# BEAM EXTRACTION FROM FFAG SYNCHROTRONS


**L. W. Jones (*) and K. M. Terwilliger (*)**

Midwestern Universities Research Association, Madison, Wis. (**)

*(presented by F. T. Cole)*


## I. INTRODUCTION

Two different methods of beam extraction from FFAG synchrotrons are considered here, a fast pulsed system which brings the beam out in one turn with a high particle density in betatron phase space, suitable for injection into a storage ring; and an adiabatic method, a modification of the Piccioni - Wright system [1], which can give essentially continuous extraction from fixed field synchrotrons with somewhat lower betatron phase space density.

## II. PULSED EXTRACTION

A rapid one-turn pulsed extraction system, employing a localized perturbing magnetic field turned on fast compared to a particle revolution, has been considered for use with pulsed synchrotrons [2]. It appears possible with such a system to extract with a high betatron phase space density, useful for experiments and necessary for injection into a storage ring. A similar system can be used with FFAG synchrotrons with possible difficulties due to the extreme non-linearity of the FFAG magnetic fields. If large radial oscillations are employed in the extraction process, for example, the radial and vertical betatron phase areas are likely to distort excessively and reduce the effective phase density of the extracted beam. Large radial oscillations may even cause the vertical motion to become unstable and the beam to be lost before extraction. This non-linear problem can obviously be avoided by keeping all oscillations small, well inside the stability limits, for any part of the process involving motion through an appreciable distance in the non-linear fields.

Such a pulsed extraction system, as applied to a spiral sector FFAG synchrotron, has been investigated with the MURA IBM-704 computer. A pulsed magnetic field, uniform over a small azimuthal region, induces a radial betatron oscillation which is inside the stability limit but large enough to send the particles past a current sheet into a d.c. deflecting field. The impulse given by the d.c. field then directs the beam down a field-free region between the spiral magnets and out of the machine. As indicated below, the whole process causes negligible distortion of the betatron phase areas.

The parameters of the spiral sector machine considered are given in Table I. The parameters were chosen to keep the betatron oscillation frequencies reasonably far from the strong non-linear inherent sector resonances. Phase plots of the coupled radial and vertical betatron oscillations in this machine, obtained with the computer, are shown in Fig. 1. The plots are made at the azimuthal positions $N\theta = \pi, 3\pi, \ldots$, i.e., midway between the magnets when $x = 0$.

TABLE I

Spiral sector machine parameters

| | |
|---|---|
| Median plane field : $H = (1+x)^k \left[ 1 + \cos\left( N\theta - \frac{1}{w}\ln(1+x) \right) \right]$ where $x$ is in units of the radius. | |
| $N = 30$ | $\sigma_x = 0.56\pi$ |
| $k = 53$ | $\sigma_y = 0.48\pi$ |
| $1/w = 301$ | |
| $\tan\gamma = Nw = 0.1$ (spiral angle to circle) | |

---

(*) University of Michigan, Ann Arbor, Mich.


(**) Supported by the United States Atomic Energy Commission.

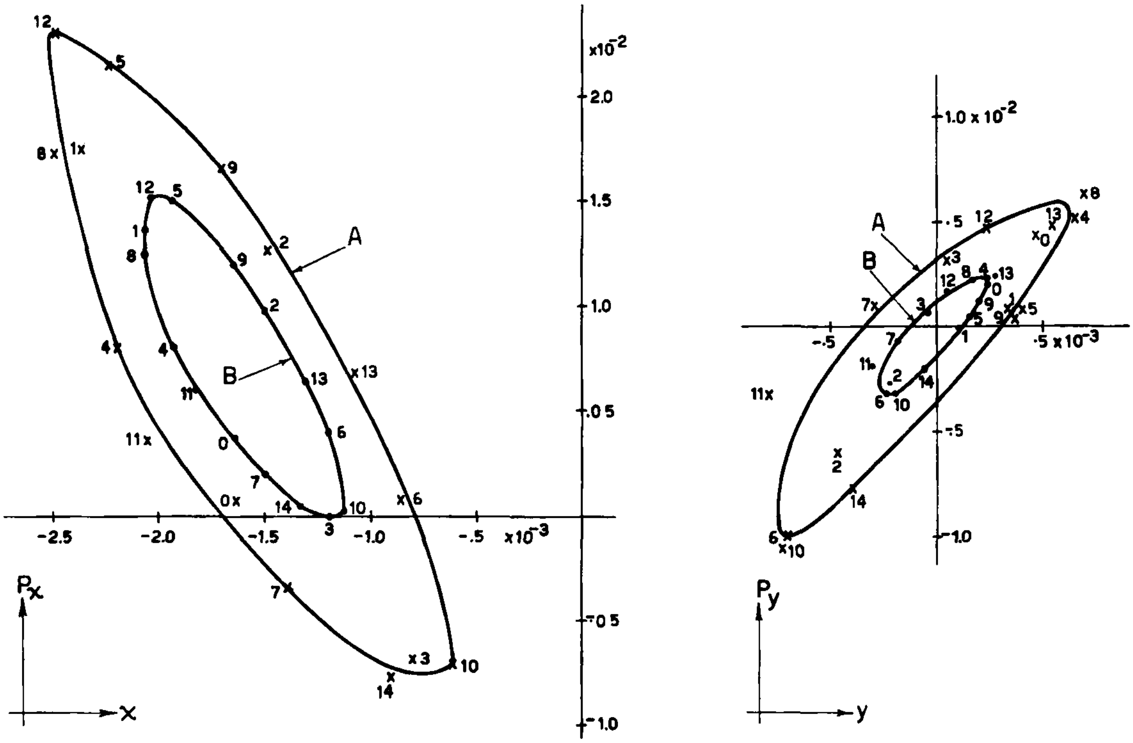


**Fig. 1** Betraton oscillation phase plots for coupled motion in the accelerator of Table I. Plots are made at periodic azimuths $\theta_m = \frac{(2m+1)\pi}{N}$ midway between the magnets. Curves labeled the same correspond to the same computer run. Displacements are measured in units of the radius, the momenta angles measured from a circle (for small angles) are in radians.

The curves rotate, of course, in going through a sector. Curves labeled the same correspond to the same computer run. Curves (A) indicate some scatter of the phase points, particularly in the $y$ plot; the motion becomes unstable if either the $x$ or $y$ amplitude is much larger than given by the curves (A). The curves (B) are reasonably well defined.

During acceleration the betatron phase volume occupied by the injected beam will damp due to the increase in momentum. If the $x$ and $y$ motions are approximately uncoupled, as in curves (B), and if there are assumed to be no losses, the $x$ and $y$ phase areas occupied by the beam will each damp by a factor $\frac{p}{p_0} = \frac{\beta\gamma}{\beta_0\gamma_0}$. Hence, with injection at 50 MeV and acceleration to 15 GeV, the separate phase areas will each decrease by a factor of 50, and the oscillation amplitudes by a factor of 7. The resulting damped $x$ phase space area at 15 GeV, from injection into (B) at 50 MeV, is shown in Fig. 2.

The radial oscillation to send the beam past the current septum into the d.c. extraction channel can be produced by a pulsed magnetic field from a coil inside the vacuum tank, the coil being on the end of a delay line. Monoenergetic 15 GeV protons can be deviated an angle $3 \times 10^{-3}$ radians by a uniform 1 kG field extended over 1.75 m, a field which can be turned on in a small fraction of a revolution. The resulting radial motion of the impulsively displaced beam will be as indicated in Fig. 2. The shapes of the scaled down $x$ and $y$ phase areas, as seen on such periodic $(\theta_m = \frac{(2m+1)\pi}{N})$ phase plots, will be essentially unaltered during the motion due to the approximate linearity of the oscillations. As shown in Fig. 2, after one sector, the beam center will move out $\Delta x = 4 \times 10^{-4}$ with a radial distance between the edges of the perturbed and unperturbed beams of $3 \times 10^{-4}$. For a machine radius of 70 m this corresponds to a net radial separation of 2 cm, enough room for a current sheet to generate the d.c. field for

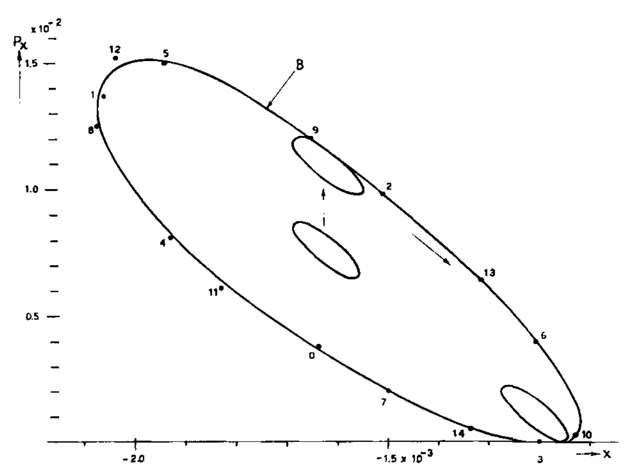


**Fig. 2** Radial motion of a displaced phase area. The large ellipse (*B*) is from Fig. 1. The small ellipse is curve (*B*) damped in going from 50 MeV to 15 GeV. The field perturbation is assumed to cause an impulsive change in $p_x$, midway between two magnets. Indicated is the position of the damped phase area one sector after the perturbation.

deflection down the free field spiral. A uniform d.c. field of 10 kG, extended over 2.6 m between the spiral magnets, will provide enough bending for this extraction (just 0.05 radians deflection are needed). This uniform field, and subsequent regions which can be made essentially field free, will contribute no non-linearities, so after extraction the phase areas will be essentially damped down replicas of the curves (B), modified of course by the distance traveled and subsequent lenses. They are reasonably shaped areas, giving an external beam with no effective decrease of density in betatron phase space, hence, a beam which could be used for re-injection into an AG storage ring.

The extraction process has been discussed for particles injected into the curves (B) with amplitudes more than a factor of two beneath the stability limits. Similar extraction should work for beams injected into the larger areas (A) with possibly some distortion.

Since the machine is fixed-field, it is possible to stack a large number of injected pulses at full energy, then extract in one turn. This might be useful for certain experiments; in particular, for colliding beams it would mean doing the RF beam stacking in the accelerating machine rather than a storage ring, possibly an easier process if there are energy matching problems on transfer, or if there are difficulties in keeping a pulsed inflector field from disturbing the previously stacked beam. The number of such stacked pulses which can be efficiently extracted in one turn is limited by the spread in their radial positions (due to their energy differences). The system presented above, assuming injection into curves (B) and the same oscillation amplitudes, could handle an extra radial increment, due to the energy spread, of $10^{-4}$ and still get the beam past a 1 cm current sheet into the extraction channel. If the pulsed deflector caused this damped radial phase area to move to curve (A) rather than (B), the system should handle, with only small distortion, a radial spread of $4 \times 10^{-4}$ with the corresponding momentum spread of $\Delta p/p = k\Delta x = 53 \times 4 \times 10^{-4} = 2\%$, nearly the full momentum range easily superposed in a storage ring [3]. Here the deflecting fields should probably have a gradient to bend the particles of different momentum through the same angle.

## III. CONTINUOUS EXTRACTION

Methods of achieving large duty factor extraction with circular accelerators generally fall into two classes. In the first class, particles gain sufficient energy per turn to spiral outside of some septum (e.g., an electrostatic peeler as in low energy cyclotrons) in one turn. The spiralling per turn at the deflector radius and azimuth can be greatly enhanced by using field perturbations as in the Tuck - Teng - Le Couteur[4, 5] synchro-cyclotron extractor. The second class employs a sharp radial discontinuity in the machine, usually an energy-loss foil which allows the particles to circulate normally until their energies are changed to allow them to strike the foil. A finite energy loss then causes them to spiral into an extraction channel or magnet. This system, discussed by Wright and by Piccioni et al. [1], has been employed on the Cosmotron and is planned for other large synchrotrons.

Due to the large value of $k$ (momentum compaction), no extractor systems of the first class have yet been developed for multi-GeV FFAG accelerators, although in principle they may be possible. The Wright - Piccioni system is not applicable to FFAG structures in the same form as used with pulsed accelerators since an energy loss always causes the particles to spiral to a smaller radius in the vacuum tank, a region which must be maintained clear of extractor channels and other structures to allow acceleration of subsequent groups of particles.

Two modifications of the Wright - Piccioni scheme are considered here for FFAG accelerators. In each an energy loss by a particle results in a trajectory which lies at a greater radius than unperturbed orbits at certain azimuths. Used together, they result in the system described as an example below, capable of producing essentially continuous extraction from an FFAG accelerator. In the first modification, use is made of the modulation, or beating, of the envelope of betatron oscillations characteristic of alternating gradient accelerators. From the phase plots of Figs. 1 and 2 it may be noted that the eccentricity of the phase ellipse is rather large; for the accelerator with the parameters given in Table I the linear (small amplitude) radial oscillations vary by a factor of three as the phase ellipse rotates through a sector. Thus, a given $x$ amplitude at one azimuth (where the phase ellipse is elongated in the $p_x$ dimension) will be one-third of the corresponding $x$ amplitude at an azimuth an integer plus one half sectors around the machine (where the phase ellipse is rotated 90°), after the particle has gone through approximately an integral number of radial oscillations. An energy-loss foil placed at the first azimuth will induce radial oscillation of a particle about a new equilibrium orbit of smaller radius. At the second azimuth, the particle will find itself radially displaced outward (from unperturbed particles) by twice the increment by which the equilibrium orbit was shifted inward (Fig. 3). In the above discussion, it is assumed that the accelerator is " scaling ", i.e. $dp/p = (k+1)\, dR/R$ at all azimuths.

The second modification makes use of the introduction of a non-scaling azimuthal field perturbation. Such a perturbation can be made to distort the equilibrium orbits such that they are tangent at certain azimuths (nodes) and separated by twice the normal radial spacing at azimuths half way between (antinodes)[3]. In an FFAG accelerator, this perturbation would be applied only to orbits near the outer (maximum energy) radius; at smaller radii the machine would " scale " as usual. The perturbation on the magnetic field may be of the form

$$B_{\text{Perturbation}} = \varepsilon(x-x_0)\cos M\theta\,, \qquad x > x_0$$
$$= 0, \qquad x < x_0\,,$$

where $M$ is an integer close to $\nu_x$, $x_0$ is some reference (unperturbed) equilibrium orbit, and $\varepsilon$ is small.

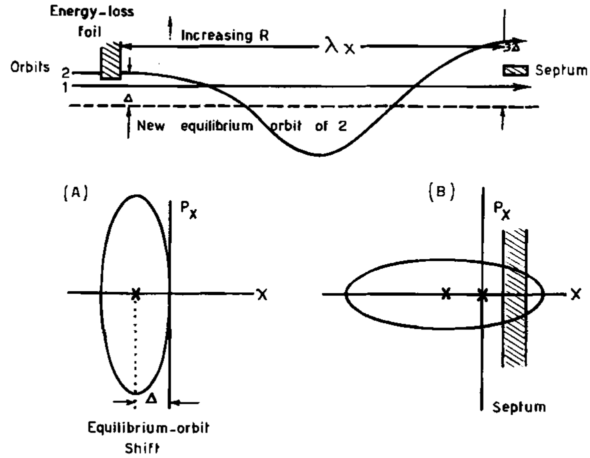


**Fig. 3** Extraction using rotation of the radial betatron phase plot. The orbits and phase plots are schematically represented. Azimuthal separation of (*A*) and (*B*) is an integral number of radial betatron wavelengths and $(P+\frac{1}{2})$ sectors of the magnet structure (where $P$ is an integer). Orbit 1 represents the path of a particle missing the foil. Orbit 2 represents the particle one turn later after it has been accelerated to a larger radius. On passing through the foil its equilibrium orbit is shifted an amount $\Delta$, enabling it to spiral outside the septum as shown. In this and subsequent illustrations the alternating gradient character of the motion and machine curvature are deleted for clarity.

Small oscillations about these perturbed equilibrium orbits should be stable and well behaved until the amplitude of the equilibrium orbit perturbation approaches the radial stability limit amplitude of oscillations about unperturbed equilibrium orbits. In applying this to an extraction system, an energy-loss foil is placed at an azimuth where the equilibrium orbits have a maximum radial separation; the extraction channel or septum is then placed at an azimuth where the equilibrium orbits are tangent and after an integral number of radial oscillations. The energy-loss foil induces a radial oscillation about the new equilibrium orbit of smaller average radius. The non-scaling perturbation then causes the particle to lie at a larger radius at the extraction channel (Fig. 4).

A major difficulty in the design of a practical extractor of the Wright - Piccioni type is due to multiple scattering within the energy-loss foil. If a particle multiple scatters out of the foil before traversing its full length, it will receive a smaller energy loss and thus strike the extraction septum and be lost. A reasonable criterion is then that the particle should strike the foil radially in from its edge an increment $\Delta_x$ such that

$$\Delta_x > S\,\langle\theta\rangle_{\text{r.m.s}}$$

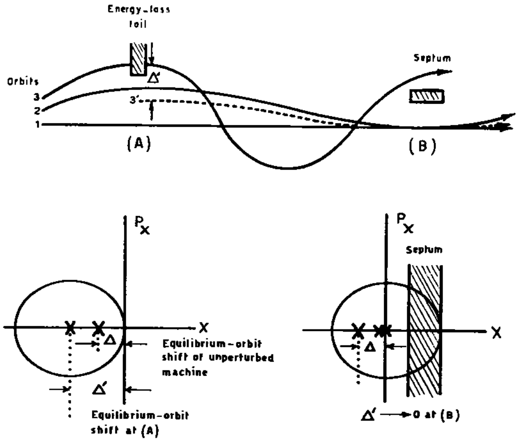


**Fig. 4** Extraction using non-scaling perturbation. The orbits and phase plots are schematically represented. Azimuthal separation of (*A*) and (*B*) are an integral number of radial betatron wavelengths and ($Q+\frac{1}{2}$) periods of the non-scaling perturbation (where $Q$ is an integer). Orbits 1, 2, and 3 represent trajectories of successive particle orbits; a particle on orbit 3 strikes the energy loss foil, causing an equilibrium orbit shift $\Delta'$ enabling it to spiral past the septum as shown.

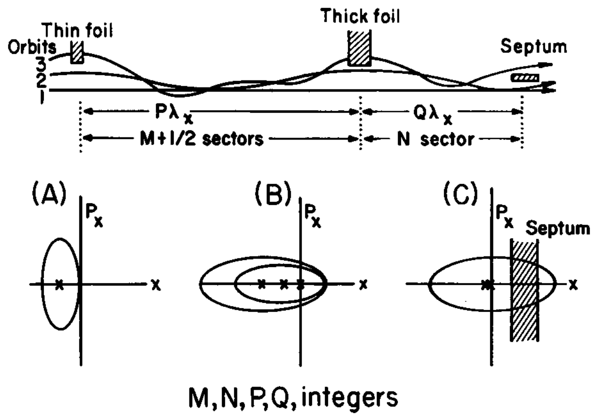


**Fig. 5** Extraction using first phase-plot rotation and second orbit non-scaling perturbation. The azimuthal separation of (*A*) and (*B*) is an integral number of radial betatron wavelengths ($P\lambda_x$), an integral number of non-scaling periodic perturbations, and a half-integral number ($M+\frac{1}{2}$) sectors. The azimuthal separation of (B) and (C) is an integral number of radial betatron wavelengths ($Q\lambda_x$), a half-integral number of non-scaling perturbations, and an integral number (N) of sectors. A particle on orbit 3 losing energy in a thin energy loss foil at (A) spirals into a thick energy loss-foil at (B) producing an even larger orbit separation at the septum azimuth (C).

where $S$ is the azimuthal foil length and $\langle\theta\rangle_{rms}$ is the root mean square multiple scattering angle.

As an example, an extractor utilizing both schemes is sketched below, applicable to the accelerator parameters of Table I. A thin energy-loss foil is located at azimuth (*a*) where the phase ellipse is vertical (small $x$) and at an antinode of the non-scaling structure. This causes particles, spiralling slowly outward due to as little as one kilovolt energy gain per turn, to lose enough energy to strike a thick " foil " at azimuth (*b*) where the phase ellipse is horizontal (large $x$) and again at an antinode of the non-scaling orbit. This causes enough energy loss to allow the particles to miss a 3 or 4 mm septum at azimuth (*c*) where the phase ellipse is horizontal and the equilibrium orbits are tangent (node). The septum is represented schematically in Fig. 5 and the relevant parameters summarized in Table II. Is should be noted that the alignment of the foils is critical to an angle of the order of $\Delta x/s$ and a radial position $\Delta x$. The extracted beam pulse length $t$ is given in terms of the energy gain per turn, $dE$ (necessary to avoid multiple scattering out of the first energy-loss foil), the energy spread of the beam, $\Delta E$, and the frequency of revolution $f$:

$$t=\frac{\Delta E}{f\,dE}$$

TABLE II

Parameters of a continuous extractor for the 15 GeV FFAG accelerator of Table I

| Azimuth | (a) | (b) | (c) |
|---|---|---|---|
| Spacing | Integral $\lambda_x$<br>Integral superperiodicity<br>Half integral number of sectors | Integral $\lambda_x$<br>Half integral superperiodicity<br>Integral number of sectors | |
| Object | 1 mm thick carbon energy-loss foil | 10 cm thick carbon energy-loss foil | Current septum 3-4 mm radial width |
| $\theta_{rms}$ | $5\times10^{-5}$ rad | $5\times10^{-4}$ rad | |
| Energy loss | 0.25 MeV | 25 MeV | |
| Radial separation from preceding turns | $10^{-4}$ mm (due to spiralling at 1 keV per turn) | 0.1 mm (due to foil (a)) | 5 mm (due to foil (b)) |

While an FFAG accelerator may have $\Delta E$ as small as 150 keV for each injected pulse (at 10 per second), the spread may be deliberately increased two or more orders of magnitude by simply introducing noise into the RF acceleration cavity (increasing the entropy of the RF phase volume of the beam). In practice a beam of $10^{14}$ protons per second could be accelerated and stacked at about 14 GeV, and RF noise introduced to increase the energy width of the stack to 1 GeV. The stack could then be brought onto the energy-loss foil slowly and uniformly over a period of a second by a phase-displacement oscillator while a new stack was being formed at 14 GeV. A beam duty factor in excess of 80% should result.

The extraction channel is not discussed beyond the current septum; magnets and magnetic channels would follow in a rather conventional manner, after the initial separation of extracted particles from circulating particles.

For simplicity, the sketches indicate particles striking the first foil as having no betatron oscillation amplitude. The operation of the system is independent of *initial* amplitudes so long as the oscillations are nearly linear. A spread of betatron amplitudes will produce the only energy spread in the external beam; approximately 100 MeV for a $\pm 1$ cm betatron oscillation amplitude.

It is reasonable to conclude, therefore, that an FFAG accelerator will be able to provide external proton beams of high quality ranging from one turn extraction of accumulated (stacked) particles to essentially continuous 10 to 50 $\mu$A beams.

LIST OF REFERENCES

1a. Wright, B. T. Magnetic deflector for the bevatron. Rev. sci. Instrum., *25*, p. 429-31, 1954.

b. Piccioni, O., Clark, D., Cool, R., Friedlander, G. and Kassner, D. External proton beam of the cosmotron. Rev. sci. Instrum., *26*, p. 232-3, 1955.

2. O'Neill, G. K. and Woods, E. J. Intersecting beam systems with storage rings. Princeton-Pennsylvania Accelerator Project Internal Report (*). GKON-12 EJ.W-2, 1959.

3. Terwilliger, K. M. Achieving higher beam densities by superposing equilibrium orbits. See below.

4. Tuck, J. L. and Teng, L. C. Synchro-cyclotron progress report III. University of Chicago, Institute of Nuclear Studies, 1950. Chapter 8.

5. Le Couteur, K. J. The regenerative deflector for synchro-cyclotrons. Proc. phys. Soc., Lond. B, *64*, p. 1073-84, 1951.

# ACHIEVING HIGHER BEAM DENSITIES BY SUPERPOSING EQUILIBRIUM ORBITS

**K. M. Terwilliger** (**)

Midwestern Universities Research Association, Madison, Wis. (***)

*(presented by F. T. Cole)*

## I. INTRODUCTION

In order to do colliding-beam experiments with a reasonable counts-to-background ratio, it is desirable to have a high beam current density. Obtaining equal numbers of beam-beam collisions and beam-residual gas collisions, for example, would require a current density of about 50 A/cm$^2$ at $10^{-8}$ mm Hg. To get this density it is usually necessary to spatially superimpose a large number of injected pulses, requiring RF beam stacking [1]. In this stacking process, however, depositing a beam pulse at a

(*) Internal memoranda not generally distributed but possibly available from author.

(**) University of Michigan, Ann Arbor, Mich.

(***) Supported by the United States Atomic Energy Commission.