HALOS OR NO HALOS : Is there "missing mass" hidden in (massive or not) halos in flat galaxies ?

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ABSTRACT : It has often be too easily accepted that a large amount of "adequate" invisible matter should be invoked to explain some phenomena observed in galactic systems. However, the existence of (massive or not) dark halos surrounding the observable matter is not so obvious, in particular as a result of the flatness of rotation curves at large radial distances.

I INTRODUCTION

Since the problem of the total mass in the Universe has appeared as crucial for its future evolution, the determination of the amount of matter contained in systems at various volume scales has been considered with particular attention. This paper is devoted to this problem for flat galaxies.

In spite of a lot of papers published during the last fifty years, the structure and the dynamics of flat galaxies (among which our Milky Way) is not clearly elucidated. It is only generally accepted that the behaviour of such systems is dominated by non-collisionnal stellar dynamics i.e. each star (and the gas) moves in the gravitationnal potential determined by all the other stars. The stars behave like a "non collisionnal gravitationnal plasma". However, this particular "non neutral plasma physics" is not well known. This is the reason why the amount and the distribution of mass in flat galactic discs are matters of discussions; an important question is : is all (or almost all the matter) in the visible stars or is there an important invisible component? Many arguments have been raised in favor of or against the presence of a (more or less) massive hidden component.

In the next section we summerize the main arguments and in the others we discuss the problem of the slope of the rotation curves. The conclusion is that the question is not so simple and that, contrary to what is too smarly stated, the observed flat rotation curves can be those of galaxies wit <u>or without</u> halos.

II WHY MASSIVE HALOS AND WHAT IN THEM ?

The arguments usually invoked in favor of the presence of a large amount of non visible mass around galactic discs are :

** Stability of such discs against the formation of bars, spiral arms, ... (Ostriker et al., 1974; Berman and Mark, 1979; Sellwood, 1983)

** Excitation of warps in galaxies (Bertin and Casertano. 1982)

** The Oort problem (Oort, 1960) : the discrepancy between the counted density at the sun "that implied by dynamics".

** The orbits of very distant stars supposed belonging to galactic systems (Hawkins, 1983)

** The orbits of satellite galaxies around the Milky Way (Lin and Lynden-Bell, 1977)

** Binary galaxy statistics (Turner, 1976)

** The stability of the Local Group (Kahn and Woltjer, 1959)

** The differential variation of the mass to light ratio with radial distance (Gilmore and Hewett, 1983)

** The flatness of rotation curves at large radial distances.

However, the existence of halos is still open to discussion because, first, same arguments do not apply in all cases, and then there are not enough correlated data showing that the same halo enables to give a correct interpretation of the data concerning same objects.

Of course, the nature of the matter these eventual halos can be made of is also open to discussion; various possibilities have been considered : gas, snowballs, dust and rocks, Jupter-like bodies, low mass stars, dead stars, neutron stars, massive neutrinos, gravitinos, monopoles, ... (discussion in a paper by Hegyi and Olive (1983)).

Hereafter we consider the problem from a dynamical point of view for individual galaxies.

III THE PROBLEM OF VISIBLE MASS AND ROTATION CURVES

The amount of mass can be determined from observational data of two types : the rotation curve and the light curve. Both can be determined for steady-state axisymmetric flat systems. In the case of non axisymmetric systems, except for very irregular systems, bars, spiral arms, rings and other similar systems can be considered as transient perturbations involving a small part of the total amount of matter, superposed on an axisymmetric average state. Then, the rotation curve is that curve which gives the velocity v(r) of any test mass eventually travelling along a circular orbit centered at the center of the galaxy and of radius r in the mean plane of the flat system versus the radial distance r.

The rotation curve can be interpreted directly in terms of the gravitationnal potential ϕ through the gravitationnal acceleration γ since, for circular motion,

$$\gamma = v^2(r)/r$$

Then, if one assumes that the stars move in the gravitationnal potential due to them, γ and ϕ can be interpreted in terms of the mass density ρ using the Poisson equation :

$$\Delta\phi = 4\pi G\rho$$

Comparing the mass density distribution to the light curve one can find the spatial variation of the mass to light ratio.

Until last years, the rotation curve was measured for fairly small radial distances, say a tenth of kiloparsec from the center of the galaxy; it was supposed that these curves should show decreasing tails according to Kepler's law ($v(r) \propto 1/\sqrt{r}$) at larger radial distances since, there the test body was probably submitted to the attraction of a point-like mass (Figs 1 and 2). Theoretical models of curves withe Keplerian tails where fitted to the data (Fig. 2), for instance using formulas like those of Brandt and Scheer (1965).

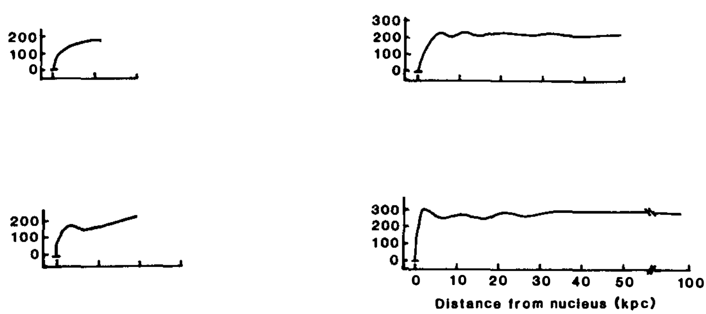
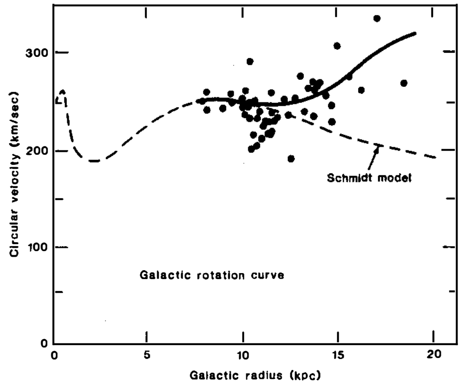


Figure 1. From Rubin, 1983. Rotation curves.



Rotation curve of the Milky Way The filled circles represent data and the solid line a fourth-order least-squares fit to the data. The dashed line is the rotation curve from the Schmidt mass model The Schmidt model fits the rotation curve from the atomic hydrogen data inside the solar circle well, but does not fit the data beyond 12 kpc from the center.

Figure 2
From Blitz
et al., 1983

Recent observations show that for much larger radial distances, the rotation curves usually remain flat : the circular velocity does not decrease like $1/\sqrt{r}$ (Figs 1 and 2). This is often interpreted as revealing the presence of a highly massive invisible halo surrounding the flat system and governing its motion. The following discussion is devoted to this question : Are halos necessary and/or sufficient to produce flat rotation curves? We shall see that, contrary to what is too often thought, the rotation curves can be flat both for systems with or without halos (massive or not).

IV PROPERTIES OF SYSTEMS WITH SPHERICAL SYMMETRY

We now investigate the rotation curves of systems with dynamics governed by a mass system with spherical symmetry.

1/ *Point mass*

First consider a point mass M at the center O of the system. In such a system, Newton's law reads

$$\phi(r) = -GM/r \qquad \gamma = -GM/r^2 = v^2/r$$
$$v(r) = \sqrt{(GM/r)} \propto 1/\sqrt{r}$$

This is illustrated by the system of the planets.

2/*Spherical mass distribution*

Now assume that the mass is distributed in space in such a way that the total mass within a sphere of radius r is m(r). Then, using the Gauss theorem, one finds

$$Gm(r)/r^2 = v^2(r)/r$$
$$v(r) = \sqrt{(Gm(r)/r)} \propto \sqrt{(m(r)/r)}$$

If m(r) tends to some limit m_o when $r \to \infty$, $v(r) \propto 1/\sqrt{r}$, but, conversely, if the rotation curve remains flat at large distances, $v(r) \simeq$ cst leads to $m(r) \propto r$.

Thus, it is important to notice that, if produced by a massive spherical halo, a flat rotation curve implies a halo with space density decreasing like $1/r^2$ and so an <u>infinite mass</u>.

Remark : A homogeneous halo (constant density) corresponds to a mass within the distance r , m(r), equal to $(4/3)\pi\rho r^3$ (where ρ denotes the constant density) and so to a circular velocity $v(r) = r\sqrt{((4/3)G\pi\rho)} \propto r$, which characterizes a rigid rotation.

V EFFECTS OF FLATTENING

Now let us compare the rotation curves produced by the mass distributed 1/ with spherical symmetry 2/ with axisymmetry in an equatorial plane according to the same law m(r) where m(r) respectively denotes the mass inside a sphere of radius r in the former case and that inside a circle of radius r in the latter case. There is a fundamental difference between these two cases. In the former case the acceleration to which a test star is submitted is equal to

$$\gamma(r) = v^2(r)/r = Gm(r)/r^2$$

so that $v^2(r) = \sqrt{(Gm(r)/r)}$

is a local function of r, whereas in the latter case v(r) depends on the whole mass distribution profile m(r) from r = 0 to r ∞, that is <u>inside</u> <u>and</u> <u>outside</u> the circle of radius r.

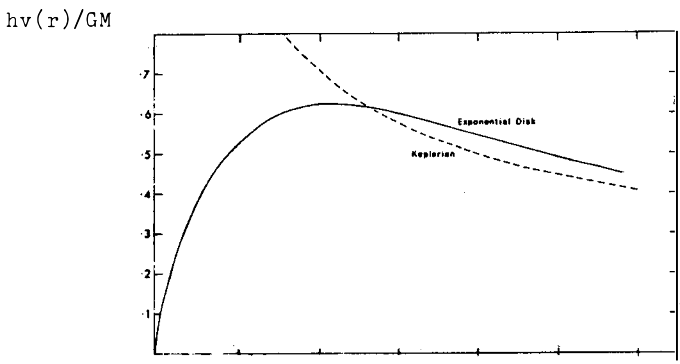
For instance, in the case of an exponential distribution of matter (space density ∝ exp(-r/h), m(r) tends to a limit and v(r) becomes Keplerian as soon as r is of the order of the length scale h, whereas in the case of an exponential disc m(r) tends to a limit at radial distances of the same order but v(r) becomes Keplerian at much larger distances (see Freeman, 1970), since

$$v^2(r) = \frac{1}{2}\left(\frac{r}{h}\right)^2 \{I_o(\frac{r}{h})K_o(\frac{r}{h}) - I_1(\frac{r}{h})K_1(\frac{r}{h})\}$$

where I_o, K_o, I_1, K_1 denote Bessel's functions. (Fig. 3)

Thus, for a given rotation curve, the ratio of the amount of matter contained contained within a radial distance r in a flat model to that in a spherically symmetric model cannot easily compared to 1. Numerical models are necessary.

Lequeux (1983) has compared the distribution of mass corresponding to simple phenomenological models of rotation curves. He has plotted the ratio Y of the mass m(r) inside a sphere of radius r in some models with rotation curves like those indexed by 1 and 2 on Fig. 4 to that in a spherical model with



Exponential disc : $m(r) \propto \{h^2 - (h^2 + hr)\exp(-\frac{r}{h})\}$

Figure 3 - Rotation curves; from Freeman, 1970

same rotation curves versus the radius a of the which separates the two first linear branches with different slopes (measured in units of the radius R at which the slope of curve 2 changes). Model 1 has a still flat rotation curve at large radial distances whereas model 2 has a Keplerian rotation curve for r > R.

Fig. 4 shows that Y is of course equal to 1 in the case of the spherical model, smaller than 1 in the other cases, and smaller in case 2 than in case 1 for any a/R between 0 and 1.

It follows that flat rotation curves can be obtained for flat systems of smaller mass than spherical systems with same rotation curves.

VI DYNAMICAL EFFECTS OF FLATTENING

Other indications of dynamical effects of flattening have recently been pointed out separately by Tohline and Kalnajs (1983).

Tohline (1983) has shown that using numerical simulations he could obtain flat rotation curves and that "cold discs" (i.e. discs of stars mainly in quasi-circular orbits) can be stabilized against bar or spiral arm formation without dark halos if dynamics is governed by interactions between two bodies

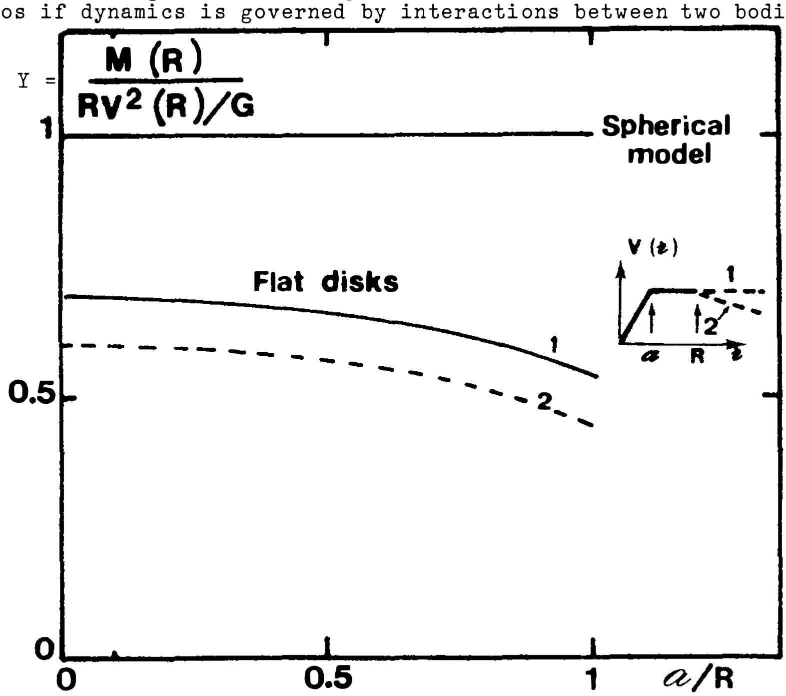


Figure 4 - The mass m(r) inside radius r for various galactic models : spherical model (horizontal line at the top), and flat disk models with rotation curves as defined in the insert Model 2 has a Keplerian rotation curve fo r > R (Lequeux, 1983)

of the form $\frac{Gm_1m_2}{r^2}\{1 + \frac{r}{a}\}$ (where m_1 and m_2 are the masses of the bodies and a is of the order of the kiloparsec), instead of the purely Newtonian interaction law. Typically the correction term is the solution of a two dimensional Poisson equation and describes the attraction between two parallel infinitely long rods.

Kalnajs (1983) could fit data for rotation curves and light curves corresponding to constant mass to light ratios, for three galaxies with only a disk component and for another with a disk and a small bulge component.

Finally, from the results mentioned in Sects V and VI we can conclude that flattening is important for dynamics. Finally, the usual arguments invoked in favor of the presence of massive halos, especially the flatness of the rotation curve at large radial distances are not sufficient and do not enable to decide clearly.

VII DYNAMICALLY SELF CONSISTENT MODELS

Thus, it is necessary to build up dynamically self consistent models in which the potential is that determined by the mass distribution through the Poisson equation and the mass distribution is governed by the motion of each star in the potential determined by all the others.

This can be done using numerical simulation of a large number of individual particles, or, as dicussed hereafter, using a statistical description of the systems based on distribution functions.

The models that will now be considered are concerned with with self consistent, axisymmetric, steady state three dimensionnal though flat star systems. We analyse the possible states : stability is not investigated.

These models are fully dynamical, i. e. each star moves in the potential du to the others and the possible orbits are populated according to distribution functions of the form Yf where

*** f is a function with a functional form satisfying conditions expressing physical properties detailed hereafter

*** y is a function equal to 1 if the maximum radial distance reached by a star is lower than some finite radius R determining the space dimension of the system, and equal to 0 in other cases.The length R is a function of the parameters appearing in f. This last condition introduces a smooth and natural cut-off in the distribution function that tends to 0 when $r \to R$.

The first paper of a series devoted to such models appeared in 1976 (Lafon, 1976). The distribution function adopted is a sum of "coarse grain" distribution functions describing "well-mixed" systems i.e. systems for which all continuous smooth functions (mean velocity, mass density, ...) at each point is independent of the "phase" of each star on its orbit but depends only on the population of the orbits reaching this point ((Lafon, 1976; Mello and Brody, 1972; Lynden-Bell, 1967). It can be expressed under the form

$$Yf(\xi,E) = Y \sum_i \alpha_i \exp(a_i E + b_i \xi)$$

with $\xi = rv/\sqrt{(2\varepsilon_o)}$ $E = (u/2\varepsilon_o)^2 + (\xi/r)^2 + (\phi(r)/\varepsilon_o)$

where u, v, $\phi(r)$, ε_o respectively denote the radial and the tengential velocity components, the gravitationnal potential, some energy unit. It is also assumed that this function satis- the following physical conditions, likely, at least as a zero order approximation, as results from observations:

$$f > 0 \qquad \partial f/\partial E < 0 \qquad \partial f/\partial \xi \neq 0$$

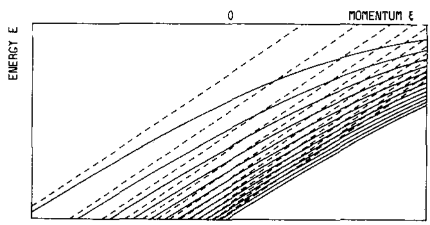
For distribution functions with one term or two terms with opposite signs in the sum, this respectively implies functions of the form (Lafon, 1976) :

$$f(\xi,E) = \alpha \exp(aE + b\xi) \qquad 1$$

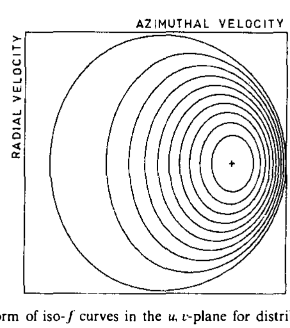
$$f(\xi,E) = \alpha \exp(b\xi)\ (\exp(a_1 E) - \exp(a_2 E)) \qquad 2$$

with $\alpha > 0$, $a < 0$ and $\alpha > 0$, $a_1 < a_2 < 0$ respectively; b was taken arbitrarily positive (changing its sign reverses the rotation of the system).

Iso-f curves corresponding to a function of the latter form are shown on Fig. 5. They look like those derived from observations.



Form of iso-f curves in the ξ, E-plane for distribution functions f



Form of iso-f curves in the u, v-plane for distribution functions f

Figure 5 - Iso-f curves from Lafon, 1976

Numerical models have been constructed for distribution functions of both forms. Typical results are shown on Figs 6 to 17. A special system of units was used used for these curves in order to allow scale changes.

*** Unit of length : arbitrary

*** Unit of potential : B, the potential at the center of the system; $B = \varepsilon_o \phi(0)$

*** Unit of mass : M, that generating a potential equal to B at the distance A; M = BA/G

*** Subsequent unit of velocity : v_o, that of a star with kinetic energy per unit mass equal to B; $v_o^2/2 = B$

All the curves correspond to systems with a total mass equal to 1. Parameters for Figs.6 to 9, 10 to 13, 14 to 17 are displayed in tables 1, 2, 3 respectively.

These results point out the following features :

*** The systems are strongly non linear and self-consistent; rotation curves very similar in the inner part are obtained for fairly different distribution functions. The inner part of the rotation curve is also insensitive to the central mass density. Same remark concerning the curve representing the percentage p(r) of the total mass within the circle of radius r. At the same time, the central density is highly sensitive to the parameters a_i, b of the distribution function.

*** The rotation curve is always flat, sometimes uprising at large radial distances, never Keplerian, without massive halo. The flatness seems linked to the smoothness of the decrease of the distribution function close to the edge of the system. The non Keplerian behaviour appears as an effect of the non circular, highly elliptic with very small ξ reaching large radial distances , which are always present and should never be neglected in the outer parts of the system : less populated non circular orbits leads to smaller circular rotation velocities.

*** This is consistent with a steep density decrease in the outer parts of the system.

Table 1 - Data for Figs 6 to 9 Lafon, 1976

Figure	Curve	a	b	Radius	Total Mass
	1	−5	3.764	1.049	1
	2	−5	3.852	1.304	1
	3	−5	3.814	1.547	1
	4	−5	3.680	1.804	1
	5	−5	3.426	2.055	1
	6	−5	2.850	2.410	1
	7	−5	1.252	2.893	1
	dashed	−5 Ng	2.850	2.397	1
	crosses	−5 Ng	2.850	3.	1.25

Table 2 - Data for Figs 10 to 13 Lafon, 1976

Figure	Curve	a_1	a_2	b	Radius	Total Mass
	1	−3	−0.5	3.868	1.203	1
	2	−3	−0.5	3.808	1.337	1
	3	−3	−0.5	3.205	1.976	1
	4	−3	−1.	4.	1.698	1
	5	−3	−1.	3.617	2.159	1

Table 3 - Data for Figs 14 to 17 Lafon, 1976

Figure	Curve	a_1	a_2	b	Radius	Total Mass
	1	−4	−0.5	4.338	2.789	1
	2	−4	−0.5	4.	3.220	1
	3	−4	−1	4.703	2.884	1
	4	−4	−1	4.127	3.704	1
	5	−4	−1	4.	3.826	1

VIII CONCLUSION

To summerize, observationnal data such as rotation curves and light curves cannot be interpreted in terms of stellar dynamics and galactic structure in a simple direct way : the "inversion problem" is complicated and must be investigated cautiously. In particular, fairly different stellar systems can correspond to curves with very similar arcs.

Then, though the presence of a (massive or not) dark halo cannot be excluded a priori, it is not possible to infer this presence from the only topology (flatness) of the rotation curve.

Additionnal models fitting observationnal data both with rotation curves and light curves related in a self consistent dynamical way are necessary in each particular case. Such models are being investigated now.

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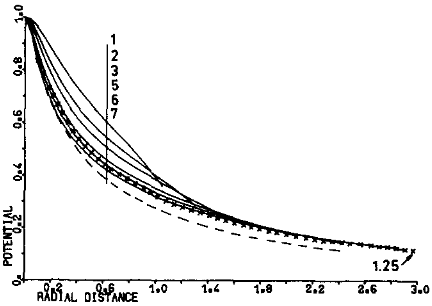
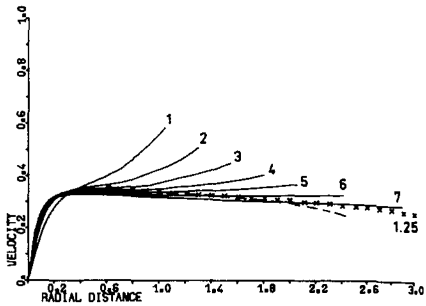


Figure 6



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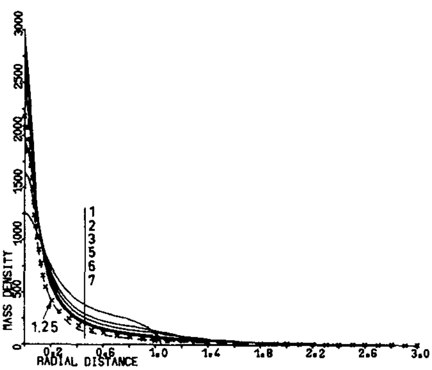

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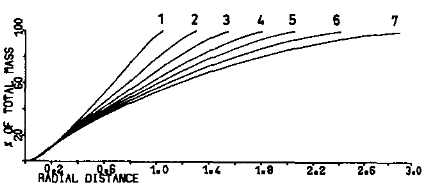


Figure 9

6 Potential versus radial distance for models based on distribution functions of the form 1 using the data given in Table 1 The dashed lines correspond to Ng's (1967) results for models with total mass equal to 1. The crosses correspond to Ng's model with mass 1.25

7 Surface density versus radial distance for models based on distribution functions of the form 1 using the data given in Table 1 The dashed lines correspond to Ng's (1967) results for models with total mass equal to 1. The crosses correspond to Ng's model with mass 1.25

8 Rotation curves for models based on distribution functions of the form 1 using the data given in Table 1 The dashed lines correspond to Ng's (1967) results for models with total mass equal to 1. The crosses correspond to Ng's model with mass 1.25

9 Percentage of total mass within a circle of radius r as a function of r for models based on distribution functions of the form 1 using the data given in Table 1 The dashed lines correspond to Ng's (1967) results for models with total mass equal to 1. The crosses correspond to Ng's model with mass 1.25

Note : NG E.W., Astrophys. J., 150, 1967, 787

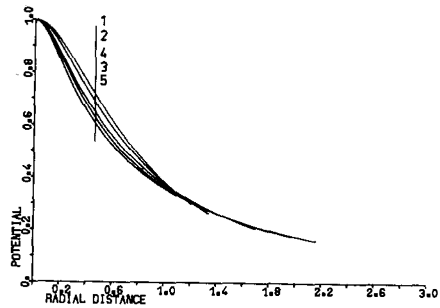
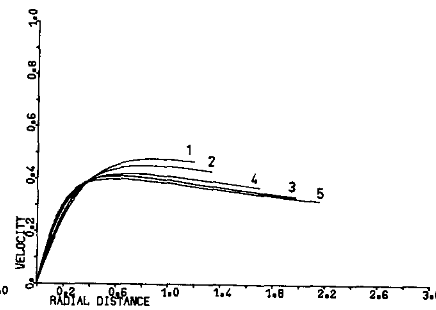


Figure 10



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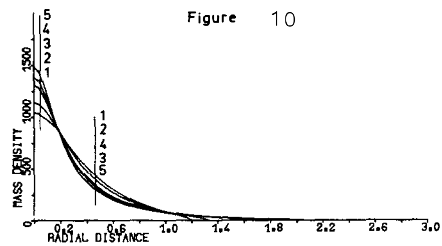


Figure 11

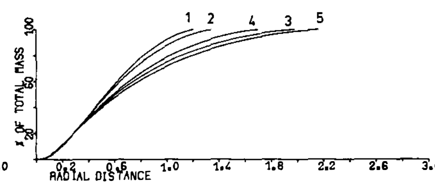


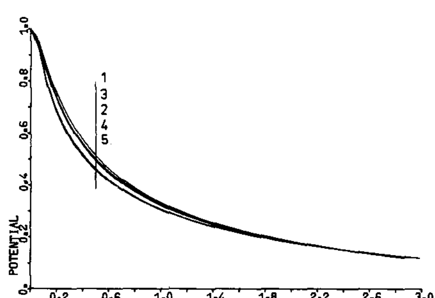
Figure 13

10 Potential versus radial distance for models based on distribution functions of the form 2 using the data given in Table 2

11 Surface density versus radial distance for models based on distribution functions of the form 2 using the data given in Table 2

12 Rotation curves for models based on distribution functions of the form 2 using the data given in Table 2

13 Percentage of total mass within a circle of radius r as a function of r for models based on distribution functions of the form 2 using ie data given in Table 2



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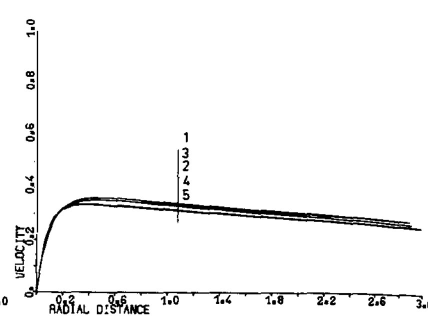

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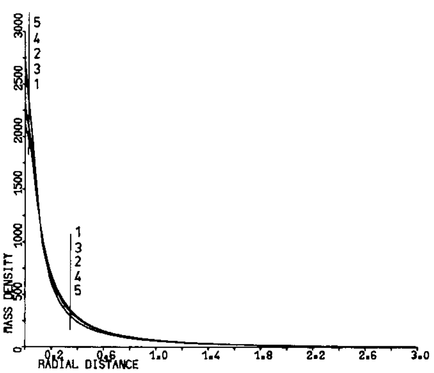

Figure 15

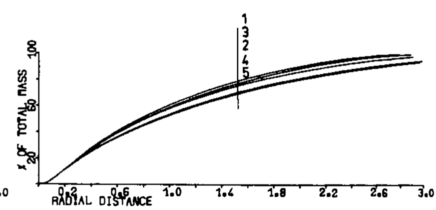

Figure 17

14 Potential versus radial distance for models based on distribution functions of the form 2 using the data given in Table 3

15 Surface density versus radial distance for models based on distribution functions of the form 2 using the data given in Table 3

16 Rotation curves for models based on distribution functions of the form 2 using the data given in Table 3

17 Percentage of total mass within a circle of radius r as a function of r for models based on distribution functions of the form 2 using the data given in Table 3