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Gaussian quantum steering for continuous variables sharing in an expanding universe

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Abstract Realistic quantum systems are affected by the expanding universe in their preparation and quantum information processing. In this paper, we study the relationship between the Gaussian quantum steering distribution and the parameters of the expanding universe. The expansion process of the universe can be described as a channel acting on a two-mode squeezed Gaussian state, with the evolution of quantum steering from the asymptotic past to the asymptotic future resulting in new distributions parameterized by cosmic parameters. We find that Gaussian quantum steering is more sensitive to the volume change of the expanding universe than the expansion rate, and the Gaussian quantum steering generated by particles with suitable mass and small momentum is more affected by the expanding universe.

1 Introduction

The concept of quantum steering originated from the Einstein–Rodolsky–Rosen paradox (EPR) [1], and with the deepening exploration of quantum systems, features such as quantum entanglement [2–9], Bell non-locality [10–16], and quantum steering have come into focus. Over the past nearly 30 years, quantum information technology has made significant progress both theoretically and experimentally. The application of quantum non-locality in quantum information processing has also made some progress, with quantum steering being an important form of quantum nonlocal correlation. For a two-body quantum system composed of subsystems Alice and Bob, if there exists a local operation on subsystem Alice that can influence the state of subsystem Bob, then it can be said that the quantum system exhibits Alice $\rightarrow$ Bob quantum steering. This property allows for remote manipulation of subsystem Bob by operating on subsystem Alice. Due to the asymmetry of quantum steering, the fact that subsystem Alice can steer subsystem Bob does not guarantee that subsystem Bob can steer subsystem Alice. If Alice can steer Bob and Bob can also steer Alice, then the system exhibits bilateral quantum steering. Conversely, if Alice can steer Bob but Bob cannot steer Alice, then the system exhibits unilateral quantum steering.The asymmetry of quantum steering refers to the unequal status of the two parties involved, while the singleness refers to the inability of quantum steering to be freely shared among multiple parties [17–27]. These unique properties of quantum steering have significant implications for quantum communication processes such as quantum key distribution (QKD), quantum teleportation, long-distance quantum state preparation, and dense coding [28–30], enhancing the efficiency and security of protocols.

Previously, Navascues and Perez-Garcia investigated quantum steering between space-like separated parties within the framework of quantum field theory [31]. Additionally, more attention has been directed towards the quantum manipulation dynamics under the influence of dynamic Casimir effects [32], Hawking radiation [33], and relativistic motion. Due to the presence of gravitational effects and relativistic phenomena in the real world, all quantum systems inevitably experience the influence of gravitational effects and relativistic phenomena during their preparation. Therefore, studying the effects of quantum steering in a relativistic framework and within an expanding universe is highly necessary. Such research can enhance the precision of quantum steering and reduce discrepancies between experimental and theoretical results.

This paper investigates the redistribution of bipartite steering for continuous variables of the scalar fields in the background of an expanding universe. We initially discuss the two-mode squeezed Gaussian state shared by Alice and Bob

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in the asymptotic past. The vacuum state in the asymptotic past evolves into a thermal state as the universe expands in the asymptotic future [34–37]. In the context of quantum information, such cosmic expansion can be described as a Gaussian channel acting on two-mode squeezed Gaussian state. The Gaussian state offers two advantages for our study: firstly, they are typical continuous-variable steering states that can approximate EPR pairs to any desired degree; secondly, these states can be generated in the laboratory and utilized to implement any quantum information with continuous variables. In the standard cosmological model, the dynamic quantum steering of continuous variables plays the role of a witness throughout the cosmic history, from the era of primordial nucleosynthesis to the epoch of large-scale structure formation. It can encode historical information about potential spacetime structures, presenting promising applications in observational cosmology. Therefore, investigating the behavior of quantum steering is crucial for understanding the history and destiny of the universe.

The organization of this paper is as follows. In Sect. 2, we quantify the strength of Gaussian quantum steering using the criterion of "Rényi-2 entropy". In Sect. 3, we describe Gaussian channels in the background of a 1 + 1 dimensional Robertson–Walker expanding universe. In Sect. 4, we investigate the effects of cosmic expansion on two-mode Gaussian steering. Finally, in Sect. 5, we provide a brief summary.

2 Gaussian variable quantum steering quantified by the Rényi-2 entropy

For the measurement of Gaussian quantum steering, we propose a new measurement scheme. Initially, we consider sharing a two-mode Gaussian state between Alice (denoted A) and Bob (denoted B). Since all Gaussian properties can be defined in terms of the symplectic form of the covariance matrix, the properties of this Gaussian state σ_{AB} can be expressed in the following form of the covariance matrix,

$$\sigma_{AB} = \begin{pmatrix} \mathcal{A} & \mathcal{C} \\ \mathcal{C}^{\mathrm{T}} & \mathcal{B} \end{pmatrix}, \tag{1}$$

when σ_{AB} satisfies condition $\sigma_{AB} + i(\Omega_{AB}) \geq 0$, it can this quantum system be described. A obtains M_A through measurement, and only when each pair of local observables R_A (obtained by measuring $\mathcal{A}$ yields r_A) can violate the joint probability $P(r_A, r_B|R_A, R_{B,\rho_{AB}}) = \sum_\lambda \wp_A \wp(r_A|R_A, \lambda) P(r_B|R_B, \rho_\lambda)$ [22], the two-body state ρ_{AB} is controllable (meaning A can steer B). In other words, when $\wp_\lambda$ ranges over all measurement outcomes, at least one pair (R_A and R_B) fails to satisfy the above equation. Here, $\wp(r_A|R_A, \lambda)$ and $\wp_\lambda$ represent probability distributions, while the conditional probability distribution is denoted by $P(r_B|R_B, \rho_\lambda)$. Therefore, only when A performs Gaussian measurements can violations occur

$$\sigma_{AB} + i(0_A \oplus \Omega_B) \geq 0. \tag{2}$$

Violating condition Eq. (2) enables the Gaussian state σ_{AB} to steer $A \to B$. Thus, violating inequality Eq. (2) is both necessary and sufficient for implementing quantum steering from $A \to B$. After A measures the quantum state, we can define Gaussian quantum steering from $A \to B$ in the following manner [38]

$$G^{A\to B}(\sigma_{AB}) := \max\left\{0, \sum_{j:\bar{v}_j^B<1} \ln(\overline{V_j}^B)\right\}, \tag{3}$$

where $\{\bar{V}_j^B\}$ represents the eigenvalues of the A submatrix of σ_{AB}. After measuring A, quantum steering from $A \to B$ disappears only when the state described by σ_{AB} is uncontrollable. However, when it fails to satisfy condition 2, quantum steering can often be quantitatively determined. When B being measured has only one mode, Gaussian quantum steering from $A \to B$ can be quantified using the following equation."

$$\begin{aligned} \mathcal{G}^{A\to B}(\sigma_{AB}) &= \max\left\{0, \frac{1}{2}\ln\frac{\det A}{\det \sigma_{AB}}\right\} \\ &= \max\{0, S(A) - S(\sigma_{AB})\}. \end{aligned} \tag{4}$$

The term $S(A) = \frac{1}{2}\ln(\det A)$ is referred to as the Rényi-2 entropy. By exchanging A and B in Eq. (4), we obtain Gaussian quantum steering from $A \to B$. Its outcome resembles Eq. (4). The distinction between quantum steering and quantum entanglement lies in the asymmetry of quantum steering. A quantum state may be controllable from A to B, but not necessarily vice versa. If a controllable state from A to B is shared between them, meaning A trusts B while B does not trust A, then through classical correlations and local measurements, it can be determined that the state is a Steerable state.

3 Description of cosmic expansion through Gaussian channels

We choose a 1 + 1 dimensional Robertson–Walker expanding universe as our background. Its metric is typically denoted by $g_{\mu\nu}$, with a representing the scale factor. By introducing conformal time η, which is linked to the cosmological time t, we can express the metric of the Robertson–Walker expanding universe as [34–37]

$$ds^2 = [a(\eta)]^2(d\eta^2 - dx^2). \tag{5}$$

The form of the scale factor is as follows

$$[a(\eta)]^2 = 1 + \epsilon(1 + \tanh(\sigma\eta)). \tag{6}$$

In Eq. (6), the parameters σ and ϵ characterize the volume and velocity of the expanding universe, respectively. When the conformal time $\eta \to -\infty$, corresponding to the universe at the infinitely distant past, the metric is $ds^2 = d\eta^2 - dx^2$, indicating flat spacetime. In the distant future, as $\eta \to +\infty$, the curvature of the universe's spacetime is determined by the metric $ds^2 = (1+2\epsilon)(d\eta^2 - dx^2)$. Therefore, in these two limits, the definition of the timelike Killing vector and the particle content of the field can be established.

In the Robertson–Walker expanding spacetime, the scalar field $\Phi(x,\eta)$ satisfies the Klein–Gordon equation.

$$(\Box + m^2)\Phi = 0, \tag{7}$$

where $\Box = \frac{1}{\sqrt{|g|}}\partial_\mu\sqrt{|g|}g^{\mu\nu}\partial_\nu$. When $\eta \to -\infty$, we solve the Klein–Gordon equation to obtain a set of modes u^{in} in the infinitely distant past ("in" region). When $\eta \to +\infty$, we solve the Klein–Gordon equation to obtain a set of modes u^{out} in the infinitely distant future ("out" region).Utilizing the inner product, we express the Bogoliubov transformation between u_k^{out} and u_k^{in}.

$$u_k^{\text{in}}(x,\eta) = \alpha_k u_k^{\text{out}}(x,\eta) + \beta_k u_{-k}^{\text{out}*}(x,\eta), \tag{8}$$

where the Bogoliubov coefficients are given by

$$\alpha_k = \sqrt{\frac{\omega_{\text{out}}}{\omega_{\text{in}}}}\frac{\Gamma([1-(i\omega_{\text{in}}/\sigma)])\Gamma(-i\omega_{\text{out}}/\sigma)}{\Gamma([1-(i\omega_+/\sigma)])\Gamma(-i\omega_+/\sigma)}, \tag{9}$$

$$\beta_k = \sqrt{\frac{\omega_{\text{out}}}{\omega_{\text{in}}}}\frac{\Gamma([1-(i\omega_{\text{in}}/\sigma)])\Gamma(i\omega_{\text{out}}/\sigma)}{\Gamma([1+(i\omega_-/\sigma)])\Gamma(i\omega_-/\sigma)}, \tag{10}$$

where Γ represents the Gamma function, $\omega_{\text{in}} = \sqrt{k^2+m^2}$ is frequency of the "in" region, $\omega_{\text{out}} = \sqrt{k^2+m^{2(1+2\epsilon)}}$ is frequency of the "out", and $\omega_\pm = \frac{1}{2}(\omega_{\text{out}} \pm \omega_{\text{in}})$ is frequency superposition of the "in" and "out" regions, respectively. Through simple calculation, the Bogoliubov coefficients satisfy $|\alpha_k|^2 - |\beta_k|^2 = 1$. For convenience, we define $\theta_k^2 = |\frac{\beta_k}{\alpha_k}|^2 = \frac{\sinh^2(\pi\frac{\omega_-}{\sigma})}{\sinh^2(\pi\frac{\omega_+}{\sigma})}$. We obtain the form of $|\alpha_k|$ and $|\beta_k|$ as

$$|\alpha_k|^2 = \frac{1}{1-\theta_k^2}, \quad |\beta_k|^2 = \frac{\theta_k^2}{1-\theta_k^2}, \tag{11}$$

Here, $|\beta_k|^2$ represents the average number of particles created in the "out" mode k. Therefore, $\theta_k^2 \to 0$ indicates that the average number of particles in mode k approaches zero, while $\theta_k^2 \to 1$ indicates that the average number of particles in mode k approaches infinity.

The annihilation and creation operators satisfy

$$b_{\text{in},k} = \alpha_k^* b_{\text{out},k} - \beta_k^* b_{\text{out},-k}^\dagger, \tag{12}$$

$$b_{\text{in},k}^\dagger = \alpha_k b_{\text{out},k}^\dagger - \beta_k b_{\text{out},-k}, \tag{13}$$

The operators $b_{\text{in},k}$ and $b_{\text{in},k}^\dagger$ act on states in the asymptotic past as the bosonic annihilation and creation operators, respectively. Similarly, $b_{\text{out},k}$ and $b_{\text{out},k}^\dagger$ act on states in the asymptotic future. Additionally, $b_{\text{out},-k}$ and $b_{\text{out},-k}^\dagger$ represent the antibosonic annihilation and creation operators, respectively. We use the condition $b_{\text{in},k}|0_k\rangle_{\text{in}} = 0$ to establish the relation between the "in" vacuum state and the "out" vacuum state. Substituting $b_{\text{in},k}$ with Eq. (12), we obtain

$$(\alpha_k^* b_{\text{out},k} - \beta_k^* b_{\text{out},-k}^\dagger)|0_k\rangle_{\text{in}} = 0. \tag{14}$$

In accordance with the normalization condition, the representation of the "in" vacuum state in the asymptotic future is as follows:

$$|0_k\rangle_{\text{in}} = \sum_{n=0}^{\infty} A_n |n_k\rangle_{\text{out}}|n_{-k}\rangle_{\text{out}}, \tag{15}$$

where $A_n = \sqrt{1-\theta_k^2}(\frac{\beta_k^*}{\alpha_k^*})^n$, n_k denotes the boson number, and n_{-k} denotes the antiboson number. This means that an initial vacuum state $|0_k\rangle_{\text{in}}$ evolves into a two-mode squeezed state in the asymptotic future. By rotating the squeezing angle and giving up the phase angle, we obtain [39–42]

$$|0_k\rangle_{\text{in}} = \sqrt{1-\theta_k^2}\sum_{n=0}^{\infty}\theta_k^n|n_k\rangle_{\text{out}}|n_{-k}\rangle_{\text{out}} = U_k|0_k\rangle|0_{-k}\rangle, \tag{16}$$

where U_k denotes a two-mode squeezing operator given by $U_k = \exp[r_k(b_{\text{out},k}^\dagger b_{\text{out},-k}^\dagger - b_{\text{out},k}b_{\text{out},-k})]$, where r_k is determined by $\cosh(r_k) = |\alpha_k|$. It's crucial to highlight that U_k is a Gaussian operation, maintaining the Gaussian nature of the input states. Consequently, Eq. (16) showcases that the expansion of a Robertson–Walker spacetime can be aptly depicted through a Gaussian channel, resembling a bosonic amplification. In the realm of phase space, the action of U_k corresponds to a symplectic transformation

$$S_k = \frac{1}{\sqrt{1-\theta_k^2}}\begin{pmatrix} I_2 & \theta_k Z_2 \\ \theta_k Z_2 & I_2 \end{pmatrix}, \tag{17}$$

where I_2 denotes the unity matrix in 2×2 space, and Z_2 denotes the third Pauli matrix.

4 The effect of the expanding universe on Gaussian steering

To investigate the quantum steering in an expanding universe, consider a quantum system composed of A and B. In the distant past, they shared a pure two-mode squeezed Gaussian state characterized by the following covariance matrix [41]

$$\sigma_{AB}^{\text{in}} = \begin{pmatrix} \cosh(2s)I_2 & \sinh(2s)Z_2 \\ \sinh(2s)Z_2 & \cosh(2s)I_2 \end{pmatrix}. \tag{18}$$

The covariance matrix the Eq. (18) of the bipartite squeezed Gaussian state, characterized by the squeezed parameter s representing the cosmic squeezing, undergoes an extension in the Robertson–Walker spacetime as time progresses and the universe expands. Post-extension, the state asymptotically transitions into four Gaussian states representing distinct modes: the bosonic mode A described by the A, the bosonic mode B described by the B, the anti-bosonic mode $\bar{A}$ described by the anti-A, and the anti-bosonic mode $\bar{B}$ described by the anti-B. The covariance matrix describing the entire system after transformation is as follows [41]

$$\sigma^{\text{out}}_{AB\bar{A}\bar{B}} = [S_{A,\bar{A}} \oplus S_{B,\bar{B}}][\sigma^{\text{in}}_{AB} \oplus I_{\bar{A}\bar{B}}][S_{A,\bar{A}} \oplus S_{B,\bar{B}}]^{\mathsf{T}}. \tag{19}$$

4.1 Bipartite Gaussian steering

For the complete covariance matrix of a four-mode Gaussian state system, the modes A and B are not directly observable. Due to the causal disconnection between the interior and exterior of a black hole's event horizon, We eliminate the modes $\bar{A}$ and $\bar{B}$ from the interior of the event horizon by tracing them out. Consequently, we obtain the covariance matrix for modes A and B as follows

$$\sigma^{\text{out}}_{AB} = \begin{pmatrix} [\cosh(2s) + \theta_k^2]I_2 & \sinh(2s)Z_2 \\ \sinh(2s)Z_2 & [\cosh(2s) + \theta_k^2]I_2 \end{pmatrix}. \tag{20}$$

Combining equations the Eqs. (20) and the (4), we obtain the following analytical expression for the quantum steering between the mode A and the mode B

$$\mathcal{G}(\sigma^{\text{out}}_{AB}) = \frac{1}{2}\ln\left(\frac{(\theta_k^2 + \cosh(2s))^2}{(\theta_k^4 + 2\theta_k^2\cosh(2s) + 1)^2}\right). \tag{21}$$

The analytical expression of quantum steering $\mathcal{G}(\sigma^{\text{out}}_{AB})$ reveals a profound interplay with cosmic expansion parameters. Notably, the expansion volume ϵ, velocity σ, and squeezing parameter s of the expanding universe significantly influence quantum steering. This implies that the effective encoding of quantum steering encapsulates the historical information of relevant cosmological parameters. Furthermore, the parameter θ in the Eqs. (9) and the (10) elucidates that the contribution of particles with different momenta and masses to quantum steering varies.

The Fig. 1a illustrates the Gaussian quantum steering $\mathcal{G}(\sigma^{\text{out}}_{AB})$ between A and B as a function of the expansion volume ϵ and the particle momentum k when $m = \sigma = s = 1$. From Fig. 1a, it can be found that $\mathcal{G}(\sigma^{\text{out}}_{AB})$ initially rapidly decreases with expansion volume ϵ, then gradually stabilizes towards a certain value. Meanwhile, $\mathcal{G}(\sigma^{\text{out}}_{AB})$ increases with increasing bosonic momentum k and approaches a constant value as k becomes sufficiently large. Additionally, by jointly observing momentum k and expansion volume ϵ, it is evident that the quantum steering between bosons with larger momenta is less affected by the cosmic expansion volume ϵ, whereas the quantum steering between bosons with smaller momenta is highly sensitive to the volume of expansion.

The Fig. 1b illustrates the Gaussian quantum steering $\mathcal{G}(\sigma^{\text{out}}_{AB})$ between A and B as a function of the expansion volume ϵ and the particle mass m when $k = \sigma = s = 1$. From Fig. 1b, it is evident that the quantum steering $\mathcal{G}(\sigma^{\text{out}}_{AB})$ initially decreases with increasing bosonic mass m, then increases, exhibiting a minimum point around $m = 0.35$. Furthermore, near the minimum point, the quantum steering is highly sensitive to changes in the mass parameter m. However, when the m is significantly larger than the minimum point, the quantum steering tends towards a constant value.

In our investigation of Gaussian steering within an expanding spacetime, we meticulously compute quantum steering across every conceivable bipartite division within the four-mode quantum system. Initially, by performing traces over the modes in sets A and B, we derive the covariance matrix $\sigma^{\text{out}}_{\bar{A}\bar{B}}$

$$\sigma^{\text{out}}_{\bar{A}\bar{B}} = \begin{pmatrix} [\theta_k^2\cosh(2s) + 1]I_2 & \theta_k^2\sinh(2s)Z_2 \\ \theta_k^2\sinh(2s)Z_2 & [\theta_k^2\cosh(2s) + 1]I_2 \end{pmatrix}. \tag{22}$$

Secondly, taking the trace over the modes B and $\bar{A}$, we get the covariance matrix between the A and the $\bar{B}$

$$\sigma^{\text{out}}_{A\bar{B}} = \begin{pmatrix} [\cosh(2s) + \theta_k^2]I_2 & \theta_k\sinh(2s)I_2 \\ \theta_k\sinh(2s)I_2 & [\theta_k^2\cosh(2s) + 1]I_2 \end{pmatrix}. \tag{23}$$

Using the Eqs. (22) and (23) and employing the Rényi-2 entropy part from the Eq. (2) to calculate the quantum steering, we find that for any parameter taking values greater than zero (as all variables possess physical significance, and therefore should be non-negative), none yield quantum steering values greater than zero. Consequently, according to the Eq. (2), both the $\mathcal{G}(\sigma^{\text{out}}_{\bar{A}\bar{B}})$ and the $\mathcal{G}(\sigma^{\text{out}}_{A\bar{B}})$ are determined to be zero. Additionally, the Fig. 2 shows the distribution of the quantum steering values we calculated. These figures separately calculate the functional relationship between the $\mathcal{G}(\sigma^{\text{out}}_{\bar{A}\bar{B}})$ and the cosmological parameters, and $\mathcal{G}(\sigma^{\text{out}}_{A\bar{B}}) = 0$ and the cosmological parameters using the Rényi-2 entropy method. Through computation and the Eq. (2), we have determined that the $\mathcal{G}(\sigma^{\text{out}}_{\bar{A}\bar{B}}) = 0$ and the $\mathcal{G}(\sigma^{\text{out}}_{A\bar{B}}) = 0$.

Considering a two-mode squeezed Gaussian state, the squeezing parameter s exerts a significant impact on the covariance matrix coefficients of the initial state (Eq. (18)). As s increases, it brings about substantial modifications to the initial state. Moreover, it is observed that even minor adjustments to s can result in considerable changes in the order of magnitude of the steering degree. In Fig. 2d, this is manifested as the greater the value of s, the further the steering degree deviates from zero. This indicates that an increase in the squeezing parameter s results in a more pronounced departure of the steering degree from its initial value, high-

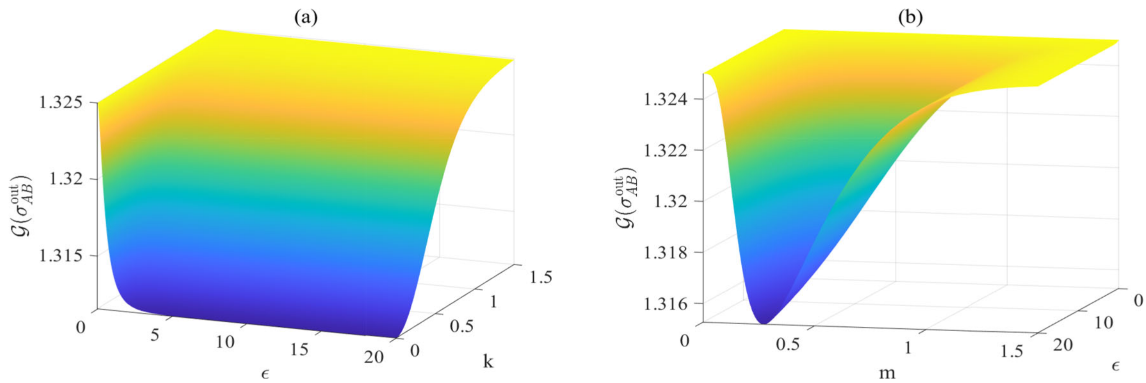

Fig. 1 **a** Quantum steering $\mathcal{G}(\sigma^{\rm out}_{AB})$ between the modes A and B as a function of the expansion volumes ϵ for different momentums k, with the fixed values of $m = \sigma = s = 1$. **b** Quantum steering $\mathcal{G}(\sigma^{\rm out}_{AB})$ as a function of the mass m for different expansion volumes ϵ, with the fixed values of $k = \sigma = s = 1$

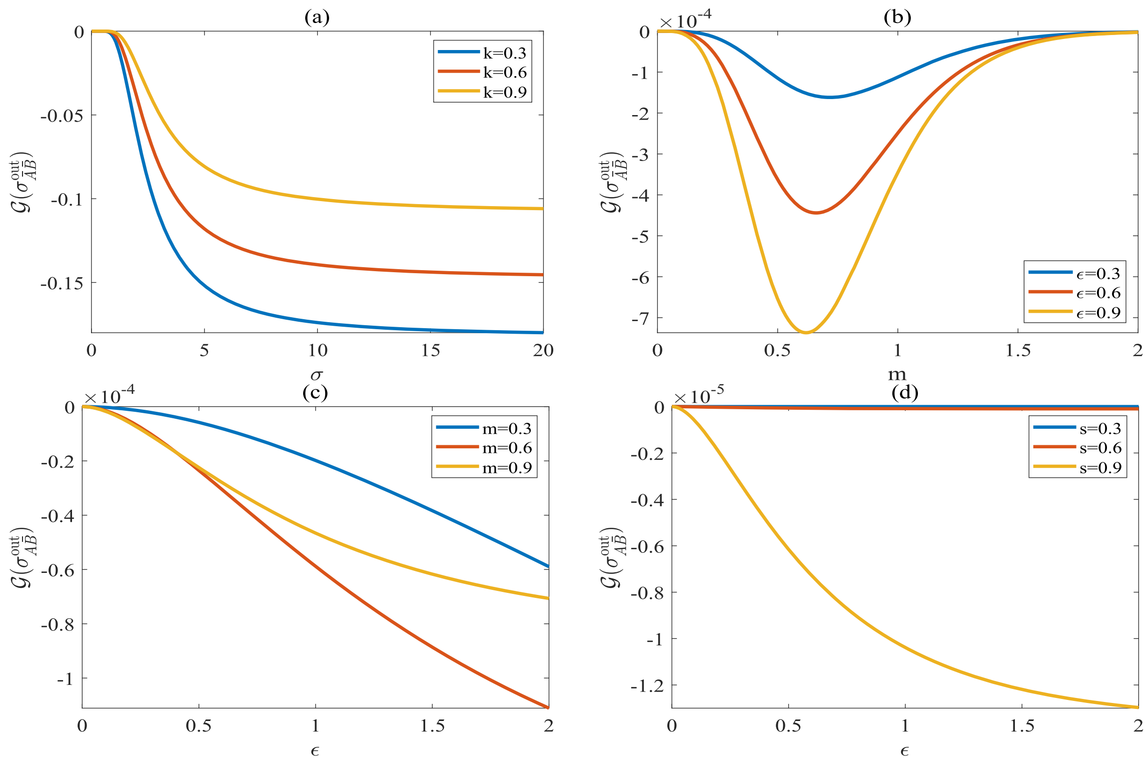

Fig. 2 **a** $\mathcal{G}(\sigma^{\rm out}_{\bar{A}\bar{B}})$ as a function of the expansion rate σ for different momentum k. **b** $\mathcal{G}(\sigma^{\rm out}_{\bar{A}\bar{B}})$ as a function of the mass m for different expansion volumes σ. **c** $\mathcal{G}(\sigma^{\rm out}_{\bar{A}\bar{B}})$ as a function of the expansion volumes σ for different mass m. **d** $\mathcal{G}(\sigma^{\rm out}_{\bar{A}\bar{B}})$ as a function of the expansion volumes σ for different squeezing parameters s

lighting the sensitivity of the system to changes by s. There is a significant order-of-magnitude difference between the states at $s = 0.3$ and $s = 0.6$.

Finally, we focus on quantum steering between the modes A and $\bar{A}$. Tracing over the modes B and $\bar{B}$, we obtain the covariance matrix $\sigma^{\rm out}_{A\bar{A}}$ for A and $\bar{A}$

$$\sigma^{\rm out}_{A\bar{A}} = \begin{pmatrix} [\cosh(2s) + \theta_k^2]I_2 & 2\theta_k \cosh^2(s) Z_2 \\ 2\theta_k \cosh^2(s) Z_2 & [\theta_k^2 \cosh(2s) + 1]I_2 \end{pmatrix}. \qquad (24)$$

Using Eqs. (4) and (24), we obtain an analytic expression of quantum steering between the modes A and $\bar{A}$

$$\mathcal{G}(\sigma^{\rm out}_{A\bar{A}}) = \frac{1}{2} \ln \left(\frac{(\theta_k^2 {\rm sech}(2s) + 1)^2}{(\theta_k^2 - 1)^4} \right). \qquad (25)$$

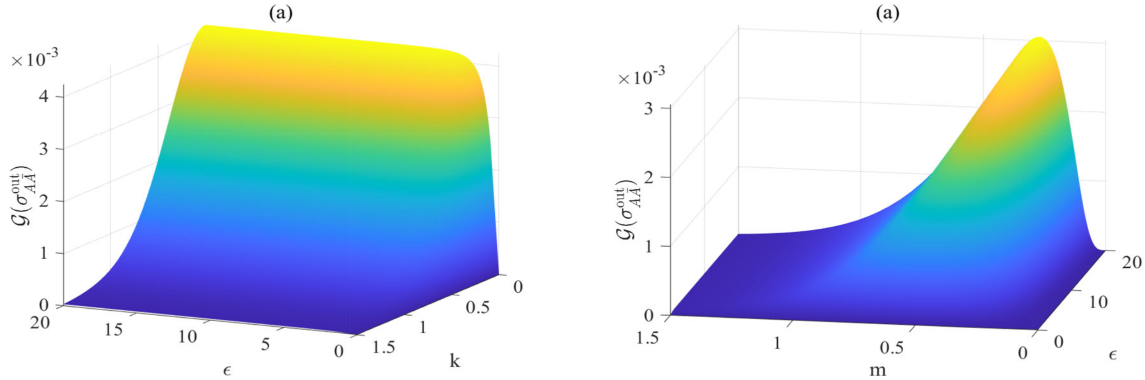

Fig. 3 **a** Quantum steering $\mathcal{G}(\sigma^{\text{out}}_{A\bar{A}})$ between the mode A and the mode $\bar{A}$ as functions of the expansion rate σ and the momentum k for the fixed values of $m = \sigma = s = 1$. **b** Quantum steering $\mathcal{G}(\sigma^{\text{out}}_{A\bar{A}})$ as functions of the mass m and the expansion volume ϵ for the fixed values of $k = \sigma = s = 1$

Through computation and the Eq. (2), we have determined that $\mathcal{G}(\sigma^{\text{out}}_{\bar{A}\bar{B}}) = 0$ and $\mathcal{G}(\sigma^{\text{out}}_{A\bar{B}}) = 0$. Our study on the presence or absence of quantum steering between the two-mode is determined through computation. Correspondingly, negative values appear in the quantum steering for some cases. Thus, we have defined and set the calculated negative results to zero. These figures provide evidence for some negative cases, representing that we have conducted precise calculations and analyses on the modes where quantum steering does not exist, thereby concluding that there is indeed no quantum steering between these modes.

From equation Eq. (25), we plot the functional relationship between the steering $\mathcal{G}(\sigma^{\text{out}}_{A\bar{A}})$ of mode $A \to \bar{A}$ in Gaussian quantum steering and various parameters. In the Fig. 3a, we observe that when $p = s = m = 1$, as the momentum k approaches infinity, the quantum steering between particles disappears. However, with the expansion of the universe volume ϵ, the quantum steering $\mathcal{G}(\sigma^{\text{out}}_{A\bar{A}})$ rapidly increases and tends towards stability. In the Fig. 3b, when $p = s = k = 1$, as the boson mass m increases, the quantum steering $\mathcal{G}(\sigma^{\text{out}}_{A\bar{A}})$ first increases and then decreases, exhibiting a maximum point. However, as m approaches infinity, the quantum steering vanishes. Meanwhile, with the increase in the expansion volume ϵ, the quantum steering between particles with quantum steering is strengthened.

By exchanging the diagonals $\mathcal{A}$ and $\mathcal{B}$ of the covariance matrices in Eqs. (20), (22)–(24), we can obtain the following four covariance matrices:

$$\sigma^{\text{out}}_{BA} = \begin{pmatrix} [\cosh(2s) + \theta_k^2] I_2 & \sinh(2s) Z_2 \\ \sinh(2s) Z_2 & [\cosh(2s) + \theta_k^2] I_2 \end{pmatrix}, \quad (26)$$

$$\sigma^{\text{out}}_{\bar{B}\bar{A}} = \begin{pmatrix} [\theta_k^2 \cosh(2s) + 1] I_2 & \theta_k^2 \sinh(2s) Z_2 \\ \theta_k^2 \sinh(2s) Z_2 & [\theta_k^2 \cosh(2s) + 1] I_2 \end{pmatrix}, \quad (27)$$

$$\sigma^{\text{out}}_{\bar{B}A} = \begin{pmatrix} [\theta_k^2 \cosh(2s) + 1] I_2 & \theta_k \sinh(2s) I_2 \\ \theta_k \sinh(2s) I_2 & [\cosh(2s) + \theta_k^2] I_2 \end{pmatrix}, \quad (28)$$

$$\sigma^{\text{out}}_{\bar{A}A} = \begin{pmatrix} [\theta_k^2 \cosh(2s) + 1] I_2 & 2\theta_k \cosh^2(s) Z_2 \\ 2\theta_k \cosh^2(s) Z_2 & [\cosh(2s) + \theta_k^2] I_2 \end{pmatrix}. \quad (29)$$

By comparison, we find that covariance matrices σ^{out}_{AB} and σ^{out}_{BA} are equal, and covariance matrices $\sigma^{\text{out}}_{\bar{A}\bar{B}}$ and $\sigma^{\text{out}}_{\bar{B}\bar{A}}$ are equal. This indicates that the quantum steering between them is equivalent. However, covariance matrices $\sigma^{\text{out}}_{A\bar{A}}$ and $\sigma^{\text{out}}_{\bar{A}A}$ are not equal. Through computation, we find $\mathcal{G}(\sigma^{\text{out}}_{\bar{A}A})$ is less than zero while $\mathcal{G}(\sigma^{\text{out}}_{A\bar{A}})$ is greater than zero. This result indicates that the quantum steering between A and $\bar{A}$ is not equivalent, and it also proves the correctness of quantum steering singularity.

5 Conclutions

We have studied the redistribution of quantum steering for continuous variables in an expanding spacetime. We get the continuous variables description for a quantum state evolution under the influence of the expansion of a Robertson–Walker spacetime. When the quantum state evolves from the asymptotic past to the asymptotic future, the dynamical steering contains historical information concerning the expanding spacetime. Four cases are considered in here: the bosonic mode A observed by Alice; the bosonic mode B observed by Bob; the antibosonic mode $\bar{A}$ observed by anti-Alice; the antibosonic mode $\bar{B}$ observed by anti-Bob. We find that quantum steering is more sensitive to the expansion rate than the expansion volume. We show the redistribution of the initial steering: quantum steering between the modes A and B decreases with the growth of the expansion rate and the expansion volume; at the same time, quantum steering between the modes A and $\bar{A}$ (or B and $\bar{B}$) can be generated by the expansion of the underlying spacetime. This means that the loss of quantum steering can be interpreted as a redistribution of the initial steering into multipartite quantum correlations. We calculate that $\mathcal{G}(\sigma^{\text{out}}_{A\bar{A}})$ is not equal to $\mathcal{G}(\sigma^{\text{out}}_{\bar{A}A})$, which directly proves that the quantum steering singularity is true in an expanding spacetime. According to the analysis of quantum steering, choosing the particles with the smaller momentum and the optimal mass is a better way to extract information about the expanding universe. These results can guide the simulation of the expanding universe in different quantum systems.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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