

$B^- \rightarrow K_1^{*-}(1270)(\rightarrow \rho^0 K^-)\ell^+\ell^-$ in LEET

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Abstract Flavor changing neutral current (FCNC) decays of the B -meson are a very useful tool for studying possible physics scenarios beyond the standard model (SM), where of the many FCNC modes radiative, purely leptonic and semi-leptonic decays of the B -meson are relatively clean tests. Within this context, the BELLE collaboration has measured the process $B \rightarrow K^*\gamma$ and also searched for the $B \rightarrow K_1(1270)\gamma$ process. Theoretical analyses of these processes are yielding similar values of the relevant form factors. In this work we have used this upper bound in studying the angular correlations for the related semi-leptonic decay mode $B^- \rightarrow K_1^{*-}(1270)(\rightarrow \rho^0 K^-)\ell^+\ell^-$, where we have used the form factors that have already been estimated for the $B \rightarrow K_1(1270)\gamma$ mode. Note that the additional form factors that are required were calculated using large energy effective theory (LEET).

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1 Introduction

Flavor changing neutral current (FCNC) decays of the B -meson are an important tool for investigating possible physics scenarios beyond the standard model (SM), where such decays are forbidden at tree level. It is for this reason that FCNC processes are very sensitive to possible small corrections that may be the result of any modification to the SM, or from some new interactions. Of the FCNC decays the radiative mode $B \rightarrow K^*\gamma$ has been experimentally measured, with a lot of theoretical work also having gone into its study. A related decay, $B \rightarrow K^*\ell^+\ell^-$, has also been observed experimentally. This latter process offers many

more observables for confrontation with the theory (such as forward-backward (FB) asymmetries, polarizations and angular correlations between the final state particles etc.), where the theoretical work for this subject has spawned many investigations. Recently the radiative mode $B \rightarrow K_2^*\gamma$ has also been observed with good limits also being set on the modes $B \rightarrow K_1(1270, 1400)\gamma$, where the K_1 are the 1^+ resonances. The numbers for these rates are comparable to those for the $K^*(890)$ resonance case and we may expect that with more data becoming available the related process $B \rightarrow K_1\ell^+\ell^-$ would be observed just as with the $K^*(890)$ case. Furthermore, as with the $K^*(890)$, such data will provide an independent opportunity to test the predictions of the SM.

Radiative and dileptonic decays of the B -meson involving $b \rightarrow s$ quark transitions are described most efficiently by an effective Hamiltonian approach which involves the relevant Wilson coefficients. Possible new physics beyond the SM will change the values of these coefficients and may possibly bring about new terms with new Wilson coefficients in the effective Hamiltonian. The decay rate of a process like $B \rightarrow K_1\ell^+\ell^-$ is related to the diagonal elements of the density matrix, but the angular distributions of the process involving the decay products of the K_1 will involve the off-diagonal elements as well. The transition amplitudes for these processes involve bilinear combinations of the Wilson coefficients, and clearly then the angular distributions involve many more combinations than the decay rate. The study of the angular distributions is thus a deeper probe of the structure of the effective Hamiltonian, and thus more suited to observing signatures of new physics beyond the SM. Similar studies involving the $K^*(890)$ have already been done in the literature (Kim and Yoshikawa [1]).

In this paper we study the angular distribution of the rare B -decay $B^- \rightarrow K_1^{*-}(1270)(\rightarrow \rho^0 K^-)\ell^+\ell^-$, which may be expected to be observed in future B -factories. We use the standard effective Hamiltonian approach, and we use the

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form factors that have already been estimated for the corresponding radiative decay $B \rightarrow K_1(1270)\gamma$ [2–4]. The additional form factors for the dileptonic channel are estimated using the large energy effective theory (LEET), which enables one to relate the additional form factors to the form factors of the radiative mode. Our results provide, just like in the case of the $K^*(890)$ resonance, an opportunity for a straightforward comparison of the basic theory with experimental results, which may be expected in the near future for this channel. Recall that the physical states $K_1^-(1270)$ and $K_1^-(1400)$ are mixtures of the $K_{1A}^-(^3P_1)$ and $K_{1B}^-(^1P_1)$ states [2]:

$$\begin{aligned} K_1^-(1270) &= K_{1A}^- \sin \theta + K_{1B}^- \cos \theta, \\ K_1^-(1400) &= K_{1A}^- \cos \theta - K_{1B}^- \sin \theta, \end{aligned} \quad (1)$$

which defines the mixing angle θ between the states. Note that for $\bar{K}^0$ the mixing angle will be the same, but for the $S = 1$ states, K^0 and K^+ , the mixing angle would change sign, since under charge conjugation the K_{1A} and K_{1B} have a relative negative sign. The actual value of θ in turn depends on the phase choice for K_{1A} and K_{1B} , which in turn decides the relative sign of the decay constants for them. We shall follow the convention of reference [2], since we shall use the values of the form factors calculated in that paper with the same convention. Other investigations have sometimes followed the same convention (for example, Cheng et al. [5]) and have sometimes taken the same sign for the decay constants (see, for example, Cheng and Yang [6] and Wang et al. [7]). We also note that if one considers the decay of B^+ , our results apply with all three momenta reversed, since we are considering a CP invariant effective Hamiltonian. Cheng et al. [2] proposed two possible solutions for this angle,¹ namely $\theta = \pm 37^\circ$ and $\pm 58^\circ$. Of these possibilities the negative values of the mixing angles predict the branching ratio (BR) of $B \rightarrow K_1(1400)\gamma$ to be larger than that of $B \rightarrow K_1(1270)\gamma$, which is disfavored from experimental data (although not ruled out). In our work, however, we have considered all possible mixing angles given in reference [2].

The paper is organized as follows. In Sect. 2 we shall give the relevant effective Hamiltonian and the LEET form factors for the process under consideration. In Sect. 3 we shall give the expressions for the differential decay rate for the semi-leptonic decay mode under consideration. Finally, we shall conclude with our results in Sect. 4.

2 The effective Hamiltonian and form factors

The short distance contribution to the decay $B \rightarrow K_1 \ell^+ \ell^-$ is governed by the quark level decay $b \rightarrow s \ell^+ \ell^-$, and where

¹For an alternative approach to the K_1 resonances see [8].

the SM operator basis can be described by the effective Hamiltonian:

$$\begin{aligned} \mathcal{H}^{\text{eff}} &= \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \left[(2C_{7R} m_b \left(\bar{s}_R i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_L \right) (\bar{\ell} \gamma_\mu \ell) \right. \\ &\quad + 2C_{7L} m_b \left(\bar{s}_L i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \right) (\bar{\ell} \gamma_\mu \ell) \\ &\quad + C_9^{\text{eff}} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell) \\ &\quad \left. + C_{10} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell) \right]. \end{aligned} \quad (2)$$

We can rewrite the above effective Hamiltonian in the following form:

$$\begin{aligned} \mathcal{H}^{\text{eff}} &= \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \left[2C_{7L} m_b \left(\bar{s}_L i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R \right) (\bar{\ell} \gamma_\mu \ell) \right. \\ &\quad + 2C_{7R} m_b \left(\bar{s}_R i \sigma_{\mu\nu} \frac{q^\nu}{q^2} b_L \right) (\bar{\ell} \gamma_\mu \ell) \\ &\quad + (C_9^{\text{eff}} - C_{10}) (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell_L) \\ &\quad \left. + (C_9^{\text{eff}} + C_{10}) (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell_R) \right], \\ &= \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} \left[-2iC_{7R} m_b \frac{q^\nu}{q^2} (T_{\mu\nu} + T_{\mu\nu}^5) (\bar{\ell} \gamma^\mu \ell) \right. \\ &\quad - 2iC_{7L} m_b \frac{q^\nu}{q^2} (T_{\mu\nu} - T_{\mu\nu}^5) (\bar{\ell} \gamma^\mu \ell) \\ &\quad + (C_9^{\text{eff}} - C_{10}) (V - A)_\mu (\bar{\ell}_L \gamma^\mu \ell_L) \\ &\quad \left. + (C_9^{\text{eff}} + C_{10}) (V - A)_\mu (\bar{\ell}_R \gamma^\mu \ell_R) \right], \end{aligned} \quad (3)$$

with

$$V_\mu = \frac{1}{2} (\bar{s} \gamma_\mu b), \quad (4)$$

$$A_\mu = \frac{1}{2} (\bar{s} \gamma_\mu \gamma_5 b), \quad (5)$$

$$T_{\mu\nu} = \frac{1}{2} (\bar{s} \sigma_{\mu\nu} b), \quad (6)$$

$$T_{\mu\nu}^5 = \frac{1}{2} (\bar{s} \sigma_{\mu\nu} \gamma_5 b). \quad (7)$$

In (3) we have used the $(V - A)$ structure for the hadronic part (except for C_7). Note that this structure does not change under the transformation $V \leftrightarrow -A$ and $T_{\mu\nu} \leftrightarrow T_{\mu\nu}^5$. Furthermore, we can relate the hadronic factors of $T_{\mu\nu}$ and $T_{\mu\nu}^5$ by using the identity²:

$$\sigma_{\mu\nu} = -\frac{i}{2} \varepsilon^{\mu\nu\rho\delta} \sigma_{\rho\delta} \gamma_5.$$

²Here we have used the convention that $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and that $\varepsilon_{0123} = 1$.

In this work we shall closely follow the notation used by Kim et al. [9] by defining the form factors of $K_1(1270)$ by [9, 10]

$$\begin{aligned} \langle K_1(p') | \bar{s} \gamma_\mu b | B(p) \rangle \\ = -f \epsilon_\mu^* - a_+ (\epsilon^* \cdot p) (p + p')_\mu \\ - a_- (\epsilon^* \cdot p) (p - p')_\mu, \end{aligned} \quad (8)$$

$$\begin{aligned} \langle K_1(p') | \bar{s} \gamma_\mu \gamma_5 b | B(p) \rangle \\ = -i g \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} (p + p')^\lambda (p - p')^\sigma \\ = 2i g \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} p^\lambda (p')^\sigma, \end{aligned} \quad (9)$$

$$\begin{aligned} \langle K_1(p') | \bar{s} \sigma_{\mu\nu} \gamma_5 b | B(p) \rangle \\ = g_+ \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (p + p')^\sigma \\ + g_- \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\lambda} (p - p')^\sigma \\ + h \epsilon_{\mu\nu\lambda\sigma} (p + p')^\lambda (p - p')^\sigma (\epsilon \cdot p), \end{aligned} \quad (10)$$

$$\begin{aligned} \langle K_1(p') | \bar{s} \sigma_{\mu\nu} b | B(p) \rangle \\ = -i g_+ [\epsilon_\nu^* (p + p')_\mu - \epsilon_\mu^* (p + p')_\nu] \\ - i g_- [\epsilon_\nu^* (p - p')_\mu - \epsilon_\mu^* (p - p')_\nu] \\ - i 2h (p_\mu p'_\nu - p_\nu p'_\mu) (\epsilon^* \cdot p). \end{aligned} \quad (11)$$

From the above equations we can observe that there are seven form factors which govern the $B \rightarrow K_1$ transition. Note that these form factors have not been tabulated previously in the literature, where Cheng et al. [2] have only tabulated the form factors required for the radiative mode, namely $B \rightarrow K_1 \gamma$. We shall here use the LEET to evaluate all the form factors required for the $B \rightarrow K_1$ transition from this radiative mode.

Cheng and Chua have parameterized the tensorial form factors for the $B \rightarrow K_1$ transition as [2]

$$\begin{aligned} \langle K_{1A,1B}(p') | \bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B(p) \rangle \\ = i \epsilon_{\mu\nu\lambda\rho} \epsilon^{*\nu} P^\lambda q^\rho Y_{A1,B1} + (\epsilon_\mu^* P \cdot q - P_\mu \epsilon^* \cdot q) Y_{A2,B2} \\ + \epsilon^* \cdot q \left[q_\mu - P_\mu \frac{q^2}{P \cdot q} \right] Y_{A3,B3}, \end{aligned} \quad (12)$$

where $P = p + p'$ and $q = p - p'$. The K_{1A} and K_{1B} states are the angular momentum eigenstates as defined in (1). Using (1) we can define the physical $B \rightarrow K_1(1270)$ form factors as

$$\begin{aligned} Y_i^{B \rightarrow K_1(1270)} = Y_{Ai}(q^2) \sin \theta + Y_{Bi}(q^2) \cos \theta, \\ i = 1, 2, 3. \end{aligned} \quad (13)$$

The parameterizations of the form factors Y_i , as used by Cheng and Chua, are given in Appendix A. These form factors can be related to the tensorial form factors given in

equations (9)–(12) by

$$Y_1^{B \rightarrow K_1(1270)} = -g_+, \quad (14)$$

$$Y_2^{B \rightarrow K_1(1270)} = -g_+ - g_- \frac{q^2}{P \cdot q}, \quad (15)$$

$$Y_3^{B \rightarrow K_1(1270)} = g_- + h(P \cdot q). \quad (16)$$

Using the LEET approach as given by Charles et al. [11] we can obtain the following relations between the form factors:

$$f = 2MEg, \quad (17)$$

$$g_+ = -gM, \quad (18)$$

$$g_- = gM, \quad (19)$$

$$a_+ = -a_- = -(g + hM), \quad (20)$$

where M is the mass of the parent hadron and E is the energy of the daughter hadron. We now define all the form factors in terms of just two independent form factors (g and h).

Using the LEET relations as given in (17)–(20) and (13)–(16), the form factors for the $B \rightarrow K_1$ transition can be related to Cheng and Chua's form factors by³

$$\begin{aligned} g_+ &= -Y_1^{B \rightarrow K_1(1270)}, \\ g_- &= Y_1^{B \rightarrow K_1(1270)}, \\ g &= \frac{Y_1^{B \rightarrow K_1(1270)}}{M}, \\ f &= 2E Y_1^{B \rightarrow K_1(1270)}, \\ h &= \frac{Y_3^{B \rightarrow K_1(1270)} - Y_1^{B \rightarrow K_1(1270)}}{M^2 - m_V^2}, \\ a_+ = -a_- &= \frac{Y_1^{B \rightarrow K_1(1270)} m_V^2 - Y_3^{B \rightarrow K_1(1270)} M^2}{M(M^2 - m_V^2)}, \end{aligned} \quad (21)$$

where M is the mass of the B -meson and m_V is the mass of the K_1 .

3 Kinematics and differential decay rate

In the following it is convenient to define our kinematics in terms of the following vectors:

$$P = p' = p_\rho + p_K, \quad Q = p_\rho - p_K,$$

$$L = p_+ + p_-, \quad N = p_+ - p_-.$$

³It is important to note that the notation of the Levi-Civita tensor in Cheng and Chua's paper [2] differs from the notation of Kim et al. [9] by a overall negative sign. We are following the notation of Kim et al.

Subsequently the decay mode $K_1 \rightarrow \rho K$ can be parameterized by the matrix element [12]

$$\begin{aligned} \mathcal{M}(K_1(p') \rightarrow \rho(p_\rho)K(p_K)) &= \frac{2g_{K_1\rho K}}{m_{K_1}m_\rho}[(p' \cdot p_\rho)(\epsilon_\rho \cdot \epsilon_{K_1}) - (p' \cdot \epsilon_\rho)(p_\rho \cdot \epsilon_{K_1})] \\ &= \frac{g_{K_1\rho K}}{m_{K_1}m_\rho}[(P^2 + P \cdot Q)(\epsilon_\rho \cdot \epsilon_{K_1}) \\ &\quad - (P \cdot \epsilon_\rho)((P + Q) \cdot \epsilon_{K_1})]. \end{aligned} \quad (22)$$

This matrix element will give the decay width [12]

$$\Gamma_{K_1} = \frac{|g_{K_1\rho K}|^2}{2\pi m_{K_1}^2} q' \left(1 + \frac{2}{3} \frac{q'^2}{m_\rho^2}\right), \quad (23)$$

with $q' = \frac{1}{2m_{K_1}}\lambda^{1/2}(m_{K_1}^2, m_\rho^2, m_K^2)$, and where p' , ϵ_{K_1} and p_ρ , ϵ_ρ are the momentum and polarization vectors of K_1 and ρ , respectively. In the following analysis we shall neglect the masses of the leptons, the kaon and the ρ , where in the above we have used $p' = P$ and $p_\rho = (P + Q)/2$.

Note also that the on-shell K_1 decays into ρ and K with ρ off-shell decaying into pions. If one were to consider the decay of K_1 into an on-shell $K\pi\pi$ and integrate out the relative momenta of the pion–pion pair, the result for any rate Γ is easily seen to be equivalent to considering the corresponding rate $\Gamma(\kappa)$ obtained by considering the decay of K_1 into a K and an off-shell ρ of mass κ and then doing the following convolution integral:

$$\Gamma = \frac{1}{\pi} \int_{4m_\pi^2}^{(m_{K_1}-m_K)^2} d\kappa \frac{m_\rho \Gamma(\kappa)}{|\kappa - m_\rho^2 + im_\rho \Gamma(\kappa)|^2}. \quad (24)$$

The effective ρ mass thus comes down to a much lower value, and the situation becomes similar to the $K^*(890)$ case. The zero mass approximation for the final particles, as used by Kim et al. [1], becomes applicable to the present case.

The final 4-body decay amplitude can be written as the sum of two amplitudes:

$$\mathcal{A} = \left[\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha m_b}{\pi L^2} \left(\frac{g_{K_1\rho K}}{m_{K_1}m_\rho} \right) \right] (\epsilon_\rho)_\beta (\mathcal{A}_R^\beta + \mathcal{A}_L^\beta), \quad (25)$$

where

$$\begin{aligned} \mathcal{A}_R^\beta &= (\bar{\ell}_R \gamma^\mu l_R) (a_R g_{\mu\nu} - b_R P_\mu L_\nu + ic_R \epsilon_{\mu\nu\alpha\beta} P^\alpha L^\beta) \\ &\quad \times \frac{g^{\nu\alpha} - P^\nu P^\alpha / m_{K_1}^2}{P^2 - m_{K_1}^2 + im_{K_1} \Gamma_{\text{tot}}} \\ &\quad \times [(P^2 + P \cdot Q) g_{\alpha\beta} - P_\beta (P + Q)_\alpha], \\ \mathcal{A}_L^\beta &= (\bar{\ell}_L \gamma^\mu l_L) (a_L g_{\mu\nu} - b_L P_\mu L_\nu + ic_L \epsilon_{\mu\nu\alpha\beta} P^\alpha L^\beta) \end{aligned} \quad (26)$$

$$\begin{aligned} &\times \frac{g^{\nu\alpha} - P^\nu P^\alpha / m_{K_1}^2}{P^2 - m_{K_1}^2 + im_{K_1} \Gamma_{\text{tot}}} \\ &\times [(P^2 + P \cdot Q) g_{\alpha\beta} - P_\beta (P + Q)_\alpha]. \end{aligned} \quad (27)$$

The a_R , b_R , c_R and a_L , b_L , c_L can be expressed as

$$\begin{aligned} a_L &= -C_7 [2(P \cdot L)g_+ + L^2(g_+ + g_-)] \\ &\quad + \frac{(C_9^{\text{eff}} - C_{10})f}{2m_b} L^2, \end{aligned} \quad (28)$$

$$b_L = -2C_7(g_+ - L^2 h) - \frac{(C_9^{\text{eff}} - C_{10})a_+}{m_b} L^2, \quad (29)$$

$$c_L = -2C_7 g_+ - \frac{(C_9^{\text{eff}} - C_{10})g}{m_b} L^2, \quad (30)$$

$$\begin{aligned} a_R &= -C_7 [2(P \cdot L)g_+ + L^2(g_+ + g_-)] \\ &\quad + \frac{(C_9^{\text{eff}} + C_{10})f}{2m_b} L^2, \end{aligned} \quad (31)$$

$$b_R = -2C_7(g_+ - L^2 h) - \frac{(C_9^{\text{eff}} + C_{10})a_+}{m_b} L^2, \quad (32)$$

$$c_R = -2C_7 g_+ - \frac{(C_9^{\text{eff}} + C_{10})g}{m_b} L^2. \quad (33)$$

As such the decay rate can be computed, with the result

$$\begin{aligned} &\frac{d^5 \Gamma}{dp^2 dl^2 d\cos\theta_K d\cos\theta_+ d\phi} \\ &= \frac{2\sqrt{\lambda}}{128 \times 256\pi^6 m_B^3} \\ &\quad \times \left| \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha m_b}{\pi L^2} \left(\frac{g_{K_1\rho K}}{m_{K_1}m_\rho} \right) \right|^2 \\ &\quad (|A_R|^2 + |A_L|^2), \end{aligned} \quad (34)$$

where

$$|A_{[L,R]}|^2 = \left(-g_{\alpha\beta} + \frac{(p_\rho)_\alpha (p_\rho)_\beta}{m_\rho^2} \right) (\mathcal{A}_{[L,R]})^\alpha (\mathcal{A}_{[L,R]}^*)^\beta, \quad (35)$$

where the various angles used above are as shown in Fig. 1. Recall that we shall present our results in terms of the vectors P , Q , L and N , by use of the transformation

$$\begin{aligned} p_\rho &\rightarrow \frac{P + Q}{2}, & p_K &\rightarrow \frac{P - Q}{2}, \\ p_+ &\rightarrow \frac{L + N}{2}, & p_- &\rightarrow \frac{L - N}{2}. \end{aligned}$$

Using the kinematics as prescribed in Kim et al. [9], that is, where we set $p = \sqrt{p_{K_1}^2}$, $l = \sqrt{(p_+ + p_-)^2}$ and

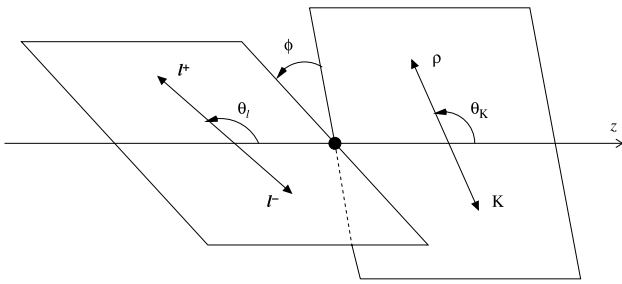


Fig. 1 The definition of the kinematical variables in the decay $B \rightarrow K_1(\rightarrow \rho K)\ell^+\ell^-$

$\lambda = \frac{1}{4}(m_B^2 - p^2 - l^2)^2 - p^2 l^2$. Furthermore, we shall introduce various angles, namely θ_K as the polar angle of the ρ momentum in the rest frame of the K_1 -meson with respect to the helicity axis, i.e. the outgoing direction of K_1 . Similarly for θ_+ , the polar angle of the positron in the dilepton CM frame with respect to the K_1 momentum. And finally ϕ is the azimuthal angle between these planes, that is, the $K_1 \rightarrow \rho K$ and $B \rightarrow K_1 \ell^+ \ell^-$ planes. In this case we have

$$|A_L|^2 = \frac{1}{(P^2 - m_{K_1}^2)^2 + (m_{K_1} \Gamma_{\text{tot}})^2} \times \frac{1}{2} [|a_L|^2 \{ 2(P^2 + 2P \cdot Q) \times ((L \cdot P)^2 - (N \cdot P)^2) + 2((N \cdot Q)^2 - (L \cdot Q)^2 + L^2(P \cdot Q) - N^2(P \cdot Q))P^2 + 4((L \cdot Q)(L \cdot P) + (N \cdot P)(N \cdot Q))(P \cdot Q) + (L^2 - N^2)(P^2 + Q^2)P^2 \} + |b_L|^2 \{ (2(L \cdot P)^2 - 2(N \cdot P)^2 + (N^2 - L^2)P^2)((P^2 + 2(P \cdot Q))(L \cdot P)^2 - (P^2 + (P \cdot Q))^2 L^2 + 2(L \cdot Q)(P \cdot Q)(L \cdot P) - (L \cdot Q)^2 P^2) \} + |c_L|^2 \{ [N^2 Q^2 P^2 - 2(N \cdot Q)^2 P^2 + (2P^4 + (4(P \cdot Q) + Q^2)P^2 + 2(P \cdot Q)^2)P^2 + 2(P \cdot Q)^2](L \cdot P)^2 + 4(N \cdot P)(N \cdot Q)(L \cdot Q) \times (L \cdot P)P^2 - 2(N \cdot P)^2(L \cdot Q)^2 P^2 - [-2(P^4 + (2(P \cdot Q) + Q^2)P^2 + (P \cdot Q)^2)(N \cdot P)^2 + 4(N \cdot Q)(P \cdot Q)(N \cdot P)P^2 + (-2P^2(N \cdot Q)^2 + N^2(P^2 Q^2 - (P \cdot Q)^2) + L^2(2P^4 + (4(P \cdot Q) + Q^2)P^2 + (P \cdot Q)^2))P^2]L^2 \} + 4\text{Re}(a_L b_L^*) \{ -(P^2 + 2(P \cdot Q))(L \cdot P)^3$$

$$- 2(L \cdot Q)(P \cdot Q)(L \cdot P)^2 + (P^2(L \cdot Q)^2 + (P^2 + (P \cdot Q))^2 L^2 + (N \cdot P)(N \cdot Q)(P \cdot Q) + (N \cdot P)^2(P^2 + 2(P \cdot Q)))(L \cdot P) + (L \cdot Q)(N \cdot P)((N \cdot P)(P \cdot Q) - (N \cdot Q)P^2) \} + 4\text{Re}(a_L c_L) \{ (N \cdot Q)(P \cdot Q)(L \cdot P)^2 + (N \cdot P) \times (L \cdot Q)^2 P^2 - ((N \cdot Q)P^2 + (N \cdot P)(P \cdot Q))(L \cdot Q)(L \cdot P) \} - 4\text{Re}(b_L c_L^*) \{ ((L \cdot Q)P^2 - (L \cdot P)(P \cdot Q)) \times (- (N \cdot Q)(L \cdot P)^2 + (L \cdot Q)(N \cdot P)(L \cdot P) + [(N \cdot Q)P^2 - (N \cdot P)(P \cdot Q)]L^2) \} + 4(\widetilde{LN \cdot P \cdot Q})(L \cdot P)(P \cdot Q) - (L \cdot Q)P^2 \} \times (\text{Im}(a_L b_L^*) + (N \cdot P)\text{Im}(b_L c_L^*)) + 4\text{Im}(a_L c_L^*)(\widetilde{LN \cdot P \cdot Q})((N \cdot Q)P^2 - (N \cdot P)(P \cdot Q)) \}, \quad (36)$$

$$|A_R|^2 = \frac{1}{(P^2 - m_{K_1}^2)^2 + (m_{K_1} \Gamma_{\text{tot}})^2} \frac{1}{2} [|a_R|^2 \times \{ 2(P^2 + 2P \cdot Q)((L \cdot P)^2 - (N \cdot P)^2) + 2((N \cdot Q)^2 - (L \cdot Q)^2 + L^2(P \cdot Q) - N^2(P \cdot Q))P^2 + 4((L \cdot Q)(L \cdot P) + (N \cdot P)(N \cdot Q))(P \cdot Q) + (L^2 - N^2)(P^2 + Q^2)P^2 \} + |b_R|^2 \{ (2(L \cdot P)^2 - 2(N \cdot P)^2 + (N^2 - L^2)P^2) \times ((P^2 + 2(P \cdot Q))(L \cdot P)^2 - (P^2 + (P \cdot Q))^2 L^2 + 2(L \cdot Q)(P \cdot Q)(L \cdot P) - (L \cdot Q)^2 P^2) \} + |c_R|^2 \{ [N^2 Q^2 P^2 - 2(N \cdot Q)^2 P^2 + (2P^4 + (4(P \cdot Q) + Q^2)P^2 + 2(P \cdot Q)^2)P^2 + 2(P \cdot Q)^2](L \cdot P)^2 + 4(N \cdot P)(N \cdot Q)(L \cdot Q)(L \cdot P)P^2 - 2(N \cdot P)^2(L \cdot Q)^2 P^2 - [-2(P^4 + (2(P \cdot Q) + Q^2)P^2 + (P \cdot Q)^2)(N \cdot P)^2 + 4(N \cdot Q)(P \cdot Q)(N \cdot P)P^2 + (-2P^2(N \cdot Q)^2 + N^2(P^2 Q^2 - (P \cdot Q)^2) + L^2(2P^4 + (4(P \cdot Q) + Q^2)P^2 + (P \cdot Q)^2))P^2]L^2 \} + 4\text{Re}(a_R b_R^*) \{ -(P^2 + 2(P \cdot Q))(L \cdot P)^3$$

$$\begin{aligned}
& -2(L.Q)(P.Q)(L.P)^2 \\
& + (P^2(L.Q)^2 + (P^2 + (P.Q))^2 L^2 \\
& + (N.P)(N.Q)(P.Q) + (N.P)^2(P^2 + 2(P.Q))) \\
& \times (L.P) + (L.Q)(N.P) \\
& \times ((N.P)(P.Q) - (N.Q)P^2) \} \\
& - 4\text{Re}(a_R c_R) \{ (N.Q)(P.Q)(L.P)^2 + (N.P) \\
& \times (L.Q)^2 P^2 - ((N.Q)P^2 + (N.P)(P.Q)) \\
& \times (L.Q)(L.P) \} + 4\text{Re}(b_R c_R^*) \{ ((L.Q)P^2 \\
& - (L.P)(P.Q))(- (N.Q)(L.P)^2 \\
& + (L.Q)(N.P)(L.P) + [(N.Q)P^2 \\
& - (N.P)(P.Q)]L^2) \} \\
& + 4(\widetilde{LNPQ})(L.P)(P.Q) - (L.Q)P^2 \\
& \times (-\text{Im}(a_R b_R^*) + (N.P)\text{Im}(b_R c_R^*)) \\
& + 4\text{Im}(a_R c_R^*)(\widetilde{LNPQ})(N.Q)P^2 \\
& - (N.P)(P.Q) \}, \quad (37)
\end{aligned}$$

where $(\widetilde{ABCD}) = \varepsilon_{\alpha\beta\gamma\delta} A^\alpha B^\beta C^\gamma D^\delta$.

The Breit–Wigner factors in the last two equations can be replaced, without much numerical error, by delta functions, as follows:

$$\frac{1}{(P^2 - m_{K_1}^2)^2 + (m_{K_1} \Gamma_{\text{tot}})^2} \rightarrow \frac{\pi}{m_\rho \Gamma_\rho} \delta(P^2 - m_\rho^2). \quad (38)$$

The integration in (36) and (37) over p^2 then becomes trivial.

As such, the total decay width can be expressed as

$$\begin{aligned}
\Gamma &= \int_{4m_\ell^2}^{(m_B - m_\rho - m_K)^2} dl^2 \int_{(m_\rho + m_K)^2}^{(m_B - \sqrt{l^2})^2} dp^2 \int_{-1}^1 d(\cos \theta_K) \\
&\times \int_{-1}^1 d(\cos \theta_+) \int_0^{2\pi} d\phi \frac{\sqrt{\lambda}}{128 \times 256\pi^8 m_B^3} \\
&\times \left| G_F V_{tb} V_{ts}^* \frac{\alpha m_b}{l^2} \right|^2 \frac{g_{K_1 \rho K}^2}{m_{K_1}^2 m_\rho^2} \times (|A_R|^2 + |A_L|^2), \quad (39)
\end{aligned}$$

with $\lambda = \frac{1}{4}(m_B^2 - p^2 - l^2)^2 - p^2 l^2$.

However, the most important correlation in this decay, which is sensitive to the effective Hamiltonian, is the dependence of the decay rate on the angle ϕ . As such, we shall concentrate on that correlation and integrate out the polar angles in (36) and (37), something that can be done analytically. The final result can be expressed in the neat form:

$$\begin{aligned}
R(l, \phi) &= \int dp^2 d(\cos \theta_K) d(\cos \theta_+) \frac{2\sqrt{\lambda}}{128 \times 256\pi^8 m_B^3} \\
&\times \left| \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha m_b}{\pi L^2} \left(\frac{g_{K_1 \rho K}}{m_{K_1} m_\rho} \right) \right|^2 \\
&\times (|A_R|^2 + |A_L|^2) \\
&= A_1 + A_2 \cos 2\phi + A_3 \sin 2\phi. \quad (40)
\end{aligned}$$

Here we have

$$\begin{aligned}
A_1 &= \sum_{i=L,R} 16l^2 p^2 (2|a_i|^2 + |B_i|^2 + 2|C_i|^2 \\
&\quad + |D_i|^2 + \text{Re}[B_i D_i^*]), \quad (41)
\end{aligned}$$

$$A_2 = \sum_{i=L,R} (2|a_i|^2 + |C_i|^2), \quad (42)$$

$$A_3 = \sum_{i=L,R} \text{Im}[C_i^* a_i], \quad (43)$$

with $C_i = \sqrt{\lambda} c_i$, $D_i = -a_i$ and $B_i = (b_i \lambda)/(pl)$.

4 Results

The form factors for the radiative mode $B \rightarrow K_1(1270)\gamma$, as given by Cheng and Chua [2], assumed that the physical states $K_1(1270)$ and $K_1(1400)$ were mixtures of the angular momentum eigenstates K_{1A} and K_{1B} , where the mixing angle between these states is not known precisely (though it is believed to be such as to cause maximal mixing between the states). The hypothesis of mixing between the two states naturally explains the suppression of one of the decay modes with respect to the other. In reference [2] the mixing angles suggested were $\theta = \pm 37^\circ, \pm 58^\circ$. The negative values of the mixing angles suggest the suppression of $B \rightarrow K_1(1270)\gamma$ as compared to $B \rightarrow K_1(1400)\gamma$, which is disfavored (although not conclusively ruled out), from the observation of the radiative decay mode $B \rightarrow K_1(1270, 1400)\gamma$ by the BELLE collaboration [13].

Although the prescription of mixing between the states helps to explain the suppression of one of the modes, as compared to the other, the form factors as given in [2] predict a lower value of the branching ratio for $B \rightarrow K_1(1270)\gamma$ as compared to the experimental results. Note that there have been many attempts [14–16] to address this issue, where these attempts essentially predict a much larger value of the zero recoil value of the form factors. For our analysis we have used the form factors as given by Cheng and Chua [2]. Our analytical results for the LEET form factors and the differential decay rate retains the same form for any possible increase in the zero recoil value of the form factors. Our results have been calculated for all possible mixing angles obtained in [2].

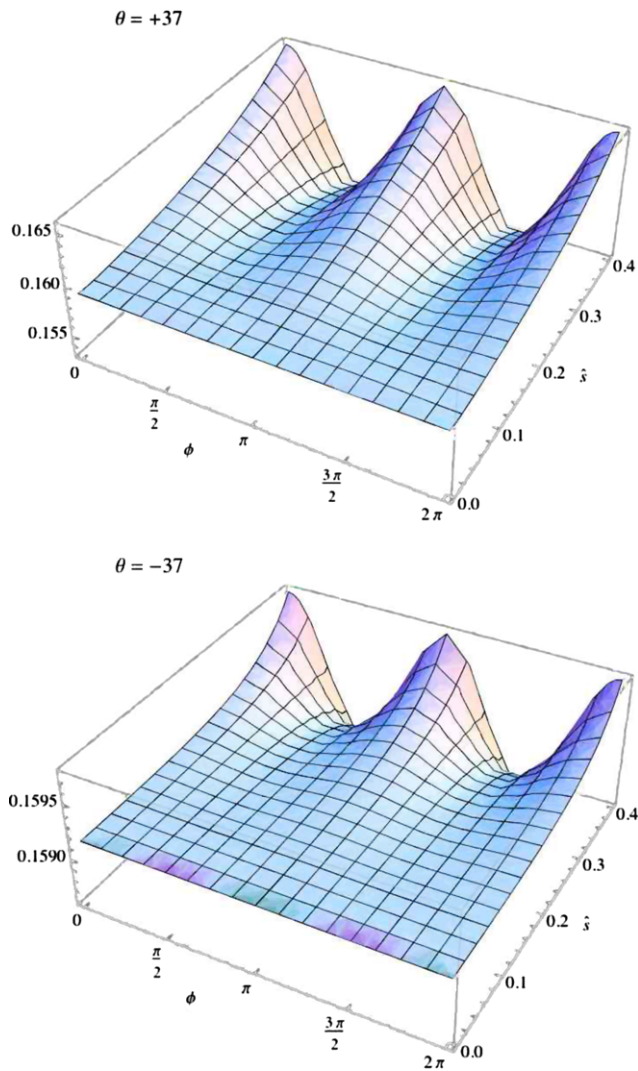


Fig. 2 Plots of $r(\hat{s}, \phi)$ for $C_{7R}/C_{7L} = 0$. The top panel is for $\theta = +37^\circ$ and the bottom panel is for $\theta = -37^\circ$. Note that $\hat{s} = s/m_B^2$

Figures 2, 3 and 4 summarize our results for the correlation between the two decay planes, where we plot the function $r(s, \phi)$, defined by $R(l, \phi)$ (as above) divided by the integral of R over ϕ . The plots are given for mixing angles equal to $+37^\circ$ and -37° . Changing these values to the other possible angle ($+58^\circ$ and -58°) does not greatly affect these results. The graphs are also given for the SM value of the parameter $x = C_{7R}/C_{7L}$, as well as two other values of x . We have included long distance effects due to $c\bar{c}$ resonances, through a modification of C_9 , as was originally done by Kruger and Sehgal [17]. Note that the use of the LEET for obtaining the vector and axial vector form factors is justified for large recoil, i.e. for relatively small values of the dileptonic invariant mass and at the moment we do not have any first principle independent determination of all the form factors required in our calculation. We therefore use the LEET based form factors and restrict ourselves to

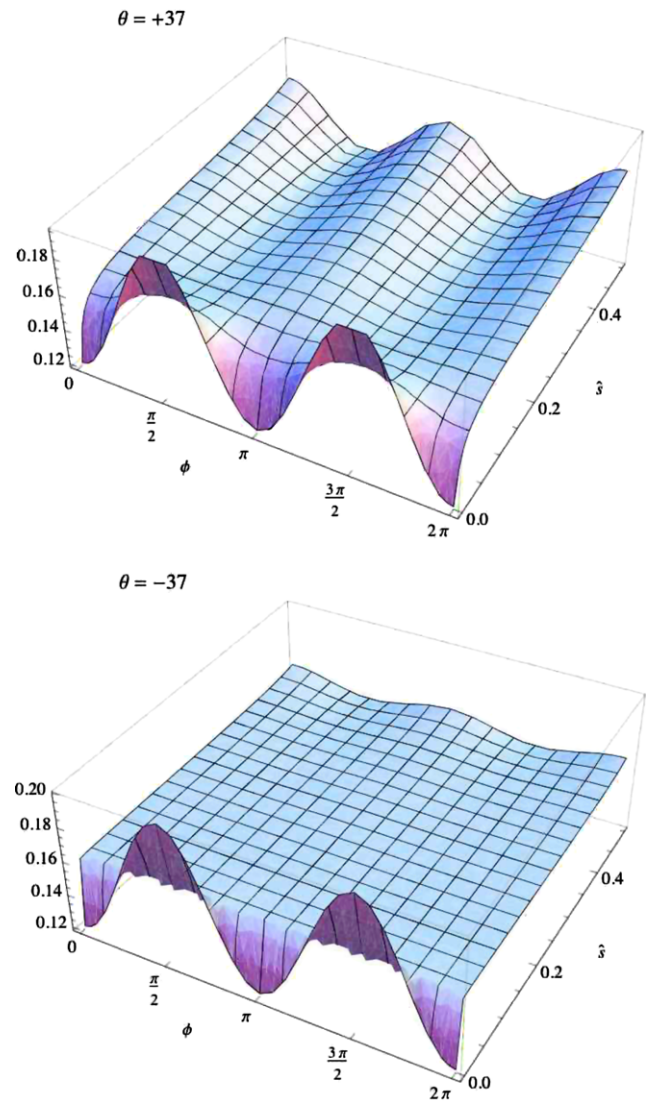


Fig. 3 Plots of $r(\hat{s}, \phi)$ for $C_{7R}/C_{7L} = -1$. The top panel is for $\theta = +37^\circ$ and the bottom panel is for $\theta = -37^\circ$. Note that $\hat{s} = s/m_B^2$

the region $s < 8 \text{ GeV}^2$. As and when form factors become available in the remaining region, our expressions can easily be reevaluated for the entire spectrum.

We therefore continue using the LEET based form factors for the entire range. As and when more accurate values are available for the low s -region our expressions can easily be reevaluated for a fresh plot.

We would like to note, at this point, that for any new physics model that can be effectively absorbed by the “standard” set of Wilson coefficients in the effective Hamiltonian, our analytic results, given in the previous sections, can be used to obtain the corresponding change in the angular correlations.

One final remark: we may integrate our differential decay rate over the final state hadrons to get the decay rate of the process $B \rightarrow K_1 \ell^+ \ell^-$. Note that by using our def-

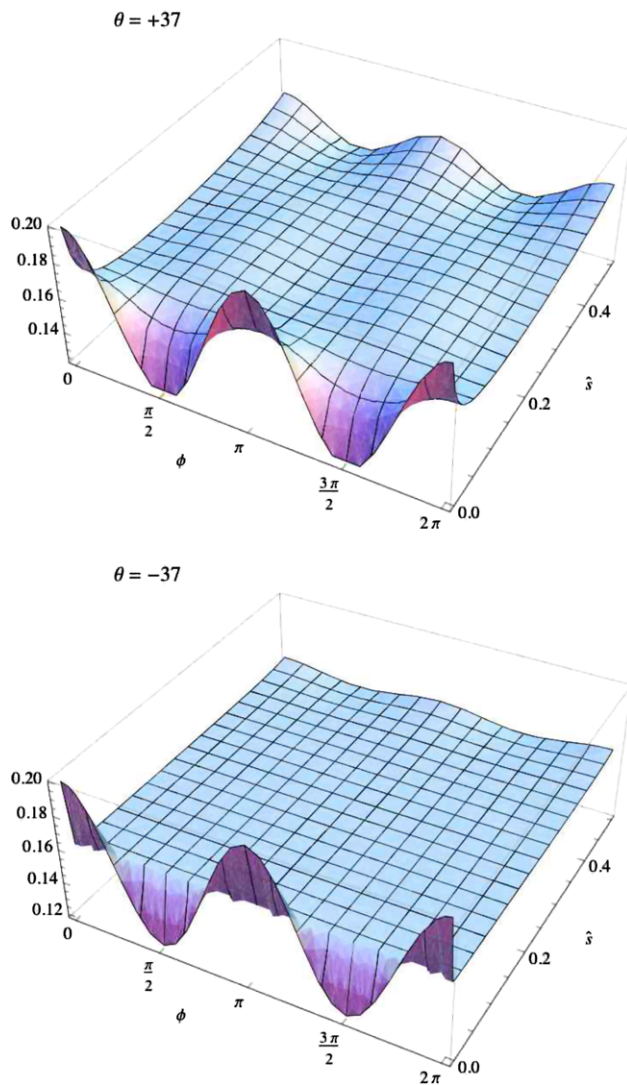


Fig. 4 Plots of $r(\hat{s}, \phi)$ for $C_{7R}/C_{7L} = +1$. The top panel is for $\theta = +37^\circ$ and the bottom panel is for $\theta = -37^\circ$. Note that $\hat{s} = s/m_B^2$

inition of the form factors we can relate these to the ones for $K^*(890)$, under the substitutions $V \leftrightarrow -A$ and $T \leftrightarrow T_5$, such that the corresponding decay rates are obtained easily (by means of this substitution). It is easily seen that the location of the zero in the FB asymmetry of this integrated decay rate is the same as in the $K^*(890)$ case, with the numerical value of the form factors being different. However, though the zeros can be related, the overall shape of the FB asymmetries could be different from $B \rightarrow K^*(890)$. Note that this has been pointed out in reference [18, 19] for the decay $B \rightarrow K^*(890)\ell^+\ell^-$, and that apart from the position of the FB asymmetry, the shape of the FB asymmetry can also be used as a very useful tool for testing beyond minimal flavor violating (MFV) models. In the case of $B \rightarrow K_1\ell^+\ell^-$, as we have remarked above, although the zero position of the FB asymmetry occurs at the same numerical value as in the $K^*(890)$ case, the shape of the FB asymmetry could be

substantially different from the $K^*(890)$ case. As such, the semi-leptonic decay mode can provide a valuable independent test for not only the SM but also for models beyond it.

To conclude, the mode which we have studied above, on SM level, can in principle be measured at present B -factories. Various angular correlations of this mode can also be studied in future SuperB-factories. The study of the various angular correlations in $B^- \rightarrow K_1^-(1270) (\rightarrow \rho^0 K^-) \ell^+ \ell^-$ can provide us with a very useful cross-checking tool for the SM and possible new physics in $b \rightarrow s\ell^+\ell^-$ transitions. In this pursuit we have given the LEET form factors for $B \rightarrow K_1(1270)\ell^+\ell^-$, which could be useful not only in testing the operator structure of the SM but also the existence of possible new physics beyond it.

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Appendix A: The form factors

Firstly, it is important to note that the additional negative sign in (19) is due to the difference in our definition of ε , as compared to that of Cheng and Chua [2], Charles et al. [11] and Kim et al. [9]. Furthermore, note that the definitions of Cheng et al. are the same as that of Charles et al., but differ from that of Kim et al. by a sign, which can be taken into account by changing the sign in (19).

The form factors, as defined in Cheng et al. (for all the form factors except Y_{B3}) [2], are

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}. \quad (\text{A.1})$$

For Y_{B3} we use [2]

$$F(q^2) = \frac{F(0)}{(1 - q^2/m_B^2)(1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2)}. \quad (\text{A.2})$$

The numerical values of the factors appearing in (A.1) and (A.2) are given in Table 1.

Table 1 The form factors for the Y_{A_i} and Y_{B_i} [2]

F	$F(0)$	a	b
Y_{A1}	0.11	0.68	0.35
Y_{A3}	0.19	1.02	0.35
Y_{B1}	0.13	1.94	1.53
Y_{B3}	-0.07	1.93	2.33

Appendix B: Input parameters

Here we present the parameters:

$$m_B = 5.279 \text{ GeV}, \quad m_{K_1} = 1.27 \text{ GeV},$$

$$m_b = 4.7 \text{ GeV}, \quad m_\rho = 0.775 \text{ GeV},$$

$$m_K = 0.494 \text{ GeV}, \quad \Gamma_{\text{tot}} = 90 \text{ MeV},$$

$$\alpha = \frac{1}{129}, \quad G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2},$$

$$|V_{tb} V_{ts}^*| = 0.0385, \quad C_7^{\text{eff}} = -0.3,$$

$$C_{10} = -4.6, \quad \text{and}$$

$$\begin{aligned} C_9 = & 4.153 + 0.381g\left(\frac{m_c}{m_b}, \frac{s}{m_B^2}\right) + 0.033g\left(1, \frac{s}{m_B^2}\right) \\ & + 0.032g\left(0, \frac{s}{m_B^2}\right) - 0.381 \times 2.3 \times \frac{3\pi}{\alpha} \\ & \times \left(\frac{\Gamma_{\psi(2S)} \mathcal{B}(\psi(2S) \rightarrow \ell^+ \ell^-) m_{\psi(2S)}}{s - m_{\psi(2S)}^2 + i m_{\psi(2S)} \Gamma_{\psi(2S)}} \right. \\ & \left. + \frac{\Gamma_{J/\psi(1S)} \mathcal{B}(J/\psi(1S) \rightarrow \ell^+ \ell^-) m_{J/\psi(1S)}}{s - m_{J/\psi(1S)}^2 + i m_{J/\psi(1S)} \Gamma_{J/\psi(1S)}} \right), \end{aligned}$$

where the function g is taken from reference [20–22]:

$$\begin{aligned} g(\hat{m}_i, \hat{s}) = & -\frac{8}{9} \ln(\hat{m}_i) + \frac{8}{27} + \frac{4}{9} \left(\frac{4\hat{m}_i^2}{\hat{s}} \right) \\ & - \frac{2}{9} \left(2 + \frac{4\hat{m}_i^2}{\hat{s}} \right) \sqrt{\left| 1 - \frac{4\hat{m}_i^2}{\hat{s}} \right|} \\ & \times \begin{cases} \left| \ln \left(\frac{1 + \sqrt{1 - 4\hat{m}_i^2/\hat{s}}}{1 - \sqrt{1 - 4\hat{m}_i^2/\hat{s}}} \right) - i\pi \right|, & 4\hat{m}_i^2 < \hat{s}, \\ 2 \arctan \frac{1}{\sqrt{4\hat{m}_i^2/\hat{s} - 1}}, & 4\hat{m}_i^2 > \hat{s}. \end{cases} \end{aligned}$$

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