Research Article

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Inflation and the principle of equivalence

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Abstract: A formal, mathematical statement of the principle of equivalence in general relativity is that one must always be able to find – at each location within a curved spacetime – the *local* free-falling frame against which one can measure the acceleration-induced time dilation and degree of curvature relative to flat spacetime. In this article, we use this theorem to prove that a de Sitter expansion, required during cosmic inflation, does not satisfy this condition and is therefore inconsistent with the PoE. To emphasize the importance – and reality – of this outcome, we contrast it with the analogous derivation for the Schwarzschild metric, which instead satisfies this requirement completely. We point out that this failure by de Sitter results from its incorrect handling of the Friedmann–Lemaître–Robertson–Walker (FLRW) lapse function, g_{tt}. Our conclusion calls into question whether a period of inflated expansion could have even been possible in the context of FLRW cosmologies, and is noteworthy in light of recent, high-precision measurements showing that inflation could not have solved the temperature horizon problem while simultaneously producing the observed primordial power spectrum.

Keywords: classical theories of gravity, cosmological theory, early universe, inflation, dark energy

1 Introduction

Inflation has become an indispensable component of the standard model [1,2] (see also previous studies [3,4]). Its implied accelerated expansion in the early Universe may have solved several inconsistencies plaguing big bang cosmology and, perhaps more importantly, may have also seeded the primordial fluctuation spectrum [5] that eventually led to the formation of large-scale structure.

In spite of the considerable effort expended in finding its underlying field and potential, however, it still lacks true predictive power. In addition, very little attention has been paid to its fundamental basis within the context of general relativity (GR), the theory that spawned it via one of the most influential solutions to Einstein's equations.

In this article, we begin to address this deficiency by examining the consistency of such an accelerated expansion with the foundational ingredient in Einstein's theory, *i.e.*, the principle of equivalence (PoE), which requires that there exist at every spacetime point a local free-falling (inertial) frame relative to which one may measure the acceleration in the comoving frame. We shall demonstrate that the adoption of the Friedmann–Lemaître–Robertson–Walker (FLRW) metric – with its constrained lapse function $g_{tt} = 1$ – to describe the inflationary spacetime is actually inconsistent with the requirements of the PoE. To emphasize de Sitter's failure in this regard, we contrast it with the analogous derivation for the Schwarzschild metric, which instead completely satisfies the PoE self-consistently. We end with a brief discussion of the consequences of this conclusion.

2 PoE

We define $x^\mu = (ct, x^1, x^2, x^3)$ to be the coordinates in the comoving frame, $g_{\mu\nu}$ the metric coefficients, and $\Gamma^\lambda{}_{\mu\nu}$ the corresponding Christoffel symbols. Most of the observations today appear to be telling us that the Universe is spatially flat [6], so we assume the spatial curvature constant, k, to be zero in the FLRW metric, which may therefore simply be written as

$$ds^2 = c^2 dt^2 - a(t)^2[(dx^1)^2 + (dx^2)^2 + (dx^3)^2], \qquad (1)$$

in terms of the expansion factor $a(t)$. In the following, we shall adopt its de Sitter form relevant during inflation, *i.e.*, with the inclusion of

$$a(t) = e^{Ht}, \qquad (2)$$

where the Hubble parameter H is constant. The metric coefficients are thus

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{pmatrix}, \qquad (3)$$

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and one can easily show from this that the non-zero Christoffel symbols in the comoving frame must be

$$\Gamma^0_{ii} = \frac{1}{c} a\dot{a} = \frac{H}{c} e^{2Ht},$$
$$\Gamma^i_{i0} = \Gamma^i_{0i} = \frac{1}{c}\frac{\dot{a}}{a} = \frac{H}{c}. \qquad (4)$$

A formal, mathematical expression of the PoE holds that there must exist at every spacetime point x_0^μ in x^μ a local free-falling (inertial) frame $\xi^\mu(x)$ that one may use to "measure" the spacetime curvature in x^μ [7]. For convenience and without loss of generality, we shall adopt the cosmological principle, in which the Universe is homogeneous and isotropic (at least on scales larger than ~300 Mpc today), and arrange the coordinates x^μ and ξ^μ to have a common origin.

In order for ξ^μ to fulfill the role played by the local, free-falling frame, they must satisfy the equations (see Eq. 3.2.11 in Weinberg's book [7])

$$\frac{\partial^2 \xi^\alpha}{\partial x^\mu \partial x^\nu} = \Gamma^\lambda{}_{\mu\nu} \frac{\partial \xi^\alpha}{\partial x^\lambda}. \qquad (5)$$

To identify these coordinates, we shall follow a procedure analogous to the transformation used to cast the Schwarzschild metric into its so-called Cartesian isotropic form [8], which we shall revisit shortly in the following section.

Of course, while velocity is "relative" in GR, acceleration is "absolute," meaning that if one can measure an acceleration with respect to one inertial frame, the same acceleration would emerge relative to any other inertial frame. So the point of finding the coordinates ξ^α is merely to identify at least one such inertial frame to test whether the metric (in this case Eq. (1)) is consistent with Eq. (5). The goal is thus to find a transformation from the frame represented by the coordinates x^μ into a local frame falling freely, whose metric has a Minkowski form (represented by the ξ^α coordinates), as we shall see shortly in Eq. (20). This procedure is facilitated by the recognition that de Sitter space is one of the six (and exactly six) applications of the FLRW metric corresponding to a constant spacetime curvature, *i.e.*, a spacetime whose metric coefficients may be written independently of time [9,10].

The fact that de Sitter space has constant spacetime curvature is not obvious from Eq. (1), but we can demonstrate this using a transformation (based on fixed observer coordinates) that renders its metric coefficients independent of time [11]. To see this, we define

$$cT = ct - \frac{1}{2} R_\mathrm{h} \ln \Phi,$$
$$R = a(t) r, \qquad (6)$$

where $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$ and the gravitational (or Hubble) radius is defined as

$$R_\mathrm{h} \equiv \frac{c}{H}. \qquad (7)$$

In addition,

$$\Phi \equiv 1 - \left(\frac{R}{R_\mathrm{h}}\right)^2. \qquad (8)$$

With this transformation, the de Sitter metric may be written

$$ds^2 = \Phi c^2 dT^2 - \Phi^{-1} dR^2 - R^2 d\Omega^2, \qquad (9)$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$. As we can easily see, all the metric coefficients in this equation are independent of the new time coordinate T and, for future reference, we define the Cartesian coordinates (X^1, X^2, X^3), such that

$$X^1 = R \sin\theta \cos\phi,$$
$$X^2 = R \sin\theta \sin\phi, \qquad (10)$$
$$X^3 = R \cos\theta.$$

Eq. (9), however, is not yet in Minkowki form, but we can now easily find a transformation from the coordinates (cT, R, θ, ϕ) into a free-falling frame. We do this by following the approach of finding the metric's Cartesian isotropic form, in which we put

$$R = \sigma \left[1 + \left(\frac{\sigma}{\sigma_\mathrm{h}}\right)^2\right]^{-1}, \qquad (11)$$

where

$$\sigma_\mathrm{h} \equiv 2R_\mathrm{h}. \qquad (12)$$

We thus have

$$dR = d\sigma \frac{P}{Q^2} \qquad (13)$$

and

$$\Phi = \frac{P^2}{Q^2}, \qquad (14)$$

in terms of the quantities

$$P \equiv \left[1 - \left(\frac{\sigma}{\sigma_\mathrm{h}}\right)^2\right],$$
$$Q \equiv \left[1 + \left(\frac{\sigma}{\sigma_\mathrm{h}}\right)^2\right]. \qquad (15)$$

The newly transformed de Sitter interval may thus be written

$$ds^2 = \left(\frac{P}{Q}\right)^2 c^2 dT^2 - \frac{1}{Q^2}(d\sigma^2 + \sigma^2 d\Omega^2) \qquad (16)$$

and, if we define the Cartesian coordinates corresponding to σ as

$$\begin{aligned} \chi^1 &= \sigma \sin\theta \cos\phi, \\ \chi^2 &= \sigma \sin\theta \sin\phi, \\ \chi^3 &= \sigma \cos\theta, \end{aligned} \tag{17}$$

we arrive at its Cartesian isotropic form,

$$\mathrm{d}s^2 = \left(\frac{P}{Q}\right)^2 c^2 \mathrm{d}T^2 - \frac{1}{Q^2}[(\mathrm{d}\chi^1)^2 + (\mathrm{d}\chi^2)^2 + (\mathrm{d}\chi^3)^2]. \tag{18}$$

The de Sitter metric written in this way allows us to see immediately what the local free-falling frame written in Cartesian coordinates looks like.

As we now apply the requirements of the PoE to the metric in Eq. (18), we invoke the condition that the local inertial frame needs to be defined only in the vicinity of x_0^μ (i.e., the local inertial frame at one such point need not be the same as those elsewhere in the spacetime), so we may put $R \approx$ constant and $\sigma \approx$ constant wherever they appear in the metric coefficients. In other words, we may assume that $P \approx P(x_0^\mu)$ and $Q \approx Q(x_0^\mu)$ near x_0^μ, which finally allows us to identify the local inertial frame coordinates:

$$\begin{aligned} \xi^0 &= \frac{P(x_0^\mu)}{Q(x_0^\mu)} cT, \\ \xi^1 &= \frac{1}{Q(x_0^\mu)} \chi^1, \\ \xi^2 &= \frac{1}{Q(x_0^\mu)} \chi^2, \\ \xi^3 &= \frac{1}{Q(x_0^\mu)} \chi^3, \end{aligned} \tag{19}$$

i.e.,

$$\mathrm{d}s^2 = (\mathrm{d}\xi^0)^2 - (\mathrm{d}\xi^1)^2 - (\mathrm{d}\xi^2)^2 - (\mathrm{d}\xi^3)^2 \tag{20}$$

at any given spacetime point x_0^μ.

It is now a simple exercise to examine whether the coordinates x^μ and ξ^μ satisfy Eq. (5). As we shall see, they do not. For example, one finds for the $\alpha = \mu = \nu = 0$ component that

$$\frac{P}{Q}\frac{2}{R_\mathrm{h}}\left(\frac{R}{R_\mathrm{h}}\right)^2 \Phi^{-2} = 0, \tag{21}$$

but this cannot be consistent for arbitrary values of R (or r). Similarly, one finds for $\alpha = \mu = \nu = i$ that

$$0 = \frac{e^{2Ht}}{R_\mathrm{h}} \frac{Q}{P} \frac{X^i}{R_\mathrm{h}}, \tag{22}$$

which, again, is inconsistent for arbitrary values of X^i. A de Sitter expansion, such as that required during inflation, therefore does not satisfy the requirements of the PoE.

Having said this, a possible caveat to this conclusion could have been that an analysis of the cosmic microwave background (CMB) anisotropies clearly shows a fluctuation spectrum that is not exactly scale free (for which the scalar spectral index would then have been exactly 1). Its exact value is instead $n_s = 0.9649 \pm 0.0042$ [6], the slight difference implying some version of a slow-roll potential [5,12], so that the Hubble parameter H during inflation could not have remained perfectly constant. The expansion during inflation is therefore not perfectly de Sitter and would have proceeded almost according to Eq. (2), but not exactly. This slight difference, however, would have had only a minimal impact, if any, on the derivations in this section, introducing essentially only small variations in the constants P and Q in Eqs (19), (21), and (22). But these changes could not prevent the inconsistencies shown in the latter two expressions, which would still require R and/or r to be zero everywhere.

A better way to understand this is that the outcome discussed here emerges for all the FLRW metrics with an equation of state different from $\rho + 3p = 0$, not just for the case being considered here (*i.e.*, $\rho + p = 0$), as formally demonstrated in the study of Melia [13]. Such previous publications have established the problem highlighted here with a forced lapse function $g_{tt} = 1$ in a more general context, so even a variable H would not change the outcome of this discussion. The benefit of the streamlined approach we are using here is that the Christoffel symbols (Eq. 4) for a constant H are so simple that one can show the result quite robustly with just straightforward analytic derivations. In other words, the problem with the use of de Sitter in the FLRW metric is not the constant H, but the fact that inflation is necessarily an accelerated expansion, which conflicts with the zero time dilation implied by $g_{tt} = 1$. The conclusions drawn in this section do not rely on the specific choice of $\rho + p = 0$ for the equation of state, but are most easily demonstrated analytically for this case.

3 Comparison with the Schwarzschild metric

Before we discuss this crucial result, let us – for comparison and to emphasize the importance of the PoE in this analysis – carry out an analogous derivation for the Schwarzschild metric and show that the outcome in the latter case is completely different. In the next section, we shall point out that a distinguishing feature between these two spacetimes is the nature of their lapse function, g_{tt}.

The Schwarzschild metric describes the (vacuum) spacetime external to a spherically symmetric mass M and is typically written in its standard form

$$ds^2 = \left(1 - \frac{r_S}{r}\right)c^2dt^2 - \left(1 - \frac{r_S}{r}\right)^{-1}dr^2 - r^2d\Omega^2, \tag{23}$$

in terms of spherical coordinates (r, θ, ϕ) and the Schwarzschild radius $r_S \equiv 2GM/c^2$.

The most direct way of finding the local free-falling frame ξ_s^μ is to introduce a new radial coordinate, ϖ, such that

$$r = \varpi\left(1 + \frac{\varpi_S}{\varpi}\right)^2, \tag{24}$$

where $\varpi_S \equiv r_S/4$. It is not difficult to show that

$$dr = \left(1 - \frac{\varpi_S}{\varpi}\right)\left(1 + \frac{\varpi_S}{\varpi}\right)d\varpi \tag{25}$$

and

$$\left(1 - \frac{r_S}{r}\right) = \left(1 - \frac{\varpi_S}{\varpi}\right)^2\left(1 + \frac{\varpi_S}{\varpi}\right)^{-2}. \tag{26}$$

Substituting these into Eq. (23), one therefore obtains

$$ds^2 = \left(1 - \frac{\varpi_S}{\varpi}\right)^2\left(1 + \frac{\varpi_S}{\varpi}\right)^{-2}c^2dt^2 - \left(1 + \frac{\varpi_S}{\varpi}\right)^4 \times [(dy^1)^2 + (dy^2)^2 + (dy^3)^2], \tag{27}$$

where $\varpi^2 \equiv (y^1)^2 + (y^2)^2 + (y^3)^2$, *i.e.*,

$$\begin{aligned} y^1 &= \varpi \sin\theta\cos\phi, \\ y^2 &= \varpi \sin\theta\sin\phi, \\ y^3 &= \varpi \cos\theta, \end{aligned} \tag{28}$$

and

$$(dy^1)^2 + (dy^2)^2 + (dy^3)^2 = d\varpi^2 + \varpi^2 d\Omega^2. \tag{29}$$

Eq. (27) is the Schwarzschild metric written in terms of Cartesian isotropic coordinates [8].

In the vicinity of any given point $x_0^\mu = (ct, r_0, \theta_0, \phi_0)$, the r and ϖ coordinates within the metric coefficients of Eq. (27) are approximately constant. We define

$$U \equiv \left(1 - \frac{\varpi_S}{\varpi}\right)\left(1 + \frac{\varpi_S}{\varpi}\right)^{-1} \tag{30}$$

and

$$V \equiv \left(1 + \frac{\varpi_S}{\varpi}\right)^2, \tag{31}$$

which are therefore also both approximately constant at x_0^μ, and thus,

$$ds^2 \approx U^2c^2dt^2 - V^2(d\varpi^2 + \varpi^2d\Omega^2). \tag{32}$$

So we may define another set of coordinates

$$\begin{aligned} \xi_s^0 &\equiv U\ ct, \\ \xi_s^1 &\equiv V\ \varpi \sin\theta\cos\phi, \\ \xi_s^2 &\equiv V\ \varpi \sin\theta\sin\phi, \\ \xi_s^3 &\equiv V\ \varpi \cos\theta. \end{aligned} \tag{33}$$

These allow us to write the Schwarzschild interval as

$$ds^2 = (d\xi_s^0)^2 - (d\xi_s^1)^2 - (d\xi_s^2)^2 - (d\xi_s^3)^2, \tag{34}$$

the familiar Minkowski form in Cartesian coordinates, which therefore represents the metric as seen within the local free-falling frame at x_0^μ, given in terms of the coordinates $\xi_s^\mu = (\xi_s^0, \xi_s^1, \xi_s^2, \xi_s^3)$. To be clear, this frame is the local free-falling frame only at that point, x_0^μ. It will be different elsewhere. Indeed, a sequence of these frames derived at progressively smaller r would reveal the infalling trajectory toward the origin, characterized by a variation in U and V with radius consistent with the free-fall velocity at each point.

It is now straightforward to show that the coordinates $x^\mu = (ct, r, \theta, \phi)$ in Eq. (23) and ξ_s^μ in Eq. (33) satisfy Eq. (5), or the equivalent expression

$$g_{\mu\nu} = \frac{\partial\xi_s^\alpha}{\partial x^\mu}\frac{\partial\xi_s^\beta}{\partial x^\nu}\eta_{\alpha\beta}, \tag{35}$$

where $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1)$ is the corresponding metric tensor in flat spacetime. To do this, we need the Christoffel symbols corresponding to the metric in Eq. (23), whose non-zero components are simply

$$\begin{aligned} &\Gamma^r{}_{tt} = \frac{1}{2}\frac{r_S}{r^2}\left(1 - \frac{r_S}{r}\right) \quad \Gamma^r{}_{rr} = -\frac{1}{2}\frac{r_S}{r^2}\left(1 - \frac{r_S}{r}\right)^{-1}, \\ &\Gamma^t{}_{tr} = \frac{1}{2}\frac{r_S}{r^2}\left(1 - \frac{r_S}{r}\right)^{-1} = \Gamma^t{}_{rt}, \\ &\Gamma^\theta{}_{r\theta} = \frac{1}{r} = \Gamma^\theta{}_{\theta r}, \\ &\Gamma^r{}_{\theta\theta} = -r\left(1 - \frac{r_S}{r}\right), \\ &\Gamma^\phi{}_{r\phi} = \frac{1}{r} = \Gamma^\phi{}_{\phi r}, \\ &\Gamma^r{}_{\phi\phi} = -r\sin^2\theta\left(1 - \frac{r_S}{r}\right), \\ &\Gamma^\theta{}_{\phi\phi} = -\sin\theta\cos\theta, \\ &\Gamma^\phi{}_{\theta\phi} = \cot\theta = \Gamma^\phi{}_{\phi\theta}. \end{aligned} \tag{36}$$

For example, in the $\alpha = \mu = \nu = 1$ component of Eq. (5), one has

$$\frac{\partial\xi_s^1}{\partial r} \approx \left(1 + \frac{\varpi_S}{\varpi}\right)\left(1 - \frac{\varpi_S}{\varpi}\right)^{-1}\sin\theta\cos\phi, \tag{37}$$

which yields

$$\frac{\partial^2\xi_s^1}{\partial r^2} \approx -\frac{1}{2}\left(\frac{r_S}{r^2}\right)\left(1-\frac{r_S}{r}\right)^{-3/2}\sin\theta\cos\phi. \tag{38}$$

By comparison,

$$\Gamma^\lambda{}_{11}\frac{\partial\xi_s^1}{\partial x^\lambda} = -\frac{1}{2}\left(\frac{r_S}{r^2}\right)\left(1-\frac{r_S}{r}\right)^{-1}\frac{\partial\xi_s^1}{\partial r}, \tag{39}$$

which becomes

$$\Gamma^\lambda{}_{11}\frac{\partial\xi_s^1}{\partial x^\lambda} = -\frac{1}{2}\left(\frac{r_S}{r^2}\right)\left(1-\frac{r_S}{r}\right)^{-3/2}\sin\theta\cos\phi, \tag{40}$$

matching Eq. (38). Repeating this exercise for any of the other components in Eq. (5) produces a similarly matching result.

This level of consistency may also be demonstrated via Eq. (35). For example, the g_{11} metric coefficient may be calculated as follows:

$$g_{11} = \frac{\partial\xi_s^\alpha}{\partial r}\frac{\partial\xi_s^\beta}{\partial r}\eta_{\alpha\beta}, \tag{41}$$

which yields

$$g_{11} = -\left(1+\frac{\varpi_S}{\varpi}\right)^4\left(\frac{\partial\varpi}{\partial r}\right)^2. \tag{42}$$

And using Eqs (25) and (26), one finally shows that

$$g_{11} = -\left(1-\frac{r_S}{r}\right)^{-1}, \tag{43}$$

as it appears in Eq. (23).

So unlike the situation we found with de Sitter in Section 2, the Schwarzschild metric is completely consistent with the PoE, based on the fact that one can find – at any location x_0^μ within this spacetime – the local, free-falling frame ξ_s^μ, relative to which one may measure the spacetime curvature associated with the metric for the accelerated observer in Eq. (23).

4 Discussion

The obvious question is why the well-studied de Sitter solution to Einstein's equations should be inconsistent with the PoE, which is, after all, the basis for the field equations themselves. Such an issue actually falls within the broader context of whether the simplifications used to derive the FLRW metric in the first place remain valid when one subsequently chooses an equation of state for the cosmic fluid in order to uniquely calculate the expansion factor $a(t)$. As discussed in previous publications, the short answer is generally **no**, except for one particular case corresponding to the zero active mass condition in GR, *i.e.*, $\rho + 3p = 0$ [13,14]. Thus, choosing an equation of state $\rho + p = 0$, producing a de Sitter expansion, or slow-roll conditions producing an expansion very close to de Sitter, appears to be inconsistent with the free-fall requirements associated with $g_{tt} = 1$, given that both of these situations produce an accelerated Hubble frame. Ultimately, this is why we see the evident, unavoidable failure resulting in Eqs (21) and (22).

The more general examination of the validity of the FLRW metric in the context of the PoE has revealed that such contradictory results disappear when the metric coefficient g_{tt} correctly accounts for the time dilation resulting from a curved spacetime, such as one finds in the Schwarzschild case. This spacetime includes the lapse function $g_{tt} = 1 - r_S/r$ and, not surprisingly, is therefore able to fully satisfy the requirements of the PoE.

But in this article, we have not needed to address such deeper considerations. We have merely used the *conventional* form of the FLRW metric applied to inflationary cosmology. The PoE expressed in the form of Eq. (5) derives from one of the most fundamental tenets in GR, notably that spacetime curvature must always be measurable relative to the local free-falling frame. And we cannot avoid the conclusion that Eq. (1), with the expansion factor in Eq. (2), is simply inconsistent with this condition.

What are we to make of this result *vis-à-vis* the observations? Inflationary cosmology has been under development for over 40 years, and the observations have not yet ruled it out (or confirmed it). But therein lies part of the problem. Inflation is frustratingly flexible, easily adjustable to match new data – at least globally, if not in fine detail. Note, for example, how poorly its underlying field and potential are understood, even after four decades of development (see, *e.g.*, a recent review in the study of Vazquez *et al.* [15]). Recent advances, however, have begun to pose some serious questions. For example, though inflation was introduced largely to fix the temperature horizon problem in the cosmic microwave background, it turns out that there are other, equally critical, horizon problems in cosmology, such as that associated with the electroweak phase transition at $t \sim 10^{-11}$ s [16]. So inflation is starting to look more like a customized process designed to address primarily one aspect of the early Universe, rather than a foundation for everything else that followed.

This is not to say that inflation no longer provides some ingredients necessary to establish the internal self-consistency of the standard model. A realistic discussion of its merits ought to include both the positives (see, *e.g.*, the study of Mukhanov [5]) and its negatives (see, *e.g.*, previous studies [17,18]), constituting an extensive discussion outside the scope of this article [19]. But to highlight some recent observational developments that should motivate the type of discussion we have presented in this article, we point to the detailed analysis of the latest *Planck* data release, which suggests that standard, slow-roll inflation could not have solved the temperature horizon problem while simultaneously producing the observed primordial power spectrum. Evidence is growing that the fluctuation spectrum seen in the CMB has a $k_{\min}$ cutoff large enough to impact the time at which inflation could have started [20–22], which, in turn, would have constrained the number of e-folds experienced by the Universe during its inflated expansion.

This implied delayed initiation to inflation ironically weakens an earlier argument that this rapid expansion would have violated the PoE. If inflation had started within the Planck domain, where quantum gravity effects were unavoidable, one would necessarily have had to include the existence of a minimal, measurable length of the order of the Planck length in the Hamiltonian, which would have violated the PoE [23]. In the post *Planck* satellite era, however, the energy of inflation is constrained to be well below the Planck energy and the "old" version of chaotic inflation has had to give way to a newer replacement with a much delayed initiation well past the Planck time [17,18]. This required delayed beginning to any de Sitter expansion would have postponed it well past a quantum beginning [24], so even this originally valid criticism could in principle be avoided now.

Nevertheless, in this "new" chaotic inflationary paradigm, the measured value of $k_{\min}$ suggests that the inflation-induced expansion would have missed solving the temperature horizon problem by a factor ~10 [12]. It is too early to tell whether such a result impacts all conceivable slow-roll inflationary potentials, or whether it creates a hurdle primarily for those proposed thus far. But the fact that inflationary theory has not yet converged to a unified, self-consistent picture after four decades of work may just be an indication that it could not have happened. In this context, the theoretical argument presented in this article does not stand in isolation. It would provide a foundation for the growing tension seen between the expectations of inflation and the new, high-precision measurements becoming available now.

5 Conclusion

Fortunately, this discussion will benefit from an influx of concrete evidence – one way or the other – in the coming years. Today, the standard model is faced with many inconsistencies and deficiencies, not solely the unresolved temperature horizon problem (see, *e.g.*, the study of Melia [19]). The campaign to measure the real-time redshift drift of distant quasars (see, *e.g.*., the study of Melia [25]) is a particularly exciting prospect, because it will probe the underlying cosmology in a truly model-independent way, and its principal outcome will be a simple yes/no answer. The ELT-HIRES [26] and the SKA Phase 2 array [27] will attempt to measure the redshift derivatives of distant sources. A measurement of *zero* drift at all redshifts, *contrasting with any variation at all*, would argue against the use of the FLRW metric to describe the spacetime of models with an accelerated expansion, such as inflationary cosmology. These observational campaigns will be able to differentiate between zero and non-zero redshift drift at a confidence level of ~3σ after only 5 years of observation, and ~5σ over a lifetime of 20 years.

A determination that the contents of the Universe exhibit zero redshift drift would point to a cosmic expansion consistent with the zero active mass condition in GR [10,14], which produces an expansion factor $a(t) = (t/t_0)$. Such a Universe does not have any horizon problems [16,28], so the inconsistency we have highlighted in this article may simply become moot if it turns out that the Universe did not experience a phase of inflated expansion after all.

This is but one of several possible alternative scenarios in which the early Universe would not have needed to inflate in order to produce the expansion we see today. Other than an adherence to the zero active mass condition in GR, the Universe may have undergone a (perhaps infinite) sequence of cyclic bounces [29,30], avoiding all horizon problems due to an equilibration before each cycle. Arguments such as those we have made in this article can help to refine the discussion more consistently with fundamental theory.

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