



Observational constraints on α -attractor inflationary models with a Higgs-like potential

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ABSTRACT

We investigate the observational viability of a class of α -attractors inflationary models in light of the most recent Cosmic Microwave Background (CMB) and Large-Scale Structure (LSS) data. By considering a double-well potential we perform a slow-roll analysis to study the behavior of this class of models, which is a continuous interpolation between the chaotic inflation for large values of α and the universal attractor, i.e., $n_s = 1 - 2/N$ and $r = \alpha 12/N^2$ for small α , where n_s is the scalar spectral index, r is the tensor-to-scalar ratio, and N is the e-fold number. In order to explore the parameter space of the model, we also perform a MCMC analysis and find $\alpha = 7.56 \pm 5.15 (1\sigma)$.

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1. Introduction

The inflationary paradigm has become a central part of modern cosmology as it provides a successful description of the first instants of the evolution of our Universe [1–3]. Since it was proposed, a considerable number of inflationary models were considered and investigated in light of observational data [4]. From the current observational perspective, the most promising scenarios rely on modified gravity theories, such as the Starobinsky model $R + R^2$ [5–7], where R is the Ricci scalar, the chaotic inflation $\lambda\phi^4$ non-minimally coupled to gravity [8–10] and a class of superconformal inflationary theories that leads to a universal attractor behavior [11–13].

Indeed, the most recent Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) data favor models which present a plateau in their inflationary potentials [14,15]. In particular, the Starobinsky and the non-minimal models exhibit a similar potential in the regime of large fields and strong couplings [16,17],

$$V(\phi) = \frac{3}{4}M^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2, \quad (1)$$

where the inflaton ϕ is given in Planck units, $M_P = 1$. The energy scale M is usually constrained by the observed value of the amplitude of scalar perturbations $A_S \simeq 2.1 \times 10^{-9}$ [18]. For large values

of the e-fold number $N \gg 1$, i.e. the number of e-foldings between the horizon crossing of modes of interest and the end of inflation, these theories lead to the same predictions for the inflationary observables,

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12}{N^2}, \quad (2)$$

where n_s is the scalar spectral index and r the tensor-to-scalar ratio. For $50 \leq N \leq 60$, the expressions above yield $0.96 \lesssim n_s \lesssim 0.97$ and $0.003 \lesssim r \lesssim 0.005$, showing a remarkable agreement with present CMB observations [14,18].

Given their ability to describe the observed data, these inflationary scenarios and their phenomenological implications have been extensively studied [17,19–25]. In particular, non-minimal inflation with the Higgs field playing the role of inflaton has received a fair amount of attention, as it can potentially connect inflationary dynamics with the low-energy phenomenology of the model. Scenarios in which extensions of the standard model drive the inflationary dynamics have also been considered in the literature [26–29].

More recently, a class of inflationary models introduced by Kallosh, Linde and Roest has received considerable attention [13, 30–40]. Based on supergravity, the so called cosmological α -attractors stand out for achieving a slow-roll phase, even for inflationary potentials of arbitrary form $V(\phi)$ [33]. Specifically, an inflationary plateau arise in the theory due to the presence of a non-canonical kinetic term for the inflaton ϕ . Such kinetic energy features a pole at $\phi^2 = 6\alpha$, which prevents any trajectory in field

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space to travel beyond this limit. On the other hand, a canonical form for the Lagrangian density can be achieved after a field redefinition. In the canonical field space the poles are displaced to infinity, “stretching” the scalar potential in the large field regime. Similarly to the Starobinsky inflation, such plateau at high energies aligns the model predictions with the observational data.

Despite of all the promising characteristics of the α -attractor class of models, only a few works address a careful statistical analysis of this scenario (see [37,41,42] for the analysis of quintessential α -attractor models). In this sense, we aim to deepen the analysis of these models, investigating its observational viability in light of the most recent CMB and LSS data [43–48]. To this end, we use the double-well function as non-canonical potential $V(\phi)$. Such a choice finds its motivation in fundamental particle physics, where this potential can be employed in the spontaneous symmetry breaking mechanism [49–51].

This work is organized as follows. Section 2 reviews the basics of α -attractors, discussing the field redefinition to obtain the canonical inflationary Lagrangian density. Section 3 presents the slow-roll analysis for the double-well potential. In Section 4 we present the observational data sets used in our statistical analysis and the main results obtained. Finally, the main conclusions of the paper are summarized in Section 5.

2. The model

The Lagrangian density for the α -attractor class of models arise in the context of supergravity theories for specific forms of the Kähler potential,

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2}R - \frac{1}{(1 - \phi^2/6\alpha)^2} \frac{(\partial\phi)^2}{2} - V(\phi) \right], \quad (3)$$

where ϕ is the inflaton field and α the free parameter of the theory, related to the curvature of the Kähler manifold [13]. The main characteristics of these models manifest due to the presence of a non-canonical kinetic term for the inflaton. It exhibits a pole at $\phi^2 = 6\alpha$, preventing any displacement in the field space from crossing this limit. On the other hand, a canonical kinetic term can be recovered through the field redefinition,

$$\phi = \sqrt{6\alpha} \tanh \frac{\varphi}{\sqrt{6\alpha}}, \quad (4)$$

according to which the inflationary Lagrangian density assumes the form,

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2}R - \frac{(\partial\varphi)^2}{2} - V(\sqrt{6\alpha} \tanh \frac{\varphi}{\sqrt{6\alpha}}) \right]. \quad (5)$$

While the field ϕ is limited by the poles $\phi = \pm\sqrt{6\alpha}$, the canonical inflaton φ is free to assume any value in field space. As result, the scalar potential $V(\sqrt{6\alpha} \tanh \frac{\varphi}{\sqrt{6\alpha}})$ acquires a plateau in the limit of large fields, $\varphi \rightarrow \pm\infty$. Such configuration is propitious to provide large field inflation, leading to the following predictions for the inflationary observables,

$$n_s = 1 - \frac{2}{N}, \quad \text{and} \quad r = \alpha \frac{12}{N^2}, \quad (6)$$

for large N and small α .

Similarly to the Starobinsky model, the α -attractors theoretical predictions are aligned with observations, at least for the slow-roll analysis. Indeed, the predictions of both scenarios coincide at leading order for $\alpha = 1$. This predictive pattern is not a coincidence, as the inflationary potential in both cases is similar at the limit of large field. Particularly, in the vicinity of $\phi = \sqrt{6\alpha}$, or equivalently

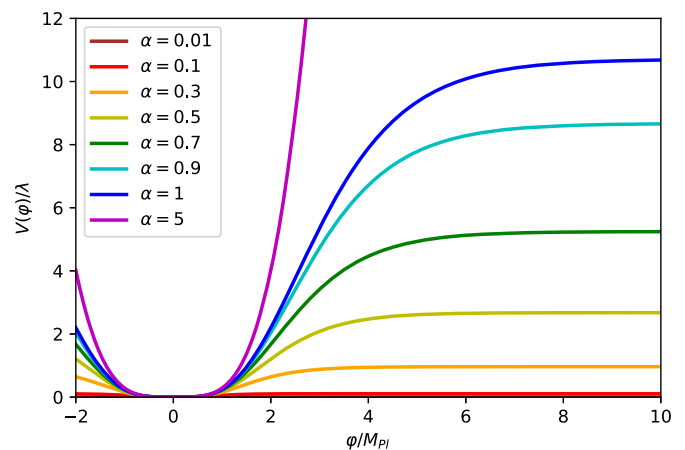


Fig. 1. The Higgs-like primordial potential as a function of the canonical field φ for different values of α .

$\varphi \rightarrow \infty$, the potential for the canonical scalar field can be written in the asymptotic form

$$V(\varphi) \simeq V_0 - 2\sqrt{6\alpha}V'_0 \exp\left(-\sqrt{\frac{2}{3\alpha}}\varphi\right). \quad (7)$$

The expression above is reduced to the Starobinsky potential (1) for $\alpha = 1$ at leading order, except for an unobservable phase shift in φ . In this sense, one might identify the Starobinsky potential as one of the solutions of the α -attractor class, reinforcing the general aspect of these inflationary models.

Despite the general aspect of the α -attractor models at small values of α , it is important to consider a specific choice of inflationary potential in order to correctly account for the predictions of the model for a broad range of α . Therefore, the slow-roll and statistical analysis of this work will focus on the double-well inflationary potential,

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2, \quad (8)$$

where ϕ is a real inflaton and v its vacuum expectation value.

This choice of primordial potential is strongly motivated in particle physics, where it can be applied in the dynamical symmetry breaking mechanism [49–51]. In particular, the scalar sector of the standard model of fundamental particles is composed of such potential, with the scalar field behaving like a doublet under the $SU(2)_L$ symmetry of the standard model, namely the Higgs doublet [52,53].

As inflation takes place in the large field regime, $\phi \simeq \sqrt{6\alpha}$, it is plausible to assume that the inflationary energy scale is much bigger than the vacuum expectation value of the theory, $\phi \gg v$. Therefore, the quartic power of the non-canonical field in the primordial potential is dominant during inflation. After the field redefinition (4), it assumes the form,

$$V(\varphi) = 9\alpha^2\lambda \left(\tanh \frac{\varphi}{\sqrt{6\alpha}} \right)^4, \quad (9)$$

hereafter referred as Higgs-like model.

Fig. 1 shows the primordial potential, as a function of the canonical field φ , for different values of α . A flat regime is obtained for $\varphi \gg \sqrt{6\alpha}$. Such behavior makes the model’s predictions deviate from those of the usual chaotic scenario. In particular, the “flattening” of the potential is more efficient for lower values of α , while the canonical chaotic inflation φ^4 is recovered in the limit $\alpha \rightarrow \infty$.

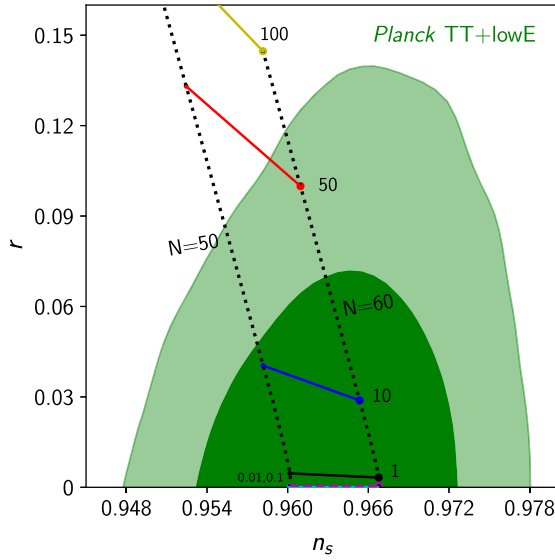


Fig. 2. The $n_s - r$ plane for the Higgs-like potential (9), considering different values of the parameter α (solid colored lines and magenta dashed line) and two values for the number of e-folds, $N = 50$ and $N = 60$ (black dotted lines). The contours are the 68% and 95% confidence level regions obtained from Planck (2018) CMB data using the pivot scale $k_* = 0.05 \text{Mpc}^{-1}$.

3. Slow-roll analysis

Inflation and its observables are described in terms of the slow-roll parameters, ϵ and η . These parameters can be computed from the inflationary potential by

$$\epsilon = \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = M_P^2 \frac{V''}{V}, \quad (10)$$

where $'$ indicates derivative with respect to φ . Inflation begins when $\epsilon, \eta \ll 1$ and comes to an end for $\epsilon, \eta \simeq 1$. In the slow-roll regime, the scalar spectral index and the tensor-to-scalar ratio have the form:

$$n_s = 1 - 6\epsilon + 2\eta \quad \text{and} \quad r = 16\epsilon. \quad (11)$$

We can finally consider the amplitude of the primordial scalar perturbations produced during inflation:

$$P_R(k_*) = A_s(k_*) = \frac{V}{24M_P^4 \pi^2 \epsilon} \Big|_{\varphi=\varphi_*}, \quad (12)$$

computed for the field strength φ_* corresponding to the energy scale at which the pivot scale, k_* , crossed the Hubble horizon. The normalization value of A_s is set by the Planck Collaboration to about 2.0933×10^{-9} for the pivot choice $k_* = 0.05 \text{Mpc}^{-1}$ [18].

The field strength at horizon crossing, φ_* , can be related to the amount of expansion the universe experienced, since the horizon crossing moment up to the end of inflation, via

$$N = \frac{1}{M_P^2} \int_{\varphi_e}^{\varphi_*} \frac{V}{V'} d\varphi. \quad (13)$$

Several uncertainties may affect the estimate of the e-fold number. The main source lies from the lack of information about the reheating process, yielding to this quantity a moderate model dependence. For the class of inflationary models which this work is concerned, the number of e-folds can be estimated between 50 – 60 [21,37,38,54–58].

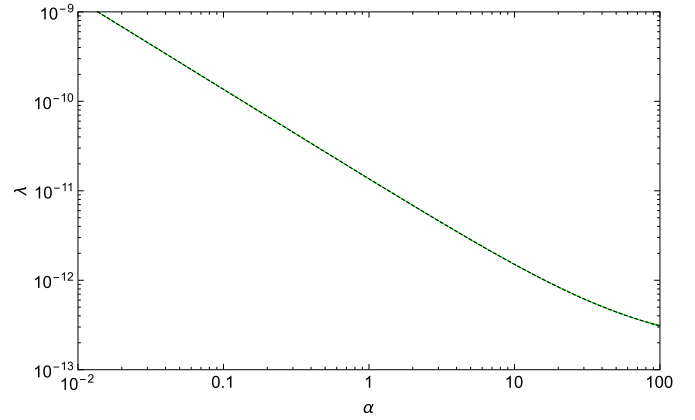


Fig. 3. λ vs α computed for $N = 55$. The green curve interpolates the numerical data points while the dashed curve shows the analytical fit to the numerical data points.

Once the number of e-folds is set, the expression in (13) can be solved for the field strength at horizon crossing, φ_* . The resulting scale of energy is then employed to compute the model predictions for the inflationary observables. Such procedure is not possible analytically, requiring the use of numerical methods. In particular, we set the number of e-folds in the range $50 \leq N \leq 60$, in order to compare the model predictions with the Planck data.

Fig. 2 shows the behavior of the scalar spectral index and the tensor-to-scalar ratio for the model (9), considering different values of the parameter α and the number of e-folds ranging from $N = 50$ to $N = 60$. The results obtained here are in agreement with the ones presented in [13], for the family of α -attractors models of type ϕ^p . In particular, a continuous interpolation between the chaotic inflation at large α and the universal attractor (6) at small α can be noticed from that figure. For $N = 55$, the consistency at 68% confidence level (C.L.) between the model predictions and Planck data occurs for $\alpha \lesssim 28$.

4. Analysis and results

In order to investigate the observational feasibility of this class of inflationary models, we modify the latest version of the Code for Anisotropies in the Microwave Background (CAMB) [59] following the lines of the MODECODE [60], adapted to our primordial potential choice (including the parameter α). On the other hand, to calculate the spectrum of CMB temperature fluctuations from the Primordial Power Spectrum (PPS), we need to solve the equations of the inflationary dynamics, i.e. the Friedmann and Klein-Gordon equations. Additionally, we also need to find the Fourier components associated with curvature perturbations produced by the fluctuations of the scalar field φ , which can be determined numerically using the MODECODE. This code solves the Mukhanov-Sasaki equations [61]

$$u_k'' + \left(k^2 - \frac{z''}{z} \right) u_k = 0, \quad (14)$$

where $u \equiv -z\mathcal{R}$ and $z \equiv a\dot{\varphi}/H$, and a , H , \mathcal{R} are the scale factor, the Hubble parameter, and the comoving curvature perturbation, respectively. These in turn are used to determine $\mathcal{P}_{\mathcal{R}}(k)$, that is related with u_k and z via:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2. \quad (15)$$

Thus, the method consists in selecting the form of the potential $V(\varphi)$, solve the dynamical equations to obtain H and φ , and then the PPS is obtained using the solutions of the Mukhanov-Sasaki

Table 1
Priors on the cosmological parameters considered in the analysis.

Parameter	Prior Ranges
$\Omega_b h^2$	[0.005 : 0.1]
$\Omega_c h^2$	[0.001 : 0.99]
θ	[0.5 : 10.0]
τ	[0.01 : 0.8]
α	[0.01 : 100]

equations. As discussed before, we can use Eq. (12), given the potential $V(\varphi)$ (9), and invert it in order to determine the potential amplitude λ in terms of α . In particular, we set $N = 55$ to obtain the green curve in Fig. 3. Note that λ decreases with α up to the chaotic φ^4 value ($\lambda \simeq 10^{-13}$) in the limit $\alpha \rightarrow \infty$. Although an expression of $\lambda(\alpha)$ can not be obtained analytically, one could fit the numerical points to an analytical curve. We adjust the numerical results to the logarithmic power of α up to fifteenth order (represented by the dashed curve in Fig. 3),

$$\begin{aligned} \lambda(\alpha) \simeq & 1.36 \times 10^{-11} - 1.35 \times 10^{-11} \ln(\alpha) \\ & + 6.80 \times 10^{-12} \ln(\alpha)^2 - 2.28 \times 10^{-12} \ln(\alpha)^3 \\ & + 5.70 \times 10^{-13} \ln(\alpha)^4 - 1.14 \times 10^{-13} \ln(\alpha)^5 \\ & + 1.90 \times 10^{-14} \ln(\alpha)^6 - 2.72 \times 10^{-15} \ln(\alpha)^7 \\ & + 3.40 \times 10^{-16} \ln(\alpha)^8 - 3.78 \times 10^{-17} \ln(\alpha)^9 \\ & + 3.80 \times 10^{-18} \ln(\alpha)^{10} - 3.40 \times 10^{-19} \ln(\alpha)^{11} \\ & + 2.72 \times 10^{-20} \ln(\alpha)^{12} - 2.35 \times 10^{-21} \ln(\alpha)^{13} \\ & + 2.06 \times 10^{-22} \ln(\alpha)^{14} - 9.79 \times 10^{-24} \ln(\alpha)^{15}. \end{aligned} \quad (16)$$

The theoretical predictions for the Higgs-like potential (9) are shown in Fig. 4. Notice that the effect of α in the temperature angular power spectrum is a slight variation in its amplitude. However, this variation does not follow a correlation with the increment of α , instead, the amplitude oscillates to upper and lower values arbitrarily. In addition, despite the $n_s - r$ plan has shown that the value of $\alpha = 100$ is out of the 95% confidence region allowed by the data (for both $N = 50$ and $N = 60$), the theoretical predictions are close to the standard Λ CDM model. Therefore, we consider as an appropriate range for our analysis the flat prior of $0.01 < \alpha < 100$.

To explore the cosmological parameter space of the Higgs-like model and investigate the impact of the parameter α , we perform a Markov Chain Monte Carlo (MCMC) analysis, using the most recent version of CosmoMC code [62]. We also vary the usual cosmological parameters, namely, the baryon and the cold dark matter density, the ratio between the sound horizon and the angular diameter distance at decoupling, and the optical depth: $\{\Omega_b h^2, \Omega_c h^2, \theta, \tau\}$. We consider purely adiabatic initial conditions, fix the sum of neutrino masses to 0.06 eV and the universe curvature to zero, and also vary the nuisance foregrounds parameters [63]. The priors we considered on the cosmological parameters are shown in Table 1.

In order to compare the theoretical predictions of the model with the observational data, we use the CMB data from the latest release of Planck Collaboration (2018) [43]. We consider high multipoles Planck temperature data from the 100-,143-, and 217-GHz half-mission T maps, and the low multipoles data by the joint TT, EE, BB and TE likelihood, where EE and BB are the E- and B-mode CMB polarization power spectrum and TE is the cross-correlation temperature-polarization (hereafter “PLA18”). In addition, we consider an extended dataset combining the CMB data with Baryon Acoustic Oscillations (BAO) coming from the 6dF Galaxy Survey

Table 2
68% confidence constraints for the cosmological parameters of the Higgs-like model using PLA18+BAO+BK15 data. The table is divided into two sections: the upper section shows the primary parameters, while in the lower part shows the derived ones.

Parameter	Higgs-like	
	mean	best fit
Primary		
$\Omega_b h^2$	0.02212 ± 0.018	0.02208
$\Omega_c h^2$	0.1199 ± 0.0009	0.1196
θ	1.04085 ± 0.00040	1.04108
τ	0.048 ± 0.003	0.048
α	7.56 ± 5.15	9.82
Derived		
H_0	67.16 ± 0.38	67.21
Ω_m	0.316 ± 0.005	0.320
Ω_Λ	0.684 ± 0.005	0.680
n_s	0.963 ± 0.001	0.9621
$r_{0.002}$	0.023 ± 0.014	0.030

(6dFGS) [44], Sloan Digital Sky Survey (SDSS) DR7 Main Galaxy Sample galaxies [45], BOSSgalaxy samples, LOWZ and CMASS [46], and the tensor amplitude of B-mode polarization from the Keck Array and BICEP2 Collaborations [47,48], using the BICEP2/Keck field, from 95, 150, and 220 GHz maps (hereafter “BK15”).

The observational constraints obtained for the Higgs-like potential, using the Planck 2018 likelihood jointly with BAO and BICEP/Keck 2015 data, are summarized in Table 2 and in Fig. 5. Table 2 shows the constraints on the cosmological parameters for Higgs-like model. Note that both primary and derived cosmological parameters of Higgs-like model are in good agreement with Λ CDM within 1σ level [18]. From this analysis, we find $\alpha = 7.56 \pm 5.15$ at 68% C.L. and at 95% C.L. we find an upper limit of $\alpha < 17.69$, that is considerably more stringent than the slow-roll result $\alpha \lesssim 28$. Other recent analysis is found in [37,41,42].

Fig. 5 shows the confidence regions at 68% and 95% C.L. and the posterior probability distribution for the main cosmological parameters of the Higgs-like model. The bounds obtained for the matter density and Hubble parameter are in accordance with Λ CDM model [18]. However, note that the limits on the tensor-to-scalar ratio, an upper bound of $r < 0.049$ at 95% C.L., are tighter than that obtained previously for both Λ CDM model and α -attractor models [18,13,64,12]. Also, there is a positive correlation between α and r . Indeed, this fact can also be inferred from the $n_s - r$ plan shown in Fig. 2, which exhibits this behavior between α and r . Lastly, the Fig. 6 displays the CMB temperature angular power spectrum for the best-fit values of the Higgs-like model, and shows a fit to the CMB data as good as the one provided by Λ CDM model.

5. Discussion and conclusions

In this paper, we analyzed a class of inflationary models based on supergravity theories, namely, α -attractors. Specifically, we investigated the observational viability of the model for the double-well inflationary potential. Such choice is strongly motivated in the context of particle physics, presenting an ideal framework not only to successfully carry out inflation but also to connect the inflationary dynamics to low-energy phenomenology of fundamental particles. For this reason, the resulting model was called Higgs-like model.

For large values of α ($\mathcal{O}(10^2)$), the slow-roll analysis revealed that the model predictions are outside the region allowed by the data on the $n_s - r$ plan, as seen in Fig. 2. As α decreases, the predictions converge to the universal attractor values, Eq. (6). Such behavior is similar to the one predicted by a general family of cosmological attractors with $V(\varphi) \sim \varphi^n$ [13]. Extending the slow-roll study, we performed a MCMC analysis in order to explore the parameter space. The results found for the primary and derived

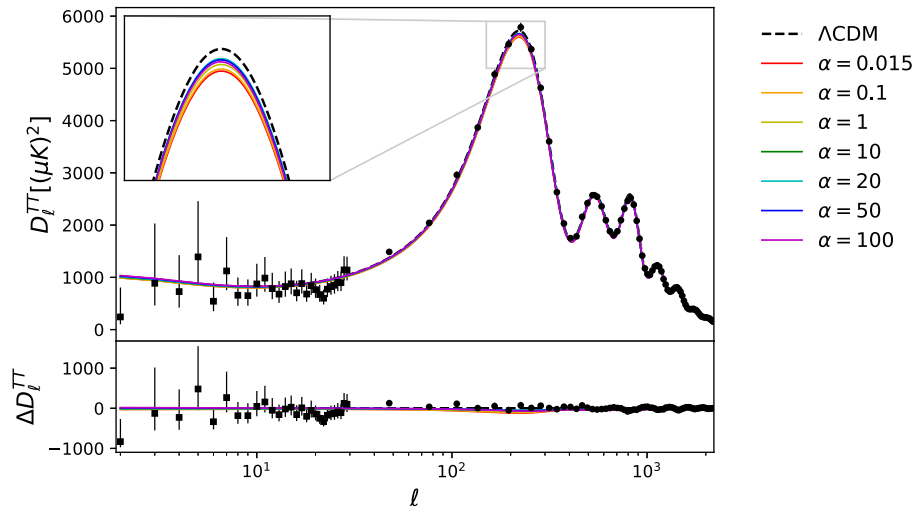


Fig. 4. The theoretical predictions for the CMB temperature angular power spectrum considering different values of α for the Higgs-like model. The data points correspond to the latest release of Planck data [18].

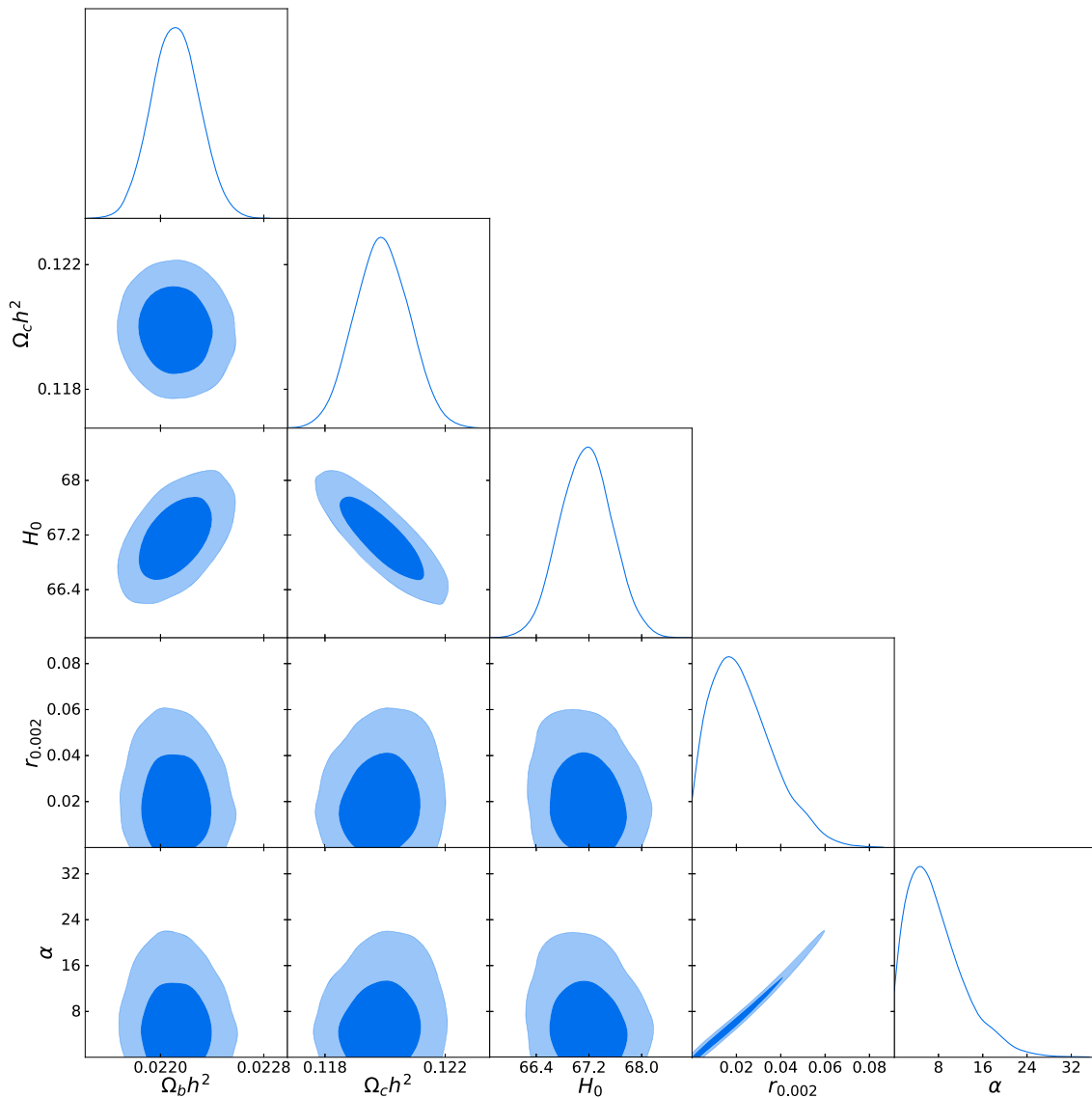


Fig. 5. 68% and 95% confidence regions for the primary and derived parameters of the Higgs-like model.

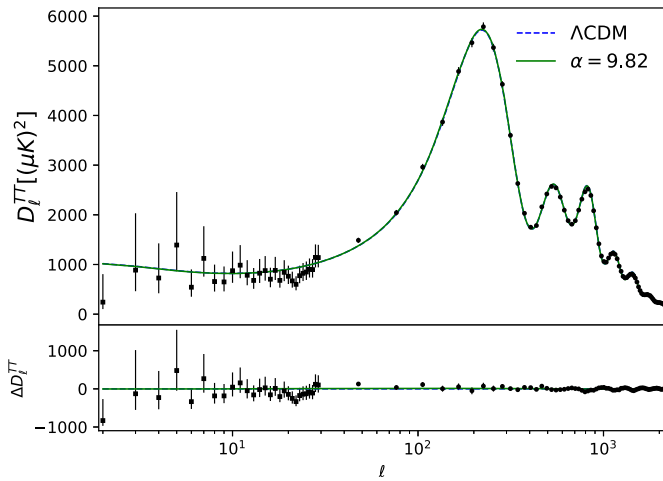


Fig. 6. The best-fit CMB temperature angular power spectra for the Higgs-like model (green curve) and Λ CDM model (blue curve). The data points correspond to the latest release of Planck (2018) data and the lower panel show the residuals with respect to the reference model (Λ CDM).

cosmological parameters show an excellent match to the latest cosmological data as well as with the predictions of the Λ CDM model.

We also obtained a tight constraint on the parameter α using an extended dataset including CMB and BAO data, i.e., $\alpha = 7.56 \pm 5.15$ (68% C.L.). Particularly from maximal supergravity, string theory, and M-theory, there are seven preferred values for the α parameter, $3\alpha = 1, 2, \dots, 7$ [37,65–67], with all of them being favored by current data (according to $\alpha < 17.69$ at 95% C.L. upper limit). Also, the best-fit value, $\alpha = 9.82$, moves the prediction of the inflationary observables, n_s and r , farther away from the universal attractor value given by Eq. (6). This may be suggesting a substantial difference in the predictive pattern of the α -attractor models for different inflationary potentials, $V(\phi)$, in contrast to what is expected from the usual slow-roll analysis [13,33].

It is worth mentioning that a similar analysis was also performed considering the Starobinsky potential (1) in the context of α -attractor models. The results showed no significant improvement of the constraints on the cosmological parameters with the bounds on α being not restrictive, which confirms the robustness of the Starobinsky model, as recently discussed in [68,69].

In conclusion, we attested the observational viability of the Higgs-like attractor inflation in light of current observational data. However, further investigation about the dependence of those predictions on the choice of non-canonical potential $V(\phi)$ is needed. We shall consider this question in a forthcoming communication.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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