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Radially excited ($n = 3$) charm mesons in heavy quark effective theory

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By exploring heavy quark effective theory we use theoretical available data for bottom mesons to analyze the masses and decays for $n = 3$ charm mesons. From the predicted masses, we study ground state strong decay modes in terms of couplings. Comparing the decays with available total decay widths, we provide upper bounds on the associated couplings. We also plot Regge trajectories for our predicted data in the (J, M^2) and (n_r, M^2) planes, and estimate higher masses ($n = 4$) by fixing Regge slopes and intercepts. These Regge trajectories and study of branching ratios are used to clarify the $D_2^*(3000)$ state's J^P as the 1^3F_2 state. The results presented may be further confirmed through upcoming experimental information.

Subject Index B36

1. Introduction

In recent decades, a noteworthy experimental development has been achieved which explored the spectrum of heavy light hadrons. Several new candidates like $D_2^*(3000)$, $D_J^*(3000)$, $D_J(3000)$, $D_3^*(2760)$, $D_1^*(2680)$, $D_2^*(2460)$, $D_J^*(2760)$, and $D_{S0}(2590)^+$ were observed by experiment facilities like LHCb, BABAR, BES III, etc., and have filled out the charm meson spectrum [1–5]. In the case of the bottom sector, many new states like $B_J(5840)$, $B_J(5960)$, $B_{S1}(5830)$, $B_{S2}^*(5840)$, $B_S^0(6063.5)$, $B_S^0(6108.8)$, $B_S^0(6114)$, and $B_S^0(6158)$ have broadened the bottom spectrum [6–8]. There has also been remarkable growth in the baryon sector, with observations of five narrow Ω_c resonances [9] and doubly charmed Ξ_{cc} resonances [10]. However, in this paper we focus on mesons only.

The masses and decay widths of charm mesons for the ground state (1S) and first excited state (1P) are well established experimentally and mentioned by the Particle Data Group (PDG) for both the strange and non-strange sectors [11]. Recently, the LHCb collaboration studied $B^- \longrightarrow D^+\pi^-\pi^+$ decays with the Dalitz plot analysis technique and reported the existence of charm resonances with spins 1, 2, and 3 at the $D^+\pi^-$ mass spectrum [1]. Their investigation found that these charm resonances come mainly from the contributions of $D_3^*(2760)$, $D_1^*(2680)$, $D_2^*(2460)$, and $D_2^*(3000)$ charmed meson decays into S-wave $D^+\pi^-$. The masses and decay

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widths obtained for these resonances are:

$$M\big(D_2^*(2460)\big) = 2463.7 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})\,\text{MeV},$$
$$\Gamma\big(D_2^*(2460)\big) = 47 \pm 0.8 \pm 0.9\,\text{MeV}/c^2;$$
$$M\big(D_1^*(2680)\big) = 2681.1 \pm 5.6(\text{stat}) \pm 4.9(\text{syst})\,\text{MeV},$$
$$\Gamma\big(D_1^*(2680)\big) = 186.7 \pm 8.5 \pm 8.6\,\text{MeV}/c^2;$$
$$M\big(D_3^*(2760)\big) = 2775.5 \pm 4.5(\text{stat}) \pm 4.5(\text{syst})\,\text{MeV},$$
$$\Gamma\big(D_3^*(2760)\big) = 95.3 \pm 9.6 \pm 7.9\,\text{MeV}/c^2;$$
$$M\big(D_2^*(3000)\big) = 3214 \pm 29(\text{stat}) \pm 33(\text{syst})\,\text{MeV},$$
$$\Gamma\big(D_2^*(2460)\big) = 186 \pm 38 \pm 34\,\text{MeV}/c^2.$$

In 2015, the LHCb group examined $B^0 \longrightarrow \bar{D}^0\pi^-\pi^+$ decays and measured the $D_0^*(2300)$ and $D_0^*(2460)$ charm meson states. They assigned the state $D_3^*(2760)$ with $J^P = 3^-$ for the first time [2]. In 2013 and 2010, remarkable achievements were recorded by the LHCb and BABAR groups, respectively [3,4]. The LHCb detector discovered two resonances, $D_J^*(2650)^0$ and $D_0^*(2760)$, with natural parity, and two resonances, $D_J(2580)^0$ and $D_J(2740)^0$, with unnatural parity by analyzing the invariant mass spectrum of $D^+\pi^-$, $D^0\pi^+$, and $D^{*+}\pi^-$. In addition, the states $D_J(3000)^0$ with unnatural parity and $D_J^*(3000)^0$ with natural parity were found in the $D^{*+}\pi^-$ and $D^+\pi^-$ mass spectra respectively. The BABAR experiment analyzed e^+e^- inclusive collisions at center-of-mass energy 10.58 GeV and observed $D_J(2560)^0$, $D_J(2600)^0$, $D_J(2600)^+$, $D_J(2750)^0$, $D_J^*(2760)^+$, and $D_J(2760)^0$. The masses and decay widths of the states confirmed by LHCb and BABAR are listed in Table 1.

Table 1. States observed by different experimental facilities with masses in MeV and decay widths in MeV/c^2.

States	BABAR (2010) [4]	LHCb (2013) [3]	LHCb (2015) [2]	LHCb (2016) [1]
$D_1(2420)^0$	M: 2420.1 ± 0.1 ± 0.8 Γ = 31.4 ± 0.5 ± 1.3	M: 2419.6 ± 0.1 ± 0.8 Γ = 35.2 ± 0.4 ± 0.9	–	–
$D_2^*(2460)^0$	M: 2462.2 ± 0.1 ± 0.8 Γ = 50.5 ± 0.6 ± 0.7	M: 2460.4 ± 0.1 ± 0.1 Γ = 45.6 ± 0.4 ± 0.1	–	–
$D(2550)^0$	M: 2539.4 ± 4.5 ± 6.8 Γ = 130 ± 12 ± 13	M: 2579.5 ± 3.4 ± 5.5 Γ = 177.5 ± 17.8 ± 46.0	–	–
$D_J^*(2600)^0$	M: 2608.7 ± 2.4 ± 2.5 Γ = 93 ± 6 ± 13	M: 2649.2 ± 3.5 ± 3.5 Γ = 140.2 ± 17.1 ± 18.6	–	M: 2681.1 ± 5.6 ± 4.9 Γ = 186.7 ± 8.5 ± 8.6
$D_3^*(2750)^0$	M: 2763.3 ± 2.3 ± 2.3 Γ = 60.9 ± 5.1 ± 3.6	M: 2760.1 ± 1.1 ± 3.7 Γ = 74.4 ± 3.4 ± 19.1	–	M: 2775.5 ± 4.5 ± 4.5 Γ = 95.3 ± 9.6 ± 7.9
$D_3^*(2750)^+$	M: 2769.7 ± 3.8 ± 1.5 Γ = 60.9	M: 2771.7 ± 1.7 ± 3.8 Γ = 66.7 ± 6.6 ± 10.5	–	–
$D_2^*(2460)^+$	–	M: 2463.1 ± 0.2 ± 0.6 Γ = 48.6 ± 1.3 ± 1.9	–	–
$D(2740)^0$	–	M: 2737.0 ± 3.5 ± 11.2 Γ = 73.2 ± 13.4 ± 25.0	–	–
$D^*(3000)^0$	–	M: 2971.8 ± 8.7 Γ = 188.1 ± 44.8	–	–
$D_J^*(3000)^0$	–	M: 3008.1 ± 4.0 Γ = 110.5 ± 11.5	–	–
$D_2^*(2460)^-$	–	–	M: 2468.6 ± 0.6 ± 0.3 Γ = 47.3 ± 1.5 ± 0.7	–
$D_3^*(2750)^-$	–	–	M: 2798 ± 7 ± 1 Γ = 105 ± 18 ± 6	–
$D_2^*(3000)^0$	–	–	–	M: 3214 ± 29 ± 33 Γ = 186 ± 38 ± 34

Different theoretical models like $^3P_0$ [12], heavy quark effective theory (HQET) [13], the quantum chromodynamics sum rule [14], and the relativized quark model [15,16] have examined all the abovementioned states, computed their masses, and suggested their J^P values. The states $D_0^*(2300)$, $D_1(2420)$, $D_1(2430)$, and $D_2^*(2460)$ were reported by the PDG in Ref. [11], and their assigned J^Ps are 1^3P_0, 1^1P_1, 1^3P_1, and 1^3P_2, respectively. The results for the $D(2550)^0$ state observed by BABAR [4] are similar to those of the $D_J(2580)^0$ state reported by LHCb [1], and are considered as a candidate for the 2^1S_0 state. The information provided by the LHCb group for the states $D_1^*(2680)$ in 2016 [1] and $D_1^*(2650)$ in 2013 [3] is similar to state $D^*(2600)$ as observed by the BABAR collaboration [4]. States $D_1^*(2680)$, $D^*(2600)$, and $D_1^*(2650)$ are probably the same particle, and theoretical studies have suggested it as the 2^3S_1 state [12,17–24]. The mass and decay width of state $D_3^*(2750)$ reported by the LHCb group in 2015 [2] and $D_J^*(2760)^0$ in 2013 [3] are close to the results for $D^*(2760)^0$ observed by the BABAR collaboration [4]. So, all of $D_3^*(2750)$, $D^*(2760)^0$, and $D_J^*(2760)^0$ may be similar states, and theoretical studies suggest them as a candidate for the 1D 3^- state [3,4,12,17–24]. Observations of $D_J(3000)$ with unnatural parity and $D_J^*(3000)$ with natural parity by LHCb [3] suggested possible assignments for state $D_J^*(3000)$ as 3^3S_1, 2^3P_2, 1^3F_2, and 1^3F_4, and for state $D_J(3000)$, 3^3S_0 and 2^3P_1. The LHCb detector observed the state $D_2^*(3000)^0$ with $J^P = 2^+$ [1] and suggested it to be a candidate for the 3^3P_2, 1^3F_2 state. Theoretical approaches also examined the $D_2^*(3000)^0$, $D_J^*(3000)$, and $D_J(3000)$ states and tried to assign their proper J^P state. Reference [25] analyzed the state $D_2^*(3000)^0$ and assigned it 1F 2^+. They also examined the state $D_2^*(3000)^0$ with the quark pair creation model by studying decays of 3^3P_2 and 2^3F_2 charmed mesons. They suggested it to be the 3^3P_2 state, but the possibility of the 2^3F_2 state was also not excluded. The states $D_J^*(3000)$ and $D_J(3000)$ were also studied in Ref. [12] and identified as 3^3P_2, 1^3F_2 respectively. Reference [26] studied the states $D_J^*(3000)$ and $D_J(3000)$, and suggested the most favorable assignment for them as 2P(0^+, 1^+) respectively.

With recent experimental information, we are motivated to look for other excited states. In this paper we predict masses for $n = 3$ charm mesons in the framework of HQET. We analyze possible ground state decay modes of the predicted charm mesons. We examine the state $D_2^*(3000)^0$ using Regge trajectory and branching ratios, and specify its spin parity state. Also, we study decays from excited to ground state via pseudoscalar mesons only and evaluate the unknown coupling constants. We construct Regge trajectories for predicted masses in the (M^2, J) and (M^2, n_r) planes, and masses are estimated for higher charm states by fixing the Regge slope and intercepts. The paper is summarized as follows: Section 2 gives a brief description of the HQET formalism and an introduction to the heavy quark symmetry parameters. Section 3 presents the numerical analysis where we predict the masses for $n = 3$ charm mesons and their decays. Section 4 gives the conclusions of the paper.

2. Theoretical formulation

The study of excited charmed mesons can be explored in the HQET framework, which is an effective tool to describe properties of heavy light mesons such as masses, decay widths, branching ratios, fractions, spin, parity, etc. [27]. This theory flourished with two approximate symmetries: heavy quark symmetry and chiral symmetry. Heavy quark symmetry is applicable in the approximation $m_Q \to \infty$. In the limit $m_Q \to \infty$, the spin of light quarks is decoupled from the spin of heavy quarks, so the total angular momentum of light quarks remains conserved. The total angular momentum of light quarks is $s_l = s_q + l$, where s_q is the spin of light quarks (1/2) and l is the total orbital momentum of light quarks. In the heavy quark limit, mesons are classified in doublets on the basis of the total angular momentum s_l of light quarks. For $l = 0$, $s_l = 1/2$, combining this with the spin of the heavy quark $s_Q = 1/2$ results in the doublet $(0^-, 1^-)$. This doublet is represented by (P, P^*). $l = 1$ corresponds to two doublets represented by $(P_0^*, P_1^{'})$ and (P_1, P_2^*), with $J_{s_l}^P = (0^+, 1^+)$ and $J_{s_l}^P = (1^+, 2^+)$ respectively. For $l = 2$ there are two doublets, denoted by (P_1^*, P_2) and $(P_2^{'}, P_3^*)$, having $J_{s_l}^P = (1^-, 2^-)$ and $J_{s_l}^P = (2^-, 3^-)$ respectively. These doublets are expressed by super effective fields H_a, S_a, and T_a [19,28,29], and expressions for the fields are:

$$H_a = \frac{1+\not{v}}{2}\left\{P_{a\mu}^*\gamma^\mu - P_a\gamma_5\right\}, \tag{1}$$

$$S_a = \frac{1+\not{v}}{2}\left[P_{1a}^{'\mu}\gamma_\mu\gamma_5 - P_{0a}^*\right], \tag{2}$$

$$T_a^\mu = \frac{1+\not{v}}{2}\left\{P_{2a}^{*\mu\nu}\gamma_\nu - P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{\gamma^\nu(\gamma^\mu - \upsilon^\mu)}{3}\right]\right\}. \tag{3}$$

Here, the H_a field belongs to $s_l^p = \frac{1}{2}^-$; S_a and T_a have $s_l^p = \frac{1}{2}^+$ and $s_l^p = \frac{3}{2}^+$ respectively. For the radial quantum number $n = 2$, these states are represented by a tilde in the notation: $\tilde{P}$, $\tilde{P}^*$, etc. For $n = 3$, the states are denoted by $\tilde{\tilde{P}}$, $\tilde{\tilde{P}}^*$, and so on. Here, a is a light quark (u, d, s) flavor index; v^μ is the heavy quark four-velocity and is conserved in strong interactions. The approximate chiral symmetry $SU(3)_L \times SU(3)_R$ is incorporated by the field of pseudoscalar mesons (π, η, k). These pseudoscalar mesons are considered as approximate Goldstone bosons and described by the matrix field $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$ and $\Sigma = \xi^2$. Here, f_π is the pion constant of 130 MeV, and $\mathcal{M}$ is expressed as

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{4}$$

By including the super fields H_a, S_a, and T_a defined in Eqs. (1)–(3) and the fields of the Goldstone bosons Σ, the effective Lagrangian of heavy light mesons is [19]:

$$\begin{aligned}\mathcal{L} = {} & i\mathrm{Tr}[\bar{H}_b v^\mu D_{\mu ba} H_a] + \frac{f_\pi^2}{8}\mathrm{Tr}[\partial^\mu\Sigma\partial_\mu\Sigma^+] + \mathrm{Tr}[\bar{S}_b(iv^\mu D_{\mu ba} - \delta_{ba}\Delta_S)S_a] \\ & + \mathrm{Tr}\left[\bar{T}_b^\alpha(iv^\mu D_{\mu ba} - \delta_{ba}\Delta_T)T_{a\alpha}\right]. \end{aligned} \tag{5}$$

Here, the operator D_μ is chirally covariant and expressed as $D_{\mu ab} = -\delta_{ab}\partial_\mu + \mathcal{V}_{\mu ab} = -\delta_{ab}\partial_\mu + \frac{1}{2}(\xi^+\partial_\mu\xi + \xi\partial_\mu\xi^+)_{ab}$, and Δ_F is the mass parameter giving the mass difference between higher-mass doublets (F) and the lowest-lying doublet (H) in terms of the spin average masses of these doublets with the same principle quantum number (n). This mass parameter can be described in terms of the spin average mass of these doublets [19]:

$$\Delta_F = \overline{M_F} - \overline{M_H} \qquad (F = S, T), \tag{6}$$

$$\overline{M_H} = \left(3m_{P^*}^Q + m_P^Q\right)/4, \tag{7}$$

$$\overline{M_S} = \left(3m_{P_1^{'}}^Q + m_{P_0^*}^Q\right)/4, \tag{8}$$

$$\overline{M_T} = \left(5m^{Q}_{P_2^*} + 3m^{Q}_{P_1}\right)/8. \tag{9}$$

In the heavy quark limit, mass degeneracy between members of meson doublets breaks and the specific Lagrangian for the mass terms is

$$\mathcal{L}_{1/m_Q} = \frac{1}{2m_Q}\left[\lambda_H \mathrm{Tr}(\overline{H}_a\sigma^{\mu\nu}H_a\sigma_{\mu\nu}) + \lambda_S \mathrm{Tr}(\overline{S}_a\sigma^{\mu\nu}S_a\sigma_{\mu\nu}) + \lambda_T \mathrm{Tr}\left(\overline{T}^{\alpha}_a\sigma^{\mu\nu}T^{\alpha}_a\sigma_{\mu\nu}\right)\right]. \tag{10}$$

Here, the parameters λ_H, λ_S, and λ_T are analogous to hyperfine splittings and are defined as

$$\lambda_H = \frac{1}{8}\left(M^2_{P^*} - M^2_P\right), \qquad \lambda_S = \frac{1}{8}\left(M^2_{P_1'} - M^2_{P_0^*}\right), \qquad \lambda_T = \frac{3}{16}\left(M^2_{P_2^*} - M^2_{P_1}\right). \tag{11}$$

The mass terms in the Lagrangian represent only the first order in $1/m_Q$ terms, but higher-order terms may also be present. We limit ourselves here to the first-order corrections in $1/m_Q$. We are motivated by fact that at the scale of 1 GeV, when we study HQET, flavor symmetry spontaneously arises for the bottom and charm quarks, and hence the elegance of the flavor symmetry refers to

$$\Delta^{(c)}_F = \Delta^{(b)}_F, \qquad \lambda^{(c)}_F = \lambda^{(b)}_F. \tag{12}$$

Two-body strong interaction through a light pseudoscalar meson can be derived from the heavy meson chiral Lagrangians L_{HH}, L_{SH}, and L_{TH}, and these interaction terms are written as [30–34]:

$$L_{HH} = g_{HH}\mathrm{Tr}\left\{\overline{H}_a H_b \gamma_\mu\gamma_5 A^\mu_{ba}\right\}, \tag{13}$$

$$L_{SH} = g_{SH}\mathrm{Tr}\left\{\overline{H}_a S_b \gamma_\mu\gamma_5 A^\mu_{ba}\right\} + \mathrm{h.c.}, \tag{14}$$

$$L_{TH} = \frac{g_{TH}}{\Lambda}\mathrm{Tr}\left\{\overline{H}_a T^\mu_b (iD_\mu \not{A} + i\not{D} A_\mu)_{ba}\gamma_5\right\} + \mathrm{h.c.} \tag{15}$$

Here, the axial vector A^μ is expressed as $A^\mu = \frac{i}{2}(\xi^+\partial_\mu\xi - \xi\partial_\mu\xi^+)$. From these Lagrangians, we can determine strong decay width expressions for heavy light meson decays to the ground state along with light pseudoscalar mesons $M\,(\pi, \eta, K)$:

$(0^-, 1^-) \to (0^-, 1^-) + M(\pi, \eta, K)$:

$$\Gamma(1^- \to 1^-) = C_M \frac{g^2_{HH} M_f p^3_M}{3\pi f^2_\pi M_i}, \tag{16}$$

$$\Gamma(1^- \to 0^-) = C_M \frac{g^2_{HH} M_f p^3_M}{6\pi f^2_\pi M_i}, \tag{17}$$

$$\Gamma(0^- \to 1^-) = C_M \frac{g^2_{HH} M_f p^3_M}{2\pi f^2_\pi M_i}; \tag{18}$$

$(0^+, 1^+) \to (0^-, 1^-) + M$:

$$\Gamma(1^+ \to 1^-) = C_M \frac{g^2_{SH} M_f \left(p^2_M + m^2_M\right) p_M}{2\pi f^2_\pi M_i}, \tag{19}$$

$$\Gamma(0^+ \to 0^-) = C_M \frac{g^2_{SH} M_f \left(p^2_M + m^2_M\right) p_M}{2\pi f^2_\pi M_i}; \tag{20}$$

$(1^+, 2^+) \to (0^-, 1^-) + M$:

$$\Gamma(2^+ \to 1^-) = C_M \frac{2g^2_{TH} M_f p^5_M}{5\pi f^2_\pi \Lambda^2 M_i}, \tag{21}$$

$$\Gamma(2^+ \to 0^-) = C_M \frac{4g_{TH}^2 M_f p_M^5}{15\pi f_\pi^2 \Lambda^2 M_i}, \tag{22}$$

$$\Gamma(1^+ \to 1^-) = C_M \frac{2g_{TH}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i}; \tag{23}$$

where M_i and M_f respectively represent the initial and final momentum, Λ is the chiral symmetry-breaking scale of 1 GeV, and p_M and m_M denote the final momentum and mass of the light pseudoscalar meson. The coupling constant plays a key role in phenomenological studies of heavy light mesons. These dimensionless coupling constants describe the strengths of the transitions between the *H*–*H* field (negative–negative parity), *S*–*H* field (positive–negative parity), and *T*–*H* field (positive–negative parity). For the transition from $n = 3$ to $n = 1$, the coupling constants are given by $\tilde{\tilde{g}}_{HH}$, $\tilde{\tilde{g}}_{SH}$, $\tilde{\tilde{g}}_{TH}$, etc., and for the transition from $n = 3$ to $n = 2$ by $\tilde{g}_{HH}$, $\tilde{g}_{SH}$, $\tilde{g}_{TH}$. The coefficients C_M for different pseudoscalar particles are $C_{\pi^\pm}$, $C_{K^\pm}$, C_{K^0}, $C_{\overline{K}^0} = 1$, $C_{\pi^0} = \frac{1}{2}$, and $C_\eta = \frac{2}{3}(c\bar{u}, c\bar{d})$ or $\frac{1}{6}(c\bar{s})$. In this paper we are not including higher-order corrections of $\frac{1}{m_Q}$ to bring new couplings. We also expect that higher corrections give negligible contributions in comparison with leading-order contributions.

3. Numerical analysis

Several higher charm states like $D_2^*(3000)$, $D_J^*(3000)$, $D_J(3000)$, $D_3^*(2760)$, $D_1^*(2680)$, $D_2^*(2460)$, and $D_J^*(2760)$ were discovered by LHCb and BABAR and analyzed with different theoretical models. Theoretical studies with different theoretical approaches give different assignments to these states. Therefore, it is important to have a better theoretical understanding of higher charm states. So, in this paper we aim to compute the masses and decay widths for $n = 3$ S-wave and P-wave charm mesons with their strange partners. The masses and decay widths for $n = 2$ charm spectra have already been calculated with same framework.

3.1 *Masses*

Mass is the most important parameter in understanding the spectroscopy of heavy light mesons. To calculate the masses for $n = 3$ S-wave and P-wave charm mesons, we first determine the average masses $\overline{M_F}$, and then compute the symmetry parameters Δ_F and λ_F for the input values listed in Table 2.

The symmetry parameters for excited states can be expressed as:

$$\Delta_{\tilde{\tilde{H}}} = \overline{M}_{\tilde{\tilde{H}}} - \overline{M}_{\tilde{H}}, \qquad \Delta_{\tilde{\tilde{S}}} = \overline{M}_{\tilde{\tilde{S}}} - \overline{M}_{\tilde{\tilde{H}}}, \qquad \Delta_{\tilde{\tilde{T}}} = \overline{M}_{\tilde{\tilde{T}}} - \overline{M}_{\tilde{\tilde{H}}}, \tag{24}$$

$$\lambda_{\tilde{\tilde{H}}} = \frac{1}{8}\left(M^2_{\tilde{\tilde{P}}^*} - M^2_{\tilde{\tilde{P}}}\right), \qquad \lambda_{\tilde{\tilde{S}}} = \frac{1}{8}\left(M^2_{\tilde{\tilde{P}}'} - M^2_{\tilde{\tilde{P}}^*}\right), \qquad \lambda_{\tilde{\tilde{T}}} = \frac{3}{16}\left(M^2_{\tilde{\tilde{P}}^*_2} - M^2_{\tilde{\tilde{P}}_1}\right). \tag{25}$$

So, with the help of the heavy quark symmetry $\Delta_F^{(c)} = \Delta_F^{(b)}$, $\lambda_F^{(c)} = \lambda_F^{(b)}$, and using the calculated flavor symmetry parameters $\Delta_{\tilde{\tilde{H}}}$, $\Delta_{\tilde{\tilde{S}}}$, $\lambda_{\tilde{\tilde{H}}}$, $\lambda_{\tilde{\tilde{S}}}$, and $\lambda_{\tilde{\tilde{T}}}$, we obtain the masses for $n = 3$ S-wave and P-wave charm spectra listed in Table 3.

Comparing our calculated masses with the available theoretical information, we found that our estimated masses lie below the prediction of the relativistic quark model in Ref. [35] with a difference of 25–50 MeV. On comparing with Ref. [36], our results lie above their values. So, our computed masses for $n = 3$ charm mesons without and with strangeness are in good overall agreement with other theoretical estimates.

Table 2. Input values used in this work [35]. All values are in units of MeV.

State	J^P	$b\bar{q}$	$b\bar{s}$	$c\bar{q}$	$c\bar{s}$
3^1S_0	0^-	6362.22	6463.07	–	–
3^3S_1	1^-	6342.74	6474.55	–	–
3^3P_0	0^+	6629	6731	–	–
3^1P_1	1^+	6685	6768	–	–
3^3P_1	1^+	6650	6761	–	–
3^3P_2	2^+	6678	6780	–	–
2^1S_0	0^-	5890	5976	2581	2688
2^3S_1	1^-	5906	5992	2632	2731
2^3P_0	0^+	6221	6318	2919	3054
2^1P_1	1^+	6281	6345	3021	3154
2^3P_1	1^+	6209	6321	2932	3067
2^3P_2	2^+	6260	6359	3012	3142

Table 3. Predicted masses for radially excited charm mesons.

	Masses of $n = 3$ charm Mesons (MeV)					
	Non-strange			Strange		
$J^P(n^{2S+1}L_J)$	Predicted	[35]	[36]	Predicted	[35]	[36]
$0^-(3^1S_0)$	3030.09	3062	2904	3186.5	3219	3044
$1^-(3^3S_1)$	3064.45	3096	2947	3209.74	3242	3087
$0^+(3^3P_0)$	3243.17	3346	3050	3496.46	3541	3214
$1^+(3^1P_1)$	3356.13	3461	3082	3567.18	3618	3234
$1^+(3^3P_1)$	3281.27	3365	3085	3508.63	3519	3244
$2^+(3^3P_2)$	3337.81	3407	3142	3563.20	3580	3283

3.2 *Decay widths*

By using predicted masses, we compute the decay width for $n = 3$ charm mesons from excited to ground state with the emission of pseudoscalar particles (π, η, K) only in terms of coupling constants. The input values used for calculating the decay width are $M_{\pi^0} = 134.97$ MeV, $M_{\pi^+} = 139.57$ MeV, $M_{K^+} = 493.67$ MeV, $M_{\eta^0} = 547.85$ MeV, $M_{K^0} = 497.61$ MeV, $M_{D^0} = 1864.83$ MeV, $M_{D^\mp} = 1869.65$ MeV, $M_{D_S^\pm} = 1968.34$ MeV, $M_{D^{*0}} = 2006.85$ MeV, $M_{D^{*\pm}} = 2010.26$ MeV, $M_{D_S^{*\pm}} = 2112.20$ MeV, $M_{D_0^{*0}} = 2318$ MeV, $M_{D_{S0}^{*\pm}} = 2317.70$ MeV, $M_{D_1'^0} = 2420.80$ MeV, $M_{D_{S1}'^\pm} = 2459.50$ MeV, and the calculated masses for $n = 3$ S-wave and P-wave charm mesons mentioned in Table 3.

The computed strong decay widths in terms of the coupling constants $\tilde{\tilde{g}}_{HH}$, $\tilde{\tilde{g}}_{SH}$, and $\tilde{\tilde{g}}_{TH}$ for radially excited charm mesons are presented in Tables 4 and 5. Without enough experimental data it is not possible to determine the values of the coupling constants from heavy quark symmetry alone, but upper bounds for these couplings are mentioned in Tables 4 and 5. We took limited decay modes, and only to the ground state. We believe that a particular state like $D(3030)$ gives a total decay width of $7783.54\tilde{\tilde{g}}^2_{HH}$ when compared with the total decay widths mentioned by other theoretical papers; we provide an upper bound on $\tilde{\tilde{g}}_{HH}$ value. Now, if we take additional modes, the value of $\tilde{\tilde{g}}_{HH}$ will be less than 0.12 ($\tilde{\tilde{g}}_{HH} < 0.12$). So, these upper bounds may give important information on other associated charm states. Large fractions of the decay widths of any excited state are dominated by modes that include the ground state.

Table 4. Decay widths of obtained masses for $n = 3$ charm mesons.

States	J^P	Decay modes	Decay widths (MeV)
$D(3030.09)$	0^-	$D^{*\pm}\pi^-$	$3703.68\tilde{\tilde{g}}^2_{HH}$
		$D^{*0}\pi^0$	$1867.45\tilde{\tilde{g}}^2_{HH}$
		$D^{*0}\eta^0$	$378.62\tilde{\tilde{g}}^2_{HH}$
		$D_s^{*+}K^-$	$1833.79\tilde{\tilde{g}}^2_{HH}$
		Total	$7783.54\tilde{\tilde{g}}^2_{HH}$
Coupling constant upper bound: $\tilde{\tilde{g}}_{HH} < 0.12$			
$D(3064.45)$	1^-	$D^+\pi^-$	$1668.5\tilde{\tilde{g}}^2_{HH}$
		$D^0\pi^0$	$841.07\tilde{\tilde{g}}^2_{HH}$
		$D^{*+}\pi^-$	$2665.54\tilde{\tilde{g}}^2_{HH}$
		$D^{*0}\pi^0$	$1343.38\tilde{\tilde{g}}^2_{HH}$
		$D^0\eta^0$	$208.05\tilde{\tilde{g}}^2_{HH}$
		$D^{*0}\eta^0$	$282.52\tilde{\tilde{g}}^2_{HH}$
		$D_s^+K^-$	$1031.87\tilde{\tilde{g}}^2_{HH}$
		D_sK^0	$1233.43\tilde{\tilde{g}}^2_{HH}$
		$D_s^+K^-$	$1031.87\tilde{\tilde{g}}^2_{HH}$
		$D_s^{*+}K^-$	$1389.41\tilde{\tilde{g}}^2_{HH}$
		Total	$9430.27\tilde{\tilde{g}}^2_{HH}$
Coupling constant upper bound: $\tilde{\tilde{g}}_{HH} < 0.11$			
$D(3243.17)$	0^+	$D^+\pi^-$	$6893.15\tilde{\tilde{g}}^2_{SH}$
		$D^0\pi^0$	$3464.3\tilde{\tilde{g}}^2_{SH}$
		$D^0\eta^0$	$1145.25\tilde{\tilde{g}}^2_{SH}$
		$D_s^+K^-$	$6061.58\tilde{\tilde{g}}^2_{SH}$
		Total	$17564.28\tilde{\tilde{g}}^2_{SH}$
Coupling constant upper bound: $\tilde{\tilde{g}}_{SH} < 0.09$			
$D(3356.13)$	1^+	$D^{*0}\pi^0$	$3528.38\tilde{\tilde{g}}^2_{SH}$
		$D^{*+}\pi^-$	$7028.54\tilde{\tilde{g}}^2_{SH}$
		$D^{*0}\eta^0$	$1160.11\tilde{\tilde{g}}^2_{SH}$
		$D_s^{*+}K^-$	$6053.09\tilde{\tilde{g}}^2_{SH}$
		Total	$17770.12\tilde{\tilde{g}}^2_{SH}$
Coupling constant upper bound: $\tilde{\tilde{g}}_{SH} < 0.08$			
$D(3281.27)$	1^+	$D^{*0}\pi^0$	$4258.74\tilde{\tilde{g}}^2_{TH}$
		$D^{*+}\pi^-$	$8426.74\tilde{\tilde{g}}^2_{TH}$
		$D^{*0}\eta^0$	$854.16\tilde{\tilde{g}}^2_{TH}$
		$D_s^{*+}K^-$	$3968.1\tilde{\tilde{g}}^2_{TH}$
		Total	$17507.75\tilde{\tilde{g}}^2_{TH}$
Coupling constant upper bound: $\tilde{\tilde{g}}_{TH} < 0.10$			
$D(3337.81)$	2^+	$D^+\pi^-$	$2705.93\tilde{\tilde{g}}^2_{TH}$
		$D^0\pi^0$	$1367.81\tilde{\tilde{g}}^2_{TH}$
		$D^{*+}\pi^-$	$5977.98\tilde{\tilde{g}}^2_{TH}$
		$D^{*0}\pi^0$	$3029.18\tilde{\tilde{g}}^2_{TH}$
		$D^0\eta^0$	$312.75\tilde{\tilde{g}}^2_{TH}$
		$D^{*0}\eta^0$	$635.44\tilde{\tilde{g}}^2_{TH}$
		$D_s^+K^-$	$1564.54\tilde{\tilde{g}}^2_{TH}$

Table 4. Continued

States	J^P	Decay modes	Decay widths (MeV)
		$D_s^{*+}K^-$	$3010.47\tilde{\tilde{g}}_{TH}^2$
		Total	$18604.1\tilde{\tilde{g}}_{TH}^2$
Coupling constant upper bound: $\tilde{\tilde{g}}_{TH} < 0.079$			

Table 5. Decay widths of obtained masses for $n = 3$ strange charm mesons.

States	J^P	Ground state decay modes	Decay widths (MeV)
$D_s^+(3186.5)$	0^-	D^*K^+	$3869.08\tilde{\tilde{g}}_{HH}^2$
		$D^{*+}K^0$	$3848.36\tilde{\tilde{g}}_{HH}^2$
		$D_s^{*+}\eta^0$	$1858.32\tilde{\tilde{g}}_{HH}^2$
		Total	$9575.76\tilde{\tilde{g}}_{HH}^2$
Coupling constant upper bound: $\tilde{\tilde{g}}_{HH} < 0.09$			
$D_s^+(3209.74)$	1^-	D^+K^0	$1720.77\tilde{\tilde{g}}_{HH}^2$
		D^0K^+	$1739.82\tilde{\tilde{g}}_{HH}^2$
		$D^{*+}K^0$	$2712.35\tilde{\tilde{g}}_{HH}^2$
		$D^{*0}K^+$	$2744.24\tilde{\tilde{g}}_{HH}^2$
		$D_s^+\eta^0$	$915.91\tilde{\tilde{g}}_{HH}^2$
		$D_s^{*+}\eta^0$	$1329.13\tilde{\tilde{g}}_{HH}^2$
		$D_s^{*+}\pi^0$	$948.81\tilde{\tilde{g}}_{HH}^2$
		Total	$82111.03\tilde{\tilde{g}}_{HH}^2$
Coupling constant upper bound: $\tilde{\tilde{g}}_{HH} < 0.29$			
$D_s^+(3496.46)$	0^+	$D_S^+\pi^0$	$4517.26\tilde{\tilde{g}}_{SH}^2$
		$D_s^+\eta$	$6002.88\tilde{\tilde{g}}_{SH}^2$
		D^0K^+	$9826.27\tilde{\tilde{g}}_{SH}^2$
		D^+K^0	$9863.43\tilde{\tilde{g}}_{SH}^2$
		Total	$30209.84\tilde{\tilde{g}}_{SH}^2$
Coupling constant upper bound: $\tilde{\tilde{g}}_{SH} < 0.06$			
$D_s^+(3567.18)$	1^+	D^*K^+	$8194.67\tilde{\tilde{g}}_{SH}^2$
		$D^{*+}K^0$	$8179.21\tilde{\tilde{g}}_{SH}^2$
		$D_s^{*+}\eta^0$	$5723.21\tilde{\tilde{g}}_{HH}^2$
		$D_s^{+*}\pi^0$	$4333.65\tilde{\tilde{g}}_{SH}^2$
		Total	$26430.74\tilde{\tilde{g}}_{SH}^2$
Coupling constant upper bound: $\tilde{\tilde{g}}_{SH} < 0.07$			
$D_s^+(3508.63)$	1^+	D^*K^+	$9302.31\tilde{\tilde{g}}_{TH}^2$
		$D^{*+}K^0$	$9246.22\tilde{\tilde{g}}_{TH}^2$
		$D_s^{*+}\eta^0$	$5675.62\tilde{\tilde{g}}_{TH}^2$
		$D_s^{+*}\pi^0$	$7613.48\tilde{\tilde{g}}_{TH}^2$
		Total	$31837.63\tilde{\tilde{g}}_{TH}^2$
Coupling constant upper bound: $\tilde{\tilde{g}}_{TH} < 0.06$			

Table 5. Continued

States	J^P	Ground state decay modes	Decay widths (MeV)
$D_s^+(3563.20)$	2^+	D^*K^+	$16804.3\tilde{\tilde{g}}_{TH}^2$
		$D^{*+}K^0$	$16885.1\tilde{\tilde{g}}_{TH}^2$
		$D_s^{*+}\eta^0$	$5675.62\tilde{\tilde{g}}_{TH}^2$
		$D_s^{*+}\pi^0$	$1150.23\tilde{\tilde{g}}_{TH}^2$
		$D_s^+\eta$	$11505.4\tilde{\tilde{g}}_{SH}^2$
		D^+K^0	$22261.8\tilde{\tilde{g}}_{SH}^2$
		D^0K^+	$22204.4\tilde{\tilde{g}}_{SH}^2$
		$D_s^+\pi^0$	$11872.7\tilde{\tilde{g}}_{TH}^2$
		Total	$108359.38\tilde{\tilde{g}}_{TH}^2$
Coupling constant upper bound: $\tilde{\tilde{g}}_{TH} < 0.035$			

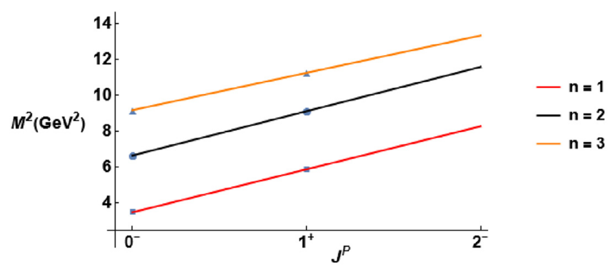

Fig. 1. Regge trajectories for non-strange charm mesons with unnatural parity.

Our work also provides a lower limit to the total decay width which gives important clues to forthcoming experimental studies. Here, we need to emphasize that the calculated total decay widths for the above states do not include contributions of decays with emission of vector mesons (ω, ρ, K^*, ϕ), since the contributions of vector mesons to total decay widths are small compared with pseudoscalar mesons. They give a contribution of ± 20 MeV [35] to the total decay widths for the states analyzed above.

3.3 *Regge trajectory*

Regge trajectories are a powerful tool for studying the spectroscopy of hadrons. The graph of total angular momentum (J) and radial quantum number (n_r) of hadrons against the square of their masses (M^2) gives information about the quantum number of a particular state, and also helps to identify recently observed states. We use the following definitions:

$$\text{The } (J, M^2) \text{ Regge trajectories:} \qquad J = \alpha M^2 + \alpha_0. \tag{26}$$

$$\text{The } (n_r, M^2) \text{ Regge trajectories:} \qquad n_r = \beta M^2 + \beta_0. \tag{27}$$

Here, α and β are slopes, and α_0 and β_0 intercepts. We construct Regge trajectories in the (J, M^2) plane with natural parity $P = (-1)^J$ and unnatural parity $P = (-1)^{J-1}$ as depicted in Figs. 1–4. Regge trajectories in the (n_r, M^2) plane are constructed in Figs. 5 and 6 using spin-averaged masses for S and P waves, where the spin-averaged masses for S-wave

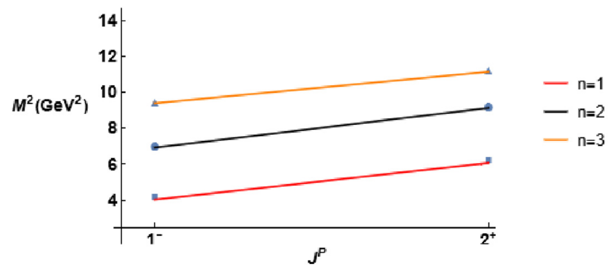

Fig. 2. Regge trajectories for non-strange charm mesons with natural parity.

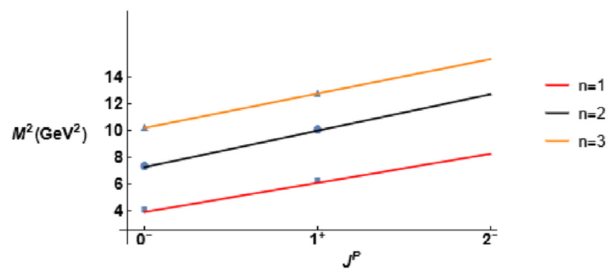

Fig. 3. Regge trajectories for strange charm mesons with unnatural parity.

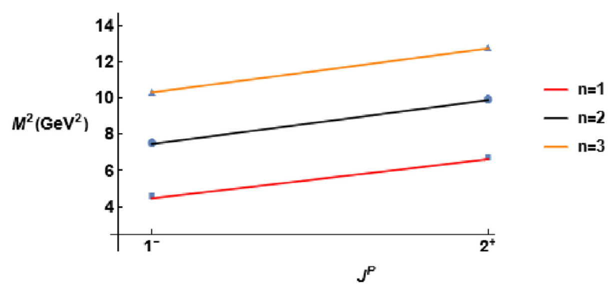

Fig. 4. Regge trajectories for strange charm mesons with natural parity.

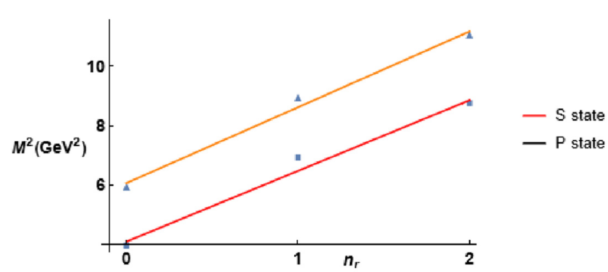

Fig. 5. Regge trajectories for spin-averaged masses for non-strange charm mesons in the (M^2, n_r) plane.

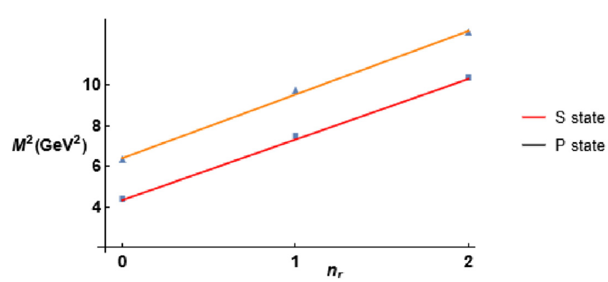

Fig. 6. Regge trajectories for spin-averaged masses for strange charm mesons in the (M^2, n_r) plane.

Table 6. Regge slopes and intercepts.

Mesons	State	Slope (β)	Intercepts (β_0)
D	0^-	0.350235	– 1.25767
	1^-	0.372595	– 1.53582
	0^+	0.41673	– 2.45654
	1^+	0.37199	– 2.18897
	1^+	0.404165	– 2.42289
	2^+	0.92575	– 2.39406
D_S	0^-	0.317195	– 1.18762
	1^-	0.339684	– 1.45611
	0^+	0.291202	– 1.53521
	1^+	0.299011	– 1.78088
	1^+	0.332348	– 2.02826
	2^+	0.325032	– 2.07554

Table 7. Higher non-strange masses lying in Regge lines in the (n_r, M^2) plane.

State	J^P	Masses (MeV)	Ref. [12]
4^1S_0	0^-	3484.65	3468
4^3S_1	1^-	3489.07	3497
4^3P_0	0^+	3618.52	3697
4^1P_1	1^+	3733.82	3709
4^3P_1	1^+	3662.99	3681
4^3P_2	1^+	3716.67	3701

Table 8. Higher strange masses lying in Regge lines in the (n_r, M^2) plane.

State	J^P	Masses (MeV)	Ref. [12]
4^1S_0	0^-	3633.46	3547
4^3S_1	1^-	3621.93	3575
4^3P_0	0^+	3946.4	3764
4^1P_1	1^+	3998.62	3778
4^3P_1	1^+	3889.67	3764
4^3P_2	1^+	3951.65	3783

is given by $S = \left(3m^Q_{P^*} + m^Q_P\right)/4$ and for P-wave by $P = \left(3m^Q_{P_1'} + m^Q_{P_0^*} + 5m^Q_{P_2^*} + 3m^Q_{P_1}\right)/12$. In Figs. 1–6 the masses for $n = 1$ are taken from Ref. [11], for $n = 2$ the masses are taken from Ref. [35], and for $n = 3$ we take our calculated masses. Our calculated data fit on the Regge lines with good accuracy. By fixing the slopes and intercepts of these Regge trajectories (Table 6), we calculate the higher masses listed in Tables 7 and 8. Using the Regge trajectories, we also assign a quantum number to state $D_2^*(3000)$. The state $D_2^*(3000)$ was reported by the LHCb Collaboration in 2016 by studying $B^- \to D^+\pi^-\pi^-$ decay. This state has been analyzed by different theoretical models and different assignments suggested. Using the relativistic quark model, Ref. [25] studied the $D_2^*(3000)$ state and assigned it to be the 1^3F_2 state. Also used was the $^3P_0$ model, which suggested it to

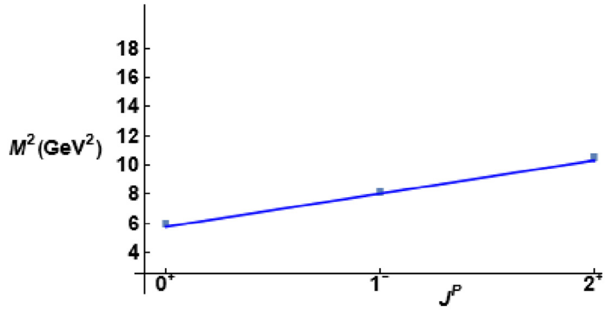

Fig. 7. Regge lines in the (M^2, J^P) plane to identify $D_2(3000)$.

be 3^3P_2. There is an ambiguity in J^P for the state $D_2^*(3000)$. Here, we assigned J^P for $D_2^*(3000)$ using Regge trajectories in the (J, M^2) plane as shown in Figs. 7 and 2, and suggest the 1^3F_2 and 2^3P_2 state respectively. In Fig. 7, the mass for 1^3P_0 is taken from Ref. [11], the mass for 1^3D_1 is from Ref. [35], and we take $D_2^*(3000)$ as 1^3F_2. To clarify its assignment between these two states $(2^3P_2, 1^3F_2)$, we studied the branching ratio $BR = \Gamma(D_2^*(3000) \rightarrow D^*\pi)/\Gamma(D_2^*(3000) \rightarrow D\pi)$. The value of the branching ratio for 2^3P_2 is 1.06, suggesting $D^*\pi$ to be the dominant mode rather than $D\pi$. But, the value of the branching ratio for 1^3F_2 is 0.40, suggesting $D\pi$ to be dominant mode. Experimentally, the $D^*\pi$ decay mode for $D_2^*(3000)$ is suppressed. Hence 1^3F_2 is the most favorable assignment for $D_2^*(3000)$ [13].

4. Conclusion

Heavy quark symmetry is an important tool for describing the spectroscopy of hadrons containing a single heavy quark. Using available experimental as well as theoretical data on bottom mesons and applying heavy quark symmetry, we have predicted masses for $n = 3$ charm meson spectra. With the computed masses for $n = 3$ charm mesons, we analyzed decay widths from excited to ground state with the emission of pseudoscalar mesons and expressed them in terms of coupling constants. These coupling constants are computed by comparing our decay widths with theoretical available total decay widths. The total decay widths may give an upper bound on these coupling constants, hence providing important clues to other associated states of charm mesons. Using our calculated charm masses for $n = 3$, we constructed Regge trajectories in the (J, M^2) and (n_r, M^2) planes. These Regge lines are almost linear, parallel, and equidistant. Most of our predicted data fit nicely to them. We also computed masses for $n = 4$ charm spectra by fixing the Regge slopes and intercepts in the (n_r, M^2) plane. Our calculated masses and upper bound findings may help experimentalists when looking into higher excited states.

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REFERENCES

[1] R. Aaij et al. [LHCb Collaboration], Phys. Rev. D **94**, 072001 (2016).
[2] R. Aaij et al. [LHCb Collaboration], Phys. Rev. D **92**, 32002 (2015).
[3] R. Aaij et al. [LHCb Collaboration], J. High Energy Phys. **09**, 145 (2013).
[4] P. del Amo Sanchez et al. [BABAR Collaboration], Phys. Rev. D **82**, 111101(R) (2010).
[5] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **126**, 122002 (2021).
[6] R. Aaij et al. [LHCb Collaboration], Eur. Phys. J. C **81**, 601 (2021).
[7] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D **90**, 012013 (2014).
[8] R. Aaij et al. [LHCb Collaboration], J. High Energy Phys. **1504**, 024 (2015).
[9] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **118**, 182001 (2017).
[10] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **119**, 112001 (2017).
[11] P. A. Zyla et al. [Particle Data Group], Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[12] S. Godfrey and K. Moats, Phys. Rev. D **93**, 034035 (2016).
[13] P. Gupta and A. Upadhyay, Phys. Rev. D **97**, 014015 (2018).
[14] S. Narison, Phys. Lett. B **605**, 319 (2005).
[15] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[16] M. Di Pierro and E. Eichten, Phys. Rev. D **64**, 114004 (2001).
[17] Z. G. Wang, Phys. Rev. D **83**, 014009 (2011).
[18] P. Colangelo, F. De Fazio, and S. Nicotri, Phys. Lett. B **642**, 48 (2006).
[19] P. Colangelo, F. De Fazio, F. Giannuzzi, and S. Nicotri, Phys. Rev. D **86**, 054024 (2012).
[20] A. M. Badalian and B. L. G. Bakker, Phys. Rev. D **84**, 034006 (2011).
[21] Q. F. Lu and D. M. Li, Phys. Rev. D **90**, 054024 (2014).
[22] Q. T. Song, D. Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D **92**, 074011 (2015).
[23] B. Chen, X. Liu, and A. Zhang, Phys. Rev. D **92**, 034005 (2015).
[24] Z. G. Wang, Phys. Rev. D **88**, 114003 (2013).
[25] G. L. Yu, Z. G. Wang, and Z. Y. Li, Phys. Rev. D **94**, 074024 (2016).
[26] Y. Sun, X. Liu, and T. Matsuki, Phys. Rev. D **88**, 094020 (2013).
[27] M. Neubert, Phys. Rept. **245**, 259 (1994).
[28] A. F. Falk and T. Mehen, Phys. Rev. D **53**, 231 (1996).
[29] Z.-G. Wang, Eur. Phys. J. Plus **129**, 186 (2014).
[30] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[31] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[32] P. L. Cho, Phys. Lett. B **285**, 145 (1992).
[33] T.-M. Yan, H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Yin, and H.-L. Yu, Phys. Rev. D 46, 1148 (1992); **55**, 5851 (1997) [erratum].
[34] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B **299**, 139 (1993).
[35] D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C **66**, 197 (2010).
[36] T. A. Lahde, C. Nyfalt, and D. O. Riska, Nucl. Phys. A **674**, 141 (2000).