Bounds on the triplet fermions in type-III seesaw and implications for collider searches

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Abstract

Type-III seesaw is a simple extension of the Standard Model (SM) with the $SU(2)_L$ triplet fermion with zero hypercharge. It can explain the origin of the tiny neutrino mass and flavor mixing. After the electroweak symmetry breaking the light neutrino mass is generated by the seesaw mechanism which further ensures the mixings between the light neutrino and heavy neutral lepton mass eigenstates. If the triplet fermions are around the electroweak scale having sizable mixings with the SM sector allowed by the correct gauge symmetry, they can be produced at the high energy colliders leaving a variety of characteristic signatures. Based on a simple and concrete realizations of the model we employ a general parametrization for the neutrino Dirac mass matrix and perform a parameter scan to identify the allowed regions satisfying the experimental constraints from the neutrino oscillation data, the electroweak precision measurements and the lepton-flavor violating processes, respectively considering the normal and inverted neutrino mass hierarchies. These parameter regions can be probed at the different collider experiments.

1. Introduction

The neutrino masses and the flavor mixings are some of the missing pieces in the SM which have been observed in different experiments [1–16] consistently. Such experimental results are

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allowing us to think about the Beyond the Standard Model (BSM) scenarios which can explain the neutrino oscillation phenomena. A simple realization of the neutrino mass generation scenario was inspired by the introduction of the dimension-5 Weinberg operator [17] within the SM which led to extend the SM with an SM-singlet Majorana right handed neutrinos [18–23] which can explain the neutrino oscillation data, however, there is no experimental observation of the seesaw mechanism or no definite answer of the question of the origin of the neutrino masses. As a result, variety of models have been proposed to address this open question on the origin of the neutrino masses and the nature of the neutrinos.

Type-III seesaw is amongst such proposals where the SM is extended by an $SU(2)_L$ triplet fermion with zero hypercharge to generate small neutrino mass [24] through the seesaw mechanism. The triplet fermion consists of a charge neutral multiplet and a singly charged multiplet where the neutral multiplet participates in the seesaw mechanism to generate the tiny neutrino mass and flavor mixing after the electroweak symmetry breaking. As a result the neutral multiplets can mix with the SM neutrinos and through the mixing they can interact with the SM gauge bosons. Like the neutral multiplet, the charged multiplets can also interact with the SM gauge bosons through the mixing at the time of associated with the SM leptons. Therefore high energy colliders can study the productions of such particles when interacting with the SM gauge bosons. The charged multiplets can be also produced directly (i.e., not suppressed by the light-heavy mixing angle) in pair at various colliders from SM gauge bosons mediated process. A variety of phenomenological aspects for studying the triplet fermions at the colliders have been discussed in [25–35] followed by the experimental searches at the Large Hadron Collider (LHC) [36–44].

The rich phenomenology of the type-III seesaw model has been studied in the past addressing the effective neutrino mass including the threshold effect in [45]. The stability of the scalar potential under the perturbativity bounds for a set of degenerate triplet fermions had been studied in [46] using the evolutions of the renormalization group equations. The electroweak vacuum stability for the nonzero neutrino mass, naturalness and lepton flavor violation have been studied in [47] for the two generations of the triples which can successfully reproduce the neutrino oscillation data for the normal and inverted orderings of the light neutrino mass spectra. Type-III seesaw has been motivated under an U(1) extension of the SM where a heavy resonantly produced pair of the triplet fermions can be successfully studied and followed by that a BSM neutral gauge boson can be probed. Type-III seesaw scenario has been realized in the grand unified theories where a triplet and a singlet fermions were proposed to be added in [48–50] where the triplet can reproduce the neutrino oscillation data being in the intermediate scale. Additionally a development of the type-III seesaw scenario was proposed in the SU(5) theory through the inclusion of the adjoint fermionic multiplet in [51] and further phenomenological analyses were performed in [51–53]. The supersymmetric version of this theory had been proposed in [54] followed by the nonsupersymmetric counterpart in [55] to find a renormalizable framework to investigate the origin of the small neutrino mass under the grand unification inspired SU(5) theory. Alternatively an inverse seesaw mechanism has been proposed in the type-III framework [56] adding a $U(1)_Y$ hyperchargeless singlet fermion and an $SU(2)_L$ triplet fermion in [57] using an additional U(1) gauge group with the anomaly free scenario [58–61] to the SM. There are a verity of indirect search strategies prescribed for the type-III seesaw scenario including Lepton Flavor Violation (LFV) [62–65] and nonunitarity effects to [66,67]. In this context we also mention that such studies have been made in the context of the type-I seesaw in [68–79] where only a Majorana type, heavy, and SM singlet right handed neutrino was introduced in the SM. Limits on the light heavy neutrino mixing from the Eletroweak Precision Data (EWPD) were studied in [80,81].

In this paper we study the type-III model generalizing the Dirac Yukawa coupling following the Casas-Ibarra conjecture [82] under the constraints obtained from the nonunitary effects, LFV and EWPD applying the neutrino oscillation data. In our study we consider three degenerate generations of the $SU(2)_L$ triplet fermions which are involved in the neutrino mass generations mechanism form the seesaw mechanism considering the normal and inverted hierarchies of the light neutrino masses. In the type-III seesaw the mixings between the light and heavy mass eigenstates play important roles to study the triplets at different high energy colliders, for example, proton-proton (pp), electron-positron (e^-e^+) and electron-proton (e^-p). There are some production processes where the production cross section of the triplet might not be affected by mixings, however, their branching ratios will depend upon the mixings. As an example we may consider the pair production triplets (charged multiplets in pair and charged and neutral multiplets productions) where the productions processes do not depend upon the mixing directly, however, the dependence of the mixing comes at the time of the decay of the triplets. The generation of the neutrino mass mechanism in the type-III seesaw is a type of seesaw mechanism where the Dirac Yukawa coupling is always non-diagonal which gives rise to the Flavor Non-diagonal (FND) scenario to correctly reproduce the neutrino oscillation data which will be considered in this article. Depending upon the constraints we will show the allowed parameter space which can be probed by the collider based experiments in the near future.

The paper is organized in the following way. In Sec. 2, we discuss the model and the interactions of the triplet fermions with the SM particles. In the Sec. 3 we discuss general parametrization of the Yukawa coupling and its effect on the different production modes and decay of the triplets. In the Sec. 4 we discuss about the branching ratios of the triplet fermions under the general parameters. We study the possibility of the displaced vertices from the type-III seesaw in Sec. 5. We compare the upper and lower bounds on the mixings in Sec. 6 with the current limits and discuss about their implications in the collider study. Finally conclude the article in Sec. 7.

2. Model

In the type-III seesaw model SM is extended by three generations of an $SU(2)_L$ triplet fermion (Ψ) with zero hypercharge. Inclusion of such triplets helps the generation of nonzero but tiny neutrino mass through the seesaw mechanism. The Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \text{Tr}(\overline{\Psi} i \gamma^\mu D_\mu \Psi) - \frac{1}{2} M \text{Tr}(\overline{\Psi} \Psi^c + \overline{\Psi^c} \Psi) - \sqrt{2}(\overline{\ell_L} Y_D^\dagger \Psi H + H^\dagger \overline{\Psi} Y_D \ell_L) \quad (1)$$

where D_μ represents the covariant derivative, M is the Majorana mass term. $\mathcal{L}_{\text{SM}}$ is the relevant part of the SM Lagrangian. We consider three degenerate generation of the triplets. Therefore M is proportional to $\mathbf{1}_{3\times3}$. Y_D is the Dirac Yukawa coupling between the SM lepton doublet (ℓ_L), SM Higgs doublet (H) and the triplet fermion (Ψ). For brevity, we have suppressed the generation indices. In this analysis we represent the relevant SM candidates, the triplet fermion and its charged conjugate ($\Psi^c = C\overline{\Psi}^T$) as in the following way

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad H = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \quad \Psi = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix} \text{ and}$$

$$\Psi^c = \begin{pmatrix} \Sigma^{0c}/\sqrt{2} & \Sigma^{-c} \\ \Sigma^{+c} & -\Sigma^{0c}/\sqrt{2} \end{pmatrix} \quad (2)$$

After the breaking of the electroweak symmetry ϕ^0 acquires a vacuum expectation value and we can express it as $\phi^0 = \frac{v+h}{\sqrt{2}}$ with $v = 246$ GeV. To study the mixing between the SM charged

leptons and $\Sigma^{\pm}$ we write the four degrees of freedom of each $\Sigma^{\pm}$ in terms of a Dirac spinor such as $\Sigma = \Sigma_R^- + \Sigma_R^{+c}$ where as Σ^0 are two component fermions with two degrees of freedom. The corresponding Lagrangian after the electroweak symmetry breaking can be written as

$$-\mathcal{L}_{\text{mass}} = \begin{pmatrix} \bar{e}_L & \overline{\Sigma}_L \end{pmatrix} \begin{pmatrix} m_\ell & Y_D^\dagger v \\ 0 & M \end{pmatrix} \begin{pmatrix} e_R \\ \Sigma_R \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\Sigma_R^0} \end{pmatrix} \begin{pmatrix} 0 & Y_D^T \frac{v}{\sqrt{2}} \\ Y_D \frac{v}{\sqrt{2}} & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \Sigma_R^{0c} \end{pmatrix}$$
$$+ h.c. \tag{3}$$

where m_ℓ is the Dirac type SM charged lepton mass. The 3×3 Dirac mass of the triplets can be written as

$$M_D = \frac{Y_D^T v}{\sqrt{2}}. \tag{4}$$

Diagonalizing the neutrino mass matrix in Eq. (3) we can write the light neutrino mass eigenvalue as

$$m_\nu \simeq -\frac{v^2}{2} Y_D^T M^{-1} Y_D = M_D M^{-1} M_D^T \tag{5}$$

hence the mixing between light and heavy mass eigenstates can be obtained as $\mathcal{O}(M_D M^{-1})$. Hence the light neutrino flavor eigenstate can be expressed in terms of the light and heavy mass eigenstates in the following way

$$\nu = \mathcal{A} \nu_m + V \Sigma_m \tag{6}$$

where ν_m and Σ_m represent the light and heavy mass eigenstates respectively where $V = M_D M^{-1}$ and $\mathcal{A} = \left(1 - \frac{1}{2}\tilde{\epsilon}\right) V_{\text{PMNS}}$ with $\tilde{\epsilon} = V^* V^T$ and V_{PMNS} is the 3×3 neutrino mixing matrix which diagonalizes the light neutrino mass matrix as

$$V_{\text{PMNS}}^T m_\nu V_{\text{PMNS}} = \text{diag}(m_1, m_2, m_3). \tag{7}$$

Due to the presence of $\tilde{\epsilon}$ the mixing matrix ($\mathcal{A}$) becomes non-unitary, $\mathcal{A}^\dagger \mathcal{A} \neq 1$. The charged current (CC) interactions can be expressed in terms of the mass eigenstates including the light heavy mixings as

$$-\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \begin{pmatrix} \bar{e} & \overline{\Sigma} \end{pmatrix} \gamma^\mu W_\mu^- P_L \begin{pmatrix} (1 + \frac{\epsilon}{2}) V_{\text{PMNS}} & -\frac{Y_D^\dagger M^{-1} v}{\sqrt{2}} \\ 0 & \sqrt{2}(1 - \frac{\epsilon'}{2}) \end{pmatrix} \begin{pmatrix} \nu \\ \Sigma^0 \end{pmatrix}$$
$$+ \frac{g}{\sqrt{2}} \begin{pmatrix} \bar{e} & \overline{\Sigma} \end{pmatrix} \gamma^\mu W_\mu^- P_R$$
$$\times \begin{pmatrix} 0 & -\sqrt{2} m_\ell Y_D^\dagger M^{-2} v \\ -\sqrt{2} M^{-1} Y_D (1 - \frac{\epsilon^*}{2}) V_{\text{PMNS}}^* & \sqrt{2}(1 - \frac{\epsilon'^*}{2}) \end{pmatrix} \begin{pmatrix} \nu \\ \Sigma^0 \end{pmatrix} \tag{8}$$

and the modified neutral current (NC) interaction for the charged sector can be written as

$$-\mathcal{L}_{\text{NC}} = \frac{g}{\cos\theta_W} \begin{pmatrix} \bar{e} & \overline{\Sigma} \end{pmatrix} \gamma^\mu Z_\mu P_L \begin{pmatrix} \frac{1}{2} - \cos^2\theta_W - \epsilon & \frac{Y_D^\dagger M^{-1} v}{2} \\ \frac{M^{-1} Y_D v}{2} & \epsilon' - \cos^2\theta_W \end{pmatrix} \begin{pmatrix} e \\ \Sigma \end{pmatrix}$$
$$+ \frac{g}{\cos\theta_W} \begin{pmatrix} \bar{e} & \overline{\Sigma} \end{pmatrix} \gamma^\mu Z_\mu P_R \begin{pmatrix} 1 - \cos^2\theta_W & m_\ell Y_D^\dagger M^{-2} v \\ M^{-2} Y_D m_\ell v & -\cos^2\theta_W \end{pmatrix} \begin{pmatrix} e \\ \Sigma \end{pmatrix}$$

$$+ \begin{pmatrix} \overline{\nu} & \overline{\Sigma^0} \end{pmatrix} \gamma^\mu Z_\mu P_L \begin{pmatrix} 1 - V_{\text{PMNS}}^\dagger \epsilon V_{\text{PMNS}} & \frac{V_{\text{PMNS}}^\dagger Y_D^\dagger M^{-1} v}{\sqrt{2}} \\ \frac{M^{-1} Y_D V_{\text{PMNS}} v}{\sqrt{2}} & \epsilon' \end{pmatrix} \begin{pmatrix} \nu \\ \Sigma^0 \end{pmatrix} \tag{9}$$

where θ_W is the Weinberg angle or weak mixing angle. Finally we write the interaction Lagrangian of the SM leptons, triplet fermions with the SM Higgs (h) boson. The interaction Lagrangian can be written as

$$-\mathcal{L}_H = \frac{g}{2M_W} \begin{pmatrix} \overline{e} & \overline{\Sigma} \end{pmatrix} h P_L \begin{pmatrix} -\frac{m_\ell}{v}(1 - 3\epsilon) & m_\ell Y_D^\dagger M^{-1} \\ Y_D(1 - \epsilon) + M^{-2} Y_D m_\ell^2 & Y_D Y_D^\dagger M^{-1} v \end{pmatrix} \begin{pmatrix} e \\ \Sigma \end{pmatrix}$$

$$+ \frac{g}{2M_W} \begin{pmatrix} \overline{e} & \overline{\Sigma} \end{pmatrix} P_R \begin{pmatrix} -\frac{m_\ell}{v}(1 - 3\epsilon^*) & M^{-1} Y_D^\dagger m_\ell \\ (1 - \epsilon^*) Y_D^\dagger + m_\ell^2 Y_D^\dagger M^{-2} & M^{-1} Y_D Y_D^\dagger v \end{pmatrix} \begin{pmatrix} e \\ \Sigma \end{pmatrix}$$

$$+ \begin{pmatrix} \overline{\nu} & \overline{\Sigma^0} \end{pmatrix} h P_L \begin{pmatrix} \frac{\sqrt{2} m_\nu}{v} & V_{\text{PMNS}}^T m_\nu Y_D^\dagger M^{-1} \\ (Y_D - \frac{Y_D \epsilon}{2} - \frac{\epsilon'^T Y_D}{2}) V_{\text{PMNS}} & \frac{Y_D Y_D^\dagger M^{-1} v}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu \\ \Sigma^0 \end{pmatrix}$$

$$+ \begin{pmatrix} \overline{e} & \overline{\Sigma^0} \end{pmatrix} P_R \begin{pmatrix} \frac{\sqrt{2} m_\nu}{v} & M^{-1} Y_D m_\nu V_{\text{PMNS}}^* \\ V_{\text{PMNS}}^* (Y_D^\dagger - \frac{\epsilon^* Y_D^\dagger}{2} - \frac{Y_D^\dagger \epsilon'^* Y_D}{2}) & \frac{M^{-1} Y_D Y_D^\dagger v}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu \\ \Sigma^0 \end{pmatrix} \tag{10}$$

The charged multiplets of the triplet fermions can interact with photons (A_μ). The corresponding Lagrangian derived from Eq. (1) can be written as
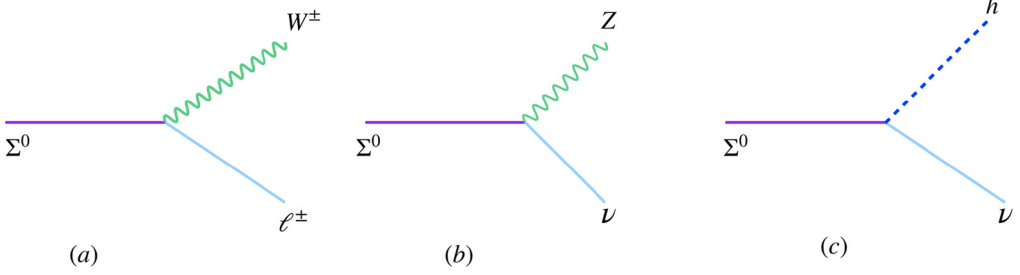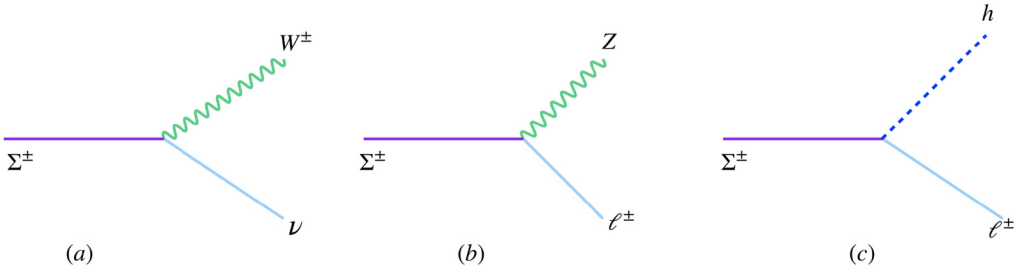
$$-\mathcal{L}_{\gamma\Sigma\Sigma} = g \sin\theta_W \begin{pmatrix} \overline{e} & \overline{\Sigma} \end{pmatrix} \gamma^\mu A_\mu P_L \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e \\ \Sigma \end{pmatrix}$$

$$+ g \sin\theta_W \begin{pmatrix} \overline{e} & \overline{\Sigma} \end{pmatrix} \gamma^\mu A_\mu P_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e \\ \Sigma \end{pmatrix}. \tag{11}$$

In the Eqs. (8)-(10) the parameters $\epsilon = \frac{v^2}{2} Y_D^\dagger M^{-2} Y_D$, $\epsilon' = \frac{v^2}{2} M^{-1} Y_D Y_D^\dagger M^{-1}$ are the small quantities according to [27,62,66]. We neglect the effects of the higher powers (above 1) of ϵ and ϵ' in the calculations. Using the Eq. (8) to Eq. (10) and the expression for the mixing ($V_{\ell\Sigma}$) we calculate the partial decay widths of (Σ^0) as

$$\Gamma(\Sigma^0 \to \ell^+ W) = \Gamma(\Sigma^0 \to \ell^- W) = \frac{g^2 |V_{\ell\Sigma}|^2}{64\pi} \left(\frac{M^3}{M_W^2}\right) \left(1 - \frac{M_W^2}{M^2}\right)^2 \left(1 + 2\frac{M_W^2}{M^2}\right)$$

$$\Gamma(\Sigma^0 \to \nu Z) = \Gamma(\Sigma^0 \to \overline{\nu} Z) = \frac{g^2 |V_{\ell\Sigma}|^2}{128\pi \cos^2\theta_W} \left(\frac{M^3}{M_Z^2}\right) \left(1 - \frac{M_Z^2}{M^2}\right)^2 \left(1 + 2\frac{M_Z^2}{M^2}\right)$$

$$\Gamma(\Sigma^0 \to \nu h) = \Gamma(\Sigma^0 \to \overline{\nu} h) = \frac{g^2 |V_{\ell\Sigma}|^2}{128\pi} \left(\frac{M^3}{M_W^2}\right) \left(1 - \frac{M_h^2}{M^2}\right)^2, \tag{12}$$

respectively for the Majorana neutrinos. The corresponding Feynman Diagrams have been shown in Fig. 1. Similarly the partial decay widths of ($\Sigma^\pm$) are calculated as

$$\Gamma(\Sigma^\pm \to \nu W) = \frac{g^2 |V_{\ell\Sigma}|^2}{32\pi} \left(\frac{M^3}{M_W^2}\right) \left(1 - \frac{M_W^2}{M^2}\right)^2 \left(1 + 2\frac{M_W^2}{M^2}\right)$$

$$\Gamma(\Sigma^\pm \to \ell Z) = \frac{g^2 |V_{\ell\Sigma}|^2}{64\pi \cos^2\theta_W} \left(\frac{M^3}{M_Z^2}\right) \left(1 - \frac{M_Z^2}{M^2}\right)^2 \left(1 + 2\frac{M_Z^2}{M^2}\right)$$

Fig. 1. Decay modes of Σ^0.

Fig. 2. Decay modes of $\Sigma^\pm$.

$$\Gamma(\Sigma^\pm \to \ell h) = \frac{g^2 |V_{\ell\Sigma}|^2}{64\pi} \left(\frac{M^3}{M_W^2}\right)\left(1 - \frac{M_h^2}{M^2}\right)^2, \tag{13}$$
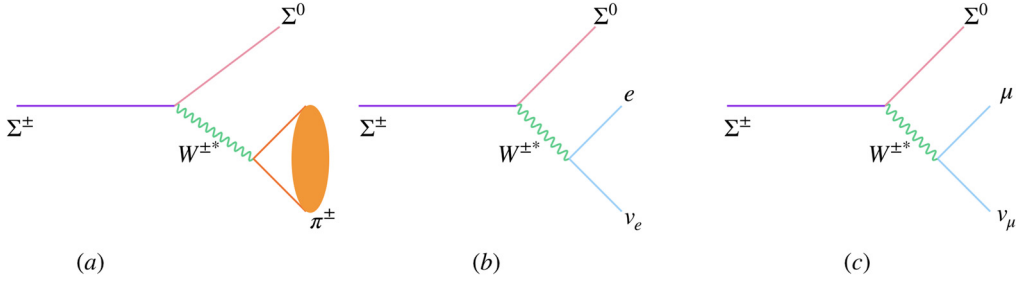
respectively. M_W, M_Z and M_h in the above expressions are the SM W, Z and Higgs boson masses respectively. The corresponding Feynman Diagrams have been shown in Fig. 2. The charged multiplet $\Sigma^\pm$ and neutral multiplet Σ^0 are degenerate in mass at the tree-level. This degeneracy is lifted up due to the radiative corrections induced by the SM gauge boson in the loop. The estimation of this mass difference ΔM is found in Ref. [83] and is given by:

$$\Delta M = \frac{\alpha_2 M}{4\pi} \left(f\left(\frac{M_W}{M}\right) - \cos^2\theta_W f\left(\frac{M_Z}{M}\right) \right) \tag{14}$$

where the function f is defined as $f(r) = \frac{r}{2}\left(2r^3\ln r - 2r + \sqrt{r^2 - 4}(r^2 + 2)\ln A\right)$ and $A = \left(r^2 - 2 - r\sqrt{r^2 - 4}\right)/2$. This mass splitting saturates at the value $\Delta M \approx 170$ MeV for mass $M > 500$ GeV. If this mass splitting ΔM is larger than pion mass, then $\Sigma^\pm$ will have the following additional decay modes [83]

$$\Gamma(\Sigma^\pm \to \Sigma^0 \pi^\pm) = \frac{2G_F^2 V_{ud}^2 \Delta M^3 f_\pi^2}{\pi} \sqrt{1 - \frac{m_\pi^2}{\Delta M^2}}$$

$$\Gamma(\Sigma^\pm \to \Sigma^0 e\nu_e) = \frac{2G_F^2 \Delta M^5}{15\pi}$$

$$\Gamma(\Sigma^\pm \to \Sigma^0 \mu\nu_\mu) = 0.12\, \Gamma(\Sigma^\pm \to \Sigma^0 e\nu_e) \tag{15}$$

which are independent of the free parameters. The corresponding Feynman Diagrams have been shown in Fig. 3. The value of the Fermi Constant, G_F, is 1.1663787×10^{-5} GeV^{-2}, the value of the CKM parameter (V_{ud}) is 0.97420 ± 0.00021 and the decay constant of the π meson, f_π,

Fig. 3. Decay modes of $\Sigma^{\pm}$ evolved from the mass splitting.

is 0.13 GeV from [84]. Notice that for vanishing mixing angles $V_{\ell\Sigma}$, the $\Sigma^{\pm}$ dominantly decay into Σ^0, hence the decay width or the decay length is determined by ΔM and is constant. On the contrary, for very large mixing angles, Σ^0 decay width (decay length) is very large (very small).

The elements of the matrices $\mathcal{A}$ and V in Eq. (6) can be constrained by the experimental data. In this analysis we take the global fit results at 3σ level [85] for the neutrino oscillation parameters:

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = \left[6.79 \times 10^{-5}\text{eV}^2, \, 8.01 \times 10^{-5}\text{eV}^2\right]$$

$$\Delta m_{23}^2 = |m_3^2 - m_2^2| = \left[2.432 \times 10^{-3}\text{eV}^2, \, 2.618 \times 10^{-3}\text{eV}^2\right]$$

$$\sin^2\theta_{12} = \left[0.275, \, 0.350\right]$$

$$\sin^2\theta_{23} = \left[0.427, \, 0.609\right]$$

$$\sin^2\theta_{13} = \left[0.02046, \, 0.02440\right]. \tag{16}$$

The 3×3 neutrino mixing matrix V_{PMNS} is given by

$$V_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13} \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho_1} & 0 \\ 0 & 0 & e^{i\rho_2} \end{pmatrix} \tag{17}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. In our analysis the Dirac CP-phase (δ_{CP}) is a free parameter running between the limit $[-\pi, \pi]$. However, in the recent experiments by NOνA [86] and T2K [87] indicate that δ_{CP} can be $-\frac{\pi}{2} \pm \frac{\pi}{2}$. Due to non-unitarity [66] the elements of $\mathcal{A}$ are severely constrained at 90% C.L.:

$$|\mathcal{A}\mathcal{A}^{\dagger}| = \begin{pmatrix} 1.001 \pm 0.002 & < 1.1 \times 10^{-6} & < 1.2 \times 10^{-3} \\ < 1.1 \times 10^{-6} & 1.002 \pm 0.002 & < 1.2 \times 10^{-3} \\ < 1.2 \times 10^{-3} & < 1.2 \times 10^{-3} & 1.002 \pm 0.002 \end{pmatrix}. \tag{18}$$

The diagonal elements of Eq. (18) are obtained from the precision studies of the SM weak boson where as the SM prediction is 1. The off-diagonal entries of Eq. (18) are the upper bounds obtained from the cLFV studies, for example, the constraints on the 12 and 21 elements of

Eq. (18) are coming from the $\mu \to 3e$ process [88], the constraints on the 23 and 32 elements are coming from the $\tau \to 3\mu$ process and finally the constraints on the 13 and 31 elements are originated from the $\tau \to 3e$ process respectively. These bounds are taken from [66]. The diagonal elements are obtained from LEP [84,89]. As a result we have $\mathcal{A}\mathcal{A}^\dagger \simeq 1 - \tilde{\epsilon}$ and we can calculate the constraints on $\tilde{\epsilon}$ from Eq. (18) as

$$|\tilde{\epsilon}| = \begin{pmatrix} 0.001 \pm 0.002 & < 1.1 \times 10^{-6} & < 1.2 \times 10^{-3} \\ < 1.1 \times 10^{-6} & 0.002 \pm 0.002 & < 1.2 \times 10^{-3} \\ < 1.2 \times 10^{-3} & < 1.2 \times 10^{-3} & 0.002 \pm 0.002 \end{pmatrix}, \tag{19}$$

where we have used the central values for the diagonal elements. Note that the stringent bound is given by the 12-element which is originated from the $\mu \to 3e$ cLFV process.

3. Bounds on the mixing angles under the general parametrization and its effect on the decay of the triplet fermions

In this analysis we generalize of the Dirac Yukawa mass matrix of Eq. (4) using the Casas-Ibarra [82] conjecture as follows

$$M_D^{\text{NH/IH}} = V_{\text{PMNS}}^* \sqrt{D_{\text{NH/IH}}}\, O \sqrt{M}, \tag{20}$$

where O is a general orthogonal matrix and it can be written as

$$O = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[x] & \sin[x] \\ 0 & -\sin[x] & \cos[x] \end{pmatrix} \begin{pmatrix} \cos[y] & 0 & \sin[y] \\ 0 & 1 & 0 \\ -\sin[y] & 0 & \cos[y] \end{pmatrix} \begin{pmatrix} \cos[z] & \sin[z] & 0 \\ -\sin[z] & \cos[z] & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{21}$$

where the angles x, y, z are the complex numbers. Now using $\tilde{\epsilon} = (V^* V^T)_{\text{NH/IH}}$, $(V_{\alpha i})_{\text{NH/IH}} = M_{D_{\text{NH/IH}}} M^{-1}$ and Eqs. (5) and (7) for the two different hierarchies we can write

$$\tilde{\epsilon}^{\text{NH/IH}} = V_{\text{PMNS}} \sqrt{D_{\text{NH/IH}}}\, O^* M^{-1} O^T \sqrt{D_{\text{NH/IH}}} V_{\text{PMNS}}^\dagger, \tag{22}$$

where NH is the normal hierarchy ($m_3 > m_2 > m_1$) and IH is the inverted hierarchy ($m_2 > m_1 > m_3$). The light neutrino mass eigenvalue matrices ($\sqrt{D_{\text{NH/IH}}}$) for the NH and IH cases are written as

$$\sqrt{D^{\text{NH}}} = \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2^{\text{NH}}} & 0 \\ 0 & 0 & \sqrt{m_3^{\text{NH}}} \end{pmatrix}, \quad \sqrt{D^{\text{NH}}} = \begin{pmatrix} \sqrt{m_1^{\text{IH}}} & 0 & 0 \\ 0 & \sqrt{m_2^{\text{IH}}} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \tag{23}$$

where $m_2^{\text{NH}} = \sqrt{\Delta m_{12}^2 + m_1^2}$, $m_3^{\text{NH}} = \sqrt{\Delta m_{23}^2 + (m_2^{\text{NH}})^2}$, $m_2^{\text{IH}} = \sqrt{\Delta m_{23}^2 + m_3^2}$ and $m_1^{\text{IH}} = \sqrt{(m_2^{\text{IH}})^2 - \Delta m_{12}^2}$ for the NH and IH respectively. In both cases, the triplet mass matrix is defined as $M = M(\mathbf{1}_{3\times3})$ which is proportional to a 3×3 unit matrix for the three degenerate triplets. In Eq. (23) the lightest mass eigenvalue is a free parameter and bounded from the PLANCK data [90] and $m_1(m_3)$ is the lightest light neutrino mass eigenvalue for the NH (IH) case. In this analysis δ_{CP} and $\rho_{1,2}$ vary between $[-\pi, \pi]$. In this context we mention that seesaw mechanism has been extensively studied utilizing the general parametrization under the Casas-Ibarra conjecture in [91–101] and following that to study the vacuum stability in type-III seesaw with two generations of the triplet fermions using the Casas-Ibarra conjecture has been studied in [47], however,
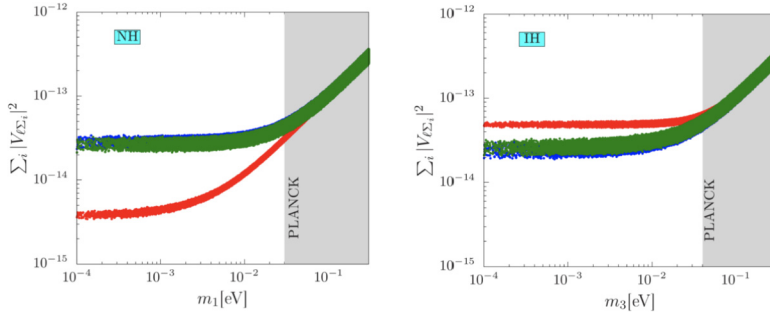
Fig. 4. Bounds on $\Sigma_i |V_{\ell\Sigma_i}|^2$ as a function of the $m_1(m_3)$ NH (IH) case in the left (right) panel for fixed SM lepton flavors. The red band represents electron (e), the blue band represents the muon (μ) and the green band represents the tau (τ). In this case we consider $O = \mathbf{1}_{3\times3}$ as a identity matrix. The same nature will be obtained from case when O is a real orthogonal matrix. We fix the triplet mass $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

in our analysis we study three degenerate triplets under the constraints obtained from the indirect searches. We have three different choices for the orthogonal matrix in Eq. (21) as follows:

(i) O is a identity matrix, $O = \mathbf{1}_{3\times3}$. In this case Eq. (20) will be

$$M_D^{\text{NH/IH}} = V_{\text{PMNS}}^* \sqrt{D_{\text{NH/IH}}} \sqrt{M}. \tag{24}$$

This will further affect the light-heavy mixing. In this case there is no dependence on x, y, z.

(ii) O is a real orthogonal matrix with diagonal and off-diagonal entries, (x, y, z) are real and vary between $[-\pi, \pi]$

(iii) O is a complex orthogonal matrix where x, y, z are the complex numbers, i.e., $x_i + iy_i$ and $-\pi \leq x_i, y_i \leq \pi$. Needless to say, the application of the non-unitarity effects will restrict the unboundedness of the complex quantities in the trigonometric functions.

For the cases (i) and (ii) using the two hierarchies of the neutrino masses (NH and IH) we calculate the modulus square of the mixing between a triplet and the corresponding lepton flavors. Then fixing the lepton flavor, we sum over the triplets as

$$\Sigma_i |V_{\ell\Sigma_i}|^2 = |V_{\ell\Sigma_1}|^2 + |V_{\ell\Sigma_2}|^2 + |V_{\ell\Sigma_3}|^2 \tag{25}$$

where $\ell = e, \mu$ and τ. Note that $\sum_i |V_{\ell\Sigma_i}|^2$ is same if O is identity or real orthogonal matrix. For both of these cases, $\Sigma_i |V_{\ell\Sigma_i}|^2$ have been plotted as a function of the lightest light neutrino mass eigenvalue in Fig. 4. The NH (IH) case is shown in the left (right) panel as a function of m_1 (m_3) where the electron flavor is presented by the red band and the muon and tau flavors are represented by the blue and green bands. In the NH (IH) case the bounds on the electron flavor (muon and tau flavors) are stronger for the decreasing m_1 (m_3). In this analysis we fix the triplet mass $M = 1$ TeV.

We also plot the individual mixing as a function of the $m_1(m_3)$ for the NH (IH) case in the top (bottom) panel of the Fig. 5 for case (i). We find that $|V_{\ell\Sigma_1}|^2$ for electron (red), muon (blue) and tau (green) in the NH case are related to m_1, lower the value of m_1 lowers the individual mixing in the NH case whereas in the IH case the mixings are parallel to the horizontal axis below the PLANCK limit. In both of the cases the $|V_{e\Sigma_1}|^2$ is less stronger than the other mixings. The
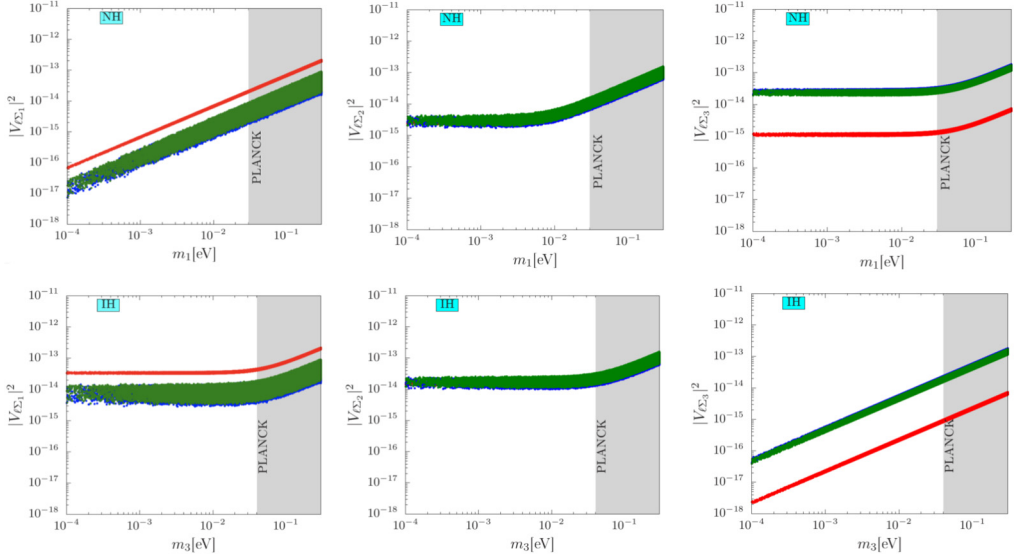
Fig. 5. Bounds on the individual mixing $|V_{\ell \Sigma_i}|^2$ for $O = \mathbf{1}_{3 \times 3}$ as a function of m_1 (m_3) in the NH (IH) case. In this case we fix the triplet flavor (Σ_i) and find the bounds on its mixing with electron (red), muon (blue) and tau (green). We have considered $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data.

mixings for other two flavors overlap with each other. The nature of the $|V_{\ell \Sigma_2}|^2$ is same for the three flavors of the leptons in both of the NH and IH cases, where all flavors overlap with each other. On the other hand for $|V_{\ell \Sigma_3}|^2$ the mixing with the electron flavor is stronger than those with the other two flavors whereas $|V_{\mu \Sigma_3}|^2$ and $|V_{\tau \Sigma_3}|^2$ overlap with each other in both of the NH and IH cases, however, in the NH case all three mixings are parallel to the horizontal axis below the PLANCK limit. On the other hand in the IH case mixing decreases with the decreasing m_3.

In the following we write down the individual mixings between the Σ_1 and the three generations of the leptons for the case of $O = \mathbf{1}_{3 \times 3}$:

$$
\begin{aligned}
|V_{e\Sigma_1}|^2 &= m_1 \frac{c_{12}^2 c_{13}^2}{M} \\
|V_{\mu\Sigma_1}|^2 &= m_1 \frac{|c_{12}s_{12} + c_{12}e^{i\delta_{\mathrm{CP}}}s_{13}s_{23}|^2}{M} \\
|V_{\tau\Sigma_1}|^2 &= m_1 \frac{|c_{12}c_{23}e^{i\delta_{\mathrm{CP}}}s_{13} - s_{12}s_{23}|^2}{M}.
\end{aligned}
\tag{26}
$$

We write down the individual mixings between the Σ_2 and the three generations of the leptons for the case of $O = \mathbf{1}_{3 \times 3}$:

$$
\begin{aligned}
|V_{e\Sigma_2}|^2 &= m_2 \frac{c_{13}^2 s_{12}^2}{M} \\
|V_{\mu\Sigma_2}|^2 &= m_2 \frac{|c_{23}e^{i\delta_{\mathrm{CP}}}s_{12}s_{13} + c_{12}s_{23}|^2}{M} \\
|V_{\tau\Sigma_2}|^2 &= m_2 \frac{|c_{12}c_{23} - e^{i\delta_{\mathrm{CP}}}s_{12}s_{13}s_{23}|^2}{M}
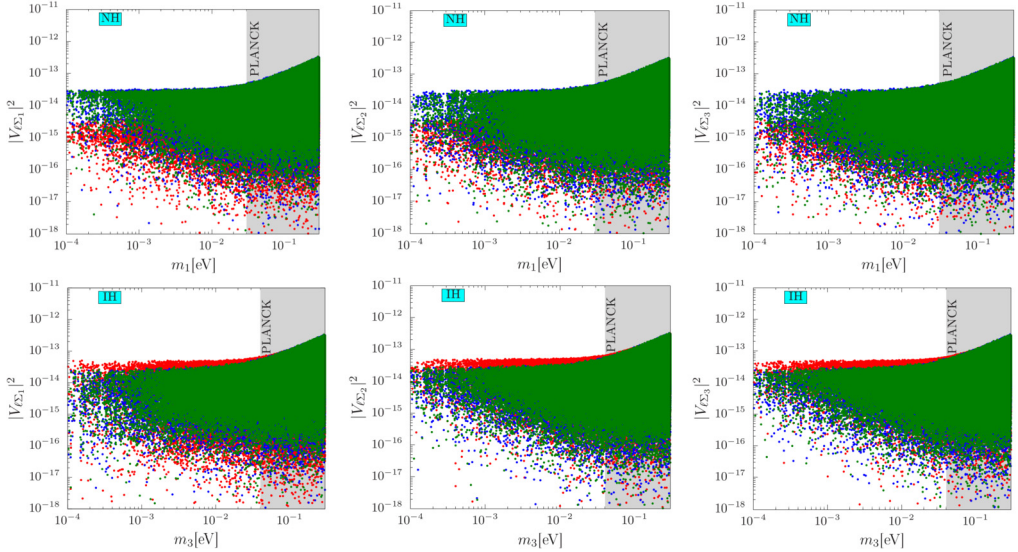\end{aligned}
\tag{27}
$$

Fig. 6. Bounds on the individual mixing $|V_{\ell \Sigma_i}|^2$ for a real orthogonal matrix O as a function of $m_1 (m_3)$ in the NH (IH) case. In this case we fix the triplet flavor (Σ_i) and find the bounds on its mixing with electron (red), muon (blue) and tau (green). We have considered $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data.

and we write down the individual mixings between the Σ_3 and the three generations of the leptons for the case of $O = \mathbf{1}_{3 \times 3}$:

$$|V_{e \Sigma_3}|^2 = m_3 \frac{s_{13}^2}{M}$$

$$|V_{\mu \Sigma_3}|^2 = m_3 \frac{c_{13}^2 s_{23}^2}{M}$$

$$|V_{\tau \Sigma_3}|^2 = m_3 \frac{c_{13}^2 c_{23}^2}{M} \tag{28}$$

Hence we can calculate $\Sigma_i |V_{\ell \Sigma_i}|^2$ from the Eqs. (26)-(28) for $i = 1, 2, 3$ and $\ell = e, \mu, \tau$.

We notice that $|V_{\ell \Sigma_1}|^2$ is proportional to m_1 and $|V_{\ell \Sigma_3}|^2$ is proportional to m_3. Hence in the NH and IH cases the corresponding individual mixings in Eqs. (26) and (28) will tend to zero as $m_1 \to 0$ for the NH and $m_3 \to 0$ for the IH cases, which is clearly visible in Fig. 5. The behavior for the other mixings in the NH ($|V_{\ell \Sigma_2}|^2$, $|V_{\ell \Sigma_3}|^2$) and IH ($|V_{\ell \Sigma_1}|^2$, $|V_{\ell \Sigma_2}|^2$) cases do not have this behavior because they depend on (m_2, m_3) for the NH and on (m_1, m_3) for the IH cases respectively. They almost independent of lightest light neutrino mass eigenvalue for the respective NH (m_1) and IH (m_3) cases slightly below the PLANCK limit.

In the similar fashion we study the case (ii) where O is a real orthogonal matrix of the form Eq. (21) where the elements are the real parameters. The corresponding parameter regions for individual mixing angles are shown in Fig. 6. We notice that the mixing $|V_{\ell \Sigma_1}|^2 (|V_{\ell \Sigma_3}|^2)$ does not go to zero even with the limit $m_1 \to 0 (m_3 \to 0)$ for the NH (IH) case. In the NH case the upper limit of the $|V_{e \Sigma_1}|^2$ parameter space stays below the other two mixings for the three generations of the triplets. This is opposite in the IH case.

In the following we write down the individual mixings between the Σ_1 and the three generations of the leptons with O as real orthogonal matrix:

$$|V_{e\Sigma_1}|^2 = \frac{1}{M_\Sigma}|c_{12}c_{13}\cos[y]\cos[z]\sqrt{m_1} -$$
$$e^{i(\delta_{CP}-\rho_2)}s_{13}\sin[y]\sqrt{m_3} - c_{13}e^{-i\rho_1}s_{12}\cos[y]\sin[z]\sqrt{m_2}|^2$$

$$|V_{\mu\Sigma_1}|^2 = \frac{1}{M_\Sigma}|-e^{i(\delta_{CP}-\rho_2)}s_{13}\cos[y]\sin[x]\sqrt{m_3}+c_{12}c_{13}\sqrt{m_2}$$
$$(-\cos[z]\sin[x]\sin[y]+\cos[x]\sin[z])+e^{-i\rho_1}c_{13}\sqrt{m_2}s_{12}$$
$$(\cos[x]\cos[z]+\sin[x]\sin[y]\sin[z])|^2$$

$$|V_{\tau\Sigma_1}|^2 = \frac{1}{M_\Sigma}|e^{i(\delta_{CP}-\rho_2)}\sqrt{m_3}s_{13}\cos[x]\cos[y]+c_{12}c_{13}\sqrt{m_1}$$
$$(\cos[x]\cos[z]\sin[y]+\sin[x]\sin[z])+c_{12}e^{-i\rho_1}$$
$$\sqrt{m_2}s_{12}(\cos[z]\sin[x]-\cos[x]\sin[y]\sin[z])|^2. \tag{29}$$

We write down the individual mixings between the Σ_2 and the three generations of the leptons with O as real orthogonal matrix:

$$|V_{e\Sigma_2}|^2 = \frac{1}{M_\Sigma}|\sqrt{m_1}(-c_{23}s_{12}-c_{12}e^{i\delta_{CP}}s_{13}s_{23})\cos[y]\cos[z]-c_{13}e^{-i\rho_2}\sqrt{m_3}s_{23}\sin[y]$$
$$-e^{-i\rho_1}\sqrt{m_2}(c_{12}c_{23}-e^{i\delta_{CP}}s_{12}s_{13}s_{23})\cos[y]\sin[z]|^2$$

$$|V_{\mu\Sigma_2}|^2 = \frac{1}{M_\Sigma}|-c_{13}e^{-i\rho_2}\sqrt{m_3}s_{23}\cos[y]\sin[x]+\sqrt{m_1}(-c_{23}s_{12}-c_{12}e^{i\delta_{CP}}s_{13}s_{23})$$
$$(-\cos[z]\sin[x]\sin[y]+\cos[x]\sin[z])+e^{-i\rho_1}\sqrt{m_2}(c_{12}c_{23}-e^{i\delta_{CP}}s_{12}s_{13}s_{23})$$
$$(\cos[x]\cos[z]+\sin[x]\sin[y]\sin[z])|^2$$

$$|V_{\tau\Sigma_2}|^2 = \frac{1}{M_\Sigma}|c_{13}e^{-i\rho_2}\sqrt{m_3}s_{23}\cos[x]\cos[y]+\sqrt{m_1}(-c_{23}s_{12}-c_{12}e^{i\delta_{CP}}s_{13}s_{23})$$
$$(\cos[x]\cos[z]\sin[y]+\sin[x]\sin[z])+e^{-i\rho_2}\sqrt{m_2}(c_{12}c_{23}-e^{i\delta_{CP}}s_{12}s_{13}s_{23})$$
$$(\cos[z]\sin[x]-\cos[x]\sin[y]\sin[z])|^2 \tag{30}$$

and we write down the individual mixings between the Σ_3 and the three generations of the leptons with O as real orthogonal matrix:

$$|V_{e\Sigma_3}|^2 = \frac{1}{M_\Sigma}|\sqrt{m_1}(-c_{12}c_{23}e^{i\delta_{CP}}s_{13}+s_{12}s_{23})\cos[y]\cos[z]-c_{13}c_{23}e^{-i\rho_2}\sqrt{m_3}\sin[y]$$
$$-e^{i\rho_1}\sqrt{m_2}(-c_{12}e^{i\delta_{CP}}s_{12}s_{13}-c_{12}s_{23})\cos[y]\sin[z])|^2$$

$$|V_{\mu\Sigma_3}|^2 = \frac{1}{M_\Sigma}|-c_{13}c_{23}e^{-i\rho_2}\cos[y]\sin[x]$$
$$+\sqrt{m_1}(-c_{12}c_{23}e^{i\delta_{CP}}s_{13}+s_{12}s_{23})(-\cos[z]\sin[x]\sin[y]+\cos[x]\sin[z])$$
$$+e^{-i\rho_1}\sqrt{m_2}(-c_{23}e^{i\delta_{CP}}s_{12}s_{13}-c_{12}s_{23})(\cos[x]\cos[z]+\sin[x]\sin[y]\sin[z])|^2$$

$$|V_{\tau\Sigma_3}|^2 = \frac{1}{M_\Sigma}|c_{13}c_{23}e^{-i\rho_2}\sqrt{m_3}\cos[x]\cos[y]+\sqrt{m_1}(-c_{12}c_{23}e^{i\delta_{CP}}s_{13}+s_{12}s_{23})$$
$$(\cos[x]\cos[z]\sin[y]+\sin[x]\sin[z])+e^{-i\rho_1}\sqrt{m_2}(-c_{23}e^{i\delta_{CP}}s_{12}s_{13}-c_{12}s_{23})$$
$$(\cos[z]\sin[x]-\cos[x]\sin[y]\sin[z])|^2. \tag{31}$$

We calculate again $\Sigma_i|V_{\ell\Sigma_i}|^2$ from the Eqs. (29)-(31) and find that this is same with the case of $O = \mathbf{1}_{3\times3}$.
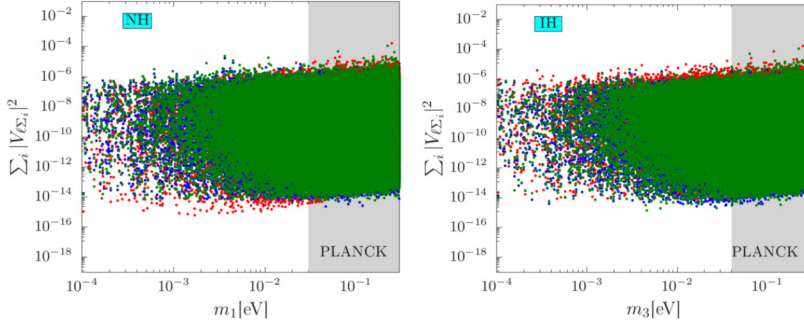
Fig. 7. Bounds on $\Sigma_i |V_{\ell \Sigma_i}|^2$ as a function of the $m_1 (m_3)$ NH (IH) case in the left (right) panel for fixed SM lepton flavors. The red band represents electron (e), the blue band represents the muon (μ) and the green band represents the tau (τ). In this case we consider O as a complex orthogonal matrix. We fix the triplet mass $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data.

We notice that unlike the case of $O = \mathbf{1}_{3 \times 3}$, now $|V_{\ell \Sigma_1}|^2 (|V_{\ell \Sigma_3}|^2)$ is a function of all three light neutrino mass eigenvalues (m_1, m_2 and m_3). Hence in the NH and IH cases the corresponding individual mixings in Eqs. (29) and (31) will not tend to zero even for $m_1 \to 0$ for the NH and $m_3 \to 0$ for the IH cases respectively when O is a real general orthogonal matrix, which is clearly visible in Fig. 6. Same argument will be applicable for the mixings $|V_{\ell \Sigma_2}|^2$ and $|V_{\ell \Sigma_3}|^2$ in the NH case and the mixings $|V_{\ell \Sigma_1}|^2$ and $|V_{\ell \Sigma_2}|^2$ in the IH case. This behavior can be observed in Fig. 6. We consider $M = 1$ TeV in this analysis.

We also study the effect of the general parametrization where O is an orthogonal matrix of the form given in Eq. (21) and using the case (iii). We take the most general form of the entries of the matrix as complex parameters. In this case running over the full set of the parameters we find that there is no special correlation between the mixings and $m_1(m_3)$ for the NH (IH) case. The interesting fact is due to Casas-Ibarra conjecture and for the complex orthogonal matrix, the maximum possible mixing is enhanced dramatically. Fixing the generation of the SM charged lepton and summing over the triplet generations, we plot the bounds on the mixings satisfying the neutrino oscillation data and the PLANCK limit for two hierarchic masses in Fig. 7 as a function of the lightest light neutrino mass in each hierarchy. We notice that the application of the Casas-Ibarra conjecture improves the mixing by several orders of magnitude under the applied constraints.

We show the individual mixing in Fig. 8 for the NH (IH) case as a function of the lightest light neutrino mass m_1 (m_3). At this point we mention that using Eq. (29)- (31) we can similarly calculate $\Sigma_i |V_{\ell \Sigma_i}|^2$ for the case (iii) using complex values of x, y and z and it will not be same as the case of (i) or (ii). $|V_{\ell \Sigma_i}|^2$ is now a complicated function of the light neutrino mass eigenvalues m_i, complex parameters x y, z and the CP violating phases $\delta_{\rm cp}$, ρ_i. Therefore the extreme smallness of lightest mass eigenvalues $m_1 \to 0(m_3 \to 0)$ will not push the mixing to zero because the rest of the two light neutrino mass eigenvalues will not allow to do that. For the individual mixing, in each panel of Fig. 8 we show the mixings between the triplet fermion and the charged lepton. The important fact of this scenario is the upper bounds of the light heavy mixing squared which can go up to an $\mathcal{O}(10^{-5})$, however, the lower bounds stay around $\mathcal{O}(10^{-18})$. We have showed the individual mixing for the NH (IH) case in the upper (lower) of Fig. 8.
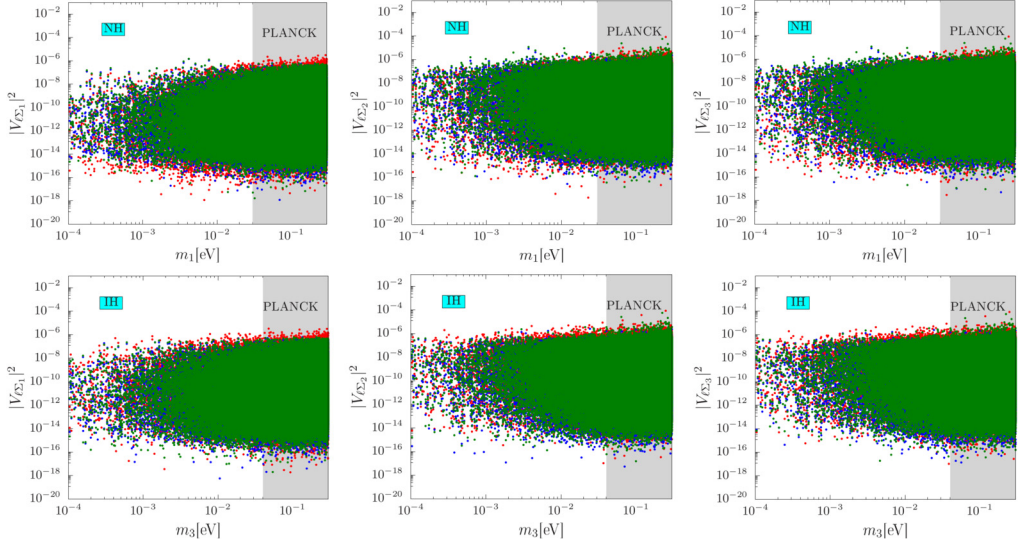
Fig. 8. Bounds on $|V_{\ell\Sigma_i}|^2$ as a function of the $m_1(m_3)$ NH (IH) case in the upper (lower) panel for fixed SM lepton flavors. The red band represents electron (e), the blue band represents the muon (μ) and the green band represents the tau (τ). In this case we consider O as a complex orthogonal matrix. The same nature will be obtained from case when O is a real orthogonal matrix. We fix the triplet mass $M = 1$ TeV. The shaded region in gray is ruled out by the PLANCK data.

The individual mixings for the orthogonal matrix O with the complex elements can be written as Eqs. (29), (30) and (31) respectively taking x, y and z as the complex quantities having real and imaginary parts.

4. Branching ratios of the triplet fermion for different choices of the orthogonal matrices

Using the three typical forms of the orthogonal matrix O, we calculate the bounds on the branching ratios of the Σ^0 and $\Sigma^\pm$ respectively. We consider a degenerate scenario for the three generations of the triplet fermions having mass at $M = 1$ TeV. Summing over the three generations of Σ_i^0 and $\Sigma_i^\pm$ separately, we obtain the total branching ratios of Σ_{Tot}^0 and $\Sigma_{\text{Tot}}^\pm$ respectively for the NH and IH cases.

We consider the leading mode of Σ_i^0 to $\ell^\pm W$ as this is the visible one. For $\Sigma_i^\pm$ we consider all the decay modes because where νW is the leading mode, $\ell^\pm Z$ and $\ell^\pm h$ are the subdominant modes but visible with the charged leptons. BR($\Sigma_{\text{Tot}}^0 \to \ell^\pm W$) for the NH (IH) case has been plotted in the top-left (top-right) panel of the Fig. 9. The muon (blue) and tau (green) modes are dominant over the electron (red) mode of the lepton flavors in the NH case. The IH case is opposite to the NH case. We find that the results are same for the orthogonal matrix as a identity and as a real matrix.

Corresponding total branching ratios BR($\Sigma_{\text{Tot}}^\pm$) for the νW, $\ell^\pm Z$ and $\ell^\pm h$ modes are shown for the NH (IH) case in the top (bottom) panel of Fig. 10 for the O as a 3×3 identity matrix. This is exactly same for the case when O is a real orthogonal matrix. The νW mode is shown in the left column. In this case we do not distinguish between the light neutrinos as they will be obtained as the missing energy. The $\ell^\pm Z$ ($\ell^\pm h$) mode has been shown in the second (third) column of the Fig. 10. For the $\ell^\pm Z$ and $\ell^\pm h$ modes we show the electron (red), muon (blue) and
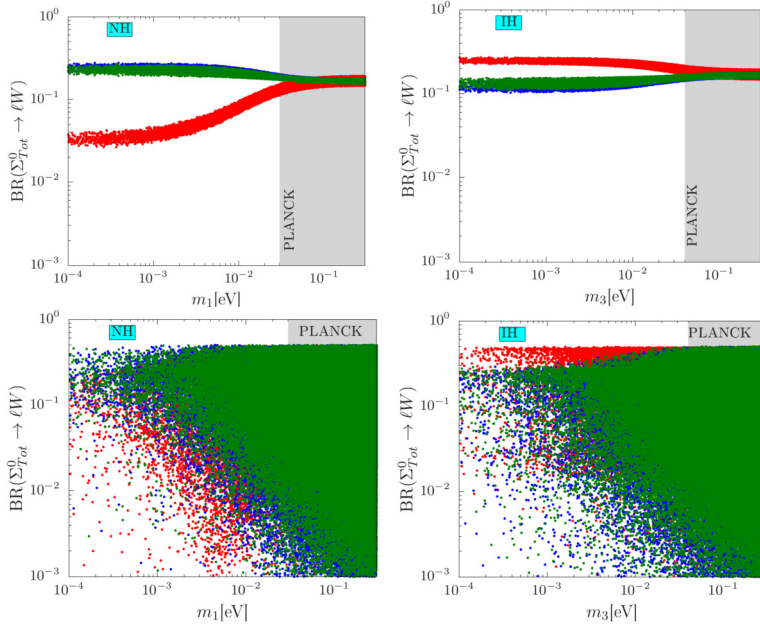
14

Fig. 9. Total branching ratio of Σ^0_{Tot} ($\sum_i \Sigma^0_i$) into the leading ℓW mode as a function of the lightest neutrino mass for the NH (m_1) and IH (m_3) cases in the left and right panels respectively. We add three generations of Σ^0_i to obtain Σ_{Tot}. The mode containing electron is represented by the red dots, that containing muon is shown by blue dots and the tau mode is represented by green dots. This result is same for the orthogonal matrix considered to be a identity matrix and a real matrix. The corresponding result is shown upper panel. We have also considered the case where O is a general orthogonal matrix. The result is shown in the lower panel. We consider $M = 1$ TeV.

tau (green) leptons separately for the NH (top row) and IH (bottom row) cases. In the NH case muon and tau regions coincide and dominate over the electron mode. In the IH case the electron mode dominates over the muon and tau modes.

We show the individual leading branching ratio of Σ^0_i in the Fig. 11 for the NH (IH) case in the top (bottom) panel for the orthogonal matrix as a identity matrix. For the first generation of the neutral multiplet of the triplet (Σ^0_1) we show that the branching ratio into the electron (red) mode dominates over the muon (blue) and tau (green) flavor for the NH and IH cases. For the second generation (Σ^0_2) all the modes coincide with each other for both of the neutrino mass hierarchy. For the third generation (Σ^0_3) the muon and tau modes coincide and they dominate over the electron mode for the NH and IH cases.

We study the individual branching ratios of the three different generations of the charged multiplet $\Sigma^\pm_1$ into νW, $\ell^\pm Z$ and $\ell^\pm h$ modes respectively where the orthogonal matrix has been considered as a identity matrix. The νW mode is dominant over the $\ell^\pm Z$ and $\ell^\pm h$ modes. In Fig. 12 we show the different decay modes of $\Sigma^\pm_i$ for the NH (IH) case in the top (bottom) panel. For the νW mode we do not distinguish between the neutrinos as the neutrinos will be considered as the missing momenta and hence we summed over all flavor of neutrinos. Therefore we have the single line in the first column for both of the NH and IH cases. In the $\ell^\pm Z$ and $\ell^\pm h$ modes we have almost the same nature in both of the neutrino mass hierarchies where the electron mode (red) dominates over the muon (blue) and tau (green) modes.
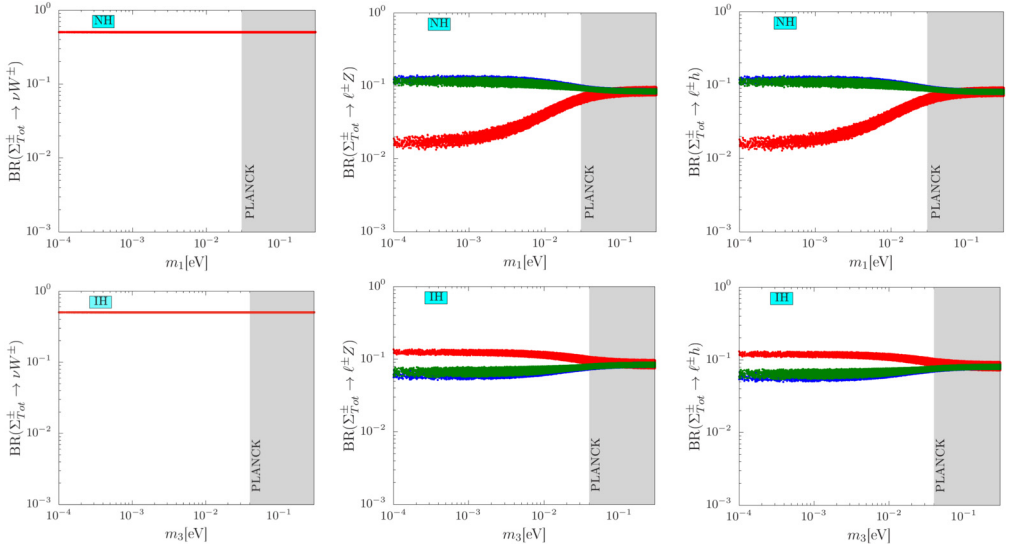
Fig. 10. Total branching ratio of $\Sigma^{\pm}$ (BR($\Sigma^{\pm}_{Tot}$)) into the leading νW (first column), subleading ℓZ (second column) and ℓh (third column) modes with respect to the lightest neutrino mass for the NH (m_1) and IH (m_3) cases in the top and bottom panels respectively. We add three generations of $\Sigma^{\pm}_i$ to obtain $\Sigma^{\pm}_{Tot}$. The mode containing electron is represented by the red dots, that containing muon is shown by blue dots and the tau mode is represented by green dots. The νW mode is indistinguishable from the point of view of the neutrinos. This result is same for the orthogonal matrix considered to be a identity matrix and a real matrix. The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.
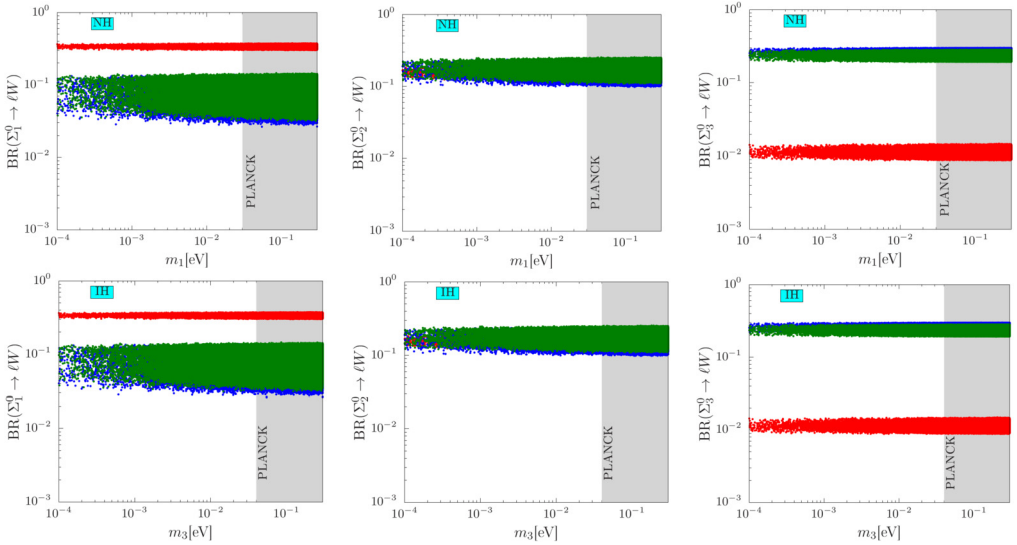

Fig. 11. Individual branching ratio of Σ^0_i into the leading ℓW mode as a function of the lightest neutrino mass for the NH (m_1) and IH (m_3) cases in the top and bottom panels respectively for the orthogonal matrix considered to be a identity matrix. The decay modes contain electron (red), muon (blue) and tau (green) for the Σ_1 (left column), Σ_2 (middle column) and Σ_3 (right column). The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.
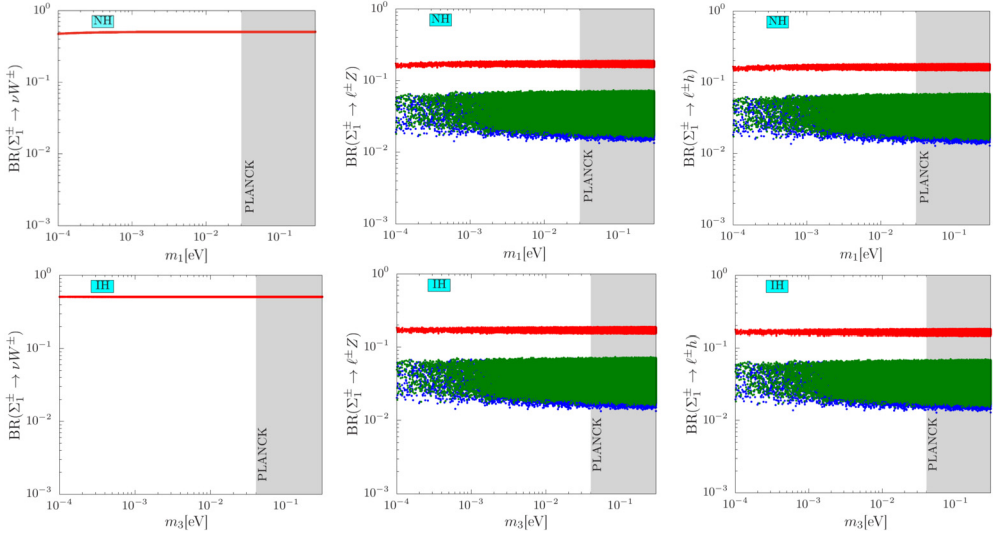
Fig. 12. Individual branching ratio of $\Sigma_1^{\pm}$ into the leading νW (left column) and subleading $\ell^{\pm} Z$ (middle column), $\ell^{\pm} h$ (right column) modes with respect to the lightest neutrino mass for the NH (m_1) and IH (m_3) cases in the top and bottom panels respectively for the orthogonal matrix considered to be a identity matrix. The decay modes contain electron (red), muon (blue) and tau (green). The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

Fig. 13. Same as Fig. 12 but now for $\Sigma_2^{\pm}$.

The behavior for the νW mode can be obtained for the second generation of the charged multiplet $\Sigma_2^{\pm}$ in the first column of the Fig. 13 for the NH and IH case. We also study the $\ell^{\pm} Z$ and $\ell^{\pm} h$ modes. We notice that the parameter regions for the three flavors coincide with each other for both the NH and IH case, see top and bottom panel of the middle column in Fig. 13.

Fig. 14. Same as Fig. 12 but now for $\Sigma_3^\pm$.

Fig. 15. Individual branching ratio of Σ_i^0 into the leading ℓW mode as a function of the lightest neutrino mass (m_1, m_3) for the two hierarchic cases (NH, IH) for the real orthogonal matrix. Red, blue and green color stand for electron, muon and tau modes, respectively. The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

The third generation charged triplet $\Sigma_3^\pm$ decaying into νW shows the same behavior as the other two generations, see top and bottom panel of first column in Fig. 14. For the $\ell^\pm Z$ and $\ell^\pm h$ modes we see a different behavior unlike the other two generations. In case of $\Sigma_3^\pm$ the muon (blue) and tau (green) modes dominate over the electron (red) mode. The corresponding parameter spaces for the NH (IH) case are shown in the top (bottom) panel of the second column in Fig. 14.

We have studied the case where O is a general real orthogonal matrix. In this case the branching ratios of the three generations of Σ_i^0 and $\Sigma_i^\pm$ are shown in Figs. 15 and 16, respectively. For Σ_i^0 we show the leading visible mode in Fig. 15. We found that the NH and IH cases show same parameter spaces for the real orthogonal matrix. For the $\Sigma_i^\pm$ we demonstrate the subdominant $\ell^\pm Z$ and $\ell^\pm h$ cases because they are the visible final states with the charged leptons.

We have also studied the case where O is a general complex orthogonal matrix. We show the total branching ratio of the neutral multiplet into the leading mode (BR($\Sigma_{\text{Tot}}^0 \to \ell W$)) in the bottom-left (bottom-right) panel of Fig. 9 for the NH (IH) cases. The muon (blue) and tau

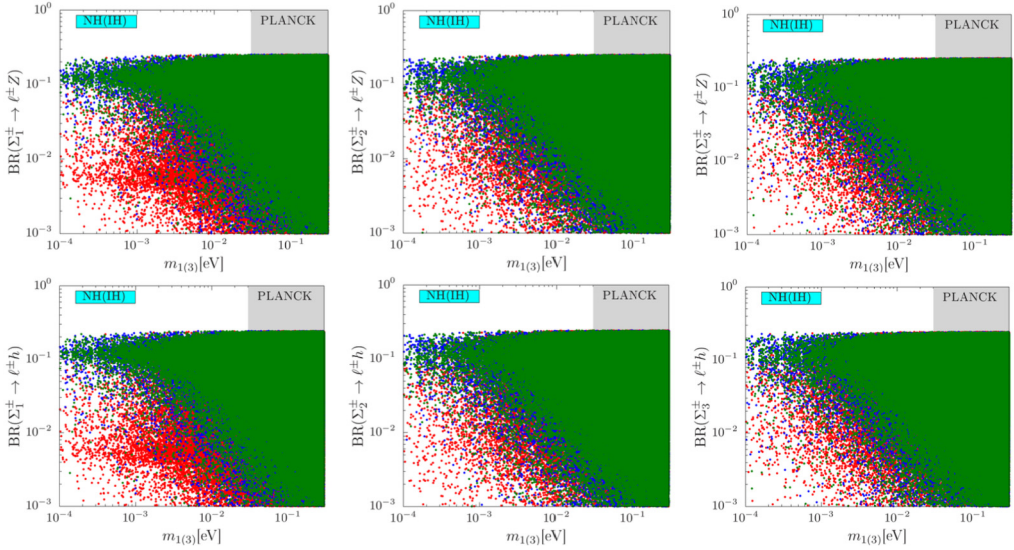Fig. 16. Individual branching ratio of $\Sigma_i^{\pm}$ into the subleading and visible $\ell^{\pm}Z$ (top panel) and $\ell^{\pm}h$ (bottom panel) modes with respect to the lightest neutrino mass (m_1, m_3) for the two hierarchic cases (NH, IH) for the real orthogonal matrix. Red, blue and green color stand for electron, muon and tau modes, respectively. The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

(green) modes are dominant over the electron (red) mode of the lepton flavors in the NH case. The IH case is opposite to the NH case. We show the individual branching ratio of the three generations of Σ_i^0 for the NH (IH) case in the upper(lower) panel of the Fig. 17. The decay of the three generations of the triplets into the electron dominates over the decay mode into the other two leptons in the IH case whereas the result is opposite in the NH case. Here we would like to comment that we do not show the individual or total branching ratio into the different modes for the $\Sigma_i^{\pm}$s because they will have exactly the same repertoire like the Σ_i^0s when O is a complex orthogonal matrix.

5. Implications on the displaced decay of the triplet fermions

We can write the proper decay lengths of the Σ_i^0 and $\Sigma_i^{\pm}$ in millimeter for the NH and IH cases as follows:

$$\mathrm{L}^{\Sigma_i^{0\,\mathrm{NH/IH}}} = \frac{1.97 \times 10^{-13}}{\Gamma_{\Sigma_i^0}^{\mathrm{NH/IH}}[\mathrm{GeV}]}[\mathrm{mm}] \text{ and } \mathrm{L}^{\Sigma_i^{\pm\,\mathrm{NH/IH}}} = \frac{1.97 \times 10^{-13}}{\Gamma_{\Sigma_i^{\pm}}^{\mathrm{NH/IH}}[\mathrm{GeV}]}[\mathrm{mm}] \quad (32)$$

where i stands for the three generations of the triplets. In this analysis we consider three types of the general orthogonal matrix (O) as described in Sec. 3. When O is an identity matrix the proper decay lengths are shown in Fig. 18 for the NH (IH) case with respect to the lightest neutrino mass $m_1(m_3)$. The decay lengths of the $\Sigma_i^0(\Sigma_i^{\pm})$ are shown in the upper (lower) panel of Fig. 18. In the NH (upper, left panel) case we see that the proper decay length of Σ_1^0 becomes inversely proportional to m_1 which has been represented in red. The proper decay lengths for the other two generations Σ_2^0 represented be blue and Σ_3^0 represented by green become constant when $m_1 < 10^{-2}$ eV. We estimate that for $m_1 = 10^{-4}$ eV, $\mathrm{L}^{\max}_{\Sigma_1^0} \sim 1.5$ mm whereas that for $\Sigma_2^0 \, (\Sigma_3^0)$
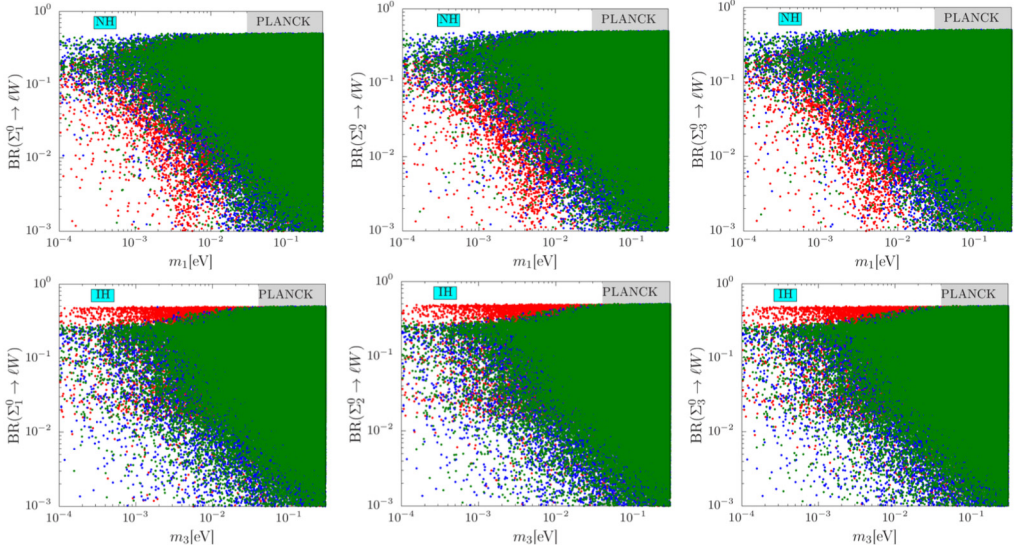
Fig. 17. Individual branching ratio of Σ_i^0 into the leading ℓW mode as a function of the lightest neutrino mass m_1 (m_3) using the NH (IH) case for the general complex orthogonal matrix in the upper (lower) panel. Red, blue and green color stand for electron, muon and tau modes, respectively. The shaded region in gray is excluded by the PLANCK data. We consider $M = 1$ TeV.

Fig. 18. Proper decay length of $\Sigma_i^0(\Sigma_i^\pm)$ for $O = \mathbf{1}_{3\times3}$ with respect to the lightest neutrino mass in the upper (lower) panel. We show the NH (IH) case in the left (right) panel using the neutrino oscillation data in Eq. (16). The first generation triplet is represented by the red band, the second generation is represented by blue band and the third generation is represented by green band respectively. We consider $M = 1$ TeV. The shaded region is excluded by the PLANCK data.

Table 1
Benchmark for the proper decay lengths of $\Sigma_1^{0,\pm}$ ($\Sigma_3^{0,\pm}$) for the NH and IH cases fitting the neutrino oscillation data in Eq. (16) when $O = \mathbf{1}_{3\times3}$. The variation of the proper decay length represents a band due to the variation of $\pm3\sigma$ the oscillation data, δ_{CP} and ρ_i. We consider $M = 1$ TeV.

Decay Length [mm]	$m_{\text{lightest}} = 10^{-6}$ eV	$m_{\text{lightest}} = 10^{-10}$ eV
$L_{\Sigma_1^0}$ (NH)	[134.13, 171.03]	[1.35×10^6, 1.74×10^6]
$L_{\Sigma_3^0}$ (IH)	[129.04, 183.71]	[1.28×10^6, 1.83×10^6]
$L_{\Sigma_1^\pm}$ (NH)	[20.29, 20.99]	[23.9321, 23.9322]
$L_{\Sigma_3^\pm}$ (IH)	[20.18, 21.17]	[23.9321, 23.9322]

is two (three) orders of magnitude less. This nature of $L_{\Sigma_1^0}$ can be realized from the Eq. (26). The mixings between Σ_1 and the SM leptons, $|V_{\ell\Sigma_1}|^2$, are proportional to m_1. Therefore when $m_1 \to 0$ the corresponding decay length of Σ_1^0 becomes very large. We have also tested this nature considering the lightest light neutrino mass m_1 (m_3) for the NH (IH) case at 10^{-6} eV and 10^{-10} eV respectively. The results are shown in the first row of Table 1. The corresponding lengths are two and six orders magnitude larger than that for $m_1 = 10^{-4}$ eV. In the IH (upper, right panel)case we have the same scenario for the Σ_3^0 where as decay lengths of Σ_1^0 and Σ_2^0 coincide. For Σ_3^0 we notice the form of $|V_{\ell\Sigma_3}|^2$, proportional to m_3, in Eq. (26). Therefore the decay length of Σ_3^0 becomes very large when m_3 is very small and the corresponding benchmarks are given in the second row of Table 1. Lower panel of Fig. 18 shows that at least for lightest neutrino mass $m_1(m_3) > 10^{-4}$ eV, the decay length of $\Sigma_1^\pm$ ($\Sigma_3^\pm$) in NH(IH) case has the same nature as the decay length of Σ_1^0 (Σ_3^0) in NH(IH) case. For the lightest neutrino mass range $m_1(m_3) \leq 10^{-4}$ eV, the behavior of the decay length of $\Sigma_1^\pm$ ($\Sigma_3^\pm$) in NH(IH) case is completely different from the decay length of Σ_1^0 (Σ_3^0) in NH(IH) case, see the third and fourth rows of Table 1. This implies that for $m_1(m_3) \leq 10^{-4}$ eV, $L_{\Sigma_1^\pm}(L_{\Sigma_3^\pm})$ is more or less constant. The reason for this is, in NH(IH) case as $m_1(m_3) \to 0$, mixing angle $|V_{\ell\Sigma_1}|^2(|V_{\ell\Sigma_3}|^2) \to 0$ and as a result the decay width for $\Sigma_1^\pm(\Sigma_3^\pm)$ will be dominated by the decay modes given in Eq. (15) which is controlled by the ΔM parameter. Hence this decay width or decay length is constant which can be noted from the benchmarks in the third or fourth row of Table 1. We notice that in this case one can obtain large decay lengths which indicate possibilities of the displaced vertex scenarios when the decay lengths are $\mathcal{O}(100 \text{ mm})$. Possible scenarios of the further long-livedness can also be observed when the decay lengths are $\mathcal{O}(10^6 \text{ mm})$.

Similarly we consider the case when O is a real orthogonal matrix. In this case the analytical form for mixings is given in Eq. (29)-(31). We notice that now $|V_{\ell\Sigma_1}|^2$ depends on all the light neutrino mass eigenvalues like m_1, m_2 and m_3. Therefore in the NH case for $m_1 \to 0$, $|V_{\ell\Sigma_1}|^2$ attains a limiting value but does not vanish. Which will be reflected in the nature of the proper decay lengths of Σ_i^0 and $\Sigma_i^\pm$ respectively. Similar behavior can be observed for the IH case when $m_3 \to 0$, $|V_{\ell\Sigma_3}|^2$ does not vanish due to its dependence on m_1 and m_2. The decay lengths of $\Sigma_i^0(\Sigma_i^\pm)$ are shown in the upper (lower) panel of Fig. 19. We find that for lightest light neutrino mass range 10^{-4} eV $\leq m_{1(3)} \leq 0.1$ eV, maximum decay length can be around 1 mm. We have also considered some benchmark scenarios for very small lightest light neutrino mass, m_1 (m_3) for the NH (IH) case. We fix m_1 (m_3) at 10^{-6} eV and 10^{-10} eV respectively and find out the corresponding decay lengths in Table 2 fitting the neutrino oscillation data from Eq. (16). In
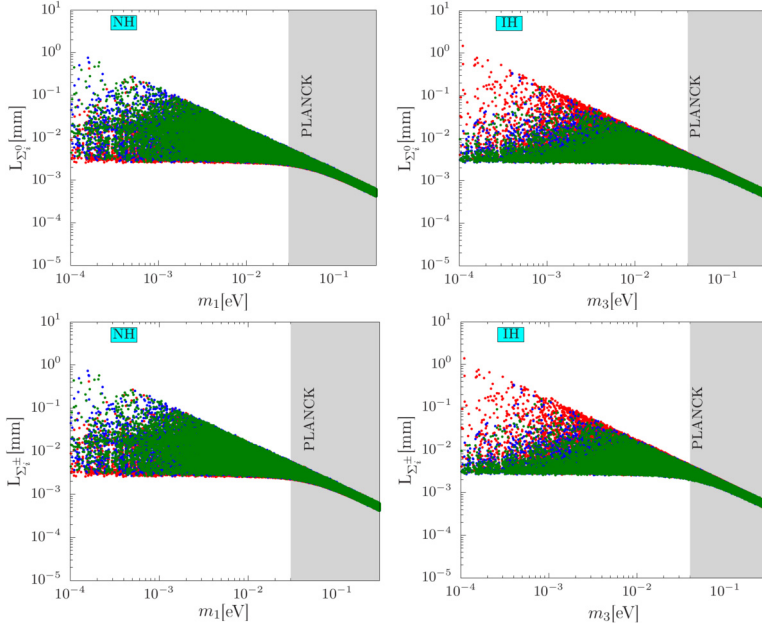
Fig. 19. Proper decay length of $\Sigma_i^0(\Sigma_i^\pm)$ when O is a real and general orthogonal matrix with respect to the lightest neutrino mass in the upper (lower) panel. We show the NH (IH) case in the left (right) panel using the neutrino oscillation data in Eq. (16). The first generation triplet is represented by the red band, the second generation is represented by blue band and the third generation is represented by the green band respectively. We consider $M = 1$ TeV. The shaded region is excluded by the PLANCK data.

Table 2

Benchmark for the proper decay lengths of $\Sigma_1^{0,\pm}$ ($\Sigma_3^{0,\pm}$) for the NH and IH cases fitting the neutrino oscillation data in Eq. (16) when O is a real and general orthogonal matrix. The variation of the proper decay length represents a band due to the variation of $\pm 3\sigma$ the oscillation data, δ_{CP}, ρ_i and the parameters of the orthogonal matrix. We consider $M = 1$ TeV.

Decay Length [mm]	$m_{lightest} = 10^{-6}$ eV	$m_{lightest} = 10^{-10}$ eV
$L_{\Sigma_1^0}$ (NH)	[0.0027, 171.1]	[0.0028, 1.74×10^6]
$L_{\Sigma_3^0}$ (IH)	[0.0026, 183.79]	[0.0026, 1.84×10^6]
$L_{\Sigma_1^\pm}$ (NH)	[0.0029, 20.84]	[0.0025, 23.93]
$L_{\Sigma_3^\pm}$ (IH)	[0.0027, 21.18]	[0.0027, 23.93]

this case we have found that the minimum decay length can be as low as $\mathcal{O}(10^{-3}$ mm) and the maximum decay length is of the same order as the case of identity orthogonal matrix O. The decay length can reach at a maximum value of $\mathcal{O}(10^6$ mm) showing the possibility of a long-lived scenario. When the decay length is $\mathcal{O}(10^{-3}$ mm), the decay of the triplet can be prompt. In that case, a comparatively large mixing can be expected.

We have also studied the effect when O is a complex orthogonal matrix. The real and imaginary parts of the elements of O vary between $[-\pi, \pi]$. Scanning over the δ_{CP} and $\rho_{1,2}$ within the interval $[-\pi, \pi]$ simultaneously we show the range of the proper decay length for some

Table 3

Benchmark for the proper decay lengths of $\Sigma_1^{0,\pm}$ ($\Sigma_3^{0,\pm}$) for the NH and IH cases fitting the neutrino oscillation data in Eq. (16) when O is a complex and general orthogonal matrix. The variation of the proper decay length represents a band due to the variation of $\pm 3\sigma$ the oscillation data, δ_{CP}, ρ_i and the parameters of the orthogonal matrix. We consider $M = 1$ TeV.

Decay Length [mm]	$m_{\text{lightest}} = 10^{-6}$ eV	$m_{\text{lightest}} = 10^{-10}$ eV
$L_{\Sigma_1^0}$ (NH)	$[1.11 \times 10^{-11}, 171.2]$	$[1.08 \times 10^{-10}, 1.74 \times 10^6]$
$L_{\Sigma_3^0}$ (IH)	$[1.74 \times 10^{-11}, 183.79]$	$[1.54 \times 10^{-11}, 1.84 \times 10^6]$
$L_{\Sigma_1^\pm}$ (NH)	$[1.32 \times 10^{-10}, 20.84]$	$[1.47 \times 10^{-10}, 23.93]$
$L_{\Sigma_3^\pm}$ (IH)	$[1.41 \times 10^{-11}, 21.18]$	$[8.23 \times 10^{-12}, 23.93]$

benchmark scenarios in Table 3 fitting the neutrino oscillation data from Eq. (16). We adopt such a method for this case because there is no special pattern observed in this case after the scan. Therefore we fix the lightest light neutrino mass m_1 (m_3) in the NH (IH) case at 10^{-6} eV and 10^{-10} eV respectively. Due to the presence of the complex orthogonal matrix there will be an improvement in the light-heavy mixings. As a result we can expect a prompt production of the triplets as expressed by the small decay lengths $\mathcal{O}(10^{-11}$ mm) which represent a large mixing. On the other hand there will be some possibilities where small mixings can be observed and due to that large decay lengths $\mathcal{O}(100$ mm) can be obtained which ensure a possible displaced vertex scenario and if the decay lengths are $\mathcal{O}(10^6$ mm) then a further long-lived case might be studied.

6. Implications on the collider searches

The triplets involved in the type-III seesaw mechanism are being studied at the LHC at different center of mass energies. We also consider to give a discussion on the possible bounds and current limits on the light-heavy mixings which are important for the neutrino mass generation mechanism. We consider these bounds to select an allowed triplet mass to study the discovery potential of the $\Sigma^\pm$ and Σ^0 at the hadron colliders at $\sqrt{s} = 13$ TeV, 27 TeV and 100 TeV respectively. Finally we briefly discuss about the discovery potentials at the electron-positron ($e^- e^+$) and electron-proton($e^- p$) colliders.

6.1. LHC bounds

A comprehensive roadmap has been pictorially represented in Fig. 20 where we have shown modes of the triplet fermions, different exclusion bounds at different stages of LHC with some salient features of the signal and model parameters. The High-Luminosity LHC (HL-LHC) and the future colliders have no results so far and kept in the gray area. At the 7 TeV LHC the CMS studied the 3 lepton final state from the $\Sigma^0 \Sigma^\pm$ process with 4.9 fb^{-1} luminosity [37] with Flavor Diagonal (FD) Yukawa coupling with equal branching ratios ($\mathcal{B}$) for the three flavors of the charged leptons, $\mathcal{B}_e = \mathcal{B}_\mu = \mathcal{B}_\tau$. CMS has also studied the cases with 100% decay into the electron and muon flavors. Finally sets a lower bound the triplet mass at $M_\Sigma < 180$ GeV. At the 8 TeV the ATLAS searched for the triplet fermion at 5.8 fb^{-1} luminosity and sets a lower bound $M_\Sigma < 245$ GeV at 95% C.L. [102] where they consider a two flavor case with electron and muon with corresponding mixings $V_e = 0.055$ (with electron) and $V_\mu = 0.063$ (with muon) and no mixing with the tau flavor. At the 8 TeV with 20.3 fb^{-1} luminosity, ATLAS studied another
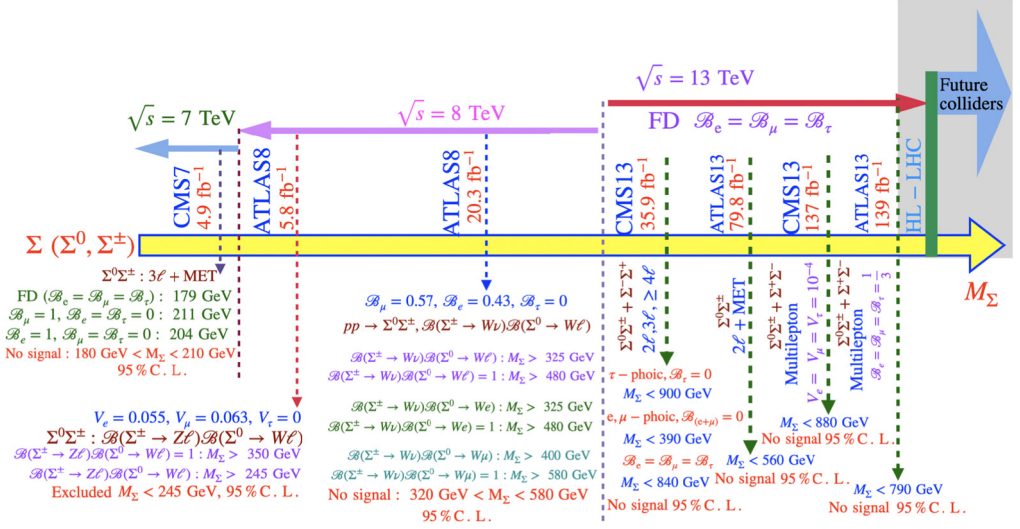
Fig. 20. The exclusion limits and the conditions on the triplet fermions from the LHC at 7 TeV, 8 TeV and 13 TeV center of mass energies. The lower bounds CMS7 [37], ATLAS8 at 5.8 fb^{-1} luminosity [102], ATLAS8 at 20.3 fb^{-1} luminosity [43], CMS13 at 35.9 fb^{-1} luminosity [40], ATLAS13 at 79.8 fb^{-1} luminosity [44], CMS13 at 137 fb^{-1} luminosity [103] and ATLAS13 at 139 fb^{-1} luminosity [104] considered the FD scenarios followed by the multilepton decay modes of the $\Sigma^{\pm}$ and Σ^0 respectively. HL-LHC and future colliders can set some stronger bounds in the near future.

scenario where the branching ratio into electron and muon is 57% and 43% respectively whereas no branching to tau flavor was considered [43]. This analysis sets a lower limit of 320 GeV on M_Σ and finally finding no signature within 320 GeV $< M_\Sigma <$ 580 GeV at the 95% C.L.

At the 13 TeV the LHC studied the flavor democratic scenario of the type-III scenario [40] where the branching ratio into the three lepton flavors is same and each branching ratio ($\mathcal{B}_\ell$) is proportional to $\frac{|V_\ell|^2}{|V_e|^2+|V_\mu|^2+|V_\tau|^2}$ where ℓ represents the charged lepton flavor according to [40] from CMS. In this article the $\Sigma^{\pm}\Sigma^0$ and $\Sigma^+\Sigma^-$ productions have been studied for the multilepton final states. The bound on the triplet mass has been obtained at 840 GeV at the 95% C.L. for 35.9 fb^{-1}. It has also been mentioned that limit on the τ-phobic case where the branching ratio of the triplet fermion is set to be zero ($\mathcal{B}_\tau = 0$) sets a limit on the triplet mass $M_\Sigma = 900$ GeV at the 90% C.L. Studying the combined triplet production mode $\Sigma^{\pm}\Sigma^0$ and $\Sigma^+\Sigma^-$ at the 13 TeV LHC with 79.8 fb^{-1} luminosity ATLAS finds a bound on M_Σ at 560 GeV [44] at the 95% C.L. using the flavor democratic scenario. In this analysis a final state includes e and μ flavors of two leptons with opposite and same sign combinations in association with the missing momentum and jets. In this case $\Sigma^{\pm}$ and Σ^0 dominantly decay into the modes containing $W^{\pm}$ and leptons ($\ell^{\mp}$ or ν). The missing momentum is mostly coming from the neutrinos and the jets are coming from the hadronic decay of the remaining $W^{\pm}$ decay obtained from the dominant decay of the triplet. Updates from both of CMS [103] and ATLAS [104] at the 13 TeV LHC using 137 fb^{-1} and 139 fb^{-1} luminosity find limits on M_Σ at 880 GeV and 790 GeV at the 95% C.L. respectively. In both of these cases a democratic flavor structure has been considered. The CMS considered a universal mixing of 10^{-4} and the ATLAS considered a scenario where the branching ratio of the triplet into each of the three flavors of the lepton is $\frac{1}{3}$.

6.2. Limits on the mixing angles

The global constraints on the triplet fermions can be found in [67]. The upper limits on the elements of symmetric $|V_{\ell\Sigma}|^2$ matrix have been calculated at the 2-σ level for the three generations of the triplets for the NH and IH cases. The upper bounds on the matrix elements for the NH case are $|V_{11}|^2 < 6.2 \times 10^{-4}$, $|V_{22}|^2 < 2.8 \times 10^{-4}$, $|V_{33}|^2 < 1.3 \times 10^{-3}$, $|V_{12}|^2 < 6.0 \times 10^{-4}$, $|V_{31}|^2 < 5.0 \times 10^{-7}$, $|V_{32}|^2 < 2.8 \times 10^{-4}$. Similarly the upper bounds on the matrix elements for the IH case are $|V_{11}|^2 < 6.4 \times 10^{-4}$, $|V_{22}|^2 < 2.2 \times 10^{-4}$, $|V_{33}|^2 < 7.8 \times 10^{-4}$, $|V_{12}|^2 < 6.0 \times 10^{-7}$, $|V_{31}|^2 < 4.6 \times 10^{-4}$, $|V_{32}|^2 < 2.6 \times 10^{-4}$.

We consider a type-III seesaw scenario where the triplets are as heavy as 1 TeV. In this scenario we first consider O as a 3×3 identity matrix to calculate the light-heavy mixing in terms of the lightest light neutrino mass. We calculate the bounds on the mixing for two hierarchic conditions of the neutrino mass, namely, NH and IH fitting the neutrino oscillation data. For identity or real orthogonal matrix O, observing the nature of the mixing summed over the triplet generation we notice that the mixing can reach up to a certain lower limit when varied with respect to the lightest light neutrino mass for the NH and IH cases under the PLANCK limit. The lower limit for the $\Sigma_i |V_{e\Sigma_i}|^2$ can go down to 3×10^{-15} whereas the upper limit can be one order of magnitude better under the PLANCK exclusion in the NH case. For the other two flavors in the NH case, the upper limit on the mixings can reach up to 5×10^{-14} whereas the lower limit is slightly better for them, namely 2×10^{-14} under the PLANCK exclusion. Alternatively if we look at the IH case, we notice that the limits on $\Sigma_i |V_{(\mu,\tau)\Sigma_i}|^2$ roughly remain the same, however, those on $\Sigma_i |V_{e\Sigma_i}|^2$ get improved. The lower and upper limits on the mixing associated with the electron flavor also improve in the IH case up to 5×10^{-14} and 6×10^{-14} respectively. These limits can be observed from Fig. 4.

If we notice the individual mixing $|V_{\ell\Sigma_i}|^2$ in Fig. 5 for identity orthogonal matrix O, we see that for the NH and IH cases the mixings $|V_{e\Sigma_1}|^2$ and $|V_{\tau\Sigma_3}|^2$ become zero as m_1 and m_3 go to zero. $|V_{\ell\Sigma_2}|^2$ has the same nature in both of the hierarchies. Below the PLANCK exclusion limit, $|V_{\ell\Sigma_2}|^2$ varies between 10^{-15} to 10^{-14} for the NH case and stays around 10^{-14} for the IH case. Following the same note, we notice that $|V_{e\Sigma_3}|^2$ stays around 10^{-15} below the PLANCK limit where as $|V_{\mu\Sigma_3}|^2$, $|V_{\tau\Sigma_3}|^2$ are one order of magnitude higher in the NH case. The scenario becomes opposite in the IH case for $|V_{\ell\Sigma_1}|^2$ where $|V_{e\Sigma_1}|^2$ stays around 2×10^{-14} and the other two mixings are one order lower than that below the PLANCK exclusion. The values could be observed from Fig. 5.

Second, we consider O as a 3×3 real orthogonal matrix. We find the same nature of $\Sigma_i |V_{\ell\Sigma_i}|^2$, as we found in the previous case and shown in Fig. 4. The individual mixing for this case is shown in Fig. 6. It is important to note that the depending upon the hierarchies the mixings are dependent upon the three light neutrino mass eigenvalues. In the NH case the mixing for the μ and τ can reach up to 2×10^{-14} and that for e flavor can reach up to 3.5×10^{-15}. This nature becomes opposite in the IH case where the maximum value of the mixing involving e flavor can go up to 4×10^{-14} where the rest of the two remain some factor below around 10^{-14}.

Third, we consider O as a 3×3 general orthogonal matrix where the entries of the matrix can be complex quantities. Using the neutrino oscillation data considering the PLANCK exclusion limit we have found that in this case there is no special correlation in the parameter space of the mixing and the lightest light neutrino mass. This happens due to the dependence of the mixing angles on the light neutrino mass eigenvalues and the complex entries of the general orthogonal matrix O. We notice that the highest mixing can reach $\mathcal{O}(10^{-5})$ depending up on the generations of the triplet and SM lepton which is very high compared to the other two choices of the
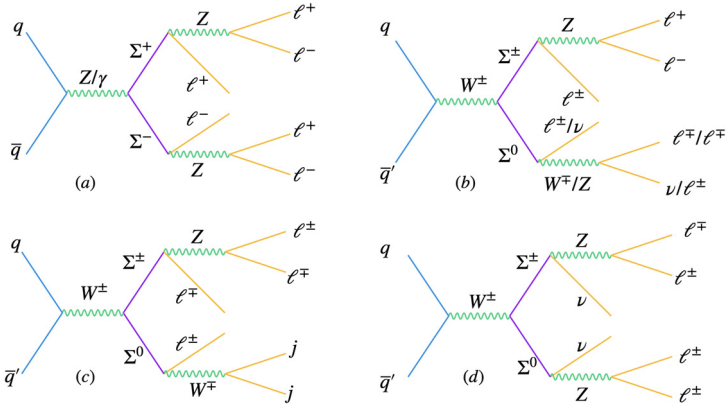
Fig. 21. Feynman diagrams for the multilepton channels at the hadron colliders: (a) 6 lepton final state, (b) 5 lepton final state with missing momentum and (c, d) 4 lepton final state. Another 4 lepton final state can be obtained from (a) when one Z from $\Sigma^{\pm}$ decays into neutrinos.

orthogonal matrices. The lower limit in the mixing in the different cases reaches around 10^{-18}. We mention that the limits on the mixing from the EWPD have been given in [81,105]. We quote limits as $|V_{e\Sigma}|^2 = 3.61 \times 10^{-4}$, $|V_{\mu\Sigma}|^2 = 2.89 \times 10^{-4}$ and $|V_{\mu\Sigma}|^2 = 7.29 \times 10^{-4}$ respectively at 90% C.L.

6.3. Multilepton channels

In this article we have shown the upper bound on the mixings as well as the lower bounds on the mixings generalizing the Yukawa coupling of the triplet fermion and applying the constraints from the direct searches. The LHC has given very strong bound on the triplet mass. At such strong bounds it becomes very crucial to study the triplets, however, interesting techniques including lepton-jet, displaced vertex, track search could be very useful for the heavier mass. These techniques can be useful in the future colliders using proton-proton, electron-positron and electron-proton in the near future. Here we focus on few multilepton final states at pp collider coming from the $\Sigma^{\pm}\Sigma^0$ and $\Sigma^{\pm}\Sigma^{\mp}$ production channels. The decay of $\Sigma^{\pm}$, Σ^0 can produce 6, 5 and 4 lepton final states. The Feynman diagrams of these channels are shown in Fig. 21.

For the 6 charged lepton final state in Fig. 21(a) the irreducible backgrounds can be obtained from ZZZ and $ZZWW$ with missing momentum and jets at the proton proton collider using MadGraph [106] followed by the hadronization [107] and detector simulation [108]. The production of this channel is independent of the mixing. The decay of the Z and W bosons into the charged leptons, namely, $Z \to \ell^+\ell^-$ and $W \to \ell\nu$ will produce 6 charged lepton final states. The leading order production cross section of ZZZ into 6 charged leptons with $\ell = \mu$ is 3.52×10^{-4} fb at the 13 TeV, 1.03×10^{-3} fb at 27 TeV and 5.3×10^{-3} fb at 100 TeV respectively. The same for the $ZZWW$ final state also produces 6 lepton final state with missing momentum and the cross section at 13 TeV is 5.02×10^{-6} fb, at 27 TeV is 2.06×10^{-5} fb and at 100 TeV is 1.52×10^{-4} fb respectively. The 6ℓ production cross section from $\Sigma^{\pm}$ with $M_{\Sigma} = 1$ TeV is 2.9×10^{-5} fb which is small to be observed at the HL-LHC era where as that at the 27 TeV is 2.3×10^{-4} fb. Finally at the 100 TeV, the 6ℓ production cross section is 3.4×10^{-3} fb. To estimate the signal and background events we applied a transverse momentum cut of the lepton with $p_T^{\ell} > 50$ GeV, pseudo-rapidity with $|\eta_{\ell}| < 2.5$, separation between the leptons in the $\eta - \phi$ plane
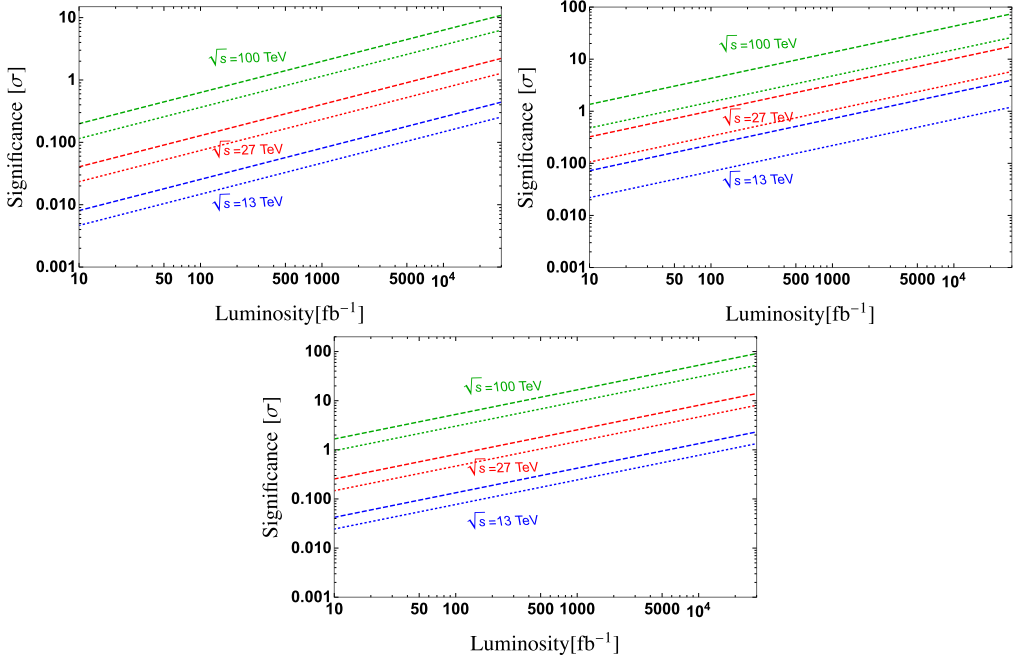
Fig. 22. The significance as a function of the luminosity. The luminosity goal for the 13 TeV LHC is 3 ab^{-1} whereas that at the 27 TeV and 100 TeV are 15 ab^{-1} and 30 ab^{-1} simultaneously. The decay of the W and Z bosons from the triplets into a single generation lepton is represented by the dotted lines whereas that for the three generation case have been shown by the dashed lines. The 13 TeV case is shown by the blue, the prospective 27 TeV and 100 TeV cases are shown by the red and green lines respectively. The 6 lepton case is shown in the top left panel, the 5 lepton case has been shown in the top right panel and the combined 4 lepton case has been shown in the bottom panel respectively.

with $\Delta R_{\ell\ell} > 0.4$, transverse momentum cut on the jets with $p_T^j > 30$ GeV, pseudo-rapidity cut on the jets with $|\eta_j| < 2.5$ and the jet-lepton separation cut in the $\eta - \phi$ plane with $\Delta R_{\ell j} > 0.4$. Additionally we consider that the missing momenta should be less than 30 GeV. We calculate the significance for the 6ℓ with $\ell = \mu$ in Fig. 22 using $\frac{\text{Signal}}{\sqrt{\text{Signal+Background}}}$, where the signal and background events have been calculated with luminosities in fb^{-1} as a free parameter. The 13 TeV and 27 TeV cases can not give significant results even at a very high luminosity whereas the projected significance can reach up to 5 σ at 16.47 ab^{-1} at the 100 TeV. If we combine three generations of the leptons from the Z decay, the situation becomes better than the single flavor case for the 100 TeV collider where 5σ significance can be reached at 5.6 ab^{-1} luminosity. The projected significance for the single flavor case has been shown by the dotted lines and that for the three flavor case can be shown by the dashed lines in the top left panel of the Fig. 22.

The production of the 5 lepton final state can be obtained from the $\Sigma^+\Sigma^0$ and $\Sigma^-\Sigma^0$ channels in Fig. 21 (b). We combine all the channels to calculate the prospective significance for $\ell = \mu$. The irreducible backgrounds are coming from ZZW and $WWWZ$ channels. The cross sections at the 13 TeV are 3.24×10^{-3} fb and 2.43×10^{-5} fb respectively. The cross sections of these backgrounds at the 27 TeV are 9.61×10^{-3} fb and 9.73×10^{-5} fb respectively. The same at the 100 TeV will be 5.00×10^{-2} fb and 6.3×10^{-4} fb respectively. Finally at the 100 TeV, the 6ℓ production cross section is 3.4×10^{-3} fb. To estimate the signal and the background events we applied same cuts as we applied in the 6 lepton case. Considering a single generation of the

leptons from the Z and W bosons can be observed at the 100 TeV collider with 5σ significance with 930 fb^{-1}. The three generation cases from the Z and W bosons can be observed at the 27 TeV collider with 2.073 ab^{-1} and at the 100 TeV collider with 111 fb^{-1} with 5σ significance. In this analysis we did not consider the misidentification of the leptons for simplicity. Generally the misidentification of the leptons at the LHC is below 1% and not known for the prospective 27 TeV and 100 TeV colliders. The projected significance for the single flavor case has been shown by the dotted lines and that for the three flavor case can be shown by the dashed lines in the top right panel of the Fig. 22.

Finally we study the 4 lepton final state from Fig. 21 (c) and (d). In addition to that Fig. 21 (a) will produce 4 lepton final state when one of the Z bosons from the $\Sigma^{\pm}$ decays into light neutrinos. We study the SM irreducible backgrounds including $W^+W^-W^+W^-$, ZZ, $ZZZW^{\pm}$, ZZZ and $ZZZZ$ to generic SM backgrounds of 4 leptons plus missing momentum accompanied by jets. The 4 lepton background from ZZ is the leading one among these channels which contributes 0.142 fb at the 13 TeV, 0.32 fb at the 27 TeV and 1.18 fb at the 100 TeV colliders respectively. The 4 lepton with missing momentum background is originated from ZZZ and $W^+W^-W^+W^-$. This combined background provides a cross section of 2.03×10^{-3} fb at 13 TeV, 6.00×10^{-3} fb at 27 TeV and 0.024 fb at 100 TeV. Additionally 4 leptons and missing momentum background can be accompanied by the jets coming from $ZZZZ$ and $ZZZW$ providing 15.8×10^{-6} fb at the 13 TeV, 6.01×10^{-5} fb at the 27 TeV and 4.16×10^{-4} fb at the 100 TeV collider. The combined signal cross sections for the 4 lepton final states with missing momentum and jets 0.003 fb at 13 TeV, 0.028 fb at 27 TeV and 0.34 fb at 100 TeV colliders respectively. To estimate the signal and the background events we applied the same cuts as we applied in the 6 lepton case. Hence the significance can reach at the 5σ level with 10.47 ab^{-1} at the 27 TeV for one generation lepton and 3.32 ab^{-1} with three generations of the leptons from the final W and Z leptonic decay. Similarly the 5σ significance for this channel can be achieved at 247 fb^{-1} with single flavor and 74.5 fb^{-1} with three flavor leptons at the 100 TeV collider. The projected significance for the single flavor case has been shown by the dotted lines and that for the three flavor case can be shown by the dashed lines in the bottom panel of the Fig. 22.

6.4. Summary at the e^-e^+ and e^-p colliders

The triplet fermions can be produced at the e^-e^+ colliders in the form of the pair production ($\Sigma^+\Sigma^-$) and single production ($\Sigma^{\pm}$ and Σ^0) in association with the SM particles. Hence, the above multilepton final states can be studied at the linear collider. Apart from the multilepton signatures, boosted objects from the $\Sigma^{\pm}$ and Σ^0 can also be studied as in linear collider the cross section remains almost same until the energy threshold. The boosted objects from the type-III seesaw have been studied in [35] with single charged lepton plus fat jet, dilepton plus fat b-jet, dilepton plus fat-jet (non-b) and two fat-jet plus missing energy for $M_{\Sigma} = 1$ TeV. Different multilepton channels have been studied in [33] for $M_{\Sigma} < 1$ TeV, however, the strong LHC bounds rule out the triplets below 880 GeV [103] and 790 GeV [104] for the flavor diagonal scenario. We have also studied the triplet production at the electron proton collider with the boosted objects [35] for the first time.

We have found that the mixing can be probed at the $\sqrt{s} = 1$ TeV and $\sqrt{s} = 3$ TeV e^-e^+ and $\sqrt{s} = 3.46$ TeV e^-p colliders well below the electroweak precision bounds [81,105] at the 5σ significance level. The boosted objects can also be studied at the hadron colliders in the near future. Apart from the electron channels, the muon and tau channels will also be interesting to study at the lepton colliders at the e^-e^+ colliders and e^-p colliders respectively.

7. Conclusions

We study the type-III seesaw model in this article where we mainly observe the allowed parameter regions for the light-heavy mixings as a function of the lightest neutrino mass. Depending upon the choice of the general Dirac Yukawa coupling of the triplet fermion with the SM lepton doublet and the Higgs doublet the allowed parameter space of the mixing changes under a variety of constraints. We also calculate the branching ratios of the neutral and charged multiplets of the triplet fermion into leading and sub-leading modes to investigate the correlation with the lightest light neutrino mass eigenvalue for two different light neutrino mass hierarchies. In a continuation we have also shown the parameter space of the proper decay length of the triplet generalizing the Dirac Yukawa coupling using three different choices. This leads to an interesting property of the displaced vertex search for the triplet fermions due to their sizable proper decay length, however, we predict that for some parameter choices prompt decay of the triplet fermion can also be possible. We evaluate the mixings, branching ratios of the triplets in the different modes and hence we predict that such parameter spaces can be probed studying the decay modes (prompt or displaced) of the triplets at the different high energy colliders in the near future.

CRediT authorship contribution statement

Arindam Das: Conceptualization, Methodology, Calculations, Writing the original draft, Reviewing and editing. **Sanjoy Mandal:** Methodology, Calculations, Simulation, Software, Reviewing and editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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