

Letter

A viable explanation of the CMB dipolar statistical anisotropy

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The presence of a dipolar statistical anisotropy in the spectrum of cosmic microwave background (CMB) fluctuations was reported by the Wilkinson Microwave Anisotropy Probe (WMAP), and has recently been confirmed in the Planck 2013 analysis of the temperature anisotropies. At the same time, the Planck 2013 results report a stringent bound on the amplitude of the local-type non-Gaussianity. We show that the non-linear effect of the dipolar anisotropy generates not only a quadrupole moment in the CMB but also a local-type non-Gaussianity. Consequently, it is not easy to build models having a large dipolar modulation and at the same time a sufficiently small quadrupole and level of local bispectral anisotropy to agree with the present data. In particular, most models proposed so far are almost excluded, or are at best marginally consistent with observational data. We present a simple alternative scenario that may explain the dipolar statistical anisotropy while satisfying the observational bounds on both the quadrupole moment and local-type non-Gaussianity.
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Subject Index E63, E81

1. Introduction

It was suggested by WMAP that the amplitude of cosmic microwave background (CMB) anisotropies has a dipolar directional dependence [1,2]. This anomalous signature has been recently confirmed by Planck [3]. Although it is possible that this anisotropy is a foreground effect, it is an interesting question whether or not we can explain this feature consistently within the paradigm of the inflationary universe.

The basic issue is that a dipolar modulation of the amplitude of the curvature perturbation requires a seed dipolar perturbation. This seed dipolar perturbation itself is known to produce quadrupole and octupole moments of CMB anisotropy, known as the Grishchuk–Zel’dovich (GZ) effect [4]. Various scenarios that explain the dipolar statistical anisotropy have been proposed [5–15] and various features have been investigated [16–18]. The proposal in Ref. [7] advocates the use of the super-curvature perturbation [19] produced in the open inflation scenario as the seed for the dipolar modulation.

The amplitudes of monopole, dipole, and quadrupole fluctuations A_ℓ ($\ell = 0, 1, 2$) caused by the super-curvature mode of a free scalar field in the small mass limit are, respectively, evaluated by

taking the small radius limit of the corresponding normalized mode functions as [19]

$$A_0 = \sqrt{\frac{3}{2\pi}} \frac{H_F^2}{m_F}, \quad A_1 = \frac{H_F}{3\sqrt{\pi}} \left(\frac{r_{\text{ls}}}{r_{\text{K}}} \right), \quad A_2 = \frac{H_F}{5\sqrt{3\pi}} \left(\frac{r_{\text{ls}}}{r_{\text{K}}} \right)^2, \quad (1)$$

where r_{ls} and r_{K} are the radii of the last scattering surface and the curvature radius in the comoving conformal coordinate, and m_{F} and H_{F} are the mass and the Hubble expansion rate in the false vacuum before bubble nucleation. Thus, one can basically avoid the constraint due to the GZ effect by choosing $r_{\text{ls}}/r_{\text{K}} \approx \sqrt{1 - \Omega_0}$ to be sufficiently small. (Although in Ref. [16] a constraint on the amplitude of the super-curvature perturbation was imposed, such a condition on the magnitude of fluctuations beyond the current horizon scale is usually thought to be unnecessary in the context of the inflationary universe. The quasi-open inflation scenario [20] is a typical example.)

However, we suspect that one important constraint has been overlooked. It is a genuine non-linear effect. If the amplitude of a dipole moment is large, a non-negligible quadrupole moment is naturally induced by non-linearity. Taking into account the constraint from the amplitude of this induced quadrupole moment, we find that almost all models proposed so far are almost excluded by observations.¹ In this paper we propose a simple alternative scenario that can naturally evade this constraint.

2. Curvaton scenario

In Refs. [5,6,16], in order to explain the observed large statistical anisotropy, the simplest curvaton scenario was studied. The only difference from the ordinary curvaton scenario is that the curvaton has a large-magnitude super-horizon fluctuation. In Ref. [7], this super-horizon fluctuation was identified with a super-curvature mode, whose amplitude can naturally be large compared with sub-curvature modes if the creation of the one-bubble open universe occurs in the sea of false vacuum whose energy scale is much higher than that inside the bubble. As this generation mechanism of a large super-horizon fluctuation seems most natural, we focus on possible scenarios in this context.

First let us discuss the model based on the simplest curvaton scenario. We denote the super-curvature fluctuation of the curvaton as $\Delta\chi$. More precisely, its distribution measured at the current horizon radius is $\Delta\chi \cos\theta$, where the axis of the angular coordinates (θ, φ) is aligned to the direction of the dipolar statistical anisotropy. A typical amplitude of $\Delta\chi$ is given by A_1 in Eq. (1), whereas $\delta\chi$ represents the ordinary mode fluctuation whose amplitude is $O(H)$, where H is the Hubble rate at the time when the relevant length scale crosses the horizon scale during the inflation. We assume $A_1 \gg H$. The inflation inside the nucleated bubble is driven by an inflaton field ϕ , but the details of the model concerning the ϕ -field are not important at all. For simplicity, we assume that the super-curvature perturbation is completely dominated by the curvaton field χ and that the inflaton ϕ is in the slow-roll regime when the relevant length scale crosses the horizon.

The observed curvature perturbation can be evaluated by using the δN -formula (see, for example, Ref. [21]),

$$\mathcal{R}_c = N_\phi \delta\phi + N_\chi (\Delta\chi + \delta\chi) + \frac{1}{2} N_{\chi\chi} (\Delta\chi + \delta\chi)^2 + \dots. \quad (2)$$

We assume that the leading-order power spectrum of the curvature perturbation is given by the terms with the first derivatives of N , except for extremely low ℓ modes like dipole or quadrupole. Then,

¹ A somewhat different mechanism that might circumvent this new constraint was proposed in Ref. [14].

we have

$$P_{\mathcal{R}_c} = (N_\phi^2 + N_\chi^2) \frac{H^2}{(2\pi)^2}.$$

The dipolar statistical anisotropy is caused by the term including $\Delta\chi$ in Eq. (2). The leading term is given by

$$\Delta P_{\mathcal{R}_c} = 2N_\chi N_{\chi\chi} \Delta\chi \frac{H^2}{(2\pi)^2}.$$

The ratio of these two quantities is constrained by the observation as [3]²

$$\frac{\Delta P_{\delta T}}{P_{\delta T}} = \frac{2N_{\chi\chi} N_\chi}{(N_\phi^2 + N_\chi^2)} \Delta\chi \approx 0.14. \quad (3)$$

Here we neglected the non-linear terms in the Sachs–Wolfe relation between \mathcal{R}_c and the observed temperature fluctuation

$$\frac{\delta T}{T} = -\frac{\mathcal{R}_c}{5} + O(\mathcal{R}_c^2), \quad (4)$$

since the effect due to this non-linearity is suppressed by $N_\chi^2/N_{\chi\chi}$ and it can be absorbed by the redefinition of $N_{\chi\chi}$.

On the other hand, this model predicts the local-type non-Gaussianity,

$$f_{\text{NL}} = \frac{5}{6} \frac{N_{\chi\chi} N_\chi^2}{(N_\phi^2 + N_\chi^2)^2} = \frac{5}{24} \left(\frac{\Delta P_{\delta T}}{P_{\delta T}} \right)^2 \frac{1}{N_{\chi\chi} \Delta\chi^2} \approx \frac{0.004}{N_{\chi\chi} \Delta\chi^2} \left(\frac{\Delta P_{\delta T}/P_{\delta T}}{0.14} \right)^2, \quad (5)$$

where we have used Eq. (3) in the second and third equalities. Furthermore, a quadrupolar anisotropy is induced by the second-order effect of $\Delta\chi$ which originates from the term $N_{\chi\chi} \Delta\chi^2$ in Eq. (2). Its magnitude is estimated as

$$\left| a_{20}^{(\Delta\chi)} \right| = \frac{2\sqrt{\pi}}{15\sqrt{5}} |N_{\chi\chi}| \Delta\chi^2. \quad (6)$$

We note that this is not the so-called Grishchuk–Zel’dovich (GZ) effect [4]. We also mention that although the functional dependence of the quadrupole is similar to that obtained in Ref. [16], our estimate gives a more robust bound while that in Ref. [16] is based on a rather crude argument concerning the convergence of perturbative expansion which cannot be directly compared with the observational data. As mentioned earlier, if $\Delta\mathcal{R}_c$ is composed of the super-curvature mode in the open inflation scenario, the constraint from the GZ effect can be evaded.

Combining Eqs. (5) and (6), we find

$$\left| a_{20}^{(\Delta\chi)} \right| |f_{\text{NL}}| \approx 4.3 \times 10^{-4} \left(\frac{\Delta P_{\delta T}/P_{\delta T}}{0.14} \right)^2. \quad (7)$$

Thus one cannot make both the quadrupole and local-type non-Gaussianity very small simultaneously. This gives a bound slightly tighter and more robust than that obtained in Eq. (59) of Ref. [16]. Here we note that the contribution of N_ϕ^2 does not relax the above constraint.

² This result, however, is of modest statistical significance, and most of the signal appears concentrated in the modulation of modes of small ℓ . We thank M. Bucher for making this point clear to us. See Ref. [3] for a more detailed discussion.

From the observed quadrupole anisotropy, we may bound the induced quadrupole as

$$\left| a_{20}^{(\Delta\chi)} \right| \lesssim \beta \sqrt{\langle a_{2m}^2 \rangle_{\text{obs}}} \equiv \beta \sqrt{C_2}, \quad (8)$$

where the observed magnitude of the squared quadrupole temperature anisotropy is $C_2 \approx 4.2 \times 10^{-11}$ and we inserted a threshold number β of order unity. This implies the constraint

$$\frac{2\sqrt{\pi}}{15\sqrt{5}} |N_{\chi\chi}| \Delta\chi^2 \lesssim \beta \sqrt{C_2}, \quad (9)$$

which should be satisfied in any kind of model. Inserting Eq. (8) into Eq. (7), we obtain

$$|f_{\text{NL}}| \gtrsim 66\beta^{-1}. \quad (10)$$

The required value of f_{NL} is already difficult to reconcile with the Planck observation ($f_{\text{NL}} = 2.7 \pm 5.8$) [22]. We emphasize that this constraint differs from the one discussed in the literature [5,6,16].

If we require the above constraint on the non-Gaussianity to be satisfied, we should take $\beta \gtrsim 6$. This implies, from Eq. (8), that $a_{20}^{(\Delta\chi)}$ must be accidentally cancelled by the completely independent CMB quadrupole contribution to give the observed small value of C_2 . Secondly, the value of $\Delta\chi$ must be fine-tuned so that $a_{20}^{(\Delta\chi)}$ is almost exactly $O(10^{-5})$. Thus it is manifest that any models of this kind require extreme fine-tuning.

3. A generic difficulty in models with two fields

We found it difficult to construct a viable model to explain the dipolar statistical anisotropy in the context of simple curvaton models using the non-linear coupling $N_{\chi\chi}$. One might wonder if using the cross term $N_{\phi\chi}$ instead of $N_{\chi\chi}$ could improve the situation. It turns out that this possibility for generating the dipolar statistical anisotropy is also almost ruled out. This is because one can show that the amplitude of the dipole anisotropy $a_{10}^{(\Delta\chi)}$ inevitably becomes too large if the dipolar statistical anisotropy is generated by the cross term $N_{\phi\chi}$.

In order to produce the dipolar statistical anisotropy, we must have $|(N_{\phi\chi}/N_\phi)\Delta\chi| \gtrsim 0.07$. However, within one expansion time during the inflation, H^{-1} , the average value of the inflaton ϕ changes by $\sim \dot{\phi}H^{-1}$, where a dot denotes the cosmic time derivative. Therefore, as long as the cross term $N_{\phi\chi}$ exists, this shift of the background value of ϕ generates N_χ ,

$$|N_\chi| \gtrsim \left| N_{\phi\chi} \frac{\dot{\phi}}{H} \right| \approx \left| \frac{N_{\phi\chi}}{N_\phi} \right|.$$

Hence the dipole temperature perturbation caused by $\Delta\chi$ becomes

$$\left| a_{10}^{(\Delta\chi)} \right| \approx \frac{1}{5} |N_\chi \Delta\chi| \gtrsim \frac{0.07}{5} \approx 0.014.$$

This constraint is one order of magnitude larger than 1.2×10^{-3} , which is obtained from the analysis of the Doppler boosting by Planck [22].

We note that the above argument also applies to a single-field model, in which the role of the χ -field is also played by the inflaton ϕ . For this case, $N_{\phi\chi}$ and $\Delta\chi$ in the above argument are replaced with $N_{\phi\phi}$ and $\Delta\phi$, respectively. Consequently, single-field models are also in conflict with observation.

If we introduce the third field, it may be possible to construct models that generate the dipolar statistical anisotropy using the non-linear terms in the δN formula. However, as it seems difficult to avoid fine-tuning in such models, we do not pursue this direction further.

4. The amplitude modulation of the fluctuations

The above consideration teaches us that it is not easy to generate the dipolar statistical anisotropy by using cross terms in the expansion of the δN formula, and simultaneously satisfy the observational constraint that the local-type non-Gaussianity is small. Therefore we pursue a different approach. We consider the possibility of modulating the amplitude of the quantum fluctuations at horizon crossing without affecting the inflationary dynamics.

First we examine a model in which the Hubble expansion rate is modulated, but the number of e -folds N is not affected by this modulation. A model Lagrangian is given by

$$L = -\frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}(\nabla\sigma)^2 - V(\phi, \sigma); \quad V(\phi, \sigma) = W(\sigma)U(\phi). \quad (11)$$

Here the σ -field is supposed to have a large amplitude of the super-curvature perturbation, but not to affect the inflationary dynamics. Namely, it is assumed that the curvature perturbation is generated by the inflaton ϕ . This means $N = N(\phi)$. On the other hand, the Hubble expansion rate H is a function of not only ϕ but also σ , $H = H(\phi, \sigma)$. For $N = N(\phi)$, this implies $H = H(N, \sigma)$. In other words, if we draw contours of $N = \text{const.}$ on the (ϕ, σ) plane, H on the contour of a given value of N may depend on σ . Then the inflaton fluctuation at around $N = N_*$, $\langle\delta\phi^2\rangle \approx H^2(N_*, \sigma)/(2\pi)^2$, also depends on σ , where N_* is the value of the e -folding number at which the comoving scale of the current Hubble horizon size left the horizon during inflation.

At first sight, for a potential of the form given by Eq. (11), this model seems viable because the e -folding number N becomes σ -independent,

$$N = \int_{\phi_f}^{\phi} \frac{d\phi}{M_{\text{pl}}^2} \frac{V(\phi, \sigma)}{V_\phi(\phi, \sigma)} = \int_{\phi_f}^{\phi} \frac{d\phi}{M_{\text{pl}}^2} \frac{U(\phi)}{U_\phi(\phi)}, \quad (12)$$

under the slow-roll approximation for ϕ , where the subscript ϕ denotes the ϕ -derivative, $V_\phi = \partial_\phi V$, and ϕ_f is the value of ϕ at the end of inflation.

Unfortunately, however, the above formula for N has non-negligible higher-order corrections. In fact, there always exists a correction of the form

$$N_{\text{corr}} \sim \frac{1}{6} \left[\ln H^2 \right]_{\phi_f}^{\phi},$$

which depends on $W(\sigma)$. Therefore the curvature perturbation induced by the super-curvature perturbation $\Delta\sigma$ becomes as large as $|N_\sigma \Delta\sigma| \approx |(W_\sigma/W)\Delta\sigma/6| \approx 0.02$, where the last equality follows from the fact that the magnitude of the dipolar anisotropy is given by $|(W_\sigma/W)\Delta\sigma/2|$. Hence this model produces too large an amplitude of the dipole anisotropy a_{10} , which is already ruled out observationally [22].

5. A viable model

Finally we propose a viable model. As before, we assume the existence of a field σ that carries a super-curvature perturbation $\Delta\sigma$. In addition, we introduce a curvaton χ . The idea is to modulate the dynamics of the curvaton χ by using the σ -field without affecting the inflationary dynamics. This can be realized by assuming that the kinetic term of χ depends on σ . A model Lagrangian takes the form

$$L = -\frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}(\nabla\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}f^2(\sigma)(\nabla\chi)^2 - \frac{1}{2}m_\chi^2\chi^2. \quad (13)$$

Owing to the function $f(\sigma)$, the amplitude of the fluctuations of χ is modulated as $\langle\delta\chi^2\rangle \approx H^2/(2\pi f(\sigma))^2$. With an appropriate choice of $f(\sigma)$, this model can easily explain the dipolar statistical anisotropy.

Assuming the hierarchy $m_\sigma^2 \gtrsim H^2 \gg m_\phi^2 \gg m_\chi^2$ inside the bubble, one can make σ decay quickly during inflation so that it will not affect the dynamics during or after inflation except for the modulation of the fluctuation amplitude $\delta\chi$ near or beyond the current Hubble horizon scale. We may also assume that the final curvature perturbation is dominated by the curvaton χ and ignore the fluctuations of the inflaton ϕ . Then, we can easily suppress the local-type non-Gaussianity. In this model the origin of the modulation is assumed to die out during the inflation. Therefore, the amplitude of the dipolar statistical anisotropy becomes smaller on smaller length scales. This seems to be favored by observational data [25,26].

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