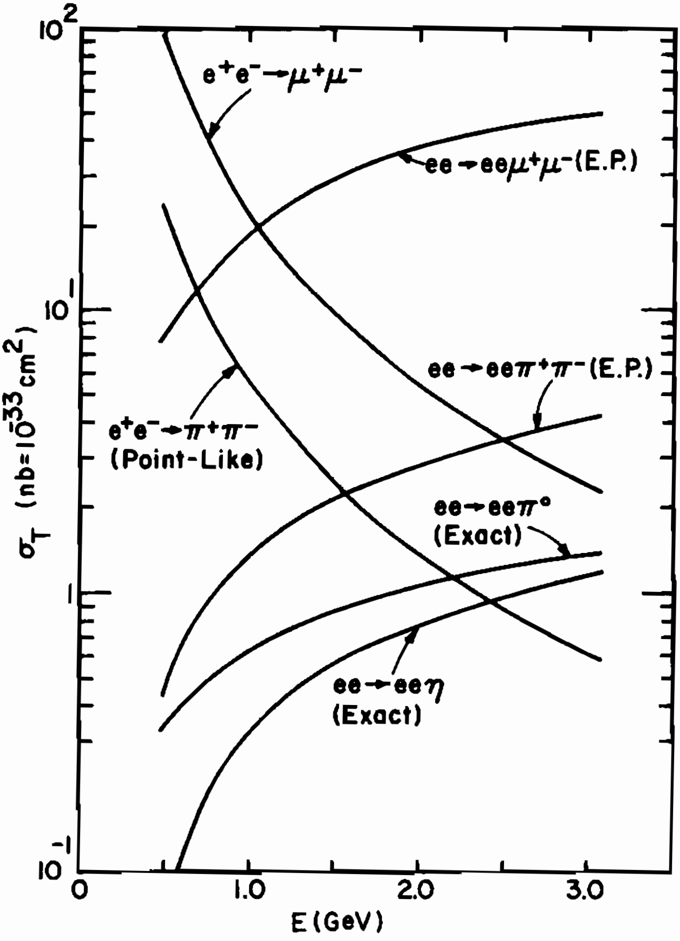


Fig. 2. The total cross sections for ee → ee hadrons (C = +). Here $E = \sqrt{s}/2$ is the colliding-beam energy. The cross sections for π^0 and η are exact and calculated without form factors. The cross sections for $\pi^+\pi^-$ and $\mu^+\mu^-$ are calculated in the equivalent photon approximation. From Ref. 14.

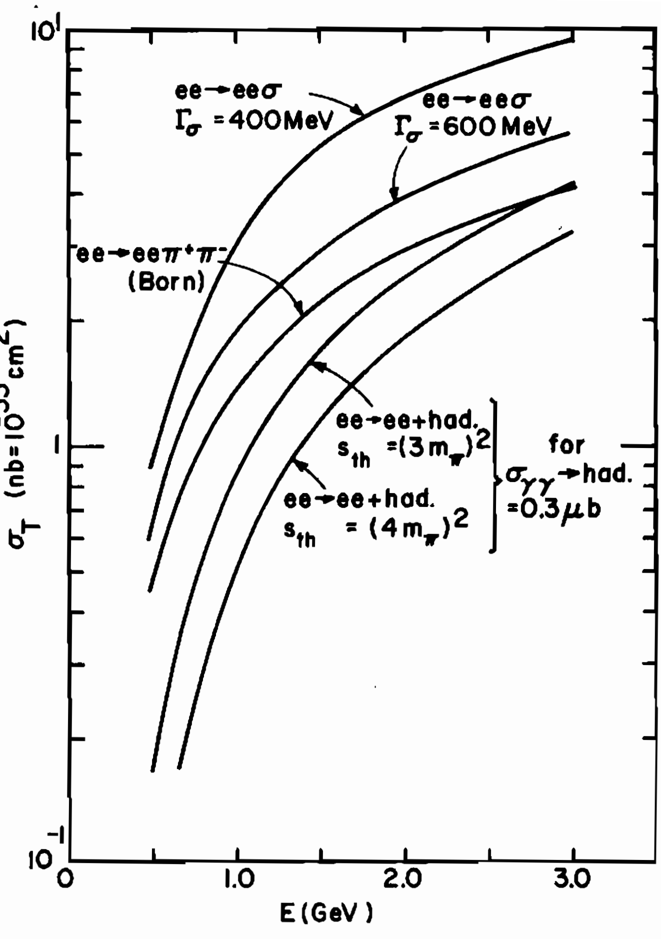
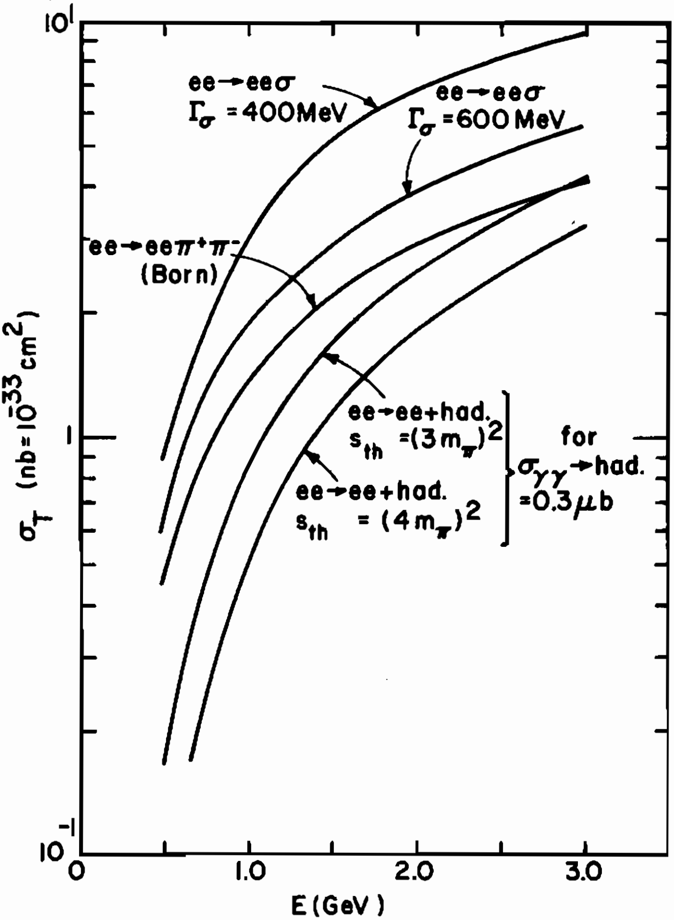


Fig. 3. Representative estimated cross sections for hadron production via the two-photon process. $E = \sqrt{s}/2$. See Refs. 14 and 16 for discussion.



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