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Induction of a Landau-Type Quantization in a Background of CPT-Odd Lorentz Symmetry Violation

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Abstract

In this article, we approach a scalar particle in a background characterized by the Lorentz symmetry violation through a non-minimal coupling in the mathematical structure of the Klein–Gordon equation, where the Lorentz symmetry violation is governed by a background vector field. For an electric field configuration and in the search for solutions of bound states, we determine the relativistic energy profile of the system, which is characterized by quantized orbits, that is, a relativistic Landau-type quantization. Then, we particularize our system and analyze it in the presence of a hard-wall potential, from which, we analytically determine its relativistic energy profile in this confining type.

Keywords: Lorentz symmetry violation; relativistic Landau quantization; hard-wall potential; bound states

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1. Introduction

The Standard Model (SM) is the best theoretical apparatus that describes in a unified way the fundamental interactions of nature, with the exception of gravitational interaction. The non-incorporation of gravitational interaction in the SM is one of the questionable problems with it. Added to this questioning is the order of magnitude discrepancy between the cosmological constant predicted by the SM and its value measured from extragalactic observations [1]. In addition, recent experimental results showed that the proton's radius is different from that which the SM predicts [2]. Furthermore, from the point of view of observational cosmology, observational data indicate that the fine structure constant is slowly changing [3,4] and there is evidence that neutrinos have mass [5]. All of the cited examples reinforce the concern to search for theoretical models that can fill these gaps in the SM.

In recent decades, several theories have been proposed in order to seek answers that the SM, so far, is not able to provide. For example, supersymmetric theory [6], non-commutativity theory [7], string theory [8] and the Lorentz symmetry violation (LSV) [9]. LSV has as its main feature privileged directions in spacetime indicating space-like or time-like anisotropies, or both, in all cases, governed by vector or tensorial fields capable of interfering with already consecrated laws or theories, consequently, violating one of the best known principles and important of modern physics proposed by Einstein [10].

One of the first theoretical manifestations of LSV appears in the description of the Higgs mechanism in string theory where the Lorentz symmetry is spontaneously broken through tensor fields [9]. This approach is then analyzed in classical electrodynamics, where the consequences range from the modification of Maxwell's equations to measurement limits in extragalactic quantities [11]. Over time, LSV has been extensively investigated, not only within field theory or particle physics but also in several other areas that are not limited only to the high energy scale, gaining great significance and strength for a possibly theoretical way of answering questions about gaps not yet answered by the SM. With this, theoretical models with fields of various natures capable of breaking the Lorentz symmetry were proposed, not to replace the standard model, but as a theoretical proposal for its improvement. These models make up what we call the Standard Model Extension (SME) [12,13]. Recently, LSV has been investigated in several branches of physics [14–28].

LSV has also been analyzed in the field of quantum mechanics. For example, in the non-relativistic context, there are studies on a harmonic oscillator subjected to a Coulomb-type potential induced by LSV [29], on a quantum particle subjected to the Aharonov–Bohm effect for bound states, which is induced by the LSV [30], on a spin-1/2 particle interacting with a quantum ring induced by LSV [31] and on a Dirac neutral particle subjected to the geometric phases induced by LSV [32,33]. In the relativistic quantum dynamics of particles, there are studies involving LSV in the Coulomb-type interaction [34], on relativistic oscillator models [35,36], LSV fields interacting with relativistic Landau gauge [37], and the possibilities of a relativistic Landau-type quantization induction by the LSV on a Dirac field [38].

In Ref. [38], Vitória and Belich have used a non-minimal coupling of LSV on the Dirac equation, where the LSV is governed by a background vector field coupled to the dual electromagnetic tensor [39]. In the current work, a possible LSV scenario is proposed, defined by a particular field configuration, which characterizes a Dirac particle interacting with an electrical field that linearly varies with the axial coordinate. In obtaining finite degree polynomials of the confluent hypergeometric series, we determine the energy levels of the system, which are analogous to the relativistic Landau levels in Minkowski spacetime [37,40,41]. We have investigated this non-minimal coupling in the spin-0 relativistic equation, where we analyzed the induction of a central Coulomb-type potential by LSV [42]. However, we disregard the terms of quadratic corrections for the Lorentz break term in the Klein-Gordon equation. Therefore, unlike Ref. [42], in this analysis we consider all the correction terms from the non-minimal coupling of the LSV in the Klein–Gordon equation, in order to describe the possible effects, from the theoretical point of view, of these neglected terms. Therefore, based on Ref. [38] we are proposing the induction of an electric dipole, which has the capacity to influence a scalar particle, in which the induction is done through a coupled background vector field, via non-minimal coupling, in the Klein–Gordon equation.

To date, no experiment has exactly confirmed the violation of Lorentz symmetry, although there are results that suggest small variations. For example, the authors of [43] explores the effects of LSV and CPT on the spectroscopy of hydrogen and its antimatter (antihydrogen) by investigating the 1–2 s and hyperfine transitions, with the aim of establishing limits on the magnitude of the background fields. Ref. [44] presents the effects of CPT violation on the physical properties of the anomalous magnetic moment, from the perspective of quantum electrodynamics, alongside data on the hyperfine structure of the muon and its anomalous magnetic moment, obtaining more precise magnitudes of the associated components of the background field. In Ref. [45], restricted maximum values for background field components for the massive, photonic, and gravitational sectors are presented.

To describe the relativistic quantum dynamics of a scalar particle in Minkowski spacetime, we must use the Klein–Gordon Equation (with $c = \hbar = 1$)

$$\Box\phi - m^2\phi = 0, \quad (1)$$

where $\Box = \partial_\mu \partial^\mu$, with $\mu = 0, i = 1, 2, 3$, and m is the rest mass of the scalar particle. Now, based on Refs. [39,42], consider the non-minimal coupling $\partial_\mu \to \partial_\mu - ig\tilde{F}_{\mu\nu}v^\nu$, such that Equation (1) is rewritten as follows

$$(\partial_\mu - ig\tilde{F}_{\mu\nu}v^\nu)(\partial^\mu - ig\tilde{F}^{\mu\kappa}v_\kappa)\phi - m^2\phi = 0, \quad (2)$$

where $g \ll 1$ is a coupling constant and $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ is the dual electromagnetic tensor, with $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, and A_μ being the gauge field. This photonic term is inspired by the four-dimensional Chern–Simons lagrangian term proposed by Carroll, Field, and Jakiw [11], where v^μ is a fixed background vector field, which is part of the photonic sector of the SME, that is, the sector of CPT-odd gauge, which in turn is characterized by the violation of CPT symmetry [13]. In Ref. [45], information about the magnitude limits of v^μ can be found, for example, for the CMB polarization, $v^z = (7.32 \pm 2.94) \times 10^{-45}$ GeV [46,47], $v^z < 10^{-19}$ GeV in hydrogen spectroscopy and [24] and $v^z = (0.57 \pm 0.70)H_0 \times 10^{-41}$ GeV in studies on astrophysical birefringence [48], where H_0 is the Hubble constant. We can go further with Equation (2):

$$\Box\phi - 2ig(\partial_\mu\phi)\tilde{F}^{\mu\nu}v_\nu - g^2\tilde{F}_{\mu\nu}v^\nu\tilde{F}^{\mu\nu}v_\nu\phi - m^2\phi = 0, \quad (3)$$

or

$$\begin{aligned}\Box\phi &+ 2ig(\vec{v}.\vec{B})\partial_0\phi + 2igv_0(\vec{B}.\vec{\nabla})\phi - 2ig(\vec{v}\times\vec{E}).\vec{\nabla}\phi + g^2v_0^2\vec{B}^2\phi - g^2(\vec{v}.\vec{B})^2\phi \\ &- g^2[(v_2^2+v_3^2)E_1^2 + (v_1^2+v_3^2)E_2^2 + (v_1^2+v_2^2)E_3^2]\phi - m^2\phi = 0, \quad (4)\end{aligned}$$

where we use $\partial_\mu\tilde{F}^{\mu\nu} = 0$. Let us consider, going forward, a Minkowski spacetime with an axial symmetry described by the following line element

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\varphi^2 + dz^2, \quad (5)$$

with $\rho = \sqrt{x^2+y^2}$. We will adopt this symmetry due to the field configuration to be adopted in this analysis, which will be given in terms of the axial coordinate. Equation (4) becomes

$$\begin{aligned}&- \partial_t^2\phi + \partial_\rho^2\phi + \frac{1}{\rho}\partial_\rho\phi + \frac{1}{\rho^2}\partial_\varphi^2\phi + \partial_z^2\phi + 2ig(\vec{v}.\vec{B})\partial_t\phi + 2igv_t(\vec{B}.\vec{\nabla})\phi - 2ig(\vec{v}\times\vec{E}).\vec{\nabla}\phi \\ &+ g^2v_t^2\vec{B}^2\phi - g^2(\vec{v}.\vec{B})^2\phi - g^2[(v_\varphi^2+v_z^2)E_\rho^2 + (v_\rho^2+v_z^2)E_\varphi^2 + (v_\rho^2+v_\varphi^2)E_z^2]\phi - m^2\phi = 0, \quad (6)\end{aligned}$$

in which, in natural units, we have $[v^\mu] = [E] = [B] = M$ and $[g] = M^{-1}$.

Equation (6) give us the Klein–Gordon equation modified by terms of CPT-odd LSV governed by a background vector field in a spacetime of axial symmetry. We can see that Equation (6) differs from the relativistic wave equation determined in Ref. [42] in that it has too many terms, that is, the quadratic terms in the vector field and in the electromagnetic field, which are neglected in Ref. [42]. Thus, Equation (6) is the most general case. In addition, we also can that Equation (6) supports several LSV possible scenarios through configurations of vector and electromagnetic fields.

LSV has profound implications for our understanding of the universe, and may lead to new physical theories and a revision of our current models, since these have been questioned due to discrepancies between predicted and observed and/or experimental results. Because LSV is linked to CPT symmetry, a symmetry linked to relativistic particles, the Dirac equation has been extensively analyzed in the most varied LSV scenarios for all sectors of the SME [49]. Thus, it is valid to investigate these anisotropic effects of LSV in the Klein–Gordon equation, since it describes relativistic spin-0 particles, such as the Higgs boson and the pion.

The structure of the rest of this paper is as follows: in Section 2, we investigate the possibility of inducing, by LSV, an electric field that characterizes the distribution of an electric dipole, which influences the relativistic quantum dynamics of a scalar particle in an analogous way to the relativistic Landau quantization. We generalize our analysis with the presence of a constant magnetic field which modifies the energy profile of the system. In Section 3, we analyze the quantum systems analyzed in the previous section, which interacts with a hard-wall potential, where we can see that the presence of this confinement potential modifies the relativistic energy levels; in Section 4, we present our conclusions.

2. Landau-Type Quantization Induced by the LSV

2.1. Space-like Background Vector Field

Consider the following vector and electromagnetic field configuration [38]:

$$v_\mu = (0,0,0,v_z); \quad \vec{E} = \frac{\vartheta\rho}{2}\hat{\rho}; \quad \vec{B} = 0, \tag{7}$$

where $v_z =$ const., characterizing a space-like background vector field [10], ϑ is a constant associated with a volumetric distribution of electric charges, with $[\vartheta] = M$, and $\hat{\rho}$ is a unit vector in the axial direction. We can note that this field configuration induces a Landau-type quantization, that is, we have a "vector potential"

$$\vec{\mathcal{A}} = \vec{v} \times \vec{E} = \frac{v_z\vartheta\rho}{2}\hat{\varphi}, \tag{8}$$

which it gives a "uniform magnetic field":

$$\vec{\mathcal{B}} = \vec{\nabla} \times \vec{\mathcal{A}} = v_z\vartheta\hat{z}, \tag{9}$$

where and $\hat{z}$ is a unit vector in the z-direction. In this case, Equation (6) is rewritten in the form

$$-\partial_t^2\phi + \partial_\rho^2\phi + \frac{1}{\rho}\partial_\rho\phi + \frac{1}{\rho^2}\partial_\varphi^2\phi + \partial_z^2\phi - iv_z g\vartheta\partial_\varphi\phi - \frac{g^2\vartheta^2 v_z^2}{4}\rho^2\phi - m^2\phi = 0. \tag{10}$$

The solution for Equation (10) is given in terms of the eigenvalues of the abelian operators $\hat{p}_z = -i\partial_z$ and $\hat{L}_z = -i\partial_\varphi$ which commute with the hamiltonian, $\hat{H} = i\partial_t$, that is,

$$\phi(\rho,\varphi,z,t) = R(\rho)\theta(\varphi)Z(z)T(t), \tag{11}$$

where $R(\rho)$ is the unknown axial wave function, $\theta(\varphi) = e^{il\varphi}$, $Z(z) = e^{ikz}$ and $T(t) = e^{-i\mathcal{E}t}$, with $l = 0, \pm1, \pm2, \ldots$, $-\infty < k < \infty$ and $\mathcal{E}$ been the angular momentum, linear momentum and relativistic energy eigenvalues, respectively. Then, by substituting Equation (11) into Equation (10), we obtain axial wave equation

$$R'' + \frac{1}{\rho}R' - \frac{l^2}{\rho^2}R - \varpi^2\rho^2 R + \alpha^2 R = 0, \tag{12}$$

with the parameters

$$\alpha^2 = \mathcal{E}^2 - m^2 - k^2 + g v_z\vartheta l; \quad \varpi = \frac{g v_z\vartheta}{2}. \tag{13}$$

Now, let us define the variable change $w = \varpi\rho^2$, then Equation (12) becomes

$$\frac{d^2R}{dw^2} + \frac{1}{w}\frac{dR}{dw} - \frac{l^2}{4w^2}R + \frac{\alpha^2}{4\varpi w}R - \frac{1}{4}R = 0. \tag{14}$$

As we are interested in well-behaved solutions to the function $w \to 0$ and $w \to \infty$ ($\rho \to 0$ and $\rho \to \infty$, respectively). We can note that, for $w \to 0$, we have the solution $R(w) \sim w^{\frac{|l|}{2}}$; for $w \to \infty$, we have the solution $R(w) \sim e^{-\frac{1}{2}w}$. Then, the ansatz for the axial wave function is

$$R(w) = w^{\frac{|l|}{2}} e^{-\frac{1}{2}w} f(w), \tag{15}$$

where $f(w)$ is an unknown function.

Then, by substituting Equation (15) into Equation (14), we obtain confluent hypergeometric Equation [50]

$$w\frac{d^2f}{dw^2} + (|l| + 1 - w)\frac{df}{dw} + \left(\frac{\alpha^2}{4\varpi} - \frac{|l|}{2} - \frac{1}{2}\right)f = 0, \tag{16}$$

where $f(w)$ is the confluent hypergeometric function: $f(w) = {}_1F_1(a, b; w)$, with

$$a = \frac{|l|}{2} + \frac{1}{2} - \frac{\alpha^2}{4\varpi}; \quad b = |l| + 1. \tag{17}$$

The solution for large values of the confluent hypergeometric power series argument is

$${}_1F_1(a, b; w) \approx \frac{\Gamma(b)}{\Gamma(a)} e^{w} w^{a-b}[1 - \mathcal{O}(|w|^{-1})]. \tag{18}$$

In order to obtain a finite degree polynomial of the confluent hypergeometric series, we must have a truncation condition for the series $A = -n = 0, 1, 2, \ldots$ [50], that is, of this condition, we obtain

$$\mathcal{E}_{k,l,n} = \pm\sqrt{m^2 + k^2 + 2m\omega_g\left(n + \frac{|l| - l + 1}{2}\right)}, \tag{19}$$

where we have defined a cyclotron-type frequency

$$\omega_g = \frac{2}{m}\varpi \leftrightarrow \omega_g = \frac{g v_z \vartheta}{m}, \tag{20}$$

where we can see that the product $g v_z \vartheta$ is analogous to the product qB_0, where q is the electric charge and B_0 is the uniform magnetic field [41].

Equation (19) represents the relativistic energy levels of a scalar particle interacting with an electric field induced by LSV governed by a constant space-like vector field. We can notice that these energy levels are analogous to the relativistic Landau levels with quantized orbits determined by a cyclotron-type frequency defined by the parameters associated with LSV (20). We can note that by taking $g \to 0$, we recover energy of a free scalar particle in the Minkowski spacetime.

By comparing Equation (20) with the relativistic energy levels obtained in Ref. [38], since in the latter, the same field configuration ass that given in Equation (7) is considered, we can note that the only difference between the results is that in the result of Ref. [38] there are correction terms associated with the spin of the fermionic particle.

We can rewrite Equation (19) as follows:

$$\mathcal{E}_{k,l,n} = m\left[1 + \frac{k^2}{m^2} + \frac{2\omega_g}{m}\left(n + \frac{|l| - l + 1}{2}\right)\right]^{\frac{1}{2}}. \tag{21}$$

Then, by using the approximation $(1+N)^\sigma \approx 1 + \sigma N$, with $N \ll 1$, and $\varepsilon_{k,l,n} = \mathcal{E}_{k,l,n} - m$ in Equation (21), we have the expression

$$\varepsilon_{k,l,n} = \frac{k^2}{2m} + \omega_g\left(n + \frac{|l| - l + 1}{2}\right), \tag{22}$$

which represents the energy spectrum of a particle under effects of an electric field induced by LSV, where it is analogous to Landau levels [51,52], with orbits quantized through a cyclotron-type frequency defined in terms of the parameters associated with LSV. By making $g \to 0$, we recover energy of a free non-relativistic particle in an isotropic medium.

Again, drawing a parallel between Equation (22) and the non-relativistic result defined in Ref. [38], we can note that the difference between both is that the result of Ref. [38] has a correction term associated with the spin of the non-relativistic fermionic particle.

2.2. Space-like Plus Time-like Background Vector Field

Here, we establish another possible LSV scenario determined by a background vector field of space-like plus time-like nature. This possible scenario is determined by a field configuration defined as follows

$$v_\mu = (-v_t, 0, 0, v_z); \quad \vec{E} = \frac{\vartheta\rho}{2}\hat{\rho}; \quad \vec{B} = B_0\hat{z}, \tag{23}$$

where v_t and B_0 are constants. This electromagnetic field configuration has been studied in induced electric dipole moment systems [53–55]. In this case, Equation (6) is rewritten in the form

$$\begin{aligned} - \quad & \partial_t^2\phi + \partial_\rho^2\phi + \frac{1}{\rho}\partial_\rho\phi + \frac{1}{\rho^2}\partial_\varphi^2\phi + \partial_z^2\phi + 2igv_zB_0\partial_t\phi + 2igv_0B_0\partial_z\phi - iv_zg\vartheta\partial_\varphi\phi \\ + \quad & g^2v_0^2B_0^2\phi - g^2v_z^2B_0^2\phi - \frac{g^2\vartheta^2v_z^2}{4}\rho^2\phi - m^2\phi = 0. \end{aligned} \tag{24}$$

By following the same steps from Equations (10)–(12), we obtain the axial differential equation

$$R'' + \frac{1}{\rho}R' - \frac{l^2}{\rho^2}R - \varpi^2\rho^2 R + \beta^2 R = 0, \tag{25}$$

where ϖ is defined in Equation (13), and we define the new parameter

$$\beta^2 = \mathcal{E}^2 - m^2 - k^2 + 2gv_zB_0\mathcal{E} - 2gv_0B_0k + g^2v_0^2B_0^2 - g^2v_z^2B_0^2 + v_z\vartheta gl. \tag{26}$$

Equation (25) is analogous to Equation (12). Then, by following the steps from Equations (12)–(16), we obtain the general solution:

$$R(w) = w^{\frac{|l|}{2}} w^{-\frac{1}{2}w}\, {}_1F_1(\bar{a}, b; w), \tag{27}$$

where b is defined in Equation (17) and

$$\bar{a} = \frac{|l|}{2} + \frac{1}{2} - \frac{\beta^2}{4\varpi}. \tag{28}$$

To obtain finite degree polynomials of the confluent hypergeometric series, we must follow the same steps of Equations (18) and (19), that is, by considering the condition

$\bar{a} = -n = 0, 1, 2, \ldots$, we find the following expression for the relativistic energy levels of system:

$$\mathcal{E}_{k,l,n} = -gv_zB_0 \pm \sqrt{m^2 + k^2 + 2m\omega_g\left(n + \frac{|l| - l + 1}{2}\right) + g^2B_0^2(2v_z^2 - v_t^2) + 2gv_tB_0k}. \quad (29)$$

We can note that the time-like anisotropies associated with the component v_t and the presence of the uniform magnetic field modify the relativistic energy profile of a scalar particle interacting with an electric field induced by the CPT-odd LSV. This modification is explicitly given in terms $\mathcal{E}_{v_z} = -gv_zB_0$ and $k_{\text{eff}} = \sqrt{k^2 + g^2B_0^2(2v_z^2 - v_t^2) + 2gv_tB_0k}$. By taking $v_t = 0$ into Equation (29), that is, only space-like anisotropies associated with the component v_z in the quantum system, we have

$$\mathcal{E}_{k,l,n} = -gv_zB_0 \pm \sqrt{m^2 + k^2 + 2m\omega_g\left(n + \frac{|l| - l + 1}{2}\right) + 2g^2B_0^2v_z^2} \quad (30)$$

which represents the relativistic Landau-type levels of a scalar particle interacting with an electric field and a uniform magnetic field, both induced by the CPT-odd LSV governed by a space-like constant vector field. Again, we can observe that the presence of the uniform magnetic field modifies the relativistic energy spectrum of the quantum system, that is, comparing Equations (21) and (30), there are the most terms $\mathcal{E}_{v_z} = -gv_zB_0$ and $k_{\text{eff}} = \sqrt{k^2 + 2g^2B_0^2v_z^2}$ into Equation (30). By taking $B_0 \to 0$ into Equation (30), we obtain Equation (21). In addition, by making $g \to 0$ in Equations (29) and (30), we recover the energy of a free scalar particle in the Minkowski spacetime.

Now, by following the same steps as Equations (21) and (22), we obtain the following expression:

$$\varepsilon_{k,l,n} = -gv_zB_0 + \frac{k^2}{2m} + \omega_g\left(n + \frac{|l| - l + 1}{2}\right) + \frac{g^2B_0^2(2v_z^2 - v_t^2)}{2m} + \frac{gv_tB_0k}{m}, \quad (31)$$

which represents the Landau-type levels of a non-relativistic particle interacting with an electric field and a uniform magnetic field, both induced by the CPT-odd LSV governed by a space-like plus time-like constant vector field. We can note that the Landau-type levels are influenced by the time-like anisotropies associated with the constant component v_t of the background vector field, that is, by comparing Equations (22) and (31), we can observe that Equation (31) has the greatest term $k_{\text{eff}} = \sqrt{k^2 + g^2B_0^2(2v_z^2 - v_t^2) + 2gv_tB_0k - 2gv_zB_0m}$. By taking $v_t = 0$ into Equation (31), that is, only space-like anisotropies associated with the component v_z in the quantum system, we have

$$\varepsilon_{k,l,n} = -gv_zB_0 + \frac{k^2}{2m} + \omega_g\left(n + \frac{|l| - l + 1}{2}\right) + \frac{g^2B_0^2v_z^2}{m}, \quad (32)$$

where we obtain the Landau-type levels of a particle interacting with an electric field and a uniform magnetic field, both induced by the CPT-odd LSV governed by a space-like constant vector field. In particular, the presence of the uniform magnetic field yields the effective quantum number $k_{\text{eff}} = \sqrt{k^2 + 2g^2B_0^2v_z^2 - 2gv_zB_0m}$. Finally, by making $B_0 \to 0$ we recover Equation (22).

3. Landau-Type Quantization Induced by the LSV in the Presence of a Hard-Wall Potential in System

In this section, we analyze the two systems treated in Section 2 in the presence of a hard-wall confining potential, that is, a scalar particle in a cylindrical box and subjected to an electromagnetic configuration immersed in a background governed by a constant vector field of space-like and time-like nature.

In recent years, the hard-wall confining potential has been studied in various quantum mechanics systems. In the relativistic case, this potential type has been investigated, for example, in the quantum behavior of a neutral fermion [56], on a scalar particle subjected to Aharanov–Bohm potential in a spacetime with torsion [41], and on Dirac and Klein–Gordon quantum oscillators in global monopole spacetime [57]; in the non-relativistic case, there are studies on the harmonic oscillator in elastic media [58] and on a quantum particle interacting with a magnetic screw dislocation [59]. It is worth remembering that the hard-wall confining potential has been extensively investigated in quantum systems in non-inertial frames since it is induced by uniform rotation in frames [60]. In this context, the hard-wall potential has been studied on scalar bosons [61], in systems of spin-0 particles and spacetime with curvature [62], on a massive scalar field by interacting with a magnetic screw dislocation [63] and in a particle described by the Duffin–Kemmer–Petiau Equation [64].

Then, let us restrict the scalar particle to a region with the following boundary condition:

$$R\left(\rho_0 = \sqrt{\frac{w_0}{\varpi}}\right) = 0, \tag{33}$$

where ρ_0 is a fixed value of axial coordinate. From the mathematical point of view, the boundary condition given in Equation (33) is the Dirichlet boundary condition; from the physical point of view, Equation (33) represents a hard-wall confining potential in the system, that is, the axial wave function vanishes at a fixed arbitrary radius ρ_0. Now, let us consider the particular cases $\alpha^2 \gg 4\varpi$ and $\beta^2 \gg 4\varpi$ where the parameters α^2 and β^2 are defined in Equations (13) and (26), respectively. The condition $\alpha^2 \gg 4\varpi$ gives us the information $\mathcal{E}^2 - m^2 \gg g v_z \vartheta(2-l) + k^2$, that is, since $-\infty < k < \infty$, k must be fixed and finite as well. From a physical point of view, the total energy is greater than the rest energy of the scalar particle, which means that the particle has significant kinetic energy, arising from the effects of LSV. From condition $\beta^2 \gg 4\varpi$, we have $\mathcal{E}^2 - m^2 + 2g v_z B_0 \mathcal{E} \gg g v_z \vartheta(2-l) + k^2 + 2g v_z B_0 k + g^2 B_0^2 (v_z^2 - v_0^2)$. Again, k must be fixed and finite and the total energy is greater than the rest energy of the scalar particle. Furthermore, our interest is in considering quadratic terms associated with the LSV, so this condition reinforces that $v_z \neq v_t$. In addition, for the parameters of the confluent hypergeometric function a to be large, a fixed value for b, that is, the fixed quantum number l, and the fixed radius ρ_0, the confluent hypergeometric function can be written in the form [65]:

$$_1F_1(a, b; w_0) \propto \cos\left(\sqrt{2w_0(b-2a)} + \frac{\pi}{4} - \frac{b\pi}{2}\right). \tag{34}$$

Going forward, let us analyze the effects of the hard-wall confining potential (33) on the systems treated in the previous Section 2.

3.1. Space-like Background Vector Field

In this case, the field configuration is given in Equation (7) and the parameters of the confluent hypergeometric function are $a = \frac{|l|}{2} + \frac{1}{2} - \frac{\alpha^2}{4\varpi}$ and $b = |l| + 1$. Then, by substituting Equations (34) and (15) into Equation (33), we have

$$\mathcal{E}_{k,l,\bar{n}} = \pm\sqrt{m^2 + k^2 + \frac{\pi^2}{\rho_0^2}\left(\bar{n} + \frac{|l|}{2} + \frac{3}{4}\right)^2 - m\omega_g l}, \tag{35}$$

where $\bar{n} = 0, 1, 2, \ldots$ are the radial modes and ω_g is defined in Equation (20).

Equation (35) represents the relativistic energy profile of a scalar particle under effects of an electric field induced by the CPT-odd LSV governed by a space-like vector field and subjected to the hard-wall confining potential. We can observe that the presence of a hard-wall potential in the quantum system modifies the relativistic Landau-type levels defined in Equation (19). This modification can be seen by comparing Equations (35) and (19). By taking $g \to 0$ ($\omega_g \to 0$) we recover the relativistic energy levels of a scalar particle under effects of hard-wall confining potential in the Minkowski spacetime [37,41].

By following the same steps as Equations (21) and (22), we obtain the energy spectrum of a non-relativistic particle in a LSV background defined by the field configuration given in Equation (7) and subjected to the hard-wall confining potential:

$$\varepsilon_{k,l,\bar{n}} = \frac{k^2}{2m} + \frac{\pi^2}{2m\rho_0^2}\left(\bar{n} + \frac{|l|}{2} + \frac{3}{4}\right)^2 - \frac{l\omega_g}{2}. \tag{36}$$

By comparing Equations (22) and (36), we can note that the presence of the hard-wall potential in the non-relativistic quantum system modifies the energy spectrum. In addition, by making $g \to 0$, we recover the energy levels of a particle subjected to the hard-wall potential in an isotropic environment [37].

3.2. Space-like Plus Time-like Background Vector Field

Now, the field configuration is given in Equation (23) and the parameters of the confluent hypergeometric function are $\bar{a} = \frac{|l|}{2} + \frac{1}{2} - \frac{\beta^2}{4\varpi}$ and $b = |l| + 1$. Then, by substituting Equations (34) and (27) into Equation (33), we have the expression

$$\mathcal{E}_{k,l,\bar{n}} = -gv_zB_0 \pm \sqrt{m^2 + k^2 + \frac{\pi^2}{\rho_0^2}\left(\bar{n} + \frac{|l|}{2} + \frac{3}{4}\right)^2 - m\omega_g l + g^2B_0^2(2v_z^2 - v_t^2) + 2gv_tB_0k}, \tag{37}$$

which represents the relativistic energy levels of a scalar particle under effects of an electric field and uniform magnetic field, both induced by the CPT-odd LSV governed by a space-like plus time-like vector field given in field configuration (23), under effects of a hard-wall confining potential. We can note that the presence of the time-like constant component of the background vector field and of the uniform magnetic field modifies the relativistic energy profile of the system, that is, by comparing Equations (35) and (37), there are the greatest terms $\mathcal{E}_{v_z} = -gv_zB_0$ and $k_{\text{eff}} = \sqrt{k^2 + g^2B_0^2(2v_z^2 - v_t^2) + 2gv_tB_0k}$ in Equation (37). By taking $v_t = 0$ into Equation (37), that is, only space-like anisotropies associated with the component v_z in the quantum system, we have

$$\mathcal{E}_{k,l,\bar{n}} = -gv_zB_0 \pm \sqrt{m^2 + k^2 + \frac{\pi^2}{\rho_0^2}\left(\bar{n} + \frac{|l|}{2} + \frac{3}{4}\right)^2 - m\omega_g l + 2g^2B_0^2v_z^2}, \tag{38}$$

which represents the relativistic energy spectrum of a scalar particle interacting with an electric field plus a uniform magnetic field both induced by the CPT-odd LSV associated with the space-like constant vector field. Again, we can observe that the presence of the uniform magnetic field modifies the relativistic energy spectrum of the quantum system, that is, by comparing Equations (35) and Equation (38), there are the greatest terms $\mathcal{E}_{v_z} = -gv_zB_0$ and $k_{\text{eff}} = \sqrt{k^2 + 2g^2B_0^2v_z^2}$ in Equation (38). In addition, by making $g \to 0$ in Equations (37) and (38), we recover the energy of a free scalar particle in the Minkowski spacetime.

By following the same steps as Equations (21) and (22), we have the expression

$$\varepsilon_{k,l,\bar{n}} = -g v_z B_0 + \frac{k^2}{2m} + \frac{\pi^2}{2m\rho_0^2}\left(\bar{n} + \frac{|l|}{2} + \frac{3}{4}\right)^2 - \frac{l\omega_g}{2} + \frac{g^2 B_0^2}{2m}(2v_z^2 - v_t^2) + \frac{g v_t B_0 k}{m}, \quad (39)$$

that is, the above expression is the energy profile of a particle under effects of electric and magnetic fields induced by the CPT-odd LSV associated with the space-like plus time-like background vector field subjected to a hard-wall confining potential. We can observe that the presence of the time-like constant component of the background vector field and of the uniform magnetic field modify the relativistic energy profile of the system; by comparing Equations (36) and (39), there is the greatest term $k_{\text{eff}} = \sqrt{k^2 + g^2 B_0^2 (2v_z^2 - v_t^2) + 2g v_t B_0 k - 2g v_z B_0 m}$ in Equation (39). By making $v_t = 0$ in Equation (39), that is, only space-like anisotropies associated with the component v_z in the quantum system, we have

$$\varepsilon_{k,l,\bar{n}} = -g v_z B_0 + \frac{k^2}{2m} + \frac{\pi^2}{2m\rho_0^2}\left(\bar{n} + \frac{|l|}{2} + \frac{3}{4}\right)^2 - \frac{l\omega_g}{2} + \frac{g^2 B_0^2 v_z^2}{m}, \quad (40)$$

which represents non-relativistic energy spectrum of a particle interacting with an electric field plus a uniform magnetic field both induced by the CPT-odd LSV associated with the space-like constant vector field. Again, we can observe that the presence of the uniform magnetic field modifies the relativistic energy spectrum of the quantum system, that is, by comparing Equations (36) and Equation (40) there is the greatest term $k_{\text{eff}} = \sqrt{k^2 + 2g^2 B_0^2 v_z^2 - 2g v_z B_0 m}$ in Equation (40). In addition, by taking $g \to 0$ in Equations (39) and (40), we recover the energy of a particle under effects of a hard-wall confining potential in an isotropic environment [37].

4. Conclusions

We have analyzed a scalar particle in an environment with CPT-odd LSV that is governed by a constant background vector field. Our analysis starts with a non-minimal coupling in the Klein–Gordon equation, where in this non-minimal coupling, there is the dual electromagnetic field coupled to the background vector field that can be of a space-like nature, time-like nature, or both at the same time, which governs anisotropies in spacetime, thus characterizing LSV in the quantum system. Given this, from a theoretical point of view, we propose a possible configuration of an electromagnetic field on an LSV background governed by a space-like vector field, which induces an electric field that varies linearly with the axial coordinate.

We show that this configuration, in this background, is analogous to the Landau gauge configuration for relativistic solutions of bound states due to the induction of a kind of "uniform magnetic field" defined in terms of the parameters associated with LSV. So, in the search for solutions of bound states, we determine the relativistic energy profile of a scalar particle in this background, which is analogous to the relativistic Landau levels, where the quantized orbits are described by a cyclotron-type frequency defined in terms of the parameters associated with LSV.

In order to generalize our system, we generalize our possible configuration proposed through the induction of a uniform magnetic field by LSV and by the presence of anisotropies governed by the constant temporal component of the background vector field that governs LSV. In this context, we show that the relativistic Landau-type levels are modified by the vector field of time-like background and by the uniform magnetic field induced.

Finally, we consider a particular case of the confluent hypergeometric function to impose the presence of a hard-wall potential in the analyzed systems. Then, we determine the relativistic energy profile of a scalar particle under the effects of LSV, which is confined in a cylindrical symmetry box, characterizing, then, the presence of the hard-wall confining potential in the quantum system. We show that the presence of this type of confinement potential drastically changes the relativistic energy profile of the systems analyzed. In all the analyzed cases, we determine the energy levels in the non-relativistic case, and we can observe that the characteristics imposed by LSV in the relativistic energy levels are similar in the non-relativistic cases.

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