

# EXTENDED JILES-ATHERTON HYSTERESIS MODEL TO ACCURATELY PREDICT FIELDS IN A RAPID CYCLING SYNCHROTRON DIPOLE MAGNET\*

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## Abstract

Particle accelerators use high field quality magnets to steer and focus beams. Normal conducting magnets commonly use soft iron for the yoke, which is subject to hysteresis effects. It is common practice to use an initialization procedure to accomplish a defined state of the magnet for which its hysteresis behavior must be known. In this article, a variation of the scalar Jiles-Atherton model with an improved physical basis called the Extended Jiles-Atherton model is employed to predict the  $B$ - $H$  trajectories in a Rapid Cycling Synchrotron magnet. Simulations are conducted in COMSOL Multiphysics using the external material feature to integrate the hysteresis model with Finite Element Method. Results from the experimental studies conducted on a magnet prototype are also presented. Finally, potential improvements in the model and extension to the case of a two-dimensional anisotropic material are discussed.

## INTRODUCTION

The Electron Ion Collider (EIC) is a planned flagship collider facility under development at Brookhaven National Laboratory [1]. To steer and focus the beams, normal-conducting and/or super-conducting magnets are needed at critical locations. The material constituting the yoke of a magnet is affected by the phenomenon of hysteresis due to which the present magnetization depends on the past magnetization, and which may further result in a remnant flux density in the core even when the applied current is zero. Accurate prediction of hysteresis trajectories in the magnetic core would facilitate making informed decisions regarding magnet excitation current waveforms.

In this work, a variation of the Jiles-Atherton model called the Extended Jiles-Atherton (EJA) model [2] is used to model hysteresis in the yoke of a dipole magnet. This model is integrated with a commercial finite element method (FEM) software. Results from magnet simulations conducted in the software are compared with those obtained from the experiment setup.

## RAPID CYCLING SYNCHROTRON DIPOLE MAGNET

The EIC facility at BNL is based on the existing Relativistic Heavy Ion Collider (RHIC) complex accompanied by new additions- an electron storage ring (ESR) and an injector, the rapid cycling synchrotron (RCS). EIC will be

capable of colliding up to 275 GeV protons with 18 GeV electrons. The RCS is expected to accelerate electrons from 400 MeV to 18 GeV. Due to the large circumference of the RHIC ring, field in the RCS dipoles is as low as 5 mT at injection. The pole tip field of the quadrupoles and sextupoles is similar.

Magnet field quality of a few units (normalized to 10000 units of the main field component) is desired. At such low fields, field quality is strongly impacted by hysteresis effects in the yoke and stray magnetic fields [3].

To study the effects of hysteresis, stray fields and low material permeability, a test dipole magnet for the RCS was constructed. A picture of this magnet is depicted in Figure 1. The magnet gap size is 120 mm and yoke length is 500 mm. The yoke was constructed using laser cut laminations made from 24 gauge (0.025 inch/ 0.64 mm) M19 C5 electrical steel which were stacked and bonded together with EB-548. The coils consist of 10 AWG insulated square copper wire, and have 80 turns each.

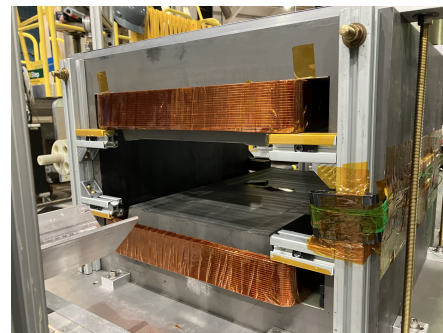


Figure 1: A picture of the test magnet.

In the next section, the EJA model for ferro/ferrimagnetic materials is discussed and material characterization is described.

## EXTENDED JILES ATHERTON MODEL FOR MAGNETIC HYSTERESIS

In this work, the Extended Jiles Atherton (EJA) model [2] was used to predict  $B$ - $H$  trajectories in the test magnet. This model introduces an energy storage factor to address the underlying cause for the prediction of non-physical behavior by Classical Jiles-Atherton (CJA) and its variant models. It was also demonstrated to be numerically more accurate in predicting hysteresis trajectories for various inputs than the CJA, Leite's JA and Modified JA models.

Unlike CJA model, the EJA model accepts flux density and its time derivative as inputs. This aspect of the model

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is advantageous in terms of computational efficiency when it is integrated with a finite element method (FEM) software for simulating electromagnetic studies that generally solves for magnetic vector potential to determine fields in the component under consideration [4].

The mathematical description of this model is as follows. Model state is chosen to be the irreversible component of magnetization,  $M_{irr}$ . Time derivative of  $M_{irr}$  is expressed as

$$\frac{dM_{irr}}{dt} = \frac{(sM_{an} - M) - (s-1)B}{k\mu_0} \left| \frac{dB}{dt} \right| \quad (1)$$

where  $\mu_0$  represents permeability of free space,  $t$  denotes time and the quantities  $M_{an}$ ,  $M$  and  $B$  denote anhysteretic magnetization, total (bulk) magnetization and flux density respectively. For improved accuracy, material parameters  $k$  and  $s$  which represent pinning site density factor and energy storage factor respectively are expressed as functions of  $M_{an}$

$$k = a_k \exp(-M_{an}^2/b_k) \quad (2)$$

$$s = c_s + a_s \exp(-M_{an}^2/b_s) \quad (3)$$

In (2) and (3),  $a_k$ ,  $b_k$ ,  $a_s$ ,  $b_s$ ,  $c_s$  are material constants. The anhysteretic permeability  $\mu_{B,an}$  is modeled as a function of  $B$  based on (36)-(38), (32) and (30) set forth in [5] in the sequential order. The anhysteretic magnetization  $M_{an}$  is then evaluated from  $\mu_{B,an}$  and  $B$  as

$$M_{an} = B \left( 1 - \frac{\mu_0}{\mu_{B,an}} \right) \quad (4)$$

The reversible component of magnetization is modeled as

$$M_{rev} = c(M_{an} - M) \quad (5)$$

where  $c$  is the coefficient of reversible magnetization. The total magnetization then becomes

$$M = M_{irr} + M_{rev} \quad (6)$$

Field intensity is finally evaluated using the relationship between the magnetic quantities

$$H = \frac{B - M}{\mu_0} \quad (7)$$

where the quantities  $B$  and  $M$  will take the unit of T and  $H$  will take the unit of A/m herein.

Material parameter identification was set up as a single-objective optimization problem in MATLAB and solved using Genetic Optimization System Engineering Toolbox (GOSET) [6]. The procedure to identify these parameters is described in detail in [2] with the exception that a set of three symmetric  $B$ - $H$  loops with different peak flux densities at a current excitation frequency of 2 Hz were used as the characterization data in lieu of a ring-down  $B$ - $H$  trajectory. This was done to avoid eddy currents in the electric steel toroid sample from contributing significantly to the total loss. Note that unlike in [2], the search space for parameters  $b_k$  and  $b_s$  was expanded to include negative values to achieve

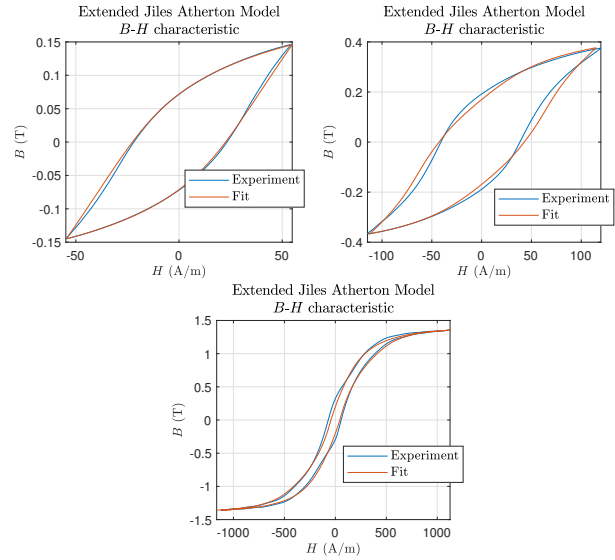


Figure 2: Measured and fitted  $B$ - $H$  characteristic.

better fits. Further, an additional term was added to field intensity in (7) to account for the increase in material loss at low flux densities as a result of laser cutting the toroid laminations [7].

Characterization hysteresis loops and corresponding fits are depicted in Figure 2 (a), (b) and (c). Anhysteresis and hysteresis model parameters for M19 C5 electrical steel obtained from the optimization study are listed in Table 1.

Table 1: Anhysteresis and Hysteresis Model Parameters

Parameter	Value			
$m$	1	2	3	4
$\alpha_m$	0.0607	0.0061	0.0037	0.0015
$\beta_m$	30.9294	76.5184	1.3172	27.2408
$\gamma_m$	1.7114	1.3268	2.7495	1.2315
$\mu_r$	4258.1456			
$a_k$	25.052			
$b_k$	-1.015			
$c$	1531.978			
$a_s$	0.171			
$b_s$	9.99e3			
$c_s$	0.8567			

In the next section, integration of this model with COMSOL Multiphysics to predict hysteresis in magnetic components (herein, a dipole magnet) is set forth.

## FINITE ELEMENT ANALYSIS WITH HYSTERESIS

Studies were simulated in the commercially available FEM software COMSOL Multiphysics 6.2. This software

has a built-in hysteresis model based on a variation of the CJA model. It also offers the option to program a user-defined material model using the ‘External Material’ functionality [8]. This feature was used to integrate the EJA model with COMSOL’s FEM. Yoke material was assumed to be isotropic. Therefore, the same set of equations was used to describe hysteresis in the three axes, and the axes were decoupled. Please note that for analysis and comparison, a 2D FEA study was conducted herein.

In the ‘external material’ function, general  $H(B)$  relation was set as the interface type because the EJA model accepts  $B$  and change in  $B$  as inputs and generates  $H$  as the output. The external material function was coded in C and compiled into a dynamic link library (DLL) [9]. The model output may also be chosen as the model state. Herein, irreversible magnetization in the three axes  $M_{irr,j} : j = \{x, y, z\}$  was chosen as the state vector.

It is also worth noting here that the simulation study is conducted in time-domain unlike the experimental data which was collected in a magneto-static fashion (discussed next). However, since the model is based on static hysteresis and neglects eddy current contribution to total material loss, and with the material resistivity set to 1 S/m in the FEM simulation, the frequency of excitation current is not expected to impact the flux density waveform at any point.

## EXPERIMENTAL SETUP

Magnetic measurements on the test magnet were carried out at BNL magnetic measurement facility using the electric mole developed a long time ago [10]. The mole has a reference radius of 31 mm and a 900 mm long dipole bucked coil.

The magnet coil was excited by a bipolar current  $I_z$  depicted in Figure 3. The current ramp rate was set to 1 A/s (1.6 mT/s). To ensure consistency in data, measurements were repeated over five cycles. Flux density at the center of the good field region  $B_{gfr}$  was approximated from this measured data. Its linear approximation was then evaluated by a line segment connecting the extreme points in the measured  $B_{gfr}$ - $I_z$  loop. Finally, deviation of the actual flux density from the linear approximation is calculated at all data points.

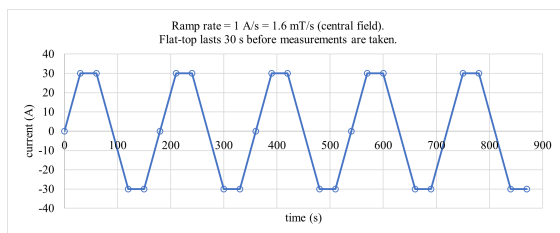


Figure 3: Current profile.

## RESULTS

A hysteresis study was also conducted in COMSOL Multiphysics wherein the magnet coil was excited by a sinusoidal

current of 30 A peak at a frequency of 1 Hz. The current was repeated over five cycles to establish steady state. Linear approximation of flux density at the centre of the good field region and corresponding deviation were similarly evaluated. The results obtained from the experimental setup and COMSOL simulation are compared in Figure 4. It can be seen that the general profile of  $\Delta B_{gfr}$  versus  $I_z$  is similar in the two cases. However, the FEM model over-estimates deviation in flux density by a maximum of  $\approx 70\%$  at  $I_z \approx 12$  A).

It is suspected that a possible reason behind this mismatch in the experimentally collected data and simulation results is that the anhysteresis model used herein is based on material permeability that is represented as a monotonically decreasing function of flux density. In reality, it has been seen [11] that the permeability initially increases with flux density, reaches a maximum value and then falls off with flux density. It is believed that incorporating this behavior of anhysteretic permeability will result in a more accurate field prediction.

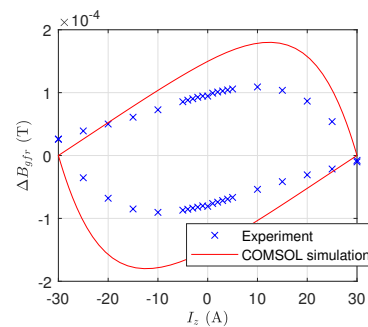


Figure 4: Comparison between results from experiment and simulation.

## CONCLUSION

In this article, simulation results from the EJA model integrated with an FEM software were compared with those obtained from experiments conducted on test dipole magnet. It was seen that the model over-predicts deviation in  $B_{gfr}$  which was attributed to the difference in actual and simulated anhysteretic material permeability. A very small percentage of the difference may also be attributed to slight anisotropy of the non-grain oriented steel used to construct the magnet yoke.

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