

**Chern-Simons Forms and Transgression Actions
or
The Universe as a Subsystem¹**

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Abstract

We review the often forgotten fact that gravitation theories invariant under local de Sitter, anti-de Sitter or Poincaré transformations can be constructed in all odd dimensions. These theories belong to the Chern–Simons family and are particular cases of the so-called Lovelock gravities, constructed as the dimensional continuations of the lower dimensional Euler classes. The supersymmetric extensions of these theories exist for the AdS and Poincaré groups, and the fields are components of a single connection for the corresponding Lie algebras. The need to regularize these theories in order to define conserved charges and thermodynamic functions in asymptotically AdS spacetimes, requires the addition of a very special boundary terms to the action. This modification turn the lagrangian into a **transgression**. Transgression forms are mathematical objects that play an important role in the Chern-Weil theorems in algebraic topology. Transgressions are invariant functions that depend on two independent connection field, that interact at the boundary of spacetime. In a naive interpretation, the presence of a second connection field with identical quantum numbers as the “physical” connection might seem embarrassing. However, there is a natural interpretation of this fact recently proposed, whereby our spacetime can be viewed as a subsystem.

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1 Gravity as a gauge theory

The Equivalence Principle, an essential ingredient of General Relativity, establishes the possibility of retaining the Lorentz symmetry of Special Relativity in every local neighborhood of a curved spacetime [1]. This is reflected by the fact that the Einstein-Hilbert action is invariant under local Lorentz rotations. However, this is something not sufficiently stressed in the standard courses on GR which, on the other hand, devote an enormous space (and time) to the invariance of the theory under general coordinate transformations.

The standard lore of GR has grossly exaggerated the importance of general coordinate invariance, as this is essentially a condition for any well posed mathematical description of nature: physical properties of nature cannot depend on coordinates that have been introduced by human beings. The origin of this lore seems to be found in Einstein's own understanding, which led to the first renderings of GR [2].

this fact and much less, of its importance. Only forty years later, with the discovery of Yang-Mills theories [3] it was possible to understand gravitation as a nonabelian gauge theory [4, 5]. The original observation that led Einstein to General Relativity (**GR**) was that the effect of the gravitational field can be eliminated by a change of reference frame to that of a freely falling observer.

The mathematical content of this assertion is that spacetime is a manifold that becomes indistinguishable from Minkowski space. An observer who has access to in a sufficiently small local neighborhood, can borrow the description of physical reality from Minkowski spacetime. In this sense, GR asserts that physics in a curved spacetime must be invariant under local Lorentz transformations. In particular, the gravitational field itself must be a gauge theory for the $SO(3,1)$ group.

Local Lorentz invariance is an exact gauge symmetry of GR, closely related to the gauge symmetries that characterize the other forces of nature. In spite of this formal similarity between gravity and the other fundamental forces of nature, there exist a number of differences, which may be at the root of the obstructions towards the quantum description of gravitational phenomena.

Thus, the principle of equivalence states that spacetime is a differentiable pseudo-Riemannian manifold M , endowed with a tangent bundle of flat Minkowski spaces at each point. Spacetime is the base manifold for a fiber bundle, where each fiber represents the Lorentz group. Note that the local Lorentz symmetry is unrelated to the freedom to make arbitrary coordinate choices on M —diffeomorphism invariance or general covariance. General covariance is neither an exclusive feature of GR, nor is it a useful physical symmetry. Invariance of a physical system under coordinate transformations is as fundamental as the invariance of ideas under a change of type fonts.

A more practical reason to avoid using general covariance as a symmetry principle is the fact that its first class generators do not form a Lie algebra but an *open* algebra, where the analogues of the structure constants are functions of the phase space variables instead of being invariants under the action of the group [6]. The structure functions of the diffeomorphism algebra are functions of the

metric of spacetime, which is dynamically determined by the classical Einstein equations. This is a drawback if one is interested in a quantum description of gravity, because it is no longer possible to use the diffeomorphism algebra as a symmetry of the quantum system.

Invariance of gravity under local Lorentz transformations is manifest when the Einstein–Hilbert action is written in the first order formalism. For example, in four dimensions

$$I_{EH} = \int d^4x \sqrt{-g} R = \int \epsilon_{abcd} R^{ab} \wedge e^c \wedge e^d . \quad (1)$$

Here $R^{ab} = d\omega^{ab} + \omega^a_c \wedge \omega^{cb}$ is the curvature two-form, ω^{ab} is the Lorentz (spin) connection, and $e^a = e^a_\mu dx^\mu$ is the local orthonormal frame (vierbein). Under a local Lorentz transformation $\Lambda^a_b(x)$, e_a and ω^{ab} transform, respectively, as a vector and as a connection

$$\begin{aligned} e^a &\rightarrow e'^a(x) = \Lambda^a_b(x) e^b(x) , \\ \omega^{ab} &\rightarrow \omega'^{ab}(x) = \Lambda^a_c(x) \Lambda^b_d(x) \omega^{cd}(x) + \Lambda^a_c(x) d\Lambda^c_d(x) . \end{aligned} \quad (2)$$

Clearly, the form of the Lagrangian (1) is quite different from the Yang–Mills (YM) one,

$$I_{YM}[A] = \frac{1}{4} \int_M \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} Tr[\mathbf{F}_{\mu\nu} \mathbf{F}_{\alpha\beta}] . \quad (3)$$

An obvious difference is that (1) is linear rather than quadratic in the curvature. More importantly, gravity requires two dynamical fields of different nature: a gauge connection for the Lorentz group, ω^a_b , and a vector under the same group, e^a . Yang–Mills theory, on the other hand, requires one dynamical field –the gauge connection \mathbf{A} –, and one of non-dynamical nature –the spacetime metric $g_{\mu\nu}$. The metric represents the background geometry, which is prescribed *a priori* and is supposed to be everywhere regular. An action of the YM type (3) for gravity would be unsatisfactory since the spacetime geometry is dynamically determined, and therefore it could not be expected to be invertible everywhere.

Each of the 1-form fields e^a , ω^a_b embodies an essential and conceptually independent aspect of geometry: the metric structure and an affine structure, parallelism. The metric establishes a way to assign a real number to different features of geometrical objects; it carries the notions of distance, area and orthogonality that we use in practical cases to compare objects. Parallelism on the other hand, is a prescription to establish an isomorphism between tangent spaces at different points of the manifold, that is, the notion of parallel transport of vectors in open neighborhoods. This is a necessary ingredient in order to define differential operators that transform consistently (gauge covariance).

Since metricity and parallelism are logically independent definitions, it is fitting to describe them by means of dynamically independent fields. Hence, the equivalence principle can be taken to mean that a D -dimensional spacetime

geometry endowed with a metric structure and a notion of parallel transport, should be described mathematically by an action principle of the form

$$I[e, \omega] = \int_M L_D(e, \omega, de, d\omega) , \quad (4)$$

where the Lagrangian L_D is a D -form constructed out of the fundamental fields and their exterior derivatives. In order to ensure the Lorentz invariance of the dynamics, it is sufficient to require the Lagrangian to be a Lorentz scalar. In order to construct L_D , the two invariant tensors of the Lorentz group,

$$\eta_{ab}, \text{ and } \epsilon_{a_1 a_2 \dots a_D} , \quad (5)$$

can also be used.

However, as we shall see, the requirement that L_D be Lorentz invariant is not strictly necessary, and it may be relaxed by demanding instead that, under a Lorentz transformation (2), the action should change at most by a surface term.

In this formulation, the metric is not assumed to be invertible a priori, as in a theory defined on a prescribed background geometry. Thus, it is conceivable that the ground state (vacuum) of gravity may correspond to a configuration with $e^a_\mu = 0$ [7]. Therefore, it would be inconsistent to introduce a structure like the Hodge dual (*) which requires the existence of a metric and its inverse, $g^{\mu\nu}$. The absence of the *-dual does not represent a limitation since all gravity theories which yield second order field equations for the metric can be obtained without the Hodge dual. In fact, by taking exterior derivatives of e^a and ω^a_b the only Lorentz tensors that can be produced are the curvature R^{ab} , and torsion $T^a = de^a + \omega^a_b \wedge e^b$ two forms. Moreover, by virtue of the Bianchi identities,

$$\begin{aligned} DR^{ab} &= dR^{ab} + \omega^a_c \wedge R^{cb} + \omega^b_c \wedge R^{ac} \equiv 0 \\ DT^a &= R^a_b \wedge e^b , \end{aligned} \quad (6)$$

it is clear that no new tensors can be generated. These tensors, together with the invariants (5) are the only ingredients at hand to build up all gravity actions in any dimension.

2 Three series

There are relatively few Lagrangians that can be written in a given spacetime dimension D , with the ingredients listed above that are Lorentz invariant D -forms. These candidate actions fall into three families.

2.1 Lovelock series

General Relativity, viewed as a dynamical theory for the metric without torsion, is generalized for a spacetime dimension $D > 4$, by the so-called Lovelock

theories of gravity [8, 9]. Their Lagrangians are the most general D -forms built out of R^{ab} and e^a . They take the form ¹

$$L = \sum_{p=0}^n \alpha_p L_{(p)} , \quad (7)$$

where $n = [(D - 1)/2]$, α_p are arbitrary coefficients and

$$L_{(p)} = \epsilon_{a_1 a_2 \dots a_D} R^{a_1 a_2} \dots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \dots e^{a_D} . \quad (8)$$

Included in the series (7) are the cosmological constant term $L_{(0)} = \epsilon_{a_1 a_2 \dots a_D} \times e^{a_1} \dots e^{a_D}$, the Einstein–Hilbert density $L_{(1)} = \epsilon_{a_1 a_2 \dots a_D} R^{a_1 a_2} e^{a_3} \dots e^{a_D}$, the Gauss-Bonnet density $L_{(2)} = \epsilon_{a_1 a_2 \dots a_D} R^{a_1 a_2} R^{a_3 a_4} e^{a_5} \dots e^{a_D}$, etc. For even D , the last term is the D -dimensional Euler density $L_{(D/2)} = \epsilon_{a_1 a_2 \dots a_D} R^{a_1 a_2} R^{a_3 a_4} \dots R^{a_{D-1} a_D}$. Each $L_{(p)}$ corresponds to the dimensional continuation to D dimensions of the $2p$ -dimensional Euler density.

Varying the action with respect to the vielbein yields a generalization of Einstein equations for arbitrary dimensions known as Lovelock equations. The variation with respect to ω^{ab} yields an equation involving torsion which is always solved by $T^a = 0$, but this is not the most general solution.

2.2 Torsional series

Torsion is not included in the Lagrangian (7), although it is not set identically equal to zero. This means that including torsion in the Lagrangian is as legitimate as including curvature. This means that there is a series of Lorentz invariant polynomials, which are not included in the Lovelock series that can be added in each dimension

$$L_D^{\text{Tor}}(e^a, \omega^{ab}) = \sum_s \beta_s P^s(e^a, R^{ab}, T^a) . \quad (9)$$

Notice that these polynomials cannot involve the totally antisymmetric symbol $\epsilon_{abc\dots}$. The explicit form of these terms is not very illuminating and takes a different form in each dimension. The construction of these polynomials, as well as a broad discussion about them, were given in [10]. These polynomials include the Pontryagin invariant $4k$ -forms, $\mathcal{P}_{4k}(R) = R_{a_2}^{a_1} \dots R_{a_1}^{a_{2k}}$, as particular cases.

There exist two additional terms in this family that can be included in four dimensions, $t = T^a T_a$ and $r = R^{ab} e_a e_b$. It turns out, however, that the combination $N_4 \equiv t - r$ is a total derivative (the Nieh–Yan invariant) and hence the two terms are equivalent Lagrangians. This type of invariants are also related to Chern-Pontryagin classes, and may also contribute to the chiral anomaly in spacetimes with torsion [11, 12, 13].

¹Hereafter, wedge product between forms is always understood.

2.3 Lorentz CS series

There is another class of actions that are not exactly invariant, but *quasi-invariant*, under local Lorentz transformations. These are the Lorentz Chern–Simons (**LCS**) forms that exist for all odd dimensions. The simplest LCS form in three dimensions is $L_3^{CS} = \omega_b^a d\omega_a^b + \frac{2}{3}\omega_b^a \omega_c^b \omega_a^c$, and, in general, in $4k - 1$ dimensions, it takes the form

$$L_{4k-1}^{CS} = [\omega(d\omega)^{2k-1}] + \gamma_1 [\omega^3(d\omega)^{2k-2}] + \gamma_2 [\omega^5(d\omega)^{2k-3}] + \dots + \gamma_{2k-1} [\omega^{4k-1}], \quad (10)$$

where the coefficients γ_s are fixed rational numbers and the bracket $[\dots]$ denotes a trace. These forms yield Lorentz invariant equations despite the fact that they involve explicitly the connection and are not truly Lorentz invariants. This "miracle" stems from the fact that the exterior derivative of CS forms are topological invariant densities. In this case, $dL_{4k-1}^{CS} = \mathcal{P}_{4k}(R) = [R^{2k}]$.

3 A very special choice

The number of Lagrangians generated in this way increases with dimension, and so does the number of arbitrary dimensionful coupling constants $\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m$. Fortunately, for $D = 4$ there is complete agreement with GR: in the absence of torsion, only the Einstein-Hilbert and the cosmological constant term can be present. What is the meaning of all those coupling constants in higher dimensions? It can be easily seen that they represent the existence of several scales in the theory, which are related to different radii of curvature in the solutions, or to different cosmological constants $\lambda_1, \lambda_2, \dots, \lambda_s$. If the cosmological constant is a problem in four dimensions, the problem is a priori much worse for $D > 4$. It can be seen that field equations admit spacetime geometries that jump discontinuously from one with $\lambda = \lambda_1$, to another with $\lambda = \lambda_2$ [14, 15].

The presence of so many dimensionful constants endows the theory with a bad prospect for its quantization. How could the theory be protected from uncontrollable ultraviolet divergences? The ideal situation is closer to the opposite extreme: a Lagrangian with no arbitrary dimensionful constants. That is the case of a Chern–Simons theory, in which all constants are fixed dimensionless rational numbers.

The good news is that in every dimension there exists a choice of coefficients $\alpha_1 \dots \alpha_n; \beta_1 \dots \beta_m$ such that all cosmological constants are the same and therefore there is only one scale in the theory. In odd dimensions, this choice is even more miraculous since all dimensionful coefficients in the action can be absorbed by means of a rescaling of the vielbein, $e^a \rightarrow l^{-1}e^a$. This choice correspond to picking the Lovelock coefficients in (7) as

$$\alpha_p = \frac{l^{2p-D}}{D-2p} \binom{\frac{D-1}{2}}{p}. \quad (11)$$

This produces a Lagrangian that describes a theory of gravity with no built-in

scale and therefore, scale-invariant. This choice has an additional bonus feature because the gauge symmetry is now enlarged from the Lorentz to the AdS group.

As it is well known, miracles don't exist; the "miracles" arising from the choice (11) are consequence of the fact that for this choice the vielbein and the Lorentz connection can be combined into a connection for the AdS group. In other words, the gauge group $SO(D-1, 1)$ has been embedded into $SO(D-1, 2)$, in the form

$$\mathbf{A} = \frac{e^a}{l} J_a + \frac{1}{2} \omega^{ab} J_{ab} , \quad (12)$$

and the action becomes a functional of this connection \mathbf{A} , and not a functional of e^a and ω^{ab} separately. The Lagrangian can now be expressed as

$$L_{2n-1}^{\text{CS}}(\mathbf{A}) = \kappa \langle \mathbf{A}(d\mathbf{A})^{n-1} + \gamma_1 \mathbf{A}^3 (d\mathbf{A})^{n-2} \dots \gamma_{n-1} \mathbf{A}^{2n-1} \rangle , \quad (13)$$

where $\langle \dots \rangle$ denotes a symmetrized trace in the matrix representation of the AdS generators $\{J_K\} = \{J_a, J_{ab}\}$, and the γ_k 's are fixed rational coefficients [16, 17] (for details and a comprehensive list of references see, e.g., [18]).

The symmetrized trace of the generators $\langle J_{K_1} \dots J_{K_s} \rangle = \Delta_{K_1 \dots K_s}$ is an invariant tensor in the Lie algebra. There are two such invariants in the rotation algebras $so(n, m)$, viz., the Levi-Civita symbol $\varepsilon_{K_1 \dots K_{n+m}}$, or the symmetric product of the $so(n, m)$ invariant metric, $\eta_{(K_1 K_2} \times \dots \times \eta_{K_{4k-1} K_{4k})}$. The first, gives rise to the Euler density and the related Lovelock series; the second produces the torsional and the Lorentz CS series [10].

A more compact form for the Chern-Simons lagrangian is provided by Cartan's homotopy formula,

$$L_{2n-1}^{\text{CS}}(\mathbf{A}) = \frac{\kappa}{n} \int_0^1 dt \langle \mathbf{A}(\mathbf{F}_t)^{n-1} \rangle , \quad (14)$$

where $\mathbf{F}_t = td\mathbf{A} + t^2\mathbf{A}^2$.

It is also possible to construct a de-Sitter invariant action, (with $SO(D, 1)$ as the gauge group) which is obtained by replacing $l^2 \rightarrow -l^2$ in (11). Finally, there is also the possibility of taking the vanishing cosmological constant limit, $l \rightarrow \infty$, which yields a theory invariant under the Poincaré group [19, 20].

It is sometimes argued that the Einstein–Hilbert action with cosmological constant in four dimensions provides a gauge theory for the (A)dS group, because its dynamical fields (e^a and ω^{ab}) are components of the (A)dS connection (12) [21, 22]. The problem with this point of view is that the (A)dS symmetry cannot be respected by the action because there is no Lagrangian for the connection \mathbf{A} , invariant under the (A)dS gauge group in four dimensions. Arguing that the symmetry is spontaneously broken is also hard to sustain since there seems to be no regime of the theory in which the symmetry can be restored.

4 Surface terms and transgressions

For mathematicians, Chern–Simons forms are not natural objects. They are not truly invariant, changing by a closed form under a gauge transformation.

In physics this is not a serious problem because close forms in the action are locally exact, and given a simple enough topology, they are just surface terms that don't affect the field equations or the conservation laws. However, there are cases of interest where life is not so simple, and invariances of the action "up to surface terms" have important physical consequences. The value of the conserved charges, and of the action itself can be renormalized by surface terms. This in turn affects the definition of thermodynamic quantities like the energy and entropy of a black hole.

A case in which this is a relevant problem is provided by anti-de Sitter space. The problem with AdS space is that although the spatial sections are infinitely extended (the radial coordinate r extends to infinity), the asymptotic region is not causally out of reach. In fact, a photon can reach infinity ($r = \infty$) and return to the source at $r = 0$ in a finite time (actually, it is the same time that a massive particle would take to travel radially and fall back to the origin). This means that $r = \infty$ resembles a physical boundary, unlike the situation in Minkowski space. On the other hand, boundary conditions sufficient to ensure that the action attains an extremum on the classical orbits, require to supplement the action by a surface term of a particular form. In asymptotically locally AdS spaces (**ALADS**) the boundary term takes the form $B_{2n}[\mathbf{A}, \bar{\mathbf{A}}]$, where the field $\bar{\mathbf{A}}$ is only defined at the surface of spacetime, whose rôle is to match the boundary conditions under which the action is to be varied [23]. This addition cures several problems at once: it provides a well-defined variation while, at the same time, it renders the charges and the on-shell value of the action finite, producing well defined thermodynamic quantities, which can also be computed by other means [24]. In particular, the energy of the vacuum turns out to be exactly the Casimir energy for AdS without additional regularizations (counterterms) or ad-hoc background subtractions [25].

As in other cases in physics, the solution to this problem comes from the requirement of gauge invariance. The field $\bar{\mathbf{A}}$ and the boundary term $B_{2n}[\mathbf{A}, \bar{\mathbf{A}}]$ correspond precisely to what is required to turn the Chern–Simons lagrangian into a *transgression form* [26, 27, 28]. Unlike CS forms, transgression forms depend on two connections, $\mathcal{T}_{2n-1}(\mathbf{A}, \bar{\mathbf{A}})$. Transgressions are gauge invariants provided \mathbf{A} and $\bar{\mathbf{A}}$ transform as connections for the same gauge group. The defining property of a transgression is that its exterior derivative is the difference of two invariant classes, for \mathbf{A} and $\bar{\mathbf{A}}$, respectively,

$$d\mathcal{T}_{2n-1}(\mathbf{A}, \bar{\mathbf{A}}) = \langle F^n(\mathbf{A}) \rangle - \langle \bar{F}^n(\bar{\mathbf{A}}) \rangle .$$

Thus, the transgression form can also be written as

$$\mathcal{T}_{2n-1} = L_{2n-1}^{CS}(\mathbf{A}) - L_{2n-1}^{CS}(\bar{\mathbf{A}}) + dB_{2n}(\mathbf{A}, \bar{\mathbf{A}}) ,$$

where $B_{2n}(\mathbf{A}, \bar{\mathbf{A}})$ is defined on a local chart over the boundary of the space-time manifold M .

5 Reinterpretation: the universe as a subsystem

If the second connection field $\bar{\mathbf{A}}$ is considered as a dynamical field on the same footing as \mathbf{A} , an extremely puzzling situation arises. First, it is an identical field (same quantum numbers, same classical dynamics); second, it does not couple to \mathbf{A} , except at the boundary of the spacetime manifold M . Third, the opposite signs of the two lagrangians means that if \mathbf{A} is physically meaningful, $\bar{\mathbf{A}}$ is a ghost field (negative energy). This would be a theory for two identical fields that don't interact except at infinity, and with non positive definite energy. This makes the interpretation of the transgression action as a physical theory for two dynamically independent fields on the same spacetime manifold untenable.

One way out of this problem would be to declare $\bar{\mathbf{A}}$ as non dynamical, some kind of reference background connection. This is hardly satisfactory, however, since it would be inconsistent to require that $\bar{\mathbf{A}}$ transform as a gauge connection in the same way as the dynamical field \mathbf{A} , and still be non-dynamical. This is also contrary to the general philosophy that everything that appears in the action and is not a constant should be varied and have dynamical equations ².

A more satisfactory interpretation comes from the observation that if $\bar{\mathbf{A}}$ is only necessarily defined at the boundary, it is sufficient to define $\bar{\mathbf{A}}$ on a different manifold, \bar{M} , that shares a common boundary with (*cobordant to*) M , $\partial M = \partial \bar{M}$. Then, the fact that the two field only interact at the boundary is a natural consequence of the fact that it is there where they meet. The action principle based on the transgression form can then be written as

$$I_{Trans}[\mathbf{A}, \bar{\mathbf{A}}] = \int_M L_{CS}(\mathbf{A}) - \int_{\bar{M}} L_{CS}(\bar{\mathbf{A}}) + \int_{\partial M} B_{2n}(\mathbf{A}, \bar{\mathbf{A}}). \quad (15)$$

The main advantage of this expression is that it allows to compute the conserved charges by direct application of Noether's theorem in covariant language and without subtractions [29, 30].

There are two more puzzling points: The manifold \bar{M} is analogous to a second Riemann sheet of the "true" spacetime M (see Fig.1); and there remains the issue of the wrong sign in one of the lagrangians. These two problems can be addressed simultaneously if the orientation of \bar{M} is reversed (see Fig.2). The resulting picture is that in our universe (M) has the physical field \mathbf{A} and there exists a neighboring universe beyond infinity (\bar{M}), where the dynamical field $\bar{\mathbf{A}}$ lives and has the same dynamics as our \mathbf{A} . The cobordant nature of the manifolds ensures that the fields \mathbf{A} and $\bar{\mathbf{A}}$ transform in the appropriate manner at the boundary, so this transformation property is not an additional assumption.

²In a note added in proof to their seminal paper [3], Yang and Mills state: "It is to be emphasized that [introducing a field as an auxiliary function to accomplish invariance, but not regarded as a field variable by itself] violates the principle of invariance. Every quantity that is not a pure numeral (like 2, or M, or any definite representation of the γ matrices) should be regarded as a dynamical variable, and should be varied in the Lagrangian to yield an equation of motion."

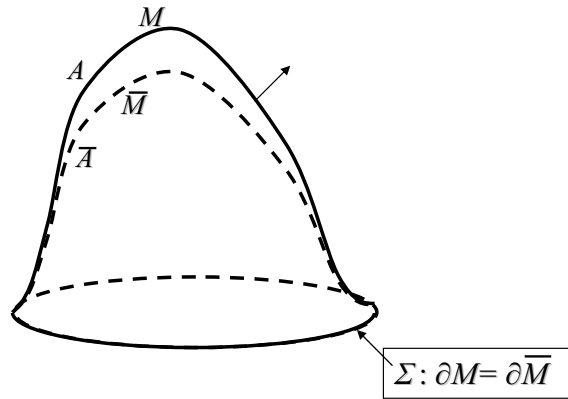


Figure 1: Two identical orientable, cobordant manifolds with a copy of the gauge connection defined on each sheet.

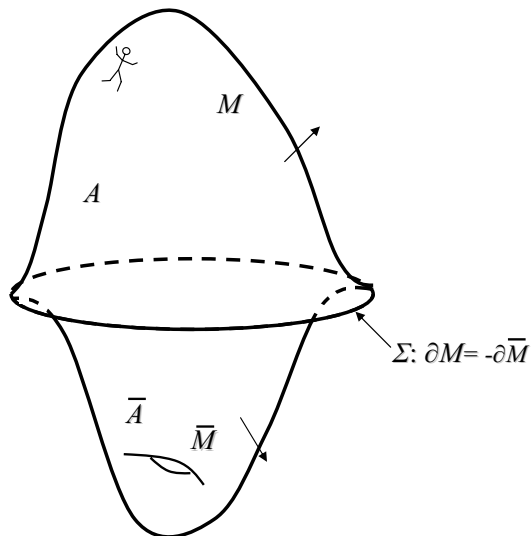


Figure 2: Two non-identical oppositely oriented cobordant manifolds with a copy of the gauge connection defined on each sheet. Our universe is one of the two pieces and is connected to the cobordant manifold on Σ (AdS infinity).

5.1 Pants and polymers

The junction of two cobordant manifolds can be generalized to couple three manifolds with three pairwise boundaries, as the “pants geometry” depicted in Fig. 3. In this case, the action takes the form

$$\begin{aligned}
 I_{Polymer}[\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3] &= \int_{M_1} L_{CS}(\mathbf{A}_1) + \int_{M_2} L_{CS}(\mathbf{A}_2) + \int_{M_3} L_{CS}(\mathbf{A}_3) \\
 &+ \int_{\Sigma_{12}} B_{2n}(\mathbf{A}_1, \mathbf{A}_2) + \int_{\Sigma_{23}} B_{2n}(\mathbf{A}_2, \mathbf{A}_3) + \int_{\Sigma_{31}} B_{2n}(\mathbf{A}_3, \mathbf{A}_1) \\
 &+ \int_{\Pi_{123}} C_{2n-1}(\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3) + \int_{\Pi_{132}} C_{2n-1}(\mathbf{A}_1, \mathbf{A}_3, \mathbf{A}_2). \quad (16)
 \end{aligned}$$

The resulting geometry can be viewed as a bifurcation or a merging of two universes or, more generally, the basic vertex for interactions in a quantum field theory of interacting geometries. The generalization to higher order “Feynman diagrams” is also straightforward and gives rise to polymers like the four point vertex depicted in Fig. 4,

$$\begin{aligned}
 I_{Polymer}[\mathbf{A}_1, \dots, \mathbf{A}_4] &= \int_{M_1} L_{CS}(\mathbf{A}_1) + \dots + \int_{M_4} L_{CS}(\mathbf{A}_4) \\
 &+ \int_{\Sigma_{12}} B_{2n}(\mathbf{A}_1, \mathbf{A}_2) + \dots + \int_{\Sigma_{41}} B_{2n}(\mathbf{A}_4, \mathbf{A}_1) \\
 &+ \int_{\Pi_{123}} C_{2n-1}(\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3) + \int_{\Pi_{132}} C_{2n-1}(\mathbf{A}_1, \mathbf{A}_3, \mathbf{A}_2) \\
 &+ \int_{\Pi_{134}} C_{2n-1}(\mathbf{A}_1, \mathbf{A}_3, \mathbf{A}_4) + \int_{\Pi_{143}} C_{2n-1}(\mathbf{A}_1, \mathbf{A}_4, \mathbf{A}_3). \quad (17)
 \end{aligned}$$

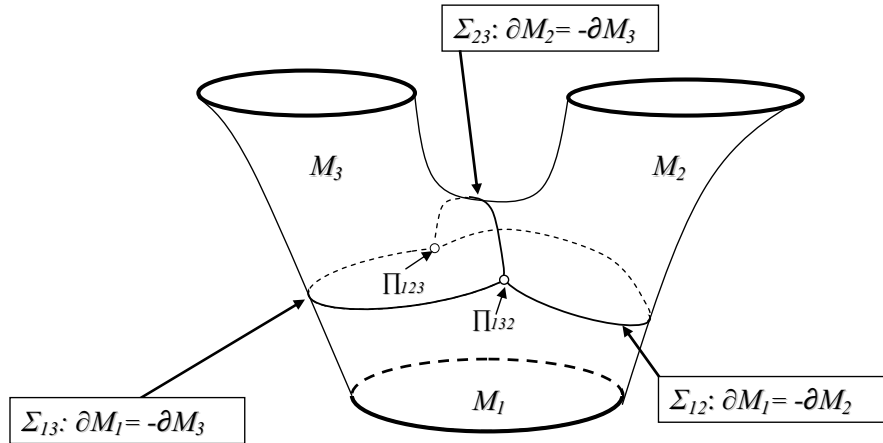


Figure 3: Pants geometry. Three or more non-identical appropriately oriented, cobordant manifolds, with a copy of the gauge connection defined on each sheet. Our universe could be one of the several pieces (e.g. M_1), connected to the cobordant manifolds on $(\Sigma_{12} \cup \Sigma_{13})$. There are additional contributions to the integrals at the intersections of boundaries, Π_{123} and Π_{132} .

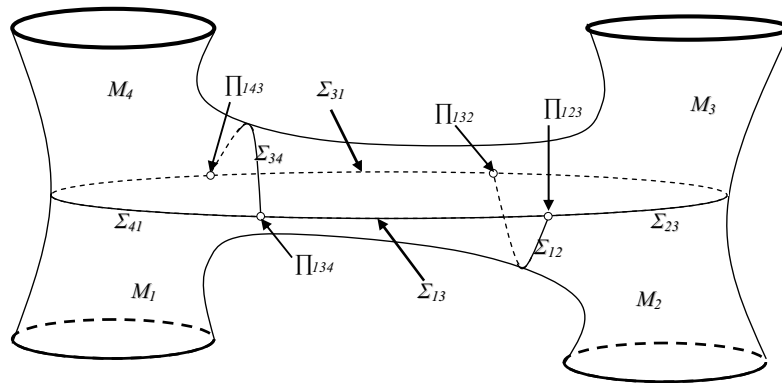


Figure 4: The generalization of the pants to other more elaborate garments is straightforward...

5.2 The universe as a defect

An interesting situation occurs when the two connections \mathbf{A} and $\bar{\mathbf{A}}$ in (15) are related by a gauge transformation, $\bar{\mathbf{A}} = \mathbf{A}^h = h^{-1}(\mathbf{A} + d)h$. In that case, the transgression for \mathbf{A} and \mathbf{A}^h , integrated over a bounded three-dimensional manifold Σ , reads [31]

$$\mathcal{T}_3(\mathcal{A}^h, \mathcal{A}) = \int_{\Sigma} \frac{1}{3} \langle (h^{-1}dh)^3 \rangle - \int_{M^2} \langle (\mathcal{A} - h^{-1}dh) \mathcal{A}^h \rangle. \quad (18)$$

This expression is the well known gauged Wess-Zumino-Witten (**g-WZW**) term found in the study of current algebra [32]. This is cute, but what could be the origin of this monster? It turns out that this monster can be seen to arise when the topological density $\langle \mathbf{F}^2 \rangle$ is integrated over a four dimensional manifold with a two-dimensional topological defect, $M^4 - M^2$ (that is, a manifold M^4 from which a two dimensional submanifold M^2 has been removed) [33]. Similarly, it can be shown that integrating $\langle \mathbf{F}^3 \rangle$ over a six dimensional manifold M^6 with a four-dimensional defect M^4 , results in a g-WZW action that defines a dynamical theory on the defect, and the induced geometry on M^4 is governed by the Einstein equations there [33].

6 Extensions and special features

6.1 Local supersymmetry

The CS theories described here have a number of important extensions, the most remarkable among them are the associated supergravities. In three dimensions, the standard Einstein-Hilbert plus cosmological constant action is a Chern-Simons theory and its supersymmetric extension has been known for many years [34]. The resulting 2+1 AdS supergravity is a gauge theory for the group $OSp(p|2; R) \otimes OSp(q|2; R)$. In five dimensions, the locally supersymmetric extension of gravity was found by Chamseddine [19], and its purely gravitational sector is the CS-AdS action described above. The generalization to higher dimensions was found in [35, 36], and the supersymmetric extensions of the Poincaré theory was presented in [37].

The supersymmetric extension of CS gravity can be obtained by introducing the necessary fermionic and bosonic fields in order to produce a connection for the gauge supergroup that extends the gauge algebra represented by \mathbf{A} , in a spacetime of a given dimension. This can be done from first principles, if one knows the semisimple superalgebras containing the d -dimensional *AdS* algebra, $SO(D-1, 2)$. Alternatively, one can start by adding to the connection (12) the supersymmetry generators Q and \bar{Q} , with the corresponding gauge fields $\bar{\psi}$ and ψ ,

$$\mathbf{A} = \frac{e^a}{l} J_a + \frac{1}{2} \omega^{ab} J_{ab} + \bar{Q} \psi + \bar{\psi} Q + \dots . \quad (19)$$

In general, the closure of the extended algebra requires extra bosonic generators and, in some cases, several copies of the fermions (this is what the dots mean

in (19)). The result is quite unique (see e.g., [37, 35, 36, 38, 39, 18]), and it is in general not the simplest standard supergravity in a given dimension. This difference reflects the fact that the super CS action obtained in this fashion is a genuine gauge theory for a connection of the corresponding superalgebra; standard supergravities, on the other hand, are not gauge theories and some of the fields are connections while the rest transform as vectors under the generators of the superalgebra. The field content for CS and standard supergravities are compared in the following table:

D	CS – AdS supergravity	Algebra	Standard supergravity
5	$e_\mu^a \omega_\mu^{ab} A_\mu \psi_\mu^\alpha \bar{\psi}_{\alpha\mu}$	$usp(2, 2 1)$	$e_\mu^a \psi_\mu^\alpha A_\mu \bar{\psi}_{\alpha\mu}$
7	$e_\mu^a \omega_\mu^{ab} A_{j\mu}^i \psi_\mu^{i\alpha}, i, j = 1, 2$	$osp(2 8)$	$e_\mu^a A_{[3]} A_{j\mu}^i \lambda^\alpha \phi \psi_\mu^{i\alpha}, i, j = 1, 2$
11	$e_\mu^a \omega_\mu^{ab} b_\mu^{abcde} \psi_\mu^\alpha$	$osp(32 1)$	$e_\mu^a A_{[3]} \psi_\mu^\alpha$

The actions obtained in this way are by construction invariant under the gauge superalgebra and coordinate diffeomorphisms. Since they include gravity, they are supergravities, albeit of a different sort. Some authors would reserve the word *supergravity* for supersymmetric theories whose gravitational sector is described by the Einstein–Hilbert Lagrangian. This narrow definition is correct in three and four dimensions, but seems unwarranted for $D > 4$ in view of the numerous possibilities beyond EH. If one wishes to be precise, the supergravities described here seem to belong to a separate class and the relation with the standard ones is still an open problem.

Under supersymmetry, the field (19) transforms as a connection, namely, $\delta\mathbf{A} = -\nabla\mathbf{\Lambda} = -(d\mathbf{\Lambda} + [\mathbf{A}, \mathbf{\Lambda}])$, where $\mathbf{\Lambda}$ is a zero-form with values in the Lie algebra, and ∇ is the exterior covariant derivative in the representation of \mathbf{A} . In particular, under a supersymmetry transformation, $\mathbf{\Lambda} = \bar{\epsilon}^i Q_i - \bar{Q}^i \epsilon_i$. For instance, in terms of the component fields of the five dimensional $usp(2, 2|1)$ theory, this means

$$\begin{aligned}
 \delta e^a &= \frac{1}{2} (\bar{\epsilon}^r \mathbf{\Gamma}^a \psi_r - \bar{\psi}^r \mathbf{\Gamma}^a \epsilon_r) , \\
 \delta \omega^{ab} &= -\frac{1}{4} (\bar{\epsilon}^r \mathbf{\Gamma}^{ab} \psi_r - \bar{\psi}^r \mathbf{\Gamma}^{ab} \epsilon_r) , \\
 \delta A_s^r &= -i (\bar{\epsilon}^r \psi_s - \bar{\psi}^r \epsilon_s) , \\
 \delta \psi_r &= -\nabla \epsilon_r , \quad \delta \bar{\psi}^r = -\nabla \bar{\epsilon}^r , \\
 \delta A &= -i (\bar{\epsilon}^r \psi_r - \bar{\psi}^r \epsilon_r) .
 \end{aligned}$$

where ∇ is the covariant derivative on the bosonic connection,

$$\nabla \epsilon_r = \left(d + \frac{1}{2} \omega^{ab} \mathbf{\Gamma}_{ab} + \frac{1}{2l} e^a \mathbf{\Gamma}_a \right) \epsilon_r - A_r^s \epsilon_s - \frac{3i}{4} A \epsilon_r .$$

These actions are invariant (up to surface terms) under these transformations, and neither on-shell conditions nor auxiliary fields are necessary to realize the symmetry. This is in contrast with the standard cases, which often require torsional on-shell conditions in order to close the symmetry algebra. Symmetries requiring on-shell conditions are likely to be troublesome, since they need not be respected in the quantum theory.

These superalgebras allow for extensions with $\mathcal{N} > 1$, and the field multiplets for them can be easily constructed in all cases. The relevant groups are $OSp(\mathcal{N}|2^{4k+1})$ for $D = 8k + 3$, $OSp(2^{4k-1}|\mathcal{N})$ for $D = 8k - 1$, and $SU(2^k, 2^k|\mathcal{N})$ for $D = 4k + 1$. Their periodic nature *mod* 8 is inherited from the well known periodicity of the Clifford algebras.

6.2 The $\Lambda \rightarrow 0^-$ limit

Since the de Sitter group does not admit supersymmetric extensions [40, 41], the supergravities described above do not exist for positive cosmological constant. For $\Lambda < 0$ these theories present no difficulties and their vanishing cosmological constant limit are well defined in the sense that some sensible form of action is obtained in this limit, in which the algebra is deformed into one that contains the Poincaré Lie algebra in the bosonic sector. This deformation is a generalized Inönü-Wigner (IW) contraction. The standard IW contraction of a semisimple algebra is, generically, a non semisimple Lie algebra of the same dimension. There exist some more interesting forms of taking the limit $\Lambda \rightarrow 0$ that may increase the dimension of the Lie algebra. These extensions arise from a different deformation in which the new structure constants are analytic functions of the old ones. The emerging algebra can have more generators than the original one, and is therefore an extension. The condition that the new algebra satisfy the Maurer-Cartan identities restricts the form of the new algebras that can be obtained in this way. Although this idea seems to have a long history in the mathematical literature [42], its use in physics is rather recent [43, 44, 45, 28] that results from the addition of new generators. If a non invertible limit is taken in the analytic functions of the original parameters, that usually results in a non semisimple algebra, which extends the original algebra by an abelian ideal.

6.3 Matching degrees of freedom

In these theories the bosonic and fermionic degrees of freedom do not occur in equal numbers. The matching present in standard supersymmetric theories³ results from two assumptions which do not hold in the present case: i) that the spacetime symmetry group is Poincaré (here it is the AdS group), and ii) the fields form a vector multiplet under supersymmetry (in CS theories all the fields are parts of a connection and therefore belong to the adjoint representation).

³It is worth mentioning that global issues –like the presence of a deficit angle– may also spoil the boson–fermion degeneracy in standard supergravity [46, 47].

6.4 Manifest M -covariant theory.

As an example, consider a CS theory for the supersymmetric extension of the Poincaré group in eleven dimensions. Following the steps outlined here, one arrives almost uniquely at a gauge invariant action for the M -algebra [38, 39]. The connection,

$$\mathbf{A} = \frac{e^a}{l} P_a + \frac{1}{2} \omega^{ab} J_{ab} + \bar{Q} \psi + b^{ab} Z_{ab} + b^{abcde} Z_{abcde} , \quad (20)$$

includes, apart from the vielbein, the Lorentz connection, and the gravitino, a second-rank and a fifth-rank antisymmetric Lorentz tensor one-forms, $b^{[2]}$ and $b^{[5]}$. The superalgebra includes the Poincaré generators (P_a, J_{ab}) , one (Majorana) supersymmetry generator Q and the “central extensions” of the M -algebra, $Z_{[2]}$ and $Z_{[5]}$,

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^a)_{\alpha\beta} P_a + (C\Gamma^{ab})_{\alpha\beta} Z_{ab} + (C\Gamma^{abcde})_{\alpha\beta} Z_{abcde} . \quad (21)$$

The generators $Z_{[2]}$, $Z_{[5]}$ commute with all but the Lorentz generators. The supersymmetric action is found to be

$$\begin{aligned} L_\alpha^M = & \epsilon_{a_1 \dots a_{11}} R^{a_1 a_2} \dots R^{a_9 a_{10}} e^{a_{11}} - \frac{1}{3} R_{abc} \bar{\psi} \Gamma^{abc} D\psi - \frac{1}{12} R_{abc} R_{de} b^{abcde} \\ & + 8 [R^2 R_{ab} - 6(R^3)_{ab}] R_{cd} \left(\bar{\psi} \Gamma^{abcd} D\psi - 6R^{[ab} b^{cd]} \right) , \end{aligned} \quad (22)$$

where $R_{abc} = \epsilon_{abca_1 \dots a_8} R^{a_1 a_2} \dots R^{a_7 a_8}$, $R^2 := R^{ab} R_{ba}$ and $(R^3)^{ab} := R^{ac} R_{cd} R^{db}$.

6.5 Special features of CS theories

CS theories are so constrained in their structure, which in turn is due to their topological lineage, that one may be easily misled to believe that the standard approaches to perturbative field theories are applicable here. In this section we summarize these extensions and the special features one must face when dealing with higher dimensional CS theories.

Only odd D . The construction outlined above cannot be extended to even dimensions. Chern classes are homogeneous polynomials in the curvature two form, and hence they are forms of even degree. Therefore, Chern–Simons forms define actions in odd dimensions only, and a similar construction does not exist in even dimensions. It is possible, however that in even dimensions a construction similar to the standard supergravity in four dimensions could be carried out, where some on-shell conditions are assumed in order to close the algebra including coordinate diffeomorphisms. An alternative route has been recently explored in [31, 33], where a four-dimensional geometry can be thought of as a defect of codimension two in a six-dimensional manifold. The four-dimensional gravitation theory is described by a gauged Wess-Zumino-Witten action, obtained from a topological theory in 6D, whose classical dynamics is indistinguishable from that of the Einstein-Hilbert action.

No higher spins. All component fields in these theories carry only one space-time index (they are 1-forms), and they are antisymmetric tensors of arbitrary rank under the Lorentz group (*i.e.*, $b_{\mu}^{ab\dots}$). Thus, they belong to representations of the rotation group whose Young tableaux have one row with two squares and a single column of a given length. This means that the fundamental fields in these theories describe states of spin 2 or less, which goes in the opposite direction of the recent interest on higher spin fields [48, 49].

Degeneracy. Chern–Simons systems in dimensions $D \geq 5$ possess remarkable dynamical features, unexpected in a field theory but often found in fluid dynamics. One of these features stems from the fact that the symplectic form is a function of the connection, and its rank depends on the configuration [50]. There are regions in phase space where the symplectic form has maximal rank (*generic* configurations), where the counting of degrees of freedom is the usual one [51]. Other regions, where the rank is smaller (*degenerate* configurations), possess fewer propagating degrees of freedom. There are even *maximally degenerate* configurations, around which the theory is topological and has no local degrees of freedom. An example of such maximally degenerate configuration is the standard vacuum of Yang–Mills theory, $\mathbf{A} = 0$.

Another unexpected feature is that degenerate systems may loose degrees of freedom in their time evolution. A simple mechanical model shows that a degenerate system may start from a nondegenerate configuration reaching a state where the degeneracy occurs in a finite time. There, some degrees of freedom cease to be dynamical and become gauge coordinates. After that, those degrees of freedom, as well as their initial data, are irreversibly lost [52].

Irregularity. An independent issue, also present in CS theories is the fact that the functional independence of the gauge generators (first class constraints) may break down for certain configurations [53], and a careful analysis is required in order to have a well defined canonical formalism [54].

7 Discussion

We have argued that GR represents a way to implement Lorentz invariance as a gauge symmetry. This is straightforward in the first order formalism, where the basic fields are the 1-forms e^a and ω^a_b . If we further demand an enlargement of the gauge symmetry from Lorentz to Poincaré, dS or AdS groups, a unique choice is singled out: Chern–Simons gravity. CS theories exist for all odd-dimensional spacetimes.

An obvious advantage of the CS construction is the economy of assumptions. The only information required to define the Lagrangian is the gauge group and the dimensionality of the manifold. The field content, the coupling constants, the dynamics of the spacetime manifold, the vacuum structure, are all outputs of the theory.

Interestingly enough, it is possible to write down an action in eleven dimensions with the symmetries dictated by the M algebra. This algebra, which corresponds to the maximal extension of the $\mathcal{N} = 1$ super Poincaré algebra, plays an important rôle in M-theory [55]. This is very suggestive and the question is unavoidable: is Chern–Simons supergravity for the M–algebra related to M–theory? Does this theory play a rôle in the M–theory diagram? There have been attempts to relate these theories. It was already suggested that M theory could be non-perturbatively equivalent to a Chern–Simons theory, though with a different symmetry group; namely, $OSp(32|1) \times OSp(32|1)$ [56]. This claim was mainly supported on arguments dealing with holography. However, the connection to eleven dimensional supergravity at low energies, to the best of our knowledge, has not been understood yet. We have seen that it is possible to extend the Poincaré algebra in such a way that the Einstein–Hilbert action comes out. However, several bosonic fields need to be introduced and their equations of motion severely constrain the system. On the other hand, standard (super) gravity is not a gauge theory of the (super) Poincaré group. Thus, it seems clear that the connection between these theories possibly demand the existence of a spontaneous symmetry breaking mechanism. One thing seems clear: a lot of interesting results are still to be uncovered.

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