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Effective Theories of Gravity

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Submitted for the degree of Doctor of Philosophy

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Declaration

I hereby declare that this thesis has not been and will not be submitted in whole or in part to another University for the award of any other degree.

Results presented in this thesis were obtained in collaboration with Prof Xavier Calmet and Prof Stanislav Alexeyev and published in the following papers:

- S.O. Alexeyev, X. Calmet, B.N. Latosh, "Gravity induced non-local effects in the standard model", Physics Letters B **776**, 2018, pages 111

- X. Calmet, B. Latosh, "Three Waves for Quantum Gravity", European Physical Journal C **78**, 2018, page 205

- X. Calmet, B. Latosh, "Dark Matter in Quantum Gravity", European Physical Journal C **78**, 2018, page 520

- B. Latosh, "Fab Four Effective Field Theory Treatment", European Physical Journal C **78**, 2018, page 991

I had provided an essential contribution to the first three papers. Research forming the last paper was reformed solely be me.

Signature:

Boris Latosh

UNIVERSITY OF SUSSEX

Boris Latosh, Doctor of Philosophy

Effective Theories of Gravity

Summary

This thesis is devoted to the study of effective field theory methods for gravitational interactions. Effective field theories were developed in the context of particle physics. They provide a consistent framework to study low energy effects of some high energy fundamental theory. Applying these methods to quantum general relativity enables one to do calculations for processes taking place at energies below the Planck mass without a detailed knowledge of the ultra-violet complete theory of quantum gravity.

Four related topics are considered in this thesis. We first study effects of quantum gravity in particle physics interactions. In particular, we focus on non-local operators involving fields of the standard model generated by quantum gravity. Bounds on the magnitude of the Wilson coefficients of non-local four fermion interactions are established.

Secondly, we calculate the production rate of gravitational waves by binary systems using effective field theory methods. New massive gravitational modes, beyond the massless graviton, appear in the low energy spectrum of the quantum gravitational effective field theory. These modes could be generated in binary inspirals.

The third direction consists in a study of dark matter candidates within this effective gravity. The non-local operators generated by quantum gravitational interactions lead to new poles in the graviton propagator. These poles describe states with a non-vanishing decay widths. These states may contribute to the contemporary dark matter content provided that their lifetime is comparable with the current age of our universe. Correspondent constraints on the dark matter candidates are established.

The last question addressed in this thesis consists in an implementation of effective field theory techniques to modified gravity models. One of the simplest stable extensions of general relativity is studied. The new interaction lying beyond general relativity significantly changes the correspondent effective theory. Implications of our results for gravitational interactions are discussed.

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Acronyms

Chapter 1

Introduction

Theory of gravity is one of the most sophisticated parts of physics. It has always played an important role in development of theoretical physics. The search for the quantum theory of gravity created numerous tools of contemporary theoretical physics such as models with auxiliary dimensions [1, 2], AdS/CFT correspondence [3, 4, 5], canonical quantum gravity [6, 7, 8], supersymmetry [9, 10, 11, 12, 13], and string theory [14, 15, 16].

At the classical level gravity is consistently described by General Relativity (GR). It provides a relevant description of multiple phenomena such as geodesic motion [17, 18], Gravitational Wave (GW) production [19, 20, 21], stellar evolution [22, 23, 24, 25], and the cosmological expansion [26, 27, 28]. However, GR is not free from disadvantages.

There is a series of phenomena that cannot be easily described within GR. At the spacial scale of galaxies a phenomenon of dark matter exists. It manifests itself through galaxy rotation curves [29, 30, 31, 32], Cosmic Microwave Background (CMB) spectrum [33, 34], and galaxy mass distributions [35]. Numerical studies also point to a necessity of introduction of dark matter for a consistent description of galaxy evolution [36, 37]. At the scale of the Universe as a whole a phenomenon of accelerated expansion, or dark energy, takes place. It has been detected independently via CMB spectrum [33, 34] and supernovae observations [38, 39]. Within GR the late-time accelerated expansion can be described with a non-vanishing cosmological constant. This solution of the late-time accelerated expansion problem is widely considered to be insufficient. Firstly, at the classical level the cosmological constant value is a free parameter that cannot be calculated [40, 41]. Secondly, a certain of issues are raised in Quantum Field Theories (QFTs) with a non-vanishing cosmological constant [42, 43, 44].

Description of the early Universe within GR also faces a few challenges. It is commonly

accepted that the Universe has passed through a stage of inflationary expansion [45]. Its consistent treatment requires either introduction of a new scalar Degree of Freedom (DoF) [46, 47] or a modification of the GR Lagrangian with higher curvature terms [48].

These facts give grounds to assume that the GR description of gravitational phenomena is relevant only at the scale of compact objects such as stars and stellar systems. Its description of large scale objects like galaxies clusters and the large-scale structure of the Universe or higher energy processes like expansion of the early Universe may not be completely relevant. Therefore, it is safe to conclude that at the classical level one should study alternative gravity models.

The branch of gravity theory devoted to alternatives or modifications of classical GR is commonly known as modified gravity [49, 50]. The manifold of alternative gravity models is constrained by the Lovelock theorem [40, 41] which is discussed in more detail in Chapter 3. The theorem defines the conditions which allow one to fix the form of the Einstein equations uniquely. To construct an alternative gravity model is to violate one of the conditions. Therefore, all modified gravity models can be classified via the Lovelock theorem. Models that introduce one additional scalar DoF to the gravity sector, also known as Scalar-Tensor Gravity (STG) models, provide one of the simplest modifications. As it is discussed in Chapter 3, it is possible to define the simplest stable alternative to GR within STG. In Chapter 7 a motivation to study this model is presented.

At the level of quantum theory GR and the other gravity models experience multiple problems that obstruct the creation of a consistent quantum gravity model. Creation of quantum gravity is a necessary step in gravity theory because of the following. The contemporary theory of matter, namely, the standard model of particle physics, is completely given in terms of quantum theory of fields. Therefore, a quantum gravity model is required to obtain a universal description of both standard model and gravitational phenomena.

Quantum GR experiences problems with renormalizability [51, 52]. Namely, at the one-loop level it can only be renormalized on-shell in an absence of matter. At the two-loop level it cannot be renormalized via standard methods. The simplest renormalizable quantum gravity model contains a ghost DoF and cannot be considered satisfactory [53, 54]. Ghost states carry negative energy thereby making the energy spectrum unbounded from below. This feature leads to an instability, as excitations can gain arbitrary large energy dumping negative energy to the ghost sector. Issues of the simplest approach to quantum gravity are discussed in Chapter 4.

Nonetheless, the necessity of quantum treatment of gravity should be addressed. Ef-

fective Field Theory (EFT), which is well-known in particle physics [55, 56], appears to be the simplest technique to account for the gravitational interaction at the quantum level. Moreover, EFT describes verifiable gravitational effects, for instance, a modification of the two-body interaction potential [57]. A detailed discussion of the EFT implementation for gravity is presented in Chapter 5.

Description of gravity in terms of quantum theory, as it was highlighted, is relevant, because an opportunity to search for quantum gravitational effects should be addressed. EFT allows one to account for the gravitational interaction within the QFT framework and to search for new gravitational effects. These effects can manifest themselves at all spacial and energy scales, provided the EFT framework can be consistently applied. At the level of particle physics gravity can affect interactions of the standard model particles such as leptons. Therefore, traces of gravity may be found in particle scattering processes and can be subjected to an empirical verification. At the level of stellar systems effective gravity can affect the GWs production rate. Recent development of terrestrial apparatus allowed one to subject GWs production to a direct verification [55, 56]. At the level of galaxies and galaxies clusters effective gravity may account for the dark matter. Finally, it is important to understand the role of various GR modifications within EFT. Particular modifications can affect the corresponding effective theory in a non-trivial way thereby ruining or improving its applicability.

The landscape of gravity theory highlights multiple perspective directions of study. This thesis addresses a narrow set of the problems related with an EFT implementation for GR, modified gravity models, and particle physics. The more detailed motivation behind the addressed problems is given in the following chapters. In Chapter 2 GR is discussed, its advantages and disadvantages are highlighted. In Chapter 3 modified gravity models are discussed and Horndeski models are highlighted. The context gives grounds to separate one particular Horndeski model suitable for an EFT treatment. Chapter 4 is devoted to a discussion of the most conservative approach to quantum gravity. It is plagued with pathologies that prevent its direct implementation for realistic quantum gravity models. In Chapter 5 EFT formalism is discussed. Finally, Chapter 6 is devoted to a discussion of a relation between gravity and the standard model of particle physics. The most perspective area for search of gravity-induced effects is highlighted.

Chapter 7 is completely devoted to the statement of the problems addressed in this thesis. I highlight four particular problems that rise in the presented context and justify their relevance for the contemporary gravity theory. The first problem covers an opportu-

nity for effective gravity to induce non-local interactions between matter states. Chapter 8 presents results of paper [58] devoted to a resolution of this problem. The second problem is related with the binary system GWs production studied within EFT. Its solution presented in paper [59] and discussed in Chapter 9. The third problem addresses an opportunity to describe the dark matter naturally within effective gravity. Corresponding results [60] are presented in Chapter 10. The last problem is related with one particular Horndeski model that admits a cosmological constant screening mechanism. Corresponding effective theory is studied and it shows a few significant differences from effective GR. Corresponding results are covered in paper [61] and presented in Chapter 11.

It should be highlighted that the choice of problems presented in this thesis is affected by considerations of consistency. Alongside the aforementioned results a few papers were published during my PhD, but they are devoted to problems of classical modified gravity [62, 63]. Some results that may be relevant in the context of this thesis were published before I entered the PhD position [64, 65, 66, 67, 68, 69]. It was decided to only present results covered in papers [58, 59, 60, 61], as they were obtained within the EFT framework and can be consistently discussed in a narrow area of gravity theory.

Chapter 2

General Relativity

GR is widely accepted as the most successful theory of gravity. It is given by the Einstein-Hilbert action:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G} \left(R - 2\Lambda \right) + \mathcal{L}_{\text{matter}} \right]. \tag{2.1}$$

Here G is the Newton constant, R is the scalar curvature, Λ is the cosmological constant, and $\mathcal{L}_{\text{matter}}$ is the matter Lagrangian. Notations used in this thesis are presented in Appendix A. Pure GR describes massless excitations with helicity ± 2 [70, 71].

The theory has a wide phenomenology. It describes effects of geodesic motion such as gravitational redshift, time dilation, and lensing [17, 18, 72]. Combined with the stellar theory it predicts existence of specific compact objects like neutron stars and black holes [22, 24, 23, 25, 73]. It also successfully describes the GWs production by binary systems, that was recently directly observed [19, 20, 21, 74, 75]. Finally, GR can be implemented for a description of the evolution of the Universe as a whole [26, 27, 28, 76].

At the same time there is a vast array of phenomena that cannot be easily described within GR. These phenomena are dark matter and the late-time accelerated expansion of the Universe, which is commonly attributed to the dark energy. It is also widely accepted that the Universe has passed through an inflationary phase of expansion which cannot be consistently described within GR.

Multiple data sources point to the existence of dark matter. First of all, galaxy observations have shown that the galaxy rotation curves are not consistent with GR predictions [29, 30, 31, 32]. Secondly, numerical simulations of the evolution of the large scale structure of the Universe have shown that cold dark matter is required for the galaxy formation to

be consistent with the empirical data [36, 37, 77, 78, 79]. Thirdly, the CMB spectrum provides independent data on the dark matter content of the Universe [33, 34]. Finally, galaxy mass distributions recovered via gravitational lensing strongly favour the collisionless dust dark matter model [35].

It is generally believed that the dark matter is a manifestation of new weakly interacting particles that lie beyond the standard model. Multiple particle physics models have various candidates for the dark matter particles [80, 81, 82]. Despite the fact that multiple observations support the existence of dark matter, its constituent particles were never detected directly [83, 84, 85, 86, 87]. Because of this there is still a room for a description of the dark matter in terms of the gravitational interaction. For instance, there are a few modified gravity models providing such a description of the dark matter [88, 89, 90].

Existence of the late-time accelerated expansion of the Universe is also well-established via multiple sources. The first empirical evidence was obtained via the supernovae observations [38, 39]. The effect was later confirmed independently via CMB spectrum [33, 34]. It is widely accepted that a non-vanishing cosmological constant provides the best fit for the empirical data. At the same time, existence of a non-vanishing cosmological constant presents a separate theoretical issue. At the classical level its value cannot be calculated, it can only be recovered via empirical data. At the level of quantum theory the cosmological constant faces a few issues that are currently unresolved [42, 43, 44].

Finally, there is strong theoretical evidence indicating that a consistent treatment of the early stages of the Universe expansion requires the introduction of an inflationary phase [46, 91]. Existing empirical data allows one to constrain inflation parameters [33, 92, 47], but it lacks the precision to uniquely define the correct inflationary model [47, 93, 94, 95].

Attempts to construct a suitable quantum gravity model also experience certain difficulties. The perturbative approach to quantum gravity [96, 97, 98] faces the problem of renormalizability. Pure quantum GR at the one-loop level can be renormalized only on-shell [51] and becomes completely non-renormalizable, if the matter content is added. At the two-loop level even pure GR becomes non-renormalizable [52]. The approach based on the canonical quantisation faces an even larger manifold of difficulties [6, 99, 7]. Perhaps, the most well-recognised problems are the problem of time and the problem of Wheeler-DeWitt equation [100, 101, 102, 103]. The problem of time is related with the complexity of the notion of time in quantum gravity. The problem of the Wheeler-DeWitt equation, which is an analogue of the Schrödinger equation, is due to the fact that it does not admit a positive norm on the space of solutions.

These features of GR give grounds to search for alternative gravity models at the classical level, to account for the dark components of the Universe, to study behaviour of gravity in the high energy regime in order to account for inflation, and to study models of quantum gravity to obtain a new fundamental description of gravity.

Chapter 3

Modified Gravity, Horndeski Models, and Fab Four

To define a gravity model is to uniquely fix its physical features. GR can be viewed as the simplest gravity model, because it is fixed by the minimal set of physical requirements. In 1971 David Lovelock found a theorem that fixes the Einstein tensor uniquely, if certain physical conditions are met [40, 41]. In the contemporary formulation the Lovelock theorem states that GR is fixed uniquely by the following conditions [104]:

1. spacetime has four dimensions

2. gravity is described by the spacetime metric

3. the action admits diffeomorphism invariance

4. the action results in the second-order field equations

To obtain an alternative gravity model is to violate a few conditions of the Lovelock theorem. Therefore, the whole manifold of modified gravity models can be classified in accordance with the Lovelock theorem [49]. For instance, gravity models in a spacetime of higher dimensions that admit second order field equations violate the first condition. Models of this type are known as Lovelock gravity [40, 105]. Gravity models that replace the Einstein-Hilbert Lagrangian with a smooth function, also known as $f(R)$ gravity, result in higher order field equations and violate the last condition of the theorem [106, 107]. Models that add an additional scalar DoF to the gravity sector are known as STG and they violate the second theorem condition [49, 50].

It should be noted, that modified gravity models can be classified on a different basis. For instance, a classification can be based on the spacial scale at which a model develops deviations from GR. In such a way, one can operate with gravity models modified either in infrared or in ultraviolet sector. The classification based on Lovelock theorem is used in this thesis, as it highlights the simplest way to introduce a modification regardless of the area in which modifications are made. In other words, Lovelock theorem points to a particular technique of modification, but not on the essence of modification. Because of this the theorem is used in the thesis, as is allows one to define the simplest mean of modification.

Phenomenology of modified gravity models is vast and it is covered in multiple reviews [49, 50, 108, 109]. They provide various descriptions of the dark matter [110, 111] and the dark energy [112, 113, 114, 109, 115]. A modification of GR is also required for a consistent description of inflation, as the most well-known inflation models belong either to STG [46] or to $f(R)$ gravity [48].

STG models should be highlighted within modified gravities, because they provide a minimal extension of GR alongside a vast phenomenology. They should be considered as, perhaps, the simplest GR modification, as they only introduce a single new DoF to the model. At the same time, this additional DoF results in a series of new phenomena. Namely, STGs can screen the existence of the new scalar field [116, 117, 118, 119], violate energy conditions [120, 121], and drive inflation [46, 122]. It is also worth noting that $f(R)$ gravity models can be mapped onto a specific subclass of STGs [123, 124, 125, 126, 106, 107].

STG models that admit second order field equations are of a special interest. Models with higher derivatives are plagued with the Ostrogradsky instability [127, 128, 129, 130]. STGs with second order field equations are free from this pathology. The class of STGs admitting second order field equations were found by Gregory Horndeski in 1974 [131]. The original result was rediscovered independently within the Generalized Galileons framework [122]. Because of this the class of STG models with second order field equations is known either as Horndeski models or, rarely, as Generalized Galileons [132, 133].

Horndeski models in the Generalized Galileons parametrization are given by the fol-

lowing Lagrangians [122]:

$$\mathcal{L}_2 = G_2 \ ,$$

$$\mathcal{L}_3 = G_3 \ \Box X \ ,$$

$$\mathcal{L}_4 = G_4 \ R + G_{4,X} \ [(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \ , \tag{3.1}$$

$$\mathcal{L}_5 = G_5 \ G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6} \ G_{5,X} \ [(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3] \ .$$

Here G_2, G_3, G_4, and G_5 are functions of the scalar field ϕ and the canonical kinetic term $X = 1/2 \ \partial_\mu\phi \ \partial^\mu\phi$; $G_{4,X}$ and $G_{5,X}$ are correspondent derivatives with respect to X; $G_{\mu\nu}$ is the Einstein tensor. It should be highlighted that only $\mathcal{L}_4$ and $\mathcal{L}_5$ terms describe a non-standard interaction between gravity and the scalar field, while terms $\mathcal{L}_2$ and $\mathcal{L}_3$ describe the scalar field self-interaction.

Horndeski models contain a special subclass known as the Fab Four [134] which is defined by its ability to screen the cosmological constant. Fab Four models completely screen an arbitrary cosmological constant on the Friedmann-Robertson-Walker (FRW) background. The screening holds even if the cosmological constant experiences a finite shift. The Fab Four class is given by the following Lagrangians:

$$\mathcal{L}_{\text{John}} = V_J(\phi) \ G_{\mu\nu} \ \nabla^\mu\phi\nabla^\nu\phi \ ,$$

$$\mathcal{L}_{\text{Paul}} = V_P(\phi) \ P^{\mu\nu\alpha\beta} \ \nabla_\mu\phi \ \nabla_\alpha\phi \ \nabla_\nu\nabla_\beta \ \phi \ ,$$

$$\mathcal{L}_{\text{George}} = V_G(\phi) \ R \ , \tag{3.2}$$

$$\mathcal{L}_{\text{Ringo}} = V_R(\phi) \ \hat{G} \ .$$

Here $\hat{G}$ is the Gauss-Bonnet term, $P^{\mu\nu\alpha\beta} = -1/2 \ \varepsilon^{\alpha\beta\lambda\tau} \ R_{\lambda\tau\sigma\rho} \ \varepsilon^{\sigma\rho\mu\nu}$ is the double-dual Riemann tensor, and V_J, V_G, V_R, V_P are interaction potentials. These Lagrangians have different screening properties. The Ringo term alone does not screen the cosmological constant, it just does not ruin the screening. The George term introduces a Brans-Dicke-like interaction which does not provide a sufficient screening on its own [134]. The Paul term demonstrates a pathological behaviour in star-like objects [135, 136]. Because of this only the John term is widely considered in the context of a cosmological constant screening.

Fab Four models on their own can hardly be considered relevant for a description of the cosmological expansion. The reason behind this is their screening features. In the contemporary Universe the observed value of the cosmological constant is non-vanishing which goes in constrast with the Fab Four features. In order to generate a non-vanishing cosmological constant Fab Four screening should be broken in the late-time Universe.

A combination of the John term and beyond Fab Four terms allows one to construct a model which can provide an adequate description of all stages of the cosmological expansion from the inflation to the late-time accelerated expansion. In the early Universe the John term provides the leading contribution, the cosmological constant is screened, and the scalar field drives the inflation. In the late-time Universe the leading contribution is given by beyond Fab Four terms, the model loses its screening features, develops a small cosmological constant, and enters the late-time acceleration expansion. One particular example of such a model is presented in paper [137] by the following action:

$$S = \int d^4 x \sqrt{-g} \left[-\frac{1}{16\pi G} \left(R - 2\Lambda \right) + \frac{1}{2} \, g^{\mu\nu} \, \nabla_\mu \phi \nabla_\nu \phi + \beta \, G^{\mu\nu} \, \nabla_\mu \phi \nabla_\nu \phi \right] . \qquad (3.3)$$

Here β is the John coupling with the dimension of an inverse mass squared. Within the modified gravity approach β should be treated as a free model parameter. In full agreement with the presented reasoning the model describes both the inflation and the late-time accelerated expansion.

This section should be summarised as follows. Firstly, classical models of gravity are worth to be studied, as they can provide some solutions for dark matter and dark energy problems in the classical regime. Dark matter and dark energy manifest themselves at scales of galaxies, galaxies clusters, and the Universe as the whole. If they are due to the gravitational interaction, then they should admit a suitable classical description given in terms of modified gravity models. Secondly, among all modified gravity models the Horndeski models should be highlighted. The reason behind this is twofold. First reason is the fact that these models are minimal, as they only introduce one additional DoF to the gravity sector. Second reason is the fact that the Horndeski models admit second order field equations, so they avoid the Ostrogradsky instability. Thus, the Horndeski models provide a minimal stable modification of GR. Finally, the Fab Four class of Horndeski models is of a special interest. The reason behind this is its ability to screen the cosmological constant on the FRW background. It is also important to highlight, that modified gravity models are not reduced to STGs alone. The aforementioned reviews [49, 50, 108, 109] provide a more detailed discussion of the contemporary state of the field. The reason behind such a choice of the research area is the belief that the Horndeski models are the simplest GR modification that may provide a suitable alternative classical gravity model.

In other words, model (3.3) provides a minimal stable modification of GR that admits a mechanism generating a small cosmological constant in the late-time Universe. In the presented context it is crucial to understand properties of the model (3.3). In order to be

viewed as a suitable alternative for GR, the model should be consistent with the empirical data at all spacial and energy scales. The original papers [134, 137] have justified its applicability in the cosmological regime. Paper [66] has shown that the simplest quantum corrections do not ruin its screening features. Because of this it is possible to consistently study it within EFT. This thesis study the behaviour of the corresponding effective model and its relevance for theory of gravity. A detailed discussion of the problem addressed in this thesis is given in Section 7.

Chapter 4

Quantum General Relativity

Attempts to construct a quantum theory of gravity date back to the 1960s [96, 97, 98, 138]. A detailed discussion of quantum gravity history can be found in [139, 140]. In this thesis only the perturbative functional integral quantisation is discussed for the sake of consistency. Other approaches to quantum gravity, for instance, the canonical quantisation program, lie beyond the scope of the thesis. Moreover, their role is discussed in detail in [6, 101, 7].

The functional integral quantisation of gravity is performed via the external field quantisation method [141]. It is assumed that the gravitational field is described by the classical background $\bar{g}_{\mu\nu}$ and the small perturbations $h_{\mu\nu}$ propagating over this background. Spacetime geometry is described by the metric $g_{\mu\nu}$ which consists of both background and perturbative contributions:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu} \ . \tag{4.1}$$

Here κ is a constant with the dimension of an inverse mass. It is introduced to give the field $h_{\mu\nu}$ the canonical mass-dimension. It is defined through the Newton constant G as follows:

$$\kappa^2 = 32\pi G \ . \tag{4.2}$$

A generic gravity model is given by the microscopic action $\mathcal{A}[g]$. The corresponding quantum model is given by the following generating functional (given up to the infinite normalizing factor for the sake of simplicity):

$$Z = \int \mathcal{D}[h_{\mu\nu}] \ \exp\left[i\, \mathcal{A}[\bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}]\right] \ . \tag{4.3}$$

Following features of this approach should be highlighted. Firstly, the approach can be considered background-independent in the following sense. Generating functional (4.3) depends of the background metric $\bar{g}_{\mu\nu}$, so do all amplitudes generated with it. But no conditions on the background are established, therefore it can be arbitrarily chosen. In that sense, the approach is not affected by a choice of the background. Secondly, as it was shown in [142] the background field quantisation technique provides a consistent way of quantisation even with a non-vanishing cosmological constant, although it does not resolve the cosmological constant problem [42, 43].

For the sake of simplicity only a flat background is considered in this thesis. Despite the fact that such a quantisation method can be applied to an arbitrary background, this would only introduce an additional complexity to the calculations. Moreover, gravity models discussed in this thesis admit the flat spacetime as a solution of the classical field equations.

Functional quantisation given by (4.3) admits the standard way of derivation of the Feynman rules [143]. It is required to introduce a formal external current $J^{\mu\nu}$ coupled to perturbations $h_{\mu\nu}$ and expand the generating functional (4.3) in a series with respect to the perturbations:

$$Z[J] = \int \mathcal{D}[h] \, \exp \left[i\mathcal{A}[\bar{g}] + i\frac{\delta\mathcal{A}[\bar{g}]}{\delta\bar{g}_{\mu\nu}} \, h_{\mu\nu} + i\frac{\delta^2\mathcal{A}[\bar{g}]}{\delta g_{\mu\nu}\delta g_{\alpha\beta}} \, h_{\mu\nu}h_{\alpha\beta} + \cdots + \, i \, h_{\mu\nu}J^{\mu\nu} \right] \, . \quad (4.4)$$

The first term in the exponent does not depend on the perturbations $h_{\mu\nu}$, so it can be included in the infinite normalization factor. The second term describes an interaction between the background and perturbations. Its structure shows that if the background $\bar{g}_{\mu\nu}$ does not deliver a minimum to the microscopic action $\mathcal{A}$, then the background itself serves as an external source generating perturbations. This feature should be understood as follows. If the background delivers a minimum to the microscopic action, then it can be considered as a suitable ground state. In that case perturbations $h_{\mu\nu}$ are free to propagate over such a background. If $\bar{g}_{\mu\nu}$ does not deliver a minimum to the microscopic action, then it should be considered as an excited state. In that case this excited state pursuits to reduce its energy, so it excites perturbations $h_{\mu\nu}$, i.e. it serves as an external source. Finally, the other terms describe multi-particle correlation functions at the tree level.

Following the standard algorithm any N-particle correlation function can be evaluated

up to an arbitrary order in perturbation theory with the following expression:

$$\langle 0|\, h_{\mu_1\nu_1}(x_1)\cdots h_{\mu_N\nu_N}(x_N)|0\rangle = \frac{-i\,\delta}{\delta J^{\mu_1\nu_1}(x_1)}\cdots\frac{-i\,\delta}{\delta J^{\mu_N\nu_N}(x_N)}\frac{1}{Z[0]}\,Z[J]\Bigg|_{J=0}. \qquad (4.5)$$

This algorithm of quantisation is discussed in more details in Appendix D.

Such an approach to quantum gravity can hardly be considered complete, as GR cannot be renormalized via the standard technique. In papers by 't Hooft, Veltman [51] and Goroff [52] it was shown that first and second-loop order corrections to the graviton self-energy cannot be normalized. In the absence of matter one-loop level corrections to GR require a counter-Lagrangian of the following form [51]:

$$\Delta\mathcal{L}_{\text{grav}} = \sqrt{-g}\left(\frac{1}{120}R^2 + \frac{7}{20}R_{\alpha\beta}R^{\alpha\beta}\right)\frac{1}{\varepsilon}. \qquad (4.6)$$

This counter-Lagrangian vanishes on-shell, as the Einstein equations are reduced to $R_{\mu\nu} = 0$. If the matter content is taken into account, the counter-Lagrangian does not vanish even on-shell, so the model cannot be renormalized with the standard approach. At the level of two-loop contribution even on-shell renormalization fails for pure GR, as the model requires a counter-Lagrangian proportional to the third power of the Riemann tensor which does not vanish on-shell [52].

Multiple attempts were made to find a more suitable way to implement the functional quantisation. Namely, an attempt was made to use the quadratic gravity model given by the following action:

$$\mathcal{A} = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}R + c_1 R^2 + c_2 R_{\mu\nu}^2\right]. \qquad (4.7)$$

Here c_1 and c_2 are free dimensionless constants. This model was first considered in [54, 53] where it was shown that it can be renormalized. At the same time, the model contains a ghost DoF, so it can hardly be considered relevant for realistic applications [130]. This model is used throughout the thesis, so a more detailed discussion of its quantization is presented in Appendix F.

These considerations give grounds for further development of quantum gravity models. The simplest approach to quantum gravity fails, so it is necessary to develop more sophisticated techniques to study gravity in the quantum regime. At the same time, there exists a framework that allows one to account for the gravitational effects in the low energy regime no matter the true theory of quantum gravity.

Chapter 5

Effective Field Theory for Quantum Gravity

EFT was widely developed and used in particle physics [55, 144]. It provides a framework that allows one to obtain an effective action describing dynamics of a physical system in the low energy regime [55, 145, 146].

The simplest approach to EFT is based on a decoupling of heavy DoFs and it is known as the Wilsonian approach [147, 148, 149]. To obtain an effective action Γ that is valid up to the energy scale Λ from the microscopic action $\mathcal{A}$ the following should be done. All DoFs should be separated on light l and heavy h once with respect to the energy scale Λ. Heavy DoFs should be excluded from initial and final states, as they cannot be excited in the low energy processes. Because of this heavy DoFs influence all processes only at the loop level. To put it otherwise, heavy DoFs should be integrated out, so the microscopic action $\mathcal{A}$ and the effective action Γ are related as follows:

$$\int \mathcal{D}[l] \exp\left[\, i \, \Gamma[l] \,\right] = \int \mathcal{D}[l] \, \mathcal{D}[h] \, \exp\left[\, i \, \mathcal{A}[l, h] \,\right]. \tag{5.1}$$

Within the Wilsonian approach the effective action Γ can be calculated explicitly, if the microscopic action $\mathcal{A}$ is known. If the microscopic action is unknown, then the effective action can be restored based on symmetry principles [146]. This approach provides a correct description of multiple phenomena [150, 151]. Moreover, it can be used to evaluate the effective action in quantum gravity models, for example, in string theory [152, 153, 154].

Another approach to EFT treats the effective action as an expansion of multi-particle correlation functions in a series with respect to the number of loops [56] and it is known as the Coleman approach. To calculate the effective action up to the N-loop order is to

evaluate all multi-particle correlation functions up to N-loop level. The simplest example of the Coleman approach is given in the original paper [56], where a model of a scalar field with a four particle interaction was considered. At the tree-level this model provides the following set of diagrams:
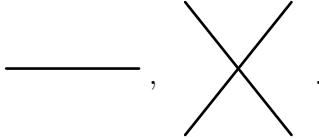
$$\text{———}, \quad \times \quad . \tag{5.2}$$

At the one-loop level the effective action receives an additional contribution given by the following series of diagrams:
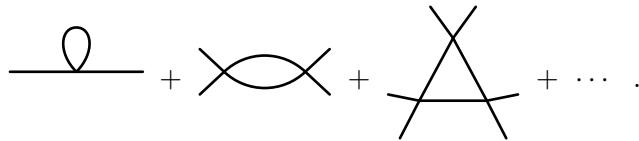
$$\text{———} + \times\!\!\prec + \triangle + \cdots . \tag{5.3}$$

In contrast to the Wilson approach the Coleman approach does not exclude any DoFs, but accounts for effects induced by quantum corrections. For instance, in the original paper [56] the effective action is used to show a spontaneously symmetry breaking induced by quantum corrections. A similar technique is used to study anomalies [155, 156, 157].

Logic of the Coleman approach can be applied to gravity models [145, 146]. In full analogy with the external field quantitation method one defines a gravity model by its microscopic action $\mathcal{A}[g_{\mu\nu}]$ given in terms of the full metric $g_{\mu\nu}$. The full metric $g_{\mu\nu}$ describes behaviour of the true quantum gravitations propagating over some background spacetime $\bar{g}$. No constraints on the background are set, so the approach can be considered background-independent.

Despite the fact that the microscopic action for gravity is unknown, one can restore the effective action via symmetry principles [145, 146, 158]. The simplest way is to study higher-order operators induced by loop corrections [145, 146]. The following procedure should be done to construct the effective model.

Firstly, in full agreement with the Wilson approach, one should integrate out high-energy gravity modes that have energies about the Planck scale. Secondly, one should define the normalization scale μ which should be placed below the Planck scale. At the normalization scale the microscopic action of the theory is defined. Within this thesis it is assumed that the microscopic action matches GR. Finally, one should preform the standard loop calculations to extend the effective action down to the low-energy regime. A diagram illustrating that algorithm is given on Fig. 5.

The one-loop effective action can be restored and leads to the following classical result
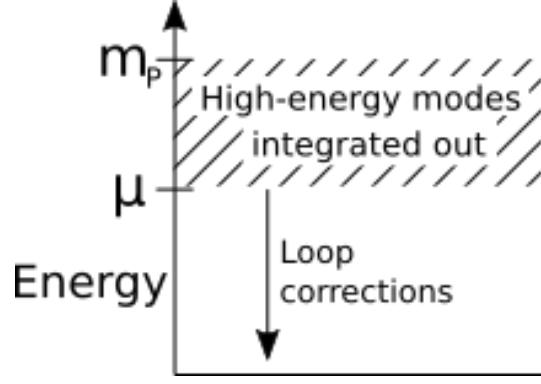
Figure 5.1: The diagram illustrating effective gravity action construction. Here m_P is the Planck scale and μ is the normalization scale.

[51]:

$$\Gamma_{\text{1-loop}} = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \right] . \tag{5.4}$$

In the same way the introduction of non-local operators is justified. Namely, in papers [159, 160] it was shown that the effective gravity action should be extended with non-local operators to the following form:

$$\Gamma_{\text{1-loop,non-local}} = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \right. \tag{5.5}$$

$$\left. + b_1 R \ln\left(\frac{\Box}{\mu^2}\right) R + b_2 R_{\mu\nu} \ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu} + b_3 R_{\mu\nu\alpha\beta} \ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu\alpha\beta} \right] .$$

This EFT implementation for gravity provides some verifiable predictions. Perhaps, the best example is given by the effective two-body interaction potential [57]:

$$V(r) = -\frac{G\, m_1\, m_2}{r} \left[1 + 3\, \frac{1}{c^2} \frac{G\,(m_1 + m_2)}{r} + \frac{41}{10\pi} \frac{1}{c^3} \frac{G\hbar}{r^2} \right] . \tag{5.6}$$

This potential was used, for instance, to estimate the influence of quantum corrections on Lagrangian points of the Earth-Moon system [161, 162] and to study gravitational light scattering [163, 164].

Existence of non-local operators in the effective action can account for a few phenomena. First of all, non-local operators result in the appearance of new poles in the graviton propagator. As discussed in Chapter 10, these new poles can be associated with excitation states and treated as the dark matter particles. At the same time, non-local operators

influence the gravitational interaction between states of the regular matter. Namely, they affect processes involving a virtual graviton exchange. As discussed in Chapter 8, their influence results in the appearance of an effective non-local interaction between standard model particles.

There are a few feature of the effective EFT for gravity that should be discussed. First of all, the unitarity of the effective theory. There are a few papers devoted to the effective field theory unitarity [165, 166, 167, 168]. In these papers it was shown the unitarity can be violated at the tree level at a certain finite energy scale which is defined by the model field content. Originally this phenomenon was associated with the effective field theory breakdown that may serve as an indication of the "new physics" required to restore the unitarity. However, in papers [169, 170] it was shown that such an interpretation can hardly be considered consistent, as the unitarity violation scale can be uncorrelated with the "new physics" scale. Moreover, a few indications were found pointing to order by order restoration on unitarity within perturbation theory. This mechanism of unitarity restoration is known as the self-healing mechanism. These results provide a consistent EFT unitarity picture, so the unitarity was not addressed in details in this thesis.

The second issue to be discussed is the poles structure of propagators evaluated within EFT. It was studies in papers [171, 172, 54] where it was shown that the effective graviton propagator receives additional poles. For models discussed in this thesis the Lambert W function plays a key role in the corresponding analysis. The poles structure is not the main focus of the thesis, nonetheless, the issue should be briefly discussed for the sake of completeness.

The Lambert W function is defined as a solution of the following equation:

$$W(z) \exp\left[W(z)\right] = z \ .$$

(5.7)

The function has two branches that are separated by a branch cut discontinuity in the complex plain. The branch cut runs by the real axis from $-\infty$ to $-1/e$. The main branch is defined by the following condition [171, 172]:

$$W(0) = 0 \ .$$

(5.8)

For real z which are $z > -1/e$ the main branch of the $W(z)$ function is real. Finally, the following approximation used in paper [60] discussed in this thesis:

$$W(x) \overset{x \sim 0}{=} \log(x) + O(x) \ .$$

(5.9)

In paper [58] the W function appears in an expression for the solution of the following equation:

$$q^2 \left(1 - \frac{NG}{120\pi} q^2 \log\left(-\frac{q^2}{\mu^2}\right) \right) = 0 \ . \tag{5.10}$$

This equation defines the position of poles of a graviton propagator within a specific EFT model discussed in Chapter 8. Here G is the gravitational constant, μ is the normalization scale, and N is a positive number related with the number of DoFs present in the model. This equation has two solutions. First one reads $q^2 = 0$ and it corresponds to the standard massless spin-2 gravitation. The second solution is given by the following expression:

$$q^2 = \frac{1}{GN} \frac{120\pi}{W\left(-\dfrac{120\pi}{\mu^2\, NG}\right)} \ . \tag{5.11}$$

Both these solutions correspond to point-like poles in the graviton propagator mentioned in Chapter 8. As it was mentioned before, the function is real if its argument is bigger then $-1/e$. Equivalently, the new poles are real if the following condition is satisfied:

$$\frac{120\pi}{\mu^2 NG} < \frac{1}{e} \ . \tag{5.12}$$

In papers [169, 170] discussed in this thesis the large N limit is essential for the self-healing mechanism. Consequently, it is safe to assume that the condition is always satisfied and the new poles appear on the real plain [171, 172].

In a similar way the poles structure is studied in paper [60] discussed in Chapter 10. In full analogy with the previous case it is shown that the graviton propagator generated by an effective gravity action containing non-local operators admits new poles. Physical implications of the existence of these poles is discussed in Chapter 10 in more details. We briefly discuss how positions of these poles were evaluated. For the sake of simplicity we only discuss the case of spin-0 mode which mass is defined by the following equation:

$$m_0^2 = \frac{-1}{(3b_1 + b_2 + b_3)\kappa^2\, W\left(\dfrac{\exp\dfrac{-3c_1 - c_2}{3b_1 + b_2 + b_3}}{(3b_1 + b_2 + b_3)\kappa^2\mu^2}\right)} \ . \tag{5.13}$$

Here c_i are couplings of local operators, b_i are couplings of non-local operators and we consider the limit $c_i > b_i$ (the reasons for such a limit is discussed in Chapter 10).

The assumption $b_i < c_i$ allows one to expand the expression in terms on the following small parameter:

$$\frac{3b_1 + b_2 + b_3}{3c_1 + c_2} \, . \tag{5.14}$$

To account for the leading corrections we use the leading term of the W function expansion, namely $W(x) \simeq \log(x)$. This allows one to obtain the following expression:

$$m_0^2 = \frac{1}{\kappa^2(3c_1 + c_2)} - i\pi \frac{3b_1 + b_2 + b_3}{\kappa^2(3c_1 + c_2)^2} \, . \tag{5.15}$$

In this expansion only two leading terms are preserved. As it is mentioned before, the W function is reals only for certain values of its argument. Therefore the complex part of the mass appears due to the W function complex part. Poles structure of spin-2 states is obtained in full analogy with this case.

Results of this section can be summarized as follows. First of all, EFT formalism provides a uniform framework for treatment of low energy quantum effects. This framework is widely used in particle physics and can be consistently apply for the gravitational interaction. Secondly, despite the fact that the microscopic action for gravity is unknown, it is possible to restore the effective action via symmetry principles. Finally, EFT can provide a verifiable description of gravitational effects taking place in the low energy regime. Therefore, for a given gravity model it is possible to restore the correspondent effective action and subject it to direct empirical verification. This is the main reason for an implementation of the EFT technique to gravity.

Chapter 6

Standard Model and Gravity

The standard model of particle physics is another extremely successful model of physics. It describes all known types of matter and all interactions except gravity with a few simple techniques. These techniques are gauge fields and spontaneous symmetry breaking. The standard model and the corresponding techniques are extensively covered in multiple sources [173, 174, 175, 176, 177]. Discussion of the particle physics lies beyond the scope of this thesis, so the standard model is to be discussed briefly for the sake of consistency.

Within the standard model the matter states, namely, leptons and quarks, are described as fermionic fields. These fields are subjected to the gauge symmetry of the group $SU(3) \times SU(2) \times U(1)$. Interactions between matter states are propagated with the corresponding gauge vector bosons which are photons, $W^{\pm}$, Z^0 bosons, and gluons. Spontaneous symmetry breaking is performed via the Higgs scalar doublet subjected to $SU(2)$ gauge symmetry.

Technique of the gauge fields is based on symmetry principles and it allows one to construct renormalizable models of vector interactions. A generic gauge model with a symmetry group G is constructed as follows. Firstly, one chooses an N dimensional representation of a group G. Secondly, one unites N fields of the same spin in a multiplet and subjects in to the gauge symmetry G. Thirdly, one constructs a covariant derivative that respects the gauge group G. To do this it is required to introduce a number of vector fields subjected to gauge transformations. The number of gauge vector fields is equal to the dimension of the representation of the group G. Finally, the gauge symmetry uniquely fixes the form of the gauge field kinetic term and the interaction sector. In such a way the gauge field technique allows one to construct an interaction model by a given symmetry group G.

The gauge group of the standard model is a product of three groups $SU(3) \times SU(2) \times U(1)$. Group $SU(3)$ is responsible for the strong interaction and its gauge vector bosons are gluons. Group $SU(2) \times U(1)$ describes the electroweak sector of the standard model. Due to the Higgs mechanism, that is to be discussed further, $SU(2) \times U(1)$ is spontaneously broken down to $SU(2)$ that describes the weak interactions carried by $W^{\pm}$ and Z^0 bosons and $U(1)$ describing the electromagnetic interaction carried by photons.

The Higgs sector is responsible for the spontaneous symmetry breaking in the standard model. It is required to introduce fermion masses in a way consistent with the gauge symmetry. Only left chiral components of fermionic fields participate in the weak interactions. The standard fermionic mass term, on the contrary, mixes the chiral components, so it is non-invariant with respect to the gauge symmetry.

Spontaneous symmetry breaking is implemented as follows. First of all, the Higgs field is introduced. It is a complex scalar doublet subjected to the gauge $SU(2)$ group. Secondly, a quartic Higgs self-interaction potential is introduced. The potential admits a spontaneous symmetry breaking, as any given minimum of the potential is non-invariant with respect to the custodial symmetry, i.e. to the symmetry of the potential itself. Finally, the Higgs field is coupled to chiral components of weak doublets in an invariant way [176, 177].

In the low energy regime the Higgs field forms a non-vanishing vacuum expectation value due to the spontaneously broken symmetry. This vacuum expectation value provides the leading contribution to the Higgs interaction with fermionic fields, thereby generating fermionic masses. The Higgs field itself acts as three massive and one massless scalars due to the Goldstone theorem [178, 179, 180]. Finally, the massive DoFs of the Higgs field are combined with the weak bosons to form massive $W^{\pm}$ and Z^0 bosons.

This is a very brief description of the standard model which is presented here for the sake of consistency. A more in-depth discussion of the standard model is irrelevant for the studies presented in this thesis, so it is not to be discussed further.

Basic physical phenomena described by the standard model, on the contrary, are relevant in the context of gravity theory. As it was highlighted, the standard model describes a vast array of phenomena, but it does not account for the gravitational interaction. EFT technique allows one to account for the gravitational interaction within the standard model and, consequently, to search for manifestations of the gravitational interaction in particle physics experiments.

The most illustrative example of the gravitational influence on the standard model

is given by the two body effective potential, discussed in the previous section. Effective gravity predicts the existence of new gravitational DoFs. Consequently, matter states can exchange not only with the standard matter gauge bosons and gravitons, but also with the new gravity DoFs. This results in a modification of the gravitational interaction potential and an alteration of particle interactions [57, 163].

The influence of non-local operators of effective gravity is of a special interest and it is one of the problems addressed in this thesis. The one-loop effective gravity action with non-local operators (5.5) should account for the gravity interaction sector of the standard model. Because of this an effective non-local interaction is induced within the standard model. For instance, one should expect the existence of non-local interactions between fermionic states, such as electrons, induced purely by gravity. This provides a room for tests of effective gravity via empirical study of non-local matter interactions. The particular problem concerning gravitationally induced non-local interactions is stated in the next section and presented in the Chapter 8.

Chapter 7

Addressed Problems

The contemporary landscape of gravity theory has multiple relevant problems. This thesis addresses a narrow set of them, as was discussed before. These problems are stated as follows.

The first problem is related with the non-local interactions of matter induced by effective gravity. As it was discussed in Chapter 6, an implementation of EFT for gravity in the context of particle physics can lead to the appearance of non-local interactions. It is shown in [159] that effective gravity receives non-local operators at the one-loop level. At the same time, the effective gravity action should describe the gravitational interaction between standard model particles.

In Chapter 8 results of paper [58] are presented and an opportunity to introduce non-local operators induced by the gravitational interaction is discussed. It is shown that the non-local operators from the effective gravity action induce non-local interactions between matter states with all spins. In particular, it is shown that the effective gravity generates an effective four-fermion non-local interaction. Interactions of that kind are constrained by the empirical data [181]. Because of this the corresponding empirical constants can be applied to the effective gravity.

The second problem is related with the binary system GWs production described within EFT. Within classical GR it is possible to describe GWs production in binary systems inspirals. Recent data on direct detection of GWs is consistent with GR at the present precision level [19, 74, 75]. Quantum effects can influence GWs production rate at the late stages of inspiral process. The modified Newtonian potential (5.6) obtained within EFT serves as an indirect evidence of this phenomenon.

The problem addressed in this thesis is the evaluation of a binary system GWs produc-

tion rate given by the local effective action (5.4). As discussed above, this effective action accounts for the leading quantum corrections generated at the one-loop level. In Chapter 9 the problem is discussed in more detail. A specific set of approximations is required to obtain analytical expressions for the GWs production rate. These approximations constrain applicability of the obtained results, but allow one to establish the characteristic spacial scale at which GWs production may develop deviations from the standard GR prediction.

Thirdly, an opportunity to describe dark matter within the effective gravity is addressed in Chapter 10. Effective action (5.4) results in a different structure of poles of the graviton propagator. These poles describe new DoFs present in the model. Non-local terms generated in the effective action (5.5) further alter the pole structure.

As discussed in paper [60] and Chapter 10, non-local terms result in poles with non-vanishing imaginary parts which indicates the existence of a non-vanishing decay width of the corresponding DoFs. At the same time, one is free to consider new DoFs of the effective action as dark matter candidates. Comparison of new effective DoFs lifetime with the age of the Universe allows one to make certain conclusions about the validity of such an interpretation. In Chapter 10 it is shown that new DoFs can contribute to the dark matter content, if certain constraints on their lifetime are established. These constraints, in turn, should be extended on the structure of the effective action (5.5).

The last problem discussed in this thesis is related with an application of the EFT technique to the modified gravity model (3.3). In Chapter 3 the STG model presented in [137] is discussed. It was highlighted that it should be considered as one of the simplest GR extensions. The model introduces a single additional DoF and one single additional interaction that screens the cosmological constant. It is crucial to understand the influence of quantum effects on this model, as they can ruin some of its original properties thereby making it irrelevant for practical applications. The influence of quantum effects on the screening properties of the model was already studied by my colleagues and me [66].

In Chapter 11 an implication of EFT for the model (3.3) is discussed. Effective theory based on GR predicts the existence of two additional DoFs at the level of one-loop effective action [145, 146, 54, 53]. It is shown that the model (3.3) predicts the existence of a bigger number of DoFs at the one-loop level. They appear due to the higher dimensional operators generated by the John interaction. Therefore the introduction of the John interaction radically changes the behaviour of the gravity theory at the quantum level. Chapter 11 contains results presented in [61].

Chapter 8

Gravity Induced Non-Local Effects in the Standard Model

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We show that the non-locality recently identified in quantum gravity using resummation techniques propagates to the matter sector of the theory. We describe these non-local effects using effective field theory techniques. We derive the complete set of non-local effective operators at order NG^2 for theories involving scalar, spinor, and vector fields. We then use recent data from the Large Hadron Collider to set a bound on the scale of space-time non-locality and find $M_\star > 3 \times 10^{-11}$ GeV.

Finding a quantum mechanical description of General Relativity, in other words, a quantum theory of gravity, remains one of the holy grails of modern theoretical physics. While it is not clear what this fundamental theory might be, we can use effective theory techniques to describe quantum gravity at energies below the Planck scale $M_P = 1/\sqrt{G}$ where G is Newton's constant. This approach is justified by the requirement that whatever the correct theory of quantum gravity might be, General Relativity must arise in its low energy limit.

We do not have much information about physics at the Planck scale as experiments at this energy scale are difficult to imagine. We, nevertheless, have indications that a unification of General Relativity and Quantum Mechanics may lead to a more complicated structure of space-time at short distances in the form of a minimal length. Indeed, there are several thought experiments [182, 183, 184, 185, 186, 187, 188] showing that, given our current understanding of Quantum Mechanics, General Relativity and causality, it is inconceivable to measure distances with a better precision than the Planck length $l_P = \sqrt{\hbar G/c^3}$ where $\hbar$ is the reduced Planck constant and c is the speed of light in vacuum. Such arguments imply a form of non-locality at short distances of the order of l_P. We will show that the scale of non-locality could actually be much larger that l_P depending on the matter content in the theory.

An important question is whether this non-locality could be found when combining quantum field theory with General Relativity as well. In [160], it was shown that General Relativity coupled to a quantum field theory generically leads to non-local effects in scalar field theories. In the current paper, we build on the results obtained in [160] and extend them to matter theories involving spinor and vector fields as well. We show that non-local effects are universal and affect all matter fields. We derive a complete set of non-local effective operators at order NG^2 where $N = N_s + 3N_f + 12N_V$ with N_s, N_f and N_V denoting respectively the number of scalar, spinor, and vector fields in the theory. Then, using recent data from the Large Hadron Collider, we set a limit on the scale of space-time non-locality.

Recently, several groups have studied perturbative linearized General Relativity coupled to matter fields. They found that perturbative unitarity can breakdown well below the reduced Planck mass [165, 166, 167, 168]. The self-healing mechanism [169, 170] demonstrates that unitarity can be recovered by resumming a series of graviton vacuum polarization diagrams in the large N limit (Fig. (8.1)), see as well [189, 190] for earlier works on large N quantum gravity. An interesting feature of this large N resummation,

Figure 8.1: Resummation of the graviton propagator.

while keeping NG small, is that the obtained resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NGq^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)},\tag{8.1}$$

where μ is the renormalization scale incorporates some of the non-perturbative physics of quantum gravity. It has poles beyond the usual one at $q^2 = 0$. Indeed, one finds [171, 191, 192] that there is a pair of complex poles at

$$q^2 \;\; = \;\; \frac{1}{GN}\frac{120\pi}{W\left(\frac{-120\pi}{\mu^2 NG}\right)}\tag{8.2}$$

where W is the Lambert function. As explained in [171], these complex poles are a sign of strong interactions. The mass and width of these objects can be calculated. It was suggested in [171] that the complex poles could be interpreted as black hole precursors. These Planckian black holes are purely quantum object and their geometry is not expected to be described accurately by the standard solutions of classical Einstein's equations. In particular, they will not decay via Hawking radiation as they are non-thermal objects. While they do not radiate, they are very short-lived objects and will decay to a few particles. Their widths are of the order of $(120\pi/GN)^{1/2}$. Because the complex poles are related by complex conjugation, one of them has an incorrect sign between its mass and its width and it corresponds to a particle propagating backwards in time. This complex pole thus leads to acausal effects which should become appreciable at energies near $(120\pi/GN)^{1/2}$. Using the in-in formalism [193, 194] it is possible to restore causality at the price of introducing non-local effects at the scale $(120\pi/GN)^{1/2}$. This was done, for example, in [159] within the context of Friedmann, Lemaître, Robertson and Walker cosmology. The Lee-Wick prescription can also be used to make sense of complex poles [195, 196]. The scale of non-locality is thus potentially much larger than l_P if there are many fields in the matter sector, i.e., if N is large.

In [160], it was shown that the resummed graviton propagator in Eq. (8.1) induces non-local effects in scalar field theories at short distances of the order of $(120\pi/GN)^{1/2}$.

We extend this work to spinor and vector fields and demonstrate that the non-local effects propagate universally in quantum field theory as would be expected from quantum black holes and the thought experiments described previously. We consider a theory with an arbitrary number of scalar fields, spinor and vector fields and calculate their two-by-two scattering gravitational amplitudes using the dressed graviton propagator (8.1). We then extract the leading order (i.e. order $G^2 N$) term of each of these amplitudes and present the results in terms of effective operators.

The stress-energy tensors for the different field species with spins 0, 1/2 and 1 are given as usual by

$$T^{\mu\nu}_{\text{scalar}} = \partial^\mu \phi \, \partial^\nu \phi - \eta^{\mu\nu} L_{\text{scalar}} , \tag{8.3}$$

$$T^{\mu\nu}_{\text{fermion}} = \frac{i}{4} \bar{\psi} \gamma^\mu \nabla^\nu \psi + \frac{i}{4} \bar{\psi} \gamma^\nu \nabla^\mu \psi - \frac{i}{4} \nabla^\mu \bar{\psi} \gamma^\nu \psi - \frac{i}{4} \nabla^\nu \bar{\psi} \gamma^\mu \psi - \eta^{\mu\nu} L_{\text{fermion}} , \tag{8.4}$$

$$T^{\mu\nu}_{\text{vector}} = -F^{\mu\sigma} F^\nu{}_\sigma + m^2 A^\mu A^\nu - \eta^{\mu\nu} L_{\text{vector}} , \tag{8.5}$$

where we have used the following free field matter Lagrangians:

$$L_{\text{scalar}} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 , \tag{8.6}$$

$$L_{\text{fermion}} = \frac{i}{2}\bar{\psi}\gamma^\sigma \nabla_\sigma \psi - \frac{i}{2}\nabla_\sigma \bar{\psi}\gamma^\sigma \psi - m\bar{\psi}\psi , \tag{8.7}$$

$$L_{\text{vector}} = -\frac{1}{4}F^2 + \frac{1}{2}m^2 A^2 . \tag{8.8}$$

We can now present the complete set of non-local operators at order NG^2. The non-local operators involving scalar fields only are given by

$$O_{\text{scalar},1} = \frac{NG^2}{30\pi} \partial_\mu\phi \, \partial_\nu\phi \, \ln\left(\frac{\Box}{\mu^2}\right) \partial^\mu\phi' \, \partial^\nu\phi' , \tag{8.9}$$

$$O_{\text{scalar},2} = -\frac{NG^2}{60\pi} \partial_\mu\phi \, \partial^\mu\phi \, \ln\left(\frac{\Box}{\mu^2}\right) \partial_\sigma\phi' \, \partial^\sigma\phi' , \tag{8.10}$$

$$O_{\text{scalar},3} = \frac{NG^2}{30\pi} L_{\text{scalar}} \, \ln\left(\frac{\Box}{\mu^2}\right) \partial_\sigma\phi' \, \partial^\sigma\phi' , \tag{8.11}$$

$$O_{\text{scalar},4} = \frac{NG^2}{30\pi} \partial_\mu\phi \, \partial^\mu\phi \, \ln\left(\frac{\Box}{\mu^2}\right) L'_{\text{scalar}} , \tag{8.12}$$

$$O_{\text{scalar},5} = -\frac{2NG^2}{15\pi} L_{\text{scalar}} \ln\left(\frac{\Box}{\mu^2}\right) L'_{\text{scalar}} . \tag{8.13}$$

The non-local operators involving spinor fields only are given by

$$O_{\text{fermion},1} = \frac{NG^2}{60\pi} \left(\frac{i}{2}\bar{\psi}\gamma^\mu\nabla^\nu\psi - \frac{i}{2}\nabla^\mu\bar{\psi}\gamma^\nu\psi \right) \left(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\beta_\mu\delta^\alpha_\nu \right) \times \tag{8.14}$$

$$\times \ln\left(\frac{\Box}{\mu^2}\right) \left(\frac{i}{2}\bar{\psi}'\gamma^\alpha\nabla^\beta\psi' - \frac{i}{2}\nabla^\alpha\bar{\psi}'\gamma^\beta\psi' \right) ,$$

$$O_{\text{fermion},2} = -\frac{NG^2}{60\pi} \left(\frac{i}{2}\bar{\psi}\gamma^\sigma\nabla_\sigma\psi - \frac{i}{2}\nabla_\sigma\bar{\psi}\gamma^\sigma\psi \right) \times \tag{8.15}$$

$$\times \ln\left(\frac{\Box}{\mu^2}\right) \left(\frac{i}{2}\bar{\psi}'\gamma^\rho\nabla_\rho\psi' - \frac{i}{2}\nabla_\rho\bar{\psi}'\gamma^\rho\psi' \right) ,$$

$$O_{\text{fermion},3} = \frac{NG^2}{30\pi} L_{\text{fermion}} \ln\left(\frac{\Box}{\mu^2}\right) \left(\frac{i}{2}\bar{\psi}'\gamma^\sigma\nabla_\sigma\psi' - \frac{i}{2}\nabla_\sigma\bar{\psi}'\gamma^\sigma\psi' \right) , \tag{8.16}$$

$$O_{\text{fermion},4} = \frac{NG^2}{30\pi} \left(\frac{i}{2}\bar{\psi}\gamma^\sigma\nabla_\sigma\psi - \frac{i}{2}\nabla_\sigma\bar{\psi}\gamma^\sigma\psi \right) \ln\left(\frac{\Box}{\mu^2}\right) L'_{\text{fermion}} , \tag{8.17}$$

$$O_{\text{fermion},5} = -\frac{2NG^2}{15\pi} L_{\text{fermion}} \ln\left(\frac{\Box}{\mu^2}\right) L'_{\text{fermion}} . \tag{8.18}$$

The non-local operators involving vector fields only are given by

$$O_{\text{vector},1} = \frac{NG^2}{30\pi} \left(F^{\mu\sigma}F_{\nu\sigma} - m^2 A^\mu A_\nu \right) \ln\left(\frac{\Box}{\mu^2}\right) \left(F'^{\mu\rho}F'_{\nu\rho} - m'^2 A'_\mu A'^\nu \right) , \tag{8.19}$$

$$O_{\text{vector},2} = -\frac{NG^2}{60\pi} \left(F^2 - m^2 A^2 \right) \ln\left(\frac{\Box}{\mu^2}\right) \left(F'^2 - m'^2 A'^2 \right) , \tag{8.20}$$

$$O_{\text{vector},3} = -\frac{NG^2}{30\pi} L_{\text{vector}} \ln\left(\frac{\Box}{\mu^2}\right) \left(F'^2 - m'^2 A'^2 \right) , \tag{8.21}$$

$$O_{\text{vector},4} = -\frac{NG^2}{30\pi} \left(F^2 - m^2 A^2 \right) \ln\left(\frac{\Box}{\mu^2}\right) L'_{\text{vector}} , \tag{8.22}$$

$$O_{\text{vector},5} = -\frac{2NG^2}{15\pi} L_{\text{vector}} \ln\left(\frac{\Box}{\mu^2}\right) L'_{\text{vector}} . \tag{8.23}$$

The non-local operators involving amplitudes with scalar and vector fields only are

given by

$$O_{\text{scalar-vector},1} = -\frac{NG^2}{30\pi}\,\partial^\mu\phi\,\partial_\nu\phi\,\ln\left(\frac{\Box}{\mu^2}\right)\left(F_{\mu\sigma}F^{\nu\sigma} - m_A^2 A_\mu A^\nu\right)\;, \qquad (8.24)$$

$$O_{\text{scalar-vector},2} = \frac{NG^2}{60\pi}\,(\partial\phi)^2\ln\left(\frac{\Box}{\mu^2}\right)(F^2 - m_A^2 A^2)\;, \qquad (8.25)$$

$$O_{\text{scalar-vector},3} = -\frac{NG^2}{30\pi}\,L_{\text{scalar}}\ln\left(\frac{\Box}{\mu^2}\right)(F^2 - m_A^2 A^2)\;, \qquad (8.26)$$

$$O_{\text{scalar-vector},4} = -\frac{NG^2}{30\pi}\,(\partial\phi)^2\ln\left(\frac{\Box}{\mu^2}\right)L_{\text{vector}}\;, \qquad (8.27)$$

$$O_{\text{scalar-vector},5} = -\frac{2NG^2}{15\pi}\,L_{\text{scalar}}\ln\left(\frac{\Box}{\mu^2}\right)L_{\text{vector}}\;. \qquad (8.28)$$

The non-local operators involving amplitudes with scalar and spinor fields only are given by

$$O_{\text{scalar-fermion},1} = \frac{NG^2}{30\pi}\,\partial_\mu\phi\,\partial_\nu\phi\ln\left(\frac{\Box}{\mu^2}\right)\left(\frac{i}{2}\bar\psi\gamma^\mu\nabla^\nu\psi - \frac{i}{2}\nabla^\mu\bar\psi\gamma^\nu\psi\right)\;, \quad (8.29)$$

$$O_{\text{scalar-fermion},2} = -\frac{NG^2}{60\pi}\,(\partial\phi)^2\ln\left(\frac{\Box}{\mu^2}\right)\left(\frac{i}{2}\bar\psi\gamma^\sigma\nabla_\sigma\psi - \frac{i}{2}\nabla_\sigma\bar\psi\gamma^\sigma\psi\right)\;, \quad (8.30)$$

$$O_{\text{scalar-fermion},3} = \frac{NG^2}{30\pi}\,L_{\text{scalar}}\ln\left(\frac{\Box}{\mu^2}\right)\left(\frac{i}{2}\bar\psi\gamma^\sigma\nabla_\sigma\psi - \frac{i}{2}\nabla_\sigma\bar\psi\gamma^\sigma\psi\right)\;, \quad (8.31)$$

$$O_{\text{scalar-fermion},4} = \frac{NG^2}{30\pi}\,(\partial\phi)^2\ln\left(\frac{\Box}{\mu^2}\right)L_{\text{fermion}}\;, \qquad (8.32)$$

$$O_{\text{scalar-fermion},5} = -\frac{2NG^2}{15\pi}\,L_{\text{scalar}}\ln\left(\frac{\Box}{\mu^2}\right)L_{\text{fermion}}\;. \qquad (8.33)$$

The non-local operators involving amplitudes with spinor and vector fields only are

given by

$$O_{\text{vector-fermion},1} = -\frac{NG^2}{30\pi}\left(\frac{i}{2}\bar{\psi}\gamma^\mu\nabla^\nu\psi - \frac{i}{2}\nabla^\mu\bar{\psi}\gamma^\nu\psi\right)\ln\left(\frac{\Box}{\mu^2}\right)\left(F_{\mu\sigma}F_\nu{}^\sigma - m_A^2 A_\mu A_\sigma\right)\ ,$$

$$(8.34)$$

$$O_{\text{vector-fermion},2} = \frac{NG^2}{60\pi}\left(\frac{i}{2}\bar{\psi}\gamma^\sigma\nabla_\sigma\psi - \frac{i}{2}\nabla_\sigma\bar{\psi}\gamma^\sigma\psi\right)\ln\left(\frac{\Box}{\mu^2}\right)\left(F^2 - m_A^2 A^2\right)\ , \qquad (8.35)$$

$$O_{\text{vector-fermion},3} = -\frac{NG^2}{30\pi}L_{\text{fermion}}\ \ln\left(\frac{\Box}{\mu^2}\right)\left(F^2 - m_A^2 A^2\right)\ , \qquad (8.36)$$

$$O_{\text{vector-fermion},4} = \frac{NG^2}{30\pi}\left(\frac{i}{2}\bar{\psi}\gamma^\sigma\nabla_\sigma\psi\right)\ln\left(\frac{\Box}{\mu^2}\right)L_{\text{vector}}\ , \qquad (8.37)$$

$$O_{\text{vector-fermion},5} = -\frac{2NG^2}{15\pi}L_{\text{fermion}}\ \ln\left(\frac{\Box}{\mu^2}\right)L_{\text{vector}}\ . \qquad (8.38)$$

By looking at these effective operators, one can see explicitly that the gravitational non-locality leads to non-local effects in the matter sector as well. This is the case for all matter fields of any spin. The non-locality is manifest due to the presence of the $\log(\Box)$ term in all of these effective operators. The universality of the non-locality in the matter sector is precisely what one expects in the context of a minimal length. The underlying argument in all minimal length demonstrations is the following. When length scales shorter than the minimal length are probed, one ends up concentrating so much energy within that region of space-time that a Planckian black hole will eventually form in that region of space-time. This is precisely what we are finding when interpreting the complex pole as a black hole precursor which is an extended object of size $(120\pi/GN)^{1/2}$. Its extension in space corresponds to the minimal length that can be probed. We conclude that space-time is smeared on distances shorter than the dynamical Planck scale given by $M_\star = M_P\sqrt{120\pi/N}$ which corresponds to the energy of the complex pole.

This non-locality prevents an observer from testing distances shorter than the corresponding length scale. It also implies that singularities cannot be probed experimentally as space-time is smeared. One may argue that the notion of space-time looses its meaning on distances smaller than $1/M_\star$. This interpretation fits well with the observations made recently in [197].

It is interesting to point out that the non-local effects in the four-fermion interactions can be probed at the Large Hadron Collider. The ATLAS collaboration has searched for

four-fermion contact interactions at $\sqrt{s} = 8$ TeV and obtained lower limits on the scale on the lepton-lepton-quark-quark contact interaction Λ between 15.4 TeV and 26.3 TeV [181]. The most restrictive bound on Λ is obtained by combining the dielectron and dimuon channels. We have contributions to these process coming from $O_{\text{fermion},1}$ and $O_{\text{fermion},2}$. We first note that the renormalization scale μ should scale with N as well, we take $\mu^2 = 120\pi/(NG)$. Since we are looking at conservative order of magnitudes, we will identify the scale generated by the derivatives in the four-fermion operators with the center of mass energy of the proton-proton collision. We are thus dealing effectively with operators of the type $\bar{q}q\bar{l}l$ which are suppressed by a factor $2NG^2/(60\pi^2)s \log(sNG/(120\pi))$. This translates into a conservative bound $N < 5 \times 10^{61}$ on the number of light fields in a hidden sector. This implies that the scale $M_\star$, which parametrizes the non-locality of space-time, is larger than 3×10^{-11} GeV. This bound is tighter than that obtained from gravitational waves and from Eötvös type pendulum experiments [192] by two orders of magnitude.

Note that our bound on the scale of space-time non-locality ($M_P\sqrt{120\pi/N}$) is much weaker than those on the Planck mass (M_P) obtained using the standard geometrical cross section (i.e. $\sigma = \pi R_S^2$ where R_S is the Schwarzschild radius) for quantum black holes [198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 67, 210, 211, 212, 213]. Collider bounds on a new object with a geometric cross section are typically of the order of 9 TeV [213]. This is not surprising as we are indeed studying different higher order effective operators. So far, we have not found, within the effective theory approach, higher dimensional operators corresponding to the geometrical cross section which has been extensively studied. The intermediate states in the propagator of the graviton, which we have studied, couple with the usual Planck mass to the particles of the standard model while in the more extensively studied models, quantum black holes are assumed to couple much stronger (i.e. with $M_P \sim$ TeV) to the particles of the standard model. Since at this stage we have not identified a mechanism which lowers the value of the Planck mass (we are using dimensional regularization in contrast to [214] where a dimensionful cutoff had been used), there is no strong gravitational effect in the TeV region to be expected.

It is worth mentioning that weakly nonlocal theories, such as the effective field theory for general relativity considered in this paper, can be also the starting point to construct an ultraviolet completion of Einstein's gravity, which turns out to be unitary at perturbative level and finite at quantum level [215, 216, 217].

In this paper, we have shown that the non-locality recently identified in quantum gravity propagates to the matter sector of the theory. We have described these non-local

effects using the tools of effective field theory. We have derived the complete set of effective operators at order NG^2 for theories involving scalar, spinor, and vector fields. We then have used recent data from the Large Hadron Collider to set a bound on the scale of space-time non-locality and found $M_\star > 3 \times 10^{-11}$ GeV.

Acknowledgements

Chapter 9

Three Waves for Quantum Gravity

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Using effective field theoretical methods, we show that besides the already observed gravitational waves, quantum gravity predicts two further massive classical fields leading to two new massive waves. We set a limit on the masses of these new modes using data from the Eöt-Wash experiment. We point out that the existence of these new states is a model independent prediction of quantum gravity. We then explain how these new classical fields could impact astrophysical processes and in particular the binary inspirals of neutron stars or black holes. We calculate the emission rate of these new states in binary inspirals astrophysical processes.

Much progress has been made in recent years in quantum gravity using effective field theory methods. These methods enable one to perform quantum gravitational calculations for processes taking place at energies below the Planck mass, or some 10^{19} GeV while remaining agnostic about the underlying theory of quantum gravity. One could argue that the first attempts in that direction were due to Feynman who has calculated quantum amplitudes using linearized general relativity [138]. Modern effective field theory techniques were introduced in the seminal works of Donoghue in the 90's [145, 218, 57]. With time, it became clear that some model independent predictions could be obtained [219, 159, 160, 58, 214, 197, 192, 220]. This approach is very generic and it could be the low energy theory for virtually any theory of quantum gravity such as e.g. string theory [15, 16], loop quantum gravity [8], asymptotically safe gravity [221, 222, 223] or super-renormalizable quantum gravity [215, 216, 217] just to name a few.

In this paper we point out that the low energy spectrum of quantum gravity must contain two new classical fields besides the massless classical graviton that has recently been observed in the form of gravitational waves [224, 225, 226]. These new states correspond to massive objects of spin-0 and spin-2. As we will show these new states are purely classical fields that could have interesting consequences for different branches of physics, from particle physics and astrophysics to cosmology.

To identify these new fields, we calculate the leading quantum gravitational corrections to the Newtonian gravitational potential using effective field theory methods. These corrections can be shown to correspond to two new classical states that must exists besides the massless spin-2 classical graviton. We set limits on the masses of these classical fields using data from the Eöt-Wash pendulum experiment [227] and we then turn our attention to astrophysical and cosmological probes of quantum gravity studying quantum gravitational contributions to the inspirals of neutron stars or black holes. We demonstrate that the new massive spin-2 and spin-0 states predicted in a model independent way by quantum gravity can modify the potential between the two astrophysical bodies and lead to testable effects. We comment on the implications of quantum gravity for inflation, dark matter and gravitational wave production in phase transition.

Although general relativity is in many regards similar to the gauge theories describing the electroweak and strong interactions, there is one basic difference which is the source of a technical difficulty with quantum gravity. The main obstacle is that the coupling constant, in the case of gravity, is a dimensional full parameter, namely Newton's constant G_N while in the case of the other interactions the fundamental coupling constant is a dimensionless

parameter. The fact that Newton's constant carries a dimension leads to problems with the renormalization of the theory of quantum gravity, at least at the perturbative level. While having a renormalizable theory is necessary to claim to have a fundamental theory of quantum gravity, and to perform calculations at energies above the Planck mass $M_P = 1/\sqrt{G_N} \sim 10^{19}$ GeV, it is now well appreciated that using effective theory techniques leads to very interesting insights into a theory of quantum gravity [145, 218, 219, 160, 58]. As a matter of fact, since all experiments, astrophysical or cosmological events we are aware of involve energies below the Planck mass, an effective theory of quantum gravity valid up to M_P may be all that we ever need.

From a technical point of view, calculations in quantum gravity using effective theory techniques are rather simple. One integrates out the quantum fluctuations of the metric to obtain an effective action. Matter fields, depending on the problem at hand and in particular on the energy involved in the problem, can also be integrated out. One is left with an effective action given by

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2}M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + c_4 \Box \mathcal{R} \right.$$
$$\left. + b_1 \mathcal{R} \log \frac{\Box}{\mu_1^2} \mathcal{R} + b_2 \mathcal{R}_{\mu\nu} \log \frac{\Box}{\mu_2^2} \mathcal{R}^{\mu\nu} + b_3 \mathcal{R}_{\mu\nu\rho\sigma} \log \frac{\Box}{\mu_3^2} \mathcal{R}^{\mu\nu\rho\sigma} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right] , \quad (9.1)$$

where $\mathcal{R}$, $\mathcal{R}_{\mu\nu}$ and $\mathcal{R}_{\mu\nu\rho\sigma}$ are respectively the Ricci scalar, the Ricci tensor and the Riemann tensor. The cosmological constant is denoted by Λ_C. The scales μ_i are renormalization scales which in principle could be different, we shall however take $\mu_i = \mu$. The Lagrangian L_{SM} contains all of the matter we know of and $M_\star$ is the energy scale up to which we can trust the effective field theory. Note that we have written down all dimension four operators which have dimensionless coupling constants and we have thus introduced a non-minimal coupling of the Higgs doublet to curvature on top of the purely gravitational terms. The term $\Box \mathcal{R}$ is a total derivative and thus does not contribute to the equation of motions. Remarkably, the values of the parameters b_i are calculable from first principles and are model independent predictions of quantum gravity, see e.g. [228] and references therein. They are related to the number of fields that have been integrated out. The non-renormalizability of the effective action is reflected in the fact that we cannot predict the coefficients c_i which, in this framework, have to be measured in experiments or observations. There will be new c_i appearing at every order in the curvature expansion performed when deriving this effective action and we thus would have to measure an infinite number of parameters. Despite this fact, the effective theory leads to falsifiable predictions as the coefficients b_i of non-local operators are, as explained previously, calculable.

The effective action contains three classical fields: the well known massless spin-2 field (the classical graviton) $h^{\mu\nu}$, a massive spin-2 classical field $k^{\mu\nu}$ and a massive classical spin-0 field σ on top of the mater fields contained in $\mathcal{L}_{SM}$. This can be see explicitly by sandwiching the Green's function of the metric in the linearized effective action between two classical sources $T^{(i)\mu\nu}$ [220]

$$256\pi^2 G_N^2 \left[\frac{T_{\mu\nu}^{(1)} T^{(2)\mu\nu} - \frac{1}{2} T^{(1)\mu}_{\mu} T^{(2)\nu}_{\nu}}{k^2} - \frac{T_{\mu\nu}^{(1)} T^{(2)\mu\nu} - \frac{1}{3} T^{(1)\mu}_{\mu} T^{(2)\nu}_{\nu}}{k^2 - \frac{2}{\kappa^2 \left(c_2 + (b_2 + 4b_3) \log\left(\frac{-k^2}{\mu^2}\right) \right)}} \right.$$
$$\left. + \frac{T^{(1)\mu}_{\mu} T^{(2)\nu}_{\nu}}{k^2 - \frac{1}{\kappa^2 \left(3c_1 + c_2 + (3b_1 + b_2 + b_3) \log\left(\frac{-k^2}{\mu^2}\right) \right)}} \right], \tag{9.2}$$

where $\kappa^2 = 32\pi G$. A careful reader will have noticed the minus sign in front of the massive spin-2 mode. This is the well known ghost due to the the term $R_{\mu\nu} R^{\mu\nu}$. However, the corresponding state $k^{\mu\nu}$ is purely classical and it does not lead to any obvious pathology. This is simply a repulsive classical force. We will show that the emission of this massive spin-2 wave leads to the production of waves with positive energy. This state simply effectively couples with a negative coupling constant M_P to matter. It is crucial to appreciate that this mode is purely classical and should not be quantized as it is obtained by integrating out the quantum fluctuations of the graviton from the original action.

Using Eq.(9.2), it is straightforward to calculate the leading second order in curvature quantum gravitational corrections to Newton's potential of a point mass m. We find:

$$\Phi(r) = -\frac{Gm}{r} \left(1 + \frac{1}{3} e^{-Re(m_0)r} - \frac{4}{3} e^{-Re(m_2)r} \right) \tag{9.3}$$

where the masses are given by

$$m_2^2 = \frac{2}{(b_2 + 4b_3)\, \kappa^2\, W\left(-\frac{2 \exp\frac{c_2}{(b_2+4b_3)}}{(b_2+4b_3)\kappa^2\mu^2} \right)}, \tag{9.4}$$

$$m_0^2 = \frac{1}{(3b_1 + b_2 + b_3)\, \kappa^2\, W\left(-\frac{\exp\frac{3c_1+c_2}{(3b_1+b_2+b_3)}}{(3b_1+b_2+b_3)\kappa^2\mu^2} \right)}, \tag{9.5}$$

and where $W(x)$ is the Lambert function. This effective Newtonian potential is a generalization of Stelle's classical result [54], it includes the non-local operators as well as the local ones and thus contains the leading quantum gravitational corrections at second order in curvature.

Note that our result is compatible with the results obtained in [218, 57, 229], we simply focus on a different limit where the coefficients of R^2 and $R_{\mu\nu}R^{\mu\nu}$ are not necessarily tiny. It is easy to show that the effective action leads to higher order corrections in G_N to the

Newtonian potential energy of two large non-relativistic masses m_1 and m_2. The quantum corrected Newtonian potential is given by

$$U(r) = -G_N \frac{m_1 m_2}{r} - 3\, G_N^2\, \frac{m_1 m_2 (m_1 + m_2)}{r^2} \tag{9.6}$$
$$-\frac{m_1 m_2}{\pi r^3}\, G_N^2\, \left(\frac{N_s}{42} + \frac{N_f}{7} + \frac{2 N_V}{7} + \frac{41}{10} \right)\, .$$

This extends the result presented in [218, 57, 229] to include the numbers N_i respectively of real scalar fields, Dirac fermions and vector fields present in the model. The number of matter fields N_i are related to the b_i which are the Wilson coefficients appearing in the effective action by the relations $N_i = b_{2,i} + 4 b_{3,i}$. Here we took the same limit as in [218, 57, 229] assuming that the c_i are very small. The corresponding terms lead to delta functions which do not contribute to the potential energy. As emphasized in [57], the second term in the potential represents the leading relativistic correction and it is not a quantum correction. Note that these corrections are appearing at order G_N^2 and are thus subleading in comparison to the contributions of the new waves appearing in $\Phi(r)$ on which we will thus focus.

The masses of the new modes correspond to pairs of complex poles in the green's functions of the massive spin-2 $k^{\mu\nu}$ and spin-0 σ states. In general, the masses are complex depending on the values of the parameters c_i, b_i and μ, in other words they contain a width. The imaginary contributions, however, vanish when adding up the contributions of these states to the Newtonian potential. It is straightforward to show that Stelle's classical result is recovered in the limit of $b_i = 0$.

It is easy to work out the coupling of $k^{\mu\nu}$ and σ to matter. We find

$$S = \int d^4x \left[\left(-\frac{1}{2} h_{\mu\nu} \Box h^{\mu\nu} + \frac{1}{2} h_\mu{}^\mu \Box h_\nu{}^\nu - h^{\mu\nu} \partial_\mu \partial_\nu h_\alpha{}^\alpha + h^{\mu\nu} \partial_\rho \partial_\nu h^\rho{}_\mu \right) \right. \tag{9.7}$$
$$+ \left(-\frac{1}{2} k_{\mu\nu} \Box k^{\mu\nu} + \frac{1}{2} k_\mu{}^\mu \Box k_\nu{}^\nu - k^{\mu\nu} \partial_\mu \partial_\nu k_\alpha{}^\alpha + k^{\mu\nu} \partial_\rho \partial_\nu k^\rho{}_\mu \right.$$
$$\left. -\frac{m_2^2}{2} \left(k_{\mu\nu} k^{\mu\nu} - k_\alpha{}^\alpha k_\beta{}^\beta \right) \right)$$
$$\left. + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{m_0^2}{2} \sigma^2 - \sqrt{8\pi G_N} (h_{\mu\nu} - k_{\mu\nu} + \frac{1}{\sqrt{3}}\, \sigma\, \eta_{\mu\nu}) T^{\mu\nu} \right]\, .$$

This result shows that quantum gravity, whatever the underlying ultra-violet theory might be, has at least three classical degrees of freedom in its low energy spectrum. The massless mode has recently been directly observed in the form of gravitational waves. While there was little doubt about their existence since the discovery of the first binary pulsar in 1974, the direct observation by the LIGO and Virgo collaborations [224, 225, 226] erased any possible remaining doubt. While the massless mode affects the distance between two

points, and thus the geometry, the massive modes are of the 5th force type and they do not affect the geometry of space-time. A 5th force will not change the proper distance between the mirrors of an interferometer such as those of LIGO or Virgo, but it could still lead to measurable displacement of the mirrors if the wavelength is shorter than the distance between the mirrors on one arm of an interferometer.

We find that the strength of the interaction between the new massive modes and matter is fixed by the gravitational coupling constant. It is crucial to appreciate that the fields $h^{\mu\nu}$, $k^{\mu\nu}$ and σ are purely classical degrees of freedom. This is why the overall negative sign of the kinetic term of $k^{\mu\nu}$ is not an issue, it simply implies that this field couples with a negative Planck mass to matter. We shall demonstrate that the corresponding massive spin-2 wave produced in binary inspiral does not violate energy conservation. Note that while $k^{\mu\nu}$ couples universally to matter, σ does not couple to massless vector fields [130, 230].

The fact that these fields are purely classical has some interesting consequences if one tries to interpret the massive modes as dark matter candidates or the inflaton in the case of the scalar field. If the massive modes constitute all of dark matter, dark matter would an emergent phenomenon. In that sense dark matter would be fundamentally different from regular matter. The same remark applies to inflation if the scalar field encompassed in the curvature squared term is responsible for the early universe exponential expansion.

We now turn our attention to the experimental bounds on the masses of the two heavy states. Newton's potential with its quantum gravitational corrections can be probed with sub-millimeter tests of the gravitational inverse-square law [227]. In the absence of accidental fine cancellations between both Yukawa terms, the current bounds imply m_0 , $m_2 > (0.03 \text{ cm})^{-1} = 6.6 \times 10^{-13} \text{GeV}$. Note that the Eöt-Wash experiment performed by Hoyle et al. [227] is probing separations between 10.77 mm and 137 μm, a cancelation between the two Yukawa terms on this range of scales seems impossible without modifying general relativity with new physics to implement a screening mechanism.

The bound on the quantum gravitational corrections to Newton's potential imply that quantum gravity could only impact the final moments of the inspiraling of binary of two neutron stars or of two black holes. Their effect will only become relevant at distances shorter than 0.03 cm. There are two possible effects. When the two astrophysical bodies are close enough, Newton's law could be affected by the propagations of the new massive modes and the new massive modes could be produced in the form of new massive waves.

The quantum gravitational correction to the orbital frequency of a inspiraling binary

system is given by

$$\omega^2 = \frac{Gm}{r^3} \left(1 + \frac{1}{3} e^{-Re(m_0)r} - \frac{4}{3} e^{-Re(m_2)r} \right) \tag{9.8}$$

where $m = m_1 + m_2$ is the total mass of the binary system. The total energy of the system is given by

$$E = -\frac{Gm\mu}{2r} \left(1 + \frac{1}{3} e^{-Re(m_0)r} - \frac{4}{3} e^{-Re(m_2)r} \right) \tag{9.9}$$

where $\mu = m_1 m_2/m$ is the reduced mass of the system. The quantum gravitationally corrected waveform can be deduced from the energy-conservation equation $\dot{E} = -P_{GW}$ where P_{GW} is the power of the quadrupole radiation of the gravitational waves corresponding to the massless spin-2 mode:

$$P_{GW} = \frac{32 G_N \mu^2 \omega^6 r^4}{5c^2} \tag{9.10}$$

which can be solve for $r(t)$ from which $\omega(t)$ can be calculated. The quantum corrected chirp signal which has frequency f_{GW} and amplitude A_{GW} can then be obtained in a straightforward manner:

$$f_{GW}(t) = \frac{\omega(t)}{\pi} \tag{9.11}$$

$$A_{GW}(t) = \frac{1}{d_L} \frac{2G_N}{c^4} 2\mu\omega(t)r^2(t), \tag{9.12}$$

where d_L is the luminosity distance of the source.

While it is easy to calculate f_{GW} and A_{GW} explicitly, it is clear that the quantum gravitational corrections to the emission of gravitational waves can only become relevant when the two objects are closer than 0.03 cm given the bound derived on the mass of the massive spin-2 object using data from the Eöt-Wash experiment [1]. This distance is well within the Schwarzschild radius of any astrophysical black hole and clearly tools from numerical relativity need to be employed to obtain a reliable computation. Note that for black holes the mass is concentrated at their center and very close to the singularity. While the horizons will have started to merge, the two singularities could be within a reasonable distance of each other. In that sense our approximation may not be so rough. In any case it is clear that incorporating our quantum gravitational effect in numerical relativity calculations [232] represents a real technical challenge as the interior of black holes is usually excised to avoid having to discuss the singularities. However, the new

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[1] The effects of the $1/r^2$ and $1/r^3$ terms discussed above, which are corrections to the propagation of the massless mode will be considered elsewhere[231].

states can only be relevant when the distance between the two black hole singularities become of the order of the inverse of the mass of the massive spin-2 object.

Besides the usual massless gravitational waves, there are two new kind of radiations, namely the massive spin-0 and spin-2 could in principle be produced in energetic astrophysical or cosmological events. However, in the case of a binary system, because the center of mass of the system is conserved, the spin-0 wave cannot be produced. On the other hand, the massive spin-2 could be emitted in the last moment of a merger when the two inspiraling objects are closer than the inverse of the mass of the massive spin-2 field. A lengthy calculation leads to a remarkable result. The energy E carried away by the massive spin-2 mode from a binary system per frequency is identical to that of massless spin-2 mode:

$$\frac{dE_{massive}}{d\omega} = \frac{G_N}{45}\,\omega^6\,\langle Q_{ij}Q^{ij}\rangle\,\theta(\omega - m_2) \tag{9.13}$$

up to a Heaviside step function which prevents the emission of massive waves when the energy of the system is below the mass threshold. Note that as usual Q_{ij} is the quadrupole moment of the binary system. The total wave emission by a binary system is thus given by

$$\frac{dE}{d\omega} = \frac{dE_{massless}}{d\omega} + \frac{dE_{massive}}{d\omega}, \tag{9.14}$$

where the first term on the righthand side is the usual general relativity result for massless gravitational waves. Once the massive channel becomes available, half of the energy is damped into the massive mode.

The massive spin-2 wave will only be produced when the two black holes are close enough from another. If we denote the distance between the black holes of masses m_A and m_B by d, we obtain the frequency of the inspiral ω:

$$\omega^2 = \frac{G_N(m_A + m_B)}{d^3}. \tag{9.15}$$

To estimate how close the two black holes have to be to generate enough energy to produce a massive wave compatible with the Eöt-Wash bound, we set $\omega = (0.03\ \text{cm})^{-1}$ and use the masses of the first merger observed by the LIGO collaboration $m_A = 36\,M_\odot$ $m_B = 29\,M_\odot$ (where $M_\odot$ is the mass of the sun). We find that for a wave of mass $(0.03\ \text{cm})^{-1}$ to be produced the two black holes would have to be at 16 cm from another. Clearly this is again well within the horizon of any astrophysical black holes and a reliable simulation will require a challenging numerical investigation. In any case, our results demonstrate

that massive spin-2 waves can be produced in the merger of astrophysical objects such as black holes and this effect must be taken into account in future numerical studies. Clearly the massive modes will only be produced in the final stage of the inspiral process at the time of the merger and ringdown. This represents a unique opportunity to probe quantum gravity with astrophysical events in a fully non-speculative manner.

Let us emphasize at this stage that we have considered binary systems in the Newtonian regime. Our main motivation was to demonstrated that first principle quantum gravitational calculations are possible. It is, however, clear that the leading order correction that we have considered here cannot be trusted in the insprial process when two astrophysical objects reach very short distances and higher order post-Newtonian corrections or, more likely, a full numerical general relativity becomes necessary. Let us also stress that we have considered the most optimistic case scenario, still compatible with the Eöt-Wash experiment, by studying masses for the new fields of the order of $(0.03 \text{ cm})^{-1}$. However, the masses of these new fields could be anywhere between $(0.03 \text{ cm})^{-1}$ and the inverse Planck length or some $(1.6 \times 10^{-35} \text{ m})^{-1}$. If numerical studies managed to consider distances equal or shorter to $(0.03 \text{ cm})^{-1}$, then gravitational signals from binary system would enable one to probe quantum gravity more accurately than the Eöt-Wash experiment.

As mentioned previously, such short distances are well within the Schwarzschild radius of any astrophysical body. This implies that mergers of neutron stars are unlikely to enable one a probe of quantum gravity. On the other hand, depending on how we think of black holes, binary systems of such objects might enable one to probe very short distance. Astrophysical black holes are the end product of the gravitational collapse of matter such as e.g. stars. Under such a collapse, matter falls towards the singularity but we expect quantum physics to smear out the singularity. In that sense, one expects the gravitational collapse of matter to lead to a very dense ring of matter at the center of the black hole. We can thus think of a black hole as an extremely dense object with matter concentrated within a Planck length of the center of the black hole. The horizon itself is not a physical object, a falling observer never notices that he passes through the horizon. It is simply a reaction of space-time to the presence of the very dense core of the black hole. While physical phenomena taking place within the horizon cannot be observed directly by an external observer, the horizon would react to a change in the matter distribution inside such an horizon. We can thus think of a black hole merger as the merger of two extremely dense astrophysical bodies. When the two dense cores get close enough, a common horizon

forms, this common horizon will keep on evolving as a the two cores continue to move towards each other inside the common horizon. This is not the standard picture which usually solely focusses on the dynamics of the horizon (indeed numerical studies usually excise space-time inside the horizon), but it must be equivalent. On the other hand, thinking of black holes as extremely dense core objects with an horizon that is a response of space-time to this dense center would enable one to study extremely short distance physics, potentially up to the Planck length. This is not doable in standard numerical studies which artificially remove the inside of black holes, purely for technical reasons. The feasibility of this alternative approach will be investigated elsewhere.

While we discussed the production of the massive waves in the context of astrophysical processes, it is also possible to envisage the production of these new quantum gravitational massive classical modes during first order phase transitions if such phases took place early on in the cosmological evolution of our universe. Clearly, the occurrence of a first order phase transition in the early universe is a speculative topic as there is no such phase transition within the electroweak standard model. Our work represents an additional complication for the study of early universe phase transitions as beyond the massless gravitational waves, the new massive modes could be produced. Indeed, the collision of bubbles and damping of plasma inhomogeneities could have generated a stochastic background of massive gravitational waves beyond the massless ones that are expected. This implies that some of the energy of these processes could be lost in massive modes. This fact has been overlooked so far when doing simulations for LISA [233].

Tests of quantum gravity often focus on exotic possibilities [234] such as the presence of Lorentz violation effects [235] or other kinds of symmetry breaking. In the case of gravitational waves, different extensions of general relativity [236, 237, 238, 239] have been considered. In this paper, we have shown that there are model independent predictions of quantum gravity which can be searched for in experiments or in observations. The main prediction is the existence of two new classical states namely a massive spin-2 and massive spin-0 classical fields. The phenomenology of these fields is clear, their interactions with matter is fixed by the underlying theory of quantum gravity. The only unknown parameters are their masses. It is thus essential to study these states and hopefully to discover them in an experiment or observation. This program is extremely conservative as any theory of quantum gravity must at least contain these two new states beyond massless gravitational waves. While we cannot calculate their masses from first principles, we have shown that there are bounds on the masses of these new classical fields. This approach to

quantum gravity opens up new directions to understand dark matter and inflation which could be emergent, i.e., purely classical, phenomena.

Acknowledgements

Chapter 10

Dark Matter in Quantum Gravity

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We show that quantum gravity, whatever its ultra-violet completion might be, could account for dark matter. Indeed, besides the massless gravitational field recently observed in the form of gravitational waves, the spectrum of quantum gravity contains two massive fields respectively of spin 2 and spin 0. If these fields are long-lived, they could easily account for dark matter. In that case, dark matter would be very light and only gravitationally coupled to the standard model particles.

While finding a unified theory of quantum field theory and general relativity remains an elusive goal, much progress has been done recently in quantum gravity using effective field theory methods [145, 218, 57, 219, 159, 160, 58, 214, 197, 192, 220, 171, 191, 172, 59]. This approach enables one to perform model independent calculations in quantum gravity. The only restriction is that only physical processes taking place at energy scales below the Planck mass can be considered. This restriction is, however, not very constraining as this is the case for all practical purposes in particle physics, astrophysics and cosmology.

In this paper, we show that quantum gravity could provide a solution to the long standing problem of dark matter. There are overwhelming astrophysical and cosmological evidences that visible matter only constitutes a small fraction of the total matter of our universe and that most of it is a new form of non-relativistic dark matter which cannot be accounted for by the standard model of particle physics. Gravity could account for dark matter in two forms. The first gravitational dark matter candidates are primordial black holes, see e.g. [240] for a recent review. They have been investigated for many years, and although the mass range for such objects to account for dark matter has shrunk quite a bit, they remain a viable option for dark matter, in particular Planckian mass black hole remnants are good dark matter candidates. Here we discuss a second class of candidates within the realm on quantum gravity. Recent work in quantum gravity has established in a model independent way that the spectrum of quantum gravity involves, beyond the massless gravitational field already observed in the form of gravitational waves, two new massive fields [171]. Their properties can be derived from the effective action for quantum gravity. We will show here that these new fields are ideal dark matter candidates.

Deriving an effective action for quantum gravity requires starting from general relativity and integrating out fluctuations of the graviton. Doing so, we obtain a classical effective action given at second order in curvature by

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2}M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + c_4 \Box \mathcal{R} \right. \tag{10.1}$$
$$\left. - b_1 \mathcal{R} \log \frac{\Box}{\mu_1^2} \mathcal{R} - b_2 \mathcal{R}_{\mu\nu} \log \frac{\Box}{\mu_2^2} \mathcal{R}^{\mu\nu} - b_3 \mathcal{R}_{\mu\nu\rho\sigma} \log \frac{\Box}{\mu_3^2} \mathcal{R}^{\mu\nu\rho\sigma} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right] ,$$

where $\mathcal{R}$, $\mathcal{R}_{\mu\nu}$ and $\mathcal{R}_{\mu\nu\rho\sigma}$ are respectively the Ricci scalar, the Ricci tensor and the Riemann tensor. The cosmological constant is denoted by Λ_C. The scales μ_i are renormalization scales which in principle could be different, we shall however take $\mu_i = \mu$. The Lagrangian L_{SM} contains all of the matter we know of and $M_\star$ is the energy scale up to which we can trust the effective field theory. The term $\Box \mathcal{R}$ is a total derivative and thus does not contribute to the equation of motions.

Remarkably, the values of the parameters b_i are calculable from first principles and are model independent predictions of quantum gravity, see e.g. [228] and references therein. They are related to the number of fields that have been integrated out. The non-renormalizability of the effective action is reflected in the fact that we cannot predict the coefficients c_i which, in this framework, have to be measured in experiments or observations. There will be new c_i appearing at every order in the curvature expansion performed when deriving this effective action and we thus would have to measure an infinite number of parameters. Despite this fact, the effective theory leads to falsifiable predictions as the coefficients b_i of non-local operators are, as explained previously, calculable.

In [220, 59], it was shown how to identify the new degrees of freedom by finding the poles of the Green's function obtained by varying the linearized version of the action given in Eq. (10.1) with respect to the metric. Besides the usual massless pole, one finds two pair of complex poles. The complex pole for the massive spin-2 object is given by

$$m_2^2 = \frac{2}{(b_2 + 4b_3)\kappa^2 W\left(-\frac{2\exp\frac{-c_2}{(b_2+4b_3)}}{(b_2+4b_3)\kappa^2\mu^2}\right)} , \tag{10.2}$$

while that of the massive spin-0 reads

$$m_0^2 = \frac{-1}{(3b_1 + b_2 + b_3)\kappa^2 W\left(\frac{\exp\frac{-3c_1-c_2}{(3b_1+b_2+b_3)}}{(3b_1+b_2+b_3)\kappa^2\mu^2}\right)} , \tag{10.3}$$

where $W(x)$ is the Lambert function and $\kappa^2 = 32\pi G$, G is Newton's constant. The b_i for the graviton are known: $b_1 = 430/(11520\pi^2)$, $b_2 = -1444/(11520\pi^2)$ and $b_3 = 434/(11520\pi^2)$. The b_i are thus small and unless the c_i are large, the masses m_2 and m_0 will be close to the Planck mass M_P and the corresponding fields will decay almost instantaneously [171]. As we are interested in the case where the new fields are light, it is useful to consider the limit where the c_i (or one of them at least) are large and $b_i \ll c_i$. In that case we can rewrite the masses as

$$m_2^2 = -\frac{2}{\kappa^2 c_2} - i\pi\frac{2}{\kappa^2 c_2^2}(b_2 + 4b_3) , \tag{10.4}$$

so we need to pick $c_2 < 0$ and

$$m_0^2 = \frac{1}{\kappa^2(3c_1 + c_2)} - i\pi\frac{1}{\kappa^2(3c_1 + c_2)^2}(3b_1 + b_2 + b_3) , \tag{10.5}$$

where we assumed that the renormalization scale $\mu \sim 1/\kappa$, i.e. we assume that the effective field theory is valid up to the reduced Planck scale. As done in [171], we can identify the mass and width of the respective field using $m_i^2 = (M_i - i\Gamma_i/2)^2$. Note that the complex

conjugate solutions $m_2^\star$ and $m_0^\star$ which lead to a positive sign between the mass and the width in the propagator can be eliminated by a proper choice of the contour integral, i.e. of boundary conditions[172], in full analogy with the usual $i\epsilon$ procedure which enables one to select the causal behavior of the Green's function.

We can now express the width in terms of the mass of the field. For the massive spin-2 field k, we find

$$M_2 = \sqrt{\frac{2}{c_2}} \frac{M_P}{2} \, , \tag{10.6}$$

$$\Gamma_2 \approx \frac{(b_2 + 4b_3)\pi}{\sqrt{2c_2^3}} M_P = \frac{73 M_2^3}{360\pi\sqrt{2}M_P^2} \, , \tag{10.7}$$

and for the massive spin-0 field σ, one has

$$M_0 \approx \sqrt{\frac{1}{(3c_1 + c_2)\kappa^2}} = \sqrt{\frac{1}{(3c_1 + c_2)}} \frac{M_P}{2} \, , \tag{10.8}$$

$$\Gamma_0 \approx \frac{(3b_1 + b_2 + b_3)\pi}{2\sqrt{(3c_1 + c_2)^3}} M_P = \frac{7 M_0^3}{72\pi M_P^2} \, , \tag{10.9}$$

where $M_P = 2.435 \times 10^{18}$ GeV is the reduced Planck mass. The widths Γ_0 and Γ_2 are the gravitational widths for the decay of the massive spin-2 and spin-0 classical modes into the classical graviton.

To obtain the total width, we need to include the decay modes into particles of the standard model. The coupling of the two states to the standard model Lagrangian has been worked out in [59]. One has

$$S = \int d^4x \left[\left(-\frac{1}{2} h_{\mu\nu} \Box h^{\mu\nu} + \frac{1}{2} h_\mu{}^\mu \Box h_\nu{}^\nu - h^{\mu\nu} \partial_\mu \partial_\nu h_\alpha{}^\alpha + h^{\mu\nu} \partial_\rho \partial_\nu h^\rho{}_\mu \right) \right. \tag{10.10}$$
$$+ \left(-\frac{1}{2} k_{\mu\nu} \Box k^{\mu\nu} + \frac{1}{2} k_\mu{}^\mu \Box k_\nu{}^\nu - k^{\mu\nu} \partial_\mu \partial_\nu k_\alpha{}^\alpha + k^{\mu\nu} \partial_\rho \partial_\nu k^\rho{}_\mu \right.$$
$$\left. - \frac{M_2^2}{2} \left(k_{\mu\nu} k^{\mu\nu} - k_\alpha{}^\alpha k_\beta{}^\beta \right) \right)$$
$$\left. + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{M_0^2}{2} \sigma^2 - \sqrt{8\pi G_N}(h_{\mu\nu} - k_{\mu\nu} + \frac{1}{\sqrt{3}} \sigma \, \eta_{\mu\nu}) T^{\mu\nu} \right].$$

We thus see that besides decaying gravitationally, the massive spin-2 and spin-0 fields can decay to standard model particles. It is straightforward to calculate the decay widths of the new massive modes into standard model particles using the results of [241].

The decay width of the scalar mode σ into massive vectors fields V, such as the W and Z bosons, is given by

$$\Gamma(\sigma \to VV) = \delta \frac{M_0^3}{48\pi M_P^2} \left(1 - 4r_V \right)^{1/2} \left(1 - 4r_V + 12r_V^2 \right), \tag{10.11}$$

where $\delta = 1/2$ for identical particles and $r_V = (m_V/M_0)^2$. The decay width of σ into fermions is given by

$$\Gamma(\sigma \to \bar{f}f) = \frac{m_f^2 M_0 N_c}{24\pi M_P^2} \left(1 - 4r_f\right)^{1/2} \left(1 - 2r_f\right) \tag{10.12}$$

with $r_f = (m_f/M_0)^2$ and $N_C = 3$ if the fermions are quarks. While σ couples to the trace of the energy-momentum tensor of the standard model and it thus does not couple to massless gauge bosons at tree level, it will couple to the photon and the gluons at one loop. In particular the decay width into two photons is given by [242, 243]

$$\Gamma(\sigma \to \gamma\gamma) = \frac{\alpha_{EM}^2 M_0^3 N_c}{768\pi^3 M_P^2} \, |c_{EM}|^2, \tag{10.13}$$

where $\alpha_{EM}=1/137$ and $c_{EM}=11/3$ if ϕ is lighter than all the fermions of the standard model. The decay width of σ into a pair of Higgs bosons is given by

$$\Gamma(\sigma \to hh) = \frac{M_0^3}{48\pi M_P^2} \left(1 - 4r_h\right)^{1/2} \left(1 + 2r_h\right)^2, \tag{10.14}$$

where $r_h = (m_h/M_0)^2$.

It is also straightforward to calculate the partial decay widths of the spin-2 object k. Its partial width to massless vector fields is given by

$$\Gamma(k \to VV) = N \frac{M_2^3}{80\pi M_P^2}, \tag{10.15}$$

where $N=1$ for photons and $N = 8$ for gluons. In the case of massive massive vector fields, one has

$$\Gamma(k \to VV) = \delta \frac{M_2^3}{40\pi M_P^2} \sqrt{1 - 4r_V} \left(\frac{13}{12} + \frac{14}{3}r_V + \frac{4}{13}r_V^2\right), \tag{10.16}$$

where $\delta = 1/2$ for identical particles, $r_V = m_V^2/M_2^2$. For the decay to fermions, we find

$$\Gamma(k \to \bar{f}f) = N_C \frac{M_2^3}{160\pi M_P^2} \left(1 - 4r_f\right)^{3/2} \left(1 + \frac{8}{3}r_f\right), \tag{10.17}$$

where $r_f = m_f^2/M_2^2$ and, as previously, $N_C = 3$ if the fermions are quarks. In the case of a decay to the Higgs boson, the partial decay width is given by

$$\Gamma(k \to hh) = \frac{M_2^3}{430\pi M_P^2} \left(1 - 4r_h\right)^{5/2}, \tag{10.18}$$

where $r_h = m_h^2/M_2^2$.

If the massive spin-0 and spin-2 fields are components of the dark matter content of the universe nowadays, their masses have to be such that none of these partial decay widths should enable these fields to decay faster than the current age of the universe. From the

requirement that the lifetime of the spin-0 σ is longer than current age of the universe, we can thus get a bound on c_2 using the gravitational decay width. We find

$$\tau = 1/\Gamma = 7.2 \times 10^{-17}\sqrt{c_2^3}\ \mathrm{GeV}^{-1} > 13.77 \times 10^9 \mathrm{y} \qquad (10.19)$$

and thus $c_2 > 4.4 \times 10^{38}$. The same reasoning leads to a similar bound on $3c_1 + c_2$. We can then deduce a maximal mass for the dark matter candidate, $M_0 < 0.16$ GeV. Note that Eöt-Wash [227] implies $c_2 < 10^{61}$, we thus have a bound $4.4 \times 10^{38} < c_2 < 10^{61}$ and 1×10^{-12} GeV $< M_0 < 0.16$ GeV. Again a similar bound applies to the combination $3c_1 + c_2$ and thus to M_2. Clearly such light dark matter candidates could not decay to the massive gauge bosons of the standard model, its charged leptons such as the electron or the quarks. They could however decay to gluons (during the deconfinement phase of the early Universe), photons and potentially neutrinos. The decay to photons might be of astrophysical relevance and could be observable by gamma-ray experiments. Note, however, that decay widths of the dark matter candidates to photons are smaller than the respective gravitational ones. It is also worth mentioning that the decay to neutrinos can be as rapid as the gravitational modes if again neutrino masses are low enough.

While we have established that quantum gravity provides two new candidates for dark matter, it remains to investigate their production mechanism. Thermal production is a possibility, but we would have to consider all higher order operators as we would need to consider temperatures larger than the Planck mass $T \geq M_P$ since these objects are gravitationally coupled to all matter fields. Also we may not want to involve temperatures above the inflation scale which we know is at most 10^{14} GeV. The weakness of the Planck-suppressed coupling hints at the possibility of out-of-equilibrium thermal production as argued in [244]. However, the mass range allowed for the dark matter particles within that framework is given by TeV$< m_{DM} < 10^{11}$ GeV [244] and it is not compatible with our ranges for the masses of our candidates. The fact that our dark matter candidates are light points towards the vacuum misalignment mechanism, see e.g. [245]. Indeed, in an expanding universe both σ and k have an effective potential in which they oscillate. The amount of dark matter produced by this mechanism becomes simply a randomly chosen initial condition for the value of the field in our patch of the universe. In [246], it was shown that the vacuum misalignment mechanism leads to the correct dark matter abundance $\rho_{DM} = 1.17$ keV/cm^3 if the dark matter field takes large values in the early universe. For example, a dark matter field with a mass in the eV region would need to take values of the order of 10^{11} GeV to account for all of the dark matter in today's universe [246].

In summary, we have shown that gravity, when quantized, provides new dark matter candidates. As these fields must live long enough to still be around in today's universe their masses must be light otherwise they would have decayed long ago. It is quite possible that gravity can account for all of dark matter in the form of primordial black holes and the new fields discussed in this paper without the need for new physics.

Acknowledgements

Chapter 11

Fab Four Effective Field Theory Treatment

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The article addresses the John interaction from Fab Four class of Horndeski models from the effective field theory point of view. Models with this interaction are heavily constrained by gravitational wave speed observations, so it is important to understand, if these constraints hold in the effective field theory framework. We show that John interaction induces new terms quadratic in curvature at the level of the effective (classical) action. These new terms generate additional low energy scalar and spin-2 gravitational degrees of freedom. Some of them have a non-vanishing decay width and some are ghosts. Discussion of these features is given.

Introduction

Modified gravity encompasses a broad range of models. Traditionally such models are classified according to the type of modification. For example, models with an additional scalar field are called scalar-tensor gravity; models whose Lagrangian is a continuous function of the scalar curvature R are called $f(R)$ gravity, etc [49]. One can also classify modified gravity models by their particle content. General Relativity (GR) describes massless spin-2 particles (i.e. gravitons) interacting both with matter and themselves. Modified gravity models change the standard GR content by adding new physical fields [230, 130]. Scalar-tensor models and $f(R)$ gravity serve as the simplest example of such a modification, as they introduce an additional spin-0 particle in the model particle spectrum [107, 247, 248]. In matter of fact, it is possible to map an $f(R)$-gravity model onto a scalar-tensor model with the Brans-Dicke parameter $\omega_{\rm BD} = 0$ [107, 124, 123]. Therefore $f(R)$ gravity and scalar-tensor models should not be treated as completely independent theories.

Clearly, the simplest way to modify GR is to introduce an additional scalar degree of freedom (DOF). However, the new DOF should not be introduced in an arbitrary manner. The action describing the new DOF must produce second order field equations as a higher derivative action either introduces ghost instabilities or describes additional DOFs (up to a proper reparametrization).

Scalar-tensor models with second order field equations are given by the Horndeski Lagrangian [131] in the Generalized Galileons parameterization [122]:

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \ , \tag{11.1}$$

$$
\begin{aligned}
\mathcal{L}_2 &= G_2 \ , \\
\mathcal{L}_3 &= -G_3 \Box X \ , \\
\mathcal{L}_4 &= G_4 R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \ , \\
\mathcal{L}_5 &= G_5 G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right] \ .
\end{aligned}
\tag{11.2}
$$

Here G_2, G_3, G_4, and G_5 are functions depending on the scalar field ϕ and the standard kinetic term $X = 1/2 \ \partial_\mu\phi\partial^\mu\phi$; G_{4X} and G_{5X} are correspondent derivatives with respect to X; $G_{\mu\nu}$ is the Einstein tensor. It is worth noting that only $\mathcal{L}_4$ and $\mathcal{L}_5$ describe a non-standard interaction between gravity and the scalar field, while terms $\mathcal{L}_2$ and $\mathcal{L}_3$ describe the scalar field self interaction.

Horndeski models contain a special subclass called Fab Four [134] which is defined by its ability to screen the cosmological constant. To be exact, Fab Four models completely

screen an arbitrary cosmological constant on the Freedman-Robertson-Walker background and such a screening holds even if the cosmological constant experience a finite shift. Fab Four class is given by the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{John}} + \mathcal{L}_{\text{George}} + \mathcal{L}_{\text{Ringo}} + \mathcal{L}_{\text{Paul}} \ , \tag{11.3}$$

$$\begin{aligned}
\mathcal{L}_{\text{John}} &= V_J(\phi) \ G_{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi \ , \\
\mathcal{L}_{\text{George}} &= V_G(\phi) \ R \ , \\
\mathcal{L}_{\text{Ringo}} &= V_R(\phi) \ \hat{G} \ , \\
\mathcal{L}_{\text{Paul}} &= V_P(\phi) \ P^{\mu\nu\alpha\beta}\nabla_{\mu}\phi\nabla_{\alpha}\phi\nabla_{\nu}\nabla_{\beta}\phi \ .
\end{aligned} \tag{11.4}$$

Here $\hat{G}$ is the Gauss-Bonnet term, $P^{\mu\nu\alpha\beta} = -1/2 \ \varepsilon^{\alpha\beta\lambda\tau}R_{\lambda\tau\sigma\rho}\varepsilon^{\sigma\rho\mu\nu}$ is the double-dual Riemann tensor, and V_J, V_G, V_R, V_P are interaction potentials. The following features of this class should be noted. The Ringo term alone does not screen the cosmological constant, it just does not ruin the screening. The George term introduces a Brans-Dicke-like coupling which may not be enough to support screening in a particular setting [134]. Therefore only the John and Paul terms drive the screening. Finally, the Paul term demonstrates a pathological behavior in star-like objects [135, 136]. Therefore, the John term is the most relevant term for the cosmological constant screening.

A combination of the John term and beyond Fab Four terms allows one to construct a model which may provide an adequate description of both the cosmological expansion while keeping the cosmological constant small. When the John term is the leading contribution, there is a screening of the cosmological constant; when the leading contribution is provided by beyond Fab Four terms the model losses its screening properties and develops a small cosmological constant. A particular example of such a model is given in [137] by the following action (we use different notations):

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}\left(R - 2\Lambda\right) + \frac{1}{2} \ g^{\mu\nu} \ \nabla_{\mu}\phi\nabla_{\nu}\phi + \beta \ G^{\mu\nu}\nabla_{\mu}\phi\nabla_{\nu}\phi\right] \ . \tag{11.5}$$

In full agreement with the aforementioned logic the model describes both inflation and the late-time accelerated expansion of the universe.

Recent direct detection of gravitational waves (GW) [224, 225, 226, 249, 249, 250] and the measurement of the GW speed [74, 75] allow one to establish sever constraints on Horndeski models [251, 252]. The authors of [251, 252] considered the propagation of tensor perturbations on a cosmological background in Horndeski models and identified them with GWs detected in the terrestrial experiment. Due to the structure of Horndeski

models the speed of perturbations strongly depends on G_4 and G_5, so in order to obtain a model with the constant GW speed one must put the following constraints on the Horndeski parameters:

$$G_{4X} = 0 \,, \qquad\qquad\qquad G_5 = \text{const}\,. \qquad\qquad (11.6)$$

Constraints (11.6) would rule out the John interaction (and Fab Four in general) from the list of relevant scalar-tensor models.

In this paper we indirectly address constraints (11.6) and their role in the context of the effective field theory treatment of gravity. We claim that although the constraints (11.6) hold for the classical Horndeski action (11.1), one cannot use the action (11.1) as a coherent effective action. One must introduce Horndeski interaction at the level of the fundamental action of the model and restore the form of the effective (i.e. classical) gravity action. We present a derivation of such an effective action in the following section. As the effective action does not match the Horndeski action (11.1), the role of the constraints (11.6) should be reconsidered. We discuss this issue in the last section.

Effective Field and John Interaction

The standard Effective Field Theory (EFT) technique is based on the following premise [146]. Let us assume that one has a physical system containing heavy h and light l degrees of freedom described by a fundamental action $\mathcal{A}[l, h]$. In the low energy regime, i.e. when energies are below the heavy degree of freedom (DOF) mass scale, the system is described by an effective action $\Gamma[l]$ given in terms of the light DOF only. In order to obtain the effective action one must integrate out the heavy DOF:

$$\int \mathcal{D}[l] \exp\left[i\,\Gamma[l]\,\right] = \int \mathcal{D}[l]\mathcal{D}[h] \exp\left[i\,\mathcal{A}[l, h]\,\right]\,. \qquad (11.7)$$

If the fundamental action is unknown, one can restore its form, as it must contain all terms permitted by general covariance, conservation laws, and other fundamental physical principles.

A similar logic holds for gravity models [145, 141]. One assumes, that gravity is described by some fundamental action $\mathcal{A}[\mathfrak{g}]$ which is given in terms of the metric $\mathfrak{g}$ describing behavior of the true quantum gravitons propagating over some background spacetime g. In order to obtain an effective action, one must integrate out all quantum gravitons:

$$\exp\left[i\Gamma[g_{\mu\nu}]\right] = \int \mathcal{D}[\mathfrak{g}] \exp\left[i\mathcal{A}[g_{\mu\nu} + \mathfrak{g}_{\mu\nu}]\right]. \qquad (11.8)$$

The effective action $\Gamma[g]$ is given in terms of the classical metric $g_{\mu\nu}$ which is generated by the underlying dynamic of quantum gravitons. Let us emphasis, that in such an implication one must integrate out light DOF unlike the before-mentioned case. Moreover, the fundamental action $\mathcal{A}$ must contain data on the matter content of the Universe, as it describes both gravity and its interaction with matter.

Despite the fact that the fundamental action for gravity is unknown, one can restore the form of the effective action. One must include R^2 and $R^2_{\mu\nu}$ terms in Γ, as the correspondent operators are generated at the level of the first matter loop [51]. Moreover, one must include nonlocal operators [159, 160, 58], as one can sum up an infinite series of matter loops in the graviton propagator. We do not discuss this feature in details, as it lies beyond the scope of this paper and it was covered in details in [145, 141, 159, 230, 58]. The standard approach to modified gravity is to consider only the classical action describing gravity without respect to the underlying quantum dynamic of the gravitational field [49, 251, 252]. This approach is coherent and appears to be fruitful in modified gravity. Within the EFT framework one is obliged to consider the classical action as the effective action, so it cannot be taken arbitrary.

In such a way we claim that if the effective (classical) action of gravity contains Horndeski interactions, then the fundamental action for gravity also must contain a Horndeski sector and vice versa. In this paper we address a particular action (11.5) as it is well motivated. We consider (11.5) as a part of the fundamental gravity action. In order to derive interaction rules we expand the action over a flat background:

$$\mathcal{A} = \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \mathcal{O}_{\mu\nu\alpha\beta} h^{\alpha\beta} - \frac{1}{2} \phi \,\Box\, \phi - \frac{\kappa}{4} h^{\mu\nu} C_{\mu\nu\alpha\beta} \partial^\alpha \phi \, \partial^\beta \phi \right. \tag{11.9}$$

$$\left. + \frac{\kappa}{2} \beta \left[\partial_\mu \partial^\sigma h_{\nu\sigma} + \partial_\nu \partial^\sigma h_{\mu\sigma} - \partial_\mu \partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu} (\partial_\alpha \partial_\beta h^{\alpha\beta} - \Box h) \right] \partial^\mu \phi \partial^\nu \phi \right] .$$

Correspondent Feynman rules are presented in the Appendix. An expansion about the flat background is due, as we are interested in the local effects. The presence of the cosmological constant can be neglected for the sake of simplicity, as it strongly affects only large-scale physics.

Following the standard procedure presented in [145, 141, 146] one can calculate the effective action based on the form of the fundamental action (11.5). The effective action

is given by the following:

$$\Gamma = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \beta\, G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right. \tag{11.10}$$

$$\left. + c_1 R^2 + c_2 R_{\mu\nu}^2 + \bar{c}_1 R \beta \Box R + \bar{c}_2 R_{\mu\nu} \beta \Box R^{\mu\nu} + \bar{\bar{c}}_1 R (\beta \Box)^2 R + \bar{\bar{c}}_2 R_{\mu\nu} (\beta \Box)^2 R^{\mu\nu} \right].$$

Here c_1, c_2, $\bar{c}_1$, $\bar{c}_2$, $\bar{\bar{c}}_1$, and $\bar{\bar{c}}_2$ are unknown dimensionless constants. First three terms in (11.10) appear in the effective action due to the fact that they present in the fundamental action. Terms R^2 and $R_{\mu\nu}^2$ appear due to the first matter loop associated with the standard interaction between gravity and matter [51]. Terms with $\beta \Box$ appear due to the presence of the John interaction and its contribution to the first order matter loop (correspondent diagrams are given in the Appendix (11.20),(11.21)). Strictly speaking, the part of action (11.10) containing the scalar field belongs to the Horndeski class, but the action also has terms missing in the Horndeski action. This implies that existing constraints (11.6) cannot be directly applied in such a framework. The dynamic of tensor perturbations over a cosmological background described by (11.10) differs from the one described by the Horndeski action, so the proper constraints should be found. This however doesn't ruin the constraint's applicability outside the EFT framework and they must be used to constraint Horndeski models in the original modified gravity framework. The effective action (11.10) itself requires a further analysis.

In full analogy with the classical results [53, 54] (see also [253] for a more detailed derivation) higher derivative terms change the content of the model. Terms R^2 and $R_{\mu\nu}^2$ introduce additional massive spin-2 and spin-0 degrees of freedom. Following the algorithm presented in [253] one can calculate a propagator of gravity modes given by the effective action (11.10):

$$G_{\mu\nu\alpha\beta}(k) = \frac{1}{k^2} \left[P^1_{\mu\nu\alpha\beta} + \frac{P^2_{\mu\nu\alpha\beta}}{1 + \frac{1}{2} \kappa^2 k^2 (c_2 - \bar{c}_2 \beta k^2 + \bar{\bar{c}}_2 (\beta k^2)^2)} \right. \tag{11.11}$$

$$- \frac{1}{2} \frac{P^0_{\mu\nu\alpha\beta} + \bar{\bar{P}}^0_{\mu\nu\alpha\beta}}{1 - \kappa^2 k^2 (3c_1 + c_2 - (3\bar{c}_1 + \bar{c}_2)\beta k^2 + (3\bar{\bar{c}}_1 + \bar{\bar{c}}_2)(\beta k^2)^2)}$$

$$\left. - \frac{1}{2} \frac{\left[3 - 4(1 - \kappa^2 k^2 (3c_1 + c_2) - (3\bar{c}_1 + \bar{c}_2)\beta k^2 + (3\bar{\bar{c}}_1 + \bar{\bar{c}}_2)(\beta k^2)^2) \right]}{1 - \kappa^2 k^2 (3c_1 + c_2 - (3\bar{c}_1 + \bar{c}_2)\beta k^2 + (3\bar{\bar{c}}_1 + \bar{\bar{c}}_2)(\beta k^2)^2)} \bar{P}^0_{\mu\nu\alpha\beta} \right].$$

In this expression operators P^2, P^0, $\bar{P}^0$, and $\bar{\bar{P}}^0$ are taken directly from [253]. Each pole in the propagator corresponds to a new particle state. Because of the similarity

between action (11.10) and the well-known Stelle action [53, 54, 253], it can be seen that propagator (11.11) describes additional scalar and spin-2 particles. The denominator of the propagator is a forth order polynomial in k^2, which means that the propagator has four complex poles. With the use of the fundamental theorem of algebra one can establish, that poles for the spin-0 mode are located in points $k^2 = \pm m_0^2$, $\pm i m_0^2$, where m_0 is a real constant; a similar statement holds for the spin-2 mode: $k^2 = \pm m_2^2$, $\pm i m_2^2$, where m_2 is another real constant. These poles corresponds to massive scalar, massive spin-2 particle, massive scalar ghost, massive spin-2 ghost, two massive spin-0 particles with non-zero decay width, and two massive spin-0 particles with non-zero decay width.

These results follow the standard EFT logic [145, 141, 159, 230]. The presence of ghost states in such models is a well-known feature. The appearance of new massive states is also a typical feature of EFT models discovered in the classical papers [53, 54]. Therefore the effective model has the standard EFT features and can be considered alongside the regular EFT models.

Discussion and conclusion

In this paper we have used effective field theory techniques to restore a form of the classical gravity action. We used a particular Horndeski model [137] as a part of the fundamental gravity action in order to generate the effective action. Such an approach is necessary to study scalar-tensor gravity models and modified gravity in general. We obtained the effective action (11.10) generated by the fundamental action (11.5). This fundamental action contains higher derivative terms, which leads to the following consequences.

First of all, the constraints (11.6) obtained in [251, 252] cannot be used within EFT framework. These constraints (11.6) were obtained from a study of tensor perturbations in Horndeski models, however the effective action (11.10) differs from the Horndeski action (11.1) and thus the dynamics of tensor perturbations is also different. Therefore the constraints (11.6) do not hold in the EFT framework, although this does not affect their relevance for the classical modified gravity framework.

Secondly, we analyzed the content of the effective action. The particle spectrum of the model is given by the propagator of the low energy gravity perturbations (11.11). The new low energy degrees of freedom are a massive scalar particle, a massive spin-2 particle, a massive scalar ghost, a massive spin-2 ghost, two massive scalar particles with non-vanishing decay width, and two massive spin-2 particles with non-vanishing decay width. The presence of ghost states and states with non-zero decay width is typical for

models of such kind [54, 145, 141, 160, 58, 197]. We prefer not to draw any conclusion on the relevance of the model based on the fact that it contains ghosts, as the issue is typical for a number of before-mentioned effective field models of gravity. We, however, argue that the presence of new gravitational degrees of freedom must affect late stages of GW production during the last stages of binary systems coalescence, as was shown in [59].

Summarizing all the results we make the following conclusions. The existence of non-trivial Horndeski interaction at the level of the fundamental action induces non-trivial corrections to low energy gravitational phenomena. The effective model discussed in this paper provides the simplest example of such phenomena. This model shares problems typical of all gravitational EFT. Finally, some well-known constraints on Horndeski models (11.6) cannot be applied to it. We finish by emphasizing that this model seems to be a rather special modification of the standard gravity EFT.

Appendix

Operator $\mathcal{O}$ used in (11.9) is given by the following expression:

$$
\begin{aligned}
\mathcal{O}_{\mu\nu\alpha\beta} = \frac{1}{2} \left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} \right) \Box - \eta_{\mu\nu}\eta_{\alpha\beta}\Box + \left(\partial_\mu\partial_\nu\eta_{\alpha\beta} + \partial_\alpha\partial_\beta\eta_{\mu\nu} \right) \\
- \frac{1}{2} \left(\partial_\alpha\partial_\mu\eta_{\beta\nu} + \partial_\alpha\partial_\nu\eta_{\beta\mu} + \partial_\beta\partial_\mu\eta_{\alpha\nu} + \partial_\beta\partial_\nu\eta_{\alpha\mu} \right) \ .
\end{aligned}
\tag{11.12}
$$

Action (11.9) generates the following Feynman rules for propagators:

$$
\text{\raisebox{0pt}{$\sim\!\!\sim\!\!\sim\!\!\sim$}} = \frac{i}{2} \frac{C_{\mu\nu\alpha\beta}}{k^2} \ , \qquad\qquad \text{\raisebox{0pt}{$-\,-\,-\,-$}} = \frac{i}{k^2} \ . \tag{11.13}
$$

Here $C_{\mu\nu\alpha\beta}$ is defined as follows:

$$
C_{\mu\nu\alpha\beta} = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta} \ . \tag{11.14}
$$

For the standard interaction between gravity and the scalar field the correspondent rule reads:
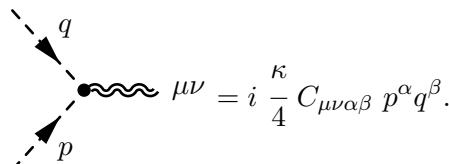
$$
\mu\nu = i\,\frac{\kappa}{4}\,C_{\mu\nu\alpha\beta}\,p^\alpha q^\beta . \tag{11.15}
$$

For John interaction the correspondent rule reads:

$$\text{(diagram)} \quad \mu\nu = i\,\frac{\kappa}{2}\,\beta\,k^2\,\mathcal{M}_{\mu\nu\alpha\beta}(k)\,p^\alpha q^\beta, \tag{11.16}$$

$$\mathcal{M}_{\mu\nu\alpha\beta}(k) \stackrel{\text{def}}{=} \eta_{\mu\nu}\eta_{\alpha\beta} - I_{\mu\nu\alpha\beta} - (\omega_{\mu\nu}\eta_{\alpha\beta} + \eta_{\mu\nu}\omega_{\alpha\beta}) \tag{11.17}$$

$$+ \frac{1}{2}(\omega_{\mu\alpha}\eta_{\nu\beta} + \omega_{\mu\beta}\eta_{\nu\alpha} + \omega_{\nu\alpha}\eta_{\mu\beta} + \omega_{\nu\beta}\eta_{\mu\alpha}),$$

$$\omega_{\mu\nu}(k) \stackrel{\text{def}}{=} \frac{k_\mu k_\nu}{k^2}. \tag{11.18}$$

The existence of the John interaction vertex provides two new one-loop level diagrams. In such a way there are three one loop level diagrams generated by action (11.5). Their divergent parts are evaluated in the dimensional-regularization scheme and read:
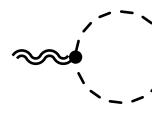
$$\text{(diagram)} \rightarrow \frac{i}{1920\pi^2}\left(R_{\mu\nu}R^{\mu\nu} + \frac{11}{4}R^2\right), \tag{11.19}$$
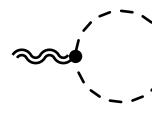
$$\text{(diagram)} \rightarrow \frac{i}{1920\pi^2}\left(R_{\mu\nu}(\beta\Box)R^{\mu\nu} + \frac{11}{4}R(\beta\Box)R\right), \tag{11.20}$$
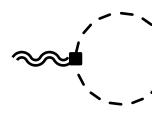
$$\text{(diagram)} \rightarrow \frac{i}{1920\pi^2}\left(R_{\mu\nu}(\beta\Box)^2 R^{\mu\nu} + \frac{11}{4}R(\beta\Box)^2 R\right). \tag{11.21}$$

Acknowledgements

Chapter 12

Conclusion

Theory of gravity plays an important role in contemporary theoretical physics. Development of gravity theory resulted in the creation of GR which is widely accepted as the theory of gravity providing the best fit for all known empirical data. At the same time, well-known phenomena of dark matter, dark energy, and inflation challenge our understanding of gravity and question GR applicability at the large spacial scales and in the high energy regime. Construction of a suitable quantum theory of gravity, on the other hand, challenges not only our understanding of gravity, but also of the quantum theory and the nature of spacetime itself.

Contemporary landscape of gravity theory highlights multiple perspective directions of research. This thesis addresses a narrow set of problems relevant within a framework of effective gravity. As pointed in Chapter 1, the choice of addressed problems is influenced by considerations of consistency.

EFT approach allows one to consistently account for gravitational effects at the level of quantum theory. The effective action for gravity can be restored in a model independent way via symmetry principles [146, 158, 145]. Because of this it accounts for quantum effects predicted by various gravity models. For instance, within string theory the effective gravity action can be recovered and studied via the standard EFT tools [152, 153]. This makes effective gravity a universal approach capable to account for a vast array of models.

The first problem covered by the thesis is related with an implementation of EFT technique to gravity in a context of particle physics. The standard model of particle physics describes all known types of matter and all interactions except gravity. EFT technique allows one to account for the gravitational interaction and search for its manifestation at the level of standard model particles interactions.

Effective gravity admits the existence of non-local operators at the level of one-loop

effective action [159]. The effective action, in turn, describes the gravitational interaction in general and, consequently, accounts for the gravitational sector of the standard model. In Chapter 8 and paper [58] it was shown, that this is, indeed, the case. Effective gravity generates non-local interactions between mater states with all spins. In particular, it generates an effective four-fermion non-local interaction. Empirical data from $\sqrt{s} = 8$ TeV LHC run[85] allows one to constrain the non-local interaction and, consequently, the effective gravity.

Results of paper [58] allow one to constrain the number N of light non-gravitational fields existing in nature. This is due to the fact that the leading contribution to the non-local interaction coupling is defined by the number of light fields of a model. The constraint reads

$$N < 5 \times 10^{61}. \tag{12.1}$$

Equivalently, one can constrain the characteristic energy scale of the non-local operators $M_\star$:

$$M_\star > 3 \times 10^{-11} \text{ GeV}. \tag{12.2}$$

This constraint improves the previous results obtained via terrestrial experiments of Eöt-Wash type [192, 227].

The second problem addressed in this thesis is related with the binary system GWs production. Recent direct detection of GWs for the first time allowed one to test GR in the strong field regime [19, 74, 224]. At the current precision level the empirical data appear to be consistent with GR [19, 20, 21], making a search for beyond GR effects manifesting in binary inspirals more relevant.

Evaluation of GWs production in a binary inspiral within the EFT framework was addressed in Chapter 9 and in paper [59]. Due to complexity and non-linearity of the gravitational interaction analytical calculations can only be performed for the initial stages of an inspiral process. Nonetheless, such calculations define the spacial scale at which gravity may develop deviations from GR.

It is well-known that the effective action for gravity contains one ghost DoF [54, 53, 145, 152]. This is a well-recognised problem that is currently unsolved. Results presented in this thesis shows that consistent calculations of the GWs production in binary inspiral are possible within EFT. As discussed in Chapter 9, within the setup suitable for a study of binary inspirals the ghost states can be understood as states carrying gravitational repulsion. To be exact, at the classical level the effective action with ghost states (9.1)

generates exactly the same field equations as a model without ghost states, but with gravitational repulsion (9.7). This allows one to treat ghost states not as states with a negative kinetic energy, but as states with a positive energy caring gravitational repulsion.

The standard algorithm of GWs production can be implemented in that particular setup [254, 26]. Because of the symmetry a binary system can only produce quadrupole radiation that is carried by spin 2 excitations. The effective gravity contains an additional massive spin 2 excitation alongside with the standard massless spin 2 excitations, i.e. gravitons. Because of this a binary system doubles its GWs production rate as soon as it obtains enough energy to excite the new modes. As discussed in Chapter 9 the mass threshold of new DoFs allows one to define the spacial scale at which a binary system starts to produce the new modes. Mass of the new modes is constrained via the empirical data on Eöt-Wash type experiments [227]. This constraint is used to evaluate the distance d between components of a realistic binary system at which new gravity modes are excited:

$$d \simeq 16 \text{ cm}. \tag{12.3}$$

Relevance of this result is twofold. Firstly, the result highlights a need for empirical study of gravity in the high energy regime. The distance d is much smaller than typical radius of a star or a black hole. Consequently, similar distances can be probed only at the later stages of binary inspirals or, equivalently, in the high energy regime. Secondly, the result points on a necessity of numerical study of effective gravity. The rate of GWs production at the late stages of binary inspirals can only be studied numerically. The distance d highlights the characteristic precision level of numerical calculations that may be suitable to account for beyond GR effects described by the effective gravity. In other words, this result sets a limit on beyond GR quantum effects manifesting at the level of binary systems.

The result also highlights a necessity for an improvement of understanding of the late stages of a binary inspiral. The spacial scale d is much smaller than the characteristic radius of a black hole or a neutron star. Because of this the production of new massive modes can be covered from a distant observer with the event horizon. Consequences of this phenomenon and its influence on the event horizon radius are not well-understood and require further research.

The third problem addressed in Chapter 10 and in paper [60] is related with an opportunity to describe the dark matter within effective gravity. It is well-known that effective gravity predicts the existence of new gravity modes both due to local and non-local operators [159, 58, 160]. An opportunity to associate new DoFs generated by non-local operators

with the dark matter particles is addressed. An introduction of non-local operators to the gravity effective action results in the appearance of new poles in the graviton propagator. Unlike local operators, they generate poles with non-vanishing imaginary parts. Such poles describe unstable particles which decay width is related with the imaginary parts of corresponding poles [255].

If DoFs induced by the non-local operators describe dark matter particles, then it is natural to expect that these particles have a lifetime comparable with the age of the Universe. Otherwise they would decay producing the standard model particles and fail to contribute to the contemporary dark matter content. This line of reasoning allows one to set constraints on the imaginary part of the new poles and, consequently, on coefficients of the effective gravity action. Namely, it is possible to establish a constraint on the number of light particles present in a model. This is due to the fact that the couplings of the non-local operators are proportional to the number of light DoFs. Equivalently, the characteristic energy scale of the non-local interaction can be constrained.

The constraints are discussed in Chapter 10 and paper [60]. Constraints are consistent with the data from Eöt-Wash type experiment, so the opportunity to describe the dark matter within gravity EFT cannot be excluded at the present precision level. Moreover, an exact mechanism generating proposed dark matter candidates is unknown. Therefore further study of this opportunity to describe dark matter is required.

Finally, an application of EFT framework to a modified gravity model is presented in Chapter 11 and paper [61]. Modified gravity models play a special role in gravity theory, as they provide a description of classical gravitational phenomena alternative to GR.

The thesis addresses one particular scalar-tensor model that belongs to the Horndeski gravity. STG models should be viewed as simplest alternatives for GR. They introduce one additional scalar DoF in the gravity sector thereby perform a minimal modification. An arbitrary STG may have higher order field equations which lead to the Ostrogradsky instability [127]. The subclass of STGs that admits second order field equations is known as the Horndeski gravity [131, 122]. In other words, Horndeski gravity should be viewed as the simplest stable extension of GR.

Models of this type are known for their wide phenomenology [62, 104, 50]. Namely, a narrow subclass of the Horndeski gravity known as the Fab Four can screen an arbitrary cosmological constant completely [134]. Moreover, the screening holds even if the cosmological constant experiences a finite shift. Because of this the Horndeski gravity may potentially provide an explanation of the small observed value of the cosmological

constant.

Based on the reasoning presented in Chapter 11 and in paper [137] a model suitable for a realistic description of the cosmological expansion was proposed. The model Lagrangian is given by (3.3) and it belongs to the Horndeski gravity. It contains one term that belongs to the Fab Four class, so the model admits the corresponding cosmological constant screening mechanism. As it was shown in paper [137] in the early universe Fab Four term dominates, the cosmological constant is screened completely, and the Universe experiences inflationary expansion. After the end of inflation the Fab Four term becomes sub-dominant, the screening fails, the model develops a small cosmological constant and enters the late-time accelerated expansion.

Research presented in Chapter 11 and paper [61] is devoted to an implication of EFT technique for the model presented in [137]. Namely, the role of the Fab Four interaction in generation of the effective action is studied. The research follows results [66] where it was shown that quantum effects do not compromise model screening features. This gives grounds for a consistent treatment of the model within the EFT framework.

It is shown that the new interaction responsible for the cosmological constant screening introduces new higher derivative operators at the one-loop effective action. The structure of the effective action is strongly affected by the introduction of the the Fab Four interaction. Namely, the corresponding effective model describes the existence of four new spin 2 DoFs and four new scalars.

This result shows that modified gravity models must not be considered irrelevant in the context of effective gravity. The existence of new higher derivative operators at the one-loop level proves that the structure of the correspondent quantum model, alongside its renormalization group flow, is strongly affected by a specific modification of GR. This gives grounds to expect a strong influence from specific GR modifications on the effects gravity.

In other words, results of this thesis are the following. Firstly, manifestations of the effective gravity are constrained at the level of particle physics in Chapter 8. Secondly, results of Chapter 9 constraint effective gravity at the level of binary systems. Thirdly, results of Chapter 10 constraint the applicability of effective gravity for description of the dark matter, equivalently, the applicability of effective gravity at the level of galaxies. Finally, results of Chapter 11 study modified gravity models within EFT framework. Because of this results presented in this thesis can be viewed as a complimentary study of the effective gravity at different spacial and energy scales within various gravity models.

The most general conclusion drawn by this thesis is the necessity for further research of gravitational effects described within the EFT framework. Results of Chapter 8 show an opportunity to study effective gravity at the level of particle physics. Results of Chapter 9 highlight a perspective study direction at the level of binary systems. Results of Chapter 10 point on an opportunity to use EFT to describe the dark mater. While results of Chapter 11 show that modifications of gravity can have a non-trivial influence on the effective gravity.

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Appendix A

Used Notations

In this thesis the following notations and conventions are used.

The flat spacetime metric reads

$$\eta_{\mu\nu} \stackrel{\text{def}}{=} \text{diag}(+---) \; . \tag{A.1}$$

Christoffel symbols, Riemann, Ricci tensors, and scalar curvature are defined as follows:

$$\Gamma^{\alpha}_{\mu\nu} \stackrel{\text{def}}{=} \frac{1}{2} \, g^{\alpha\beta} \left(\partial_{\mu} g_{\nu\beta} + \partial_{\nu} g_{\mu\beta} - \partial_{\beta} g_{\mu\nu} \right) \; ,$$

$$R_{\mu\nu}{}^{\alpha}{}_{\beta} \stackrel{\text{def}}{=} \partial_{\mu}\Gamma^{\alpha}_{\nu\beta} - \partial_{\nu}\Gamma^{\alpha}_{\mu\beta} + \Gamma^{\alpha}_{\mu\sigma}\Gamma^{\sigma}_{\nu\beta} - \Gamma^{\alpha}_{\nu\sigma}\Gamma^{\sigma}_{\mu\beta} \; , \tag{A.2}$$

$$R_{\mu\nu} \stackrel{\text{def}}{=} R_{\sigma\mu}{}^{\sigma}{}_{\nu} = R_{\alpha\mu\beta\nu}g^{\alpha\beta} \; ,$$

$$R \stackrel{\text{def}}{=} R_{\mu\nu} \, g^{\mu\nu} \; .$$

Standard definitions of the Einstein tensor $G_{\mu\nu}$, Weyl tensor $\mathcal{C}_{\mu\nu\alpha\beta}$ (in d-dimensional spacetime), and the Gauss-Bonnet term $\mathcal{G}$ are used:

$$G_{\mu\nu} \stackrel{\text{def}}{=} R_{\mu\nu} - \frac{1}{2} \, R \, g_{\mu\nu},$$

$$\mathcal{C}_{\mu\nu\alpha\beta} \stackrel{\text{def}}{=} R_{\mu\nu\alpha\beta} - \frac{1}{d-2} \left(g_{\mu\alpha}R_{\nu\beta} + g_{\nu\beta}R_{\mu\alpha} - g_{\mu\beta}R_{\nu\alpha} - g_{\nu\alpha}R_{\mu\beta} \right)$$

$$+ \frac{1}{(d-1)(d-2)} \left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha} \right)R \; , \tag{A.3}$$

$$\mathcal{G} \stackrel{\text{def}}{=} R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \; .$$

Square of the Weyl tensor is given by the following expression:

$$\mathcal{C}_{\mu\nu\alpha\beta}\mathcal{C}^{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - \frac{4}{d-2} \, R_{\mu\nu}R^{\mu\nu} + \frac{2}{(d-1)(d-2)} \, R^2. \tag{A.4}$$

In four dimensional spacetime it is related with the Gauss-Bonnet term as follows:

$$\mathcal{C}_{\mu\nu\alpha\beta}\mathcal{C}^{\mu\nu\alpha\beta} = \mathcal{G} + 2 \, R_{\mu\nu}R^{\mu\nu} - \frac{2}{3} \, R^2 \; . \tag{A.5}$$

The following generalisation of the unit for rank-2 tensors is used:

$$I_{\mu\nu\alpha\beta} \stackrel{\text{def}}{=} \frac{1}{2} \left(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} \right) .$$

(A.6)

The following tensor is often used in the flat spacetime:

$$C_{\mu\nu\alpha\beta} \stackrel{\text{def}}{=} \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta} .$$

(A.7)

Appendix B

Nieuwenhuizen Operators

The full set of projection operators in flat spacetime known as Nieuwenhuizen operators was introduced in [256]. In this paper the following momentum representation is used [253]:

$$
\begin{cases}
P^1_{\mu\nu\alpha\beta} & = \dfrac{1}{2} \left(\Theta_{\mu\alpha}\,\omega_{\nu\beta} + \Theta_{\mu\beta}\,\omega_{\nu\alpha} + \Theta_{\nu\beta}\,\omega_{\mu\alpha} + \Theta_{\nu\alpha}\,\omega_{\mu\beta} \right), \\[2mm]
P^2_{\mu\nu\alpha\beta} & = \dfrac{1}{2} \left(\Theta_{\mu\alpha}\,\Theta_{\nu\beta} + \Theta_{\mu\beta}\,\Theta_{\nu\alpha} \right) - \dfrac{1}{3}\,\Theta_{\mu\nu}\,\Theta_{\alpha\beta}, \\[2mm]
P^0_{\mu\nu\alpha\beta} & = \dfrac{1}{3}\,\Theta_{\mu\nu}\,\Theta_{\alpha\beta}, \\[2mm]
\overline{P}^0_{\mu\nu\alpha\beta} & = \omega_{\mu\nu}\,\omega_{\alpha\beta}, \\[2mm]
\overline{\overline{P}}^0_{\mu\nu\alpha\beta} & = \Theta_{\mu\nu}\omega_{\alpha\beta} + \Theta_{\alpha\beta}\omega_{\mu\nu}.
\end{cases}
\tag{B.1}
$$

Projectors Θ and ω are defined as follows:

$$
\begin{cases}
\Theta_{\mu\nu} & = \eta_{\mu\nu} - \dfrac{k_\mu k_\nu}{k^2}\,, \\[3mm]
\omega_{\mu\nu} & = \dfrac{k_\mu k_\nu}{k^2}\,.
\end{cases}
\tag{B.2}
$$

They satisfy the standard orthogonality relation:

$$
\begin{cases}
\Theta_{\mu\sigma}\,\Theta^{\sigma\nu} & = \Theta_\mu{}^\nu\,, \\[2mm]
\omega_{\mu\sigma}\,\omega^{\sigma\nu} & = \omega_\mu{}^\nu\,, \\[2mm]
\Theta_{\mu\sigma}\,\omega^{\sigma\nu} & = 0\,.
\end{cases}
\tag{B.3}
$$

In this representation Nieuwenhuizen operators P_1, P^2, P^0, and $\overline{P}^0$ form the complete orthogonal basis of projection operators. They are subjected to the following orthogonality

condition:

$$P^1_{\mu\nu\alpha\beta} + P^2_{\mu\nu\alpha\beta} + P^0_{\mu\nu\alpha\beta} + \overline{P}^0_{\mu\nu\alpha\beta} = I_{\mu\nu\alpha\beta}. \tag{B.4}$$

This condition is equivalent to the set of orthogonality relations:

$$\begin{cases} P^1_{\mu\nu\rho\sigma} P^1_{\rho\sigma\alpha\beta} = P^1_{\mu\nu}{}^{\alpha\beta} \ , \\ P^2_{\mu\nu\rho\sigma} P^2_{\rho\sigma\alpha\beta} = P^2_{\mu\nu}{}^{\alpha\beta} \ , \\ P^0_{\mu\nu\rho\sigma} P^0_{\rho\sigma\alpha\beta} = P^0_{\mu\nu}{}^{\alpha\beta} \ , \\ \overline{P}^0_{\mu\nu\rho\sigma} \overline{P}^0_{\rho\sigma\alpha\beta} = \overline{P}^0_{\mu\nu}{}^{\alpha\beta} \ . \end{cases} \qquad \begin{cases} P^1_{\mu\nu\rho\sigma} P^{2\rho\sigma\alpha\beta} = 0 \ , \\ P^2_{\mu\nu\rho\sigma} P^{0\rho\sigma\alpha\beta} = 0 \ , \\ P^0_{\mu\nu\rho\sigma} \overline{P}^{0\rho\sigma\alpha\beta} = 0 \ , \\ \overline{P}^0_{\mu\nu\rho\sigma} P^{1\rho\sigma\alpha\beta} = 0 \ , \\ P^1_{\mu\nu\rho\sigma} P^{0\rho\sigma\alpha\beta} = 0 \ , \\ P^2_{\mu\nu\rho\sigma} \overline{P}^{0\rho\sigma\alpha\beta} = 0 \ . \end{cases} \tag{B.5}$$

Operator $\overline{\overline{P}}^0$ is introduced for the sake of simplicity. It is orthogonal to operators P^1 and P^2:

$$P^1_{\mu\nu\rho\sigma} \overline{\overline{P}}^{0\,\rho\sigma\alpha\beta} = P^2_{\mu\nu\rho\sigma} \overline{\overline{P}}^{0\,\rho\sigma\alpha\beta} = 0 \ . \tag{B.6}$$

Its relation with the other operators is given by the following:

$$\begin{cases} \overline{\overline{P}}^0_{\mu\nu\rho\sigma} \overline{\overline{P}}^{0\,\rho\sigma}{}_{\alpha\beta} = 3\ (P^0_{\mu\nu\alpha\beta} + \overline{P}^0_{\mu\nu\alpha\beta})\ , \\ P^0_{\mu\nu\rho\sigma} \overline{\overline{P}}^{0\,\rho\sigma}{}_{\alpha\beta} = \overline{\overline{P}}^0_{\mu\nu\rho\sigma} \overline{P}^{0\,\rho\sigma}{}_{\alpha\beta} = \Theta_{\mu\nu} \omega_{\alpha\beta}\ , \\ \overline{P}^0_{\mu\nu\rho\sigma} \overline{\overline{P}}^{0\,\rho\sigma}{}_{\alpha\beta} = \overline{\overline{P}}^0_{\mu\nu\rho\sigma} P^{0\rho\sigma}{}_{\alpha\beta} = \omega_{\mu\nu} \Theta_{\alpha\beta}\ . \end{cases} \tag{B.7}$$

Appendix C

Small Metric Perturbations

The formalism presented in this appendix provides a framework for a treatment of small metric perturbations over a classical background. The metric of the background spacetime is denoted as $\overline{g}_{\mu\nu}$, small perturbations are described by matrix $h_{\mu\nu}$. The spacetime metric accounting for the background and perturbations is given by the following formula:

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \kappa h_{\mu\nu} \ . \tag{C.1}$$

Here κ is defined through the Newton's constant:

$$\kappa^2 = 32\pi G. \tag{C.2}$$

All indices are raised, lowered, and contracted with the background metric $\overline{g}_{\mu\nu}$. Variable $h_{\mu\nu}$ has the canonical mass dimension.

Notations from paper [51] are used. Background quantities are noted with a bar on top of a symbol; quantities linear in h are noted with an underline; quantities quadratic in h are noted with a double underline. For instance, $\overline{g}_{\mu\nu}$ notes the background part of a metric; $\underline{g}^{\mu\nu} = -\kappa h^{\mu\nu}$ notes the part of $g^{\mu\nu}$ linear in $h^{\mu\nu}$; and $\underline{\underline{g}}^{\mu\nu} = \kappa^2 h^{\mu\sigma} h_\sigma{}^\nu$ notes the part of $g^{\mu\nu}$ quadratic in $h^{\mu\nu}$.

The inverse metric is given by the following formal series:

$$g^{\mu\nu} = \overline{g}^{\mu\nu} + \sum_{n=1}^\infty (-\kappa)^n (h^n)^{\mu\nu} = \overline{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2\ h^{\mu\sigma} h_\sigma{}^\nu + O(h^3). \tag{C.3}$$

The following notations were used:

$$h^{\mu\nu} \stackrel{\text{def}}{=} h_{\alpha\beta}\ \overline{g}^{\mu\alpha} \overline{g}^{\nu\beta},$$
$$(h^n)^{\mu\nu} \stackrel{\text{def}}{=} \underbrace{h^\mu{}_{\sigma_1} h^{\sigma_1}{}_{\sigma_2} \cdots h^{\sigma\nu}}_{n \text{ matrices}}\ . \tag{C.4}$$

The series is convergent if perturbations $h_{\mu\nu}$ are small with respect to κ.

The following expression relating can be used for determinant of the metric:

$$g = \det g_{\mu\nu} = \det \exp \log g_{\mu\nu} = \exp \operatorname{tr} \log g_{\mu\nu} = \exp \operatorname{tr} \log(\bar{g}_{\mu\sigma}(\delta^\sigma_\nu + \kappa h^\sigma_\nu)) = \tag{C.5}$$

$$= \bar{g} \ \exp \operatorname{tr} \log(\delta^\mu_\nu + \kappa h^\mu_\nu) = \bar{g} \ \exp \left[\operatorname{tr} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\kappa^n}{n} (h^n)^\mu_\nu \right] = \tag{C.6}$$

$$= \bar{g} \ \exp \left[\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\kappa^n}{n} \operatorname{tr}\{(h^n)^\mu_\nu\} \right]. \tag{C.7}$$

Because of this relation, the element of invariant volume is given by the following:

$$\int d^4 x \sqrt{-g} = \int d^4 x \ \sqrt{-\bar{g}} \left[\frac{1}{2} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\kappa^n}{n} \operatorname{tr}\{(h^n)^\mu_\nu\} \right] =$$

$$= \int d^4 x \ \sqrt{-\bar{g}} \left[1 + \frac{\kappa}{2} h - \frac{\kappa^2}{4} \left(h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} h^2 \right) + O(h^2) \right] , \tag{C.8}$$

with $h \overset{\text{def}}{=} h_{\mu\nu} \bar{g}^{\mu\nu}$. Christoffel symbols express as follows:

$$\Gamma^\alpha_{\mu\nu} = \bar{\Gamma}^\alpha_{\mu\nu} + \underline{\Gamma}^\alpha_{\mu\nu} - \frac{1}{2} \sum_{n=2}^{\infty} (-\kappa)^n \left(h^{n-1} \right)^{\alpha\beta} \left(\nabla_\mu h_{\nu\beta} + \nabla_\nu h_{\mu\beta} - \nabla_\beta h_{\mu\nu} \right),$$

$$\underline{\Gamma}^\alpha_{\mu\nu} = \frac{\kappa}{2} \left[\nabla_\mu h^\alpha_\nu + \nabla_\nu h^\alpha_\mu - \nabla^\alpha h_{\mu\nu} \right], \tag{C.9}$$

$$\underline{\underline{\Gamma}}^\alpha_{\mu\nu} = -\frac{\kappa^2}{2} h^{\alpha\beta} \left[\nabla_\mu h_{\nu\beta} + \nabla_\nu h_{\mu\beta} - \nabla_\beta h_{\mu\nu} \right].$$

Covariant derivatives are calculated with respect to the background metric. The Riemann tensor components read:

$$\underline{R}_{\mu\nu}{}^\alpha{}_\beta = \nabla_\mu \underline{\Gamma}^\alpha_{\nu\beta} - \nabla_\nu \underline{\Gamma}^\alpha_{\mu\beta},$$

$$\underline{\underline{R}}_{\mu\nu}{}^\alpha{}_\beta = \nabla_\mu \underline{\underline{\Gamma}}^\alpha_{\nu\beta} - \nabla_\nu \underline{\underline{\Gamma}}^\alpha_{\mu\beta} + \underline{\Gamma}^\alpha_{\mu\sigma} \underline{\Gamma}^\sigma_{\nu\beta} - \underline{\Gamma}^\alpha_{\nu\sigma} \underline{\Gamma}^\sigma_{\mu\beta} . \tag{C.10}$$

The correspondent expressions give Riemann tensor, Ricci tensor, and scalar curvature in terms of perturbations are given by the following:

$$\underline{R}_{\mu\nu}{}^\alpha{}_\beta = \frac{\kappa}{2} \left[[\nabla_\mu, \nabla_\mu] h^\alpha{}_\beta - \nabla_\mu \nabla^\alpha h_{\nu\beta} - \nabla_\nu \nabla_\beta h_\mu{}^\alpha + \nabla_\nu \nabla^\alpha h_{\mu\beta} + \nabla_\mu \nabla_\beta h_\nu{}^\alpha \right] ,$$

$$\underline{R}_{\mu\nu} = \underline{R}_{\sigma\mu}{}^\sigma{}_\nu = \frac{\kappa}{2} [\nabla^\sigma \nabla_\mu h_{\sigma\nu} + \nabla^\sigma \nabla_\nu h_{\sigma\mu} - \nabla_\mu \nabla_\nu h - \Box h_{\mu\nu}] , \tag{C.11}$$

$$\underline{R} = \underline{R_{\mu\nu} g^{\mu\nu}} = \kappa [\nabla^{\mu\nu} h_{\mu\nu} - \Box h] - \kappa \bar{R}_{\mu\nu} h^{\mu\nu} .$$

These expressions allow one to show that the following contributions linear in small perturbations vanish over the flat background:

$$\underline{\int d^4x \ \sqrt{-g} \ R} = -\int d^4x \sqrt{-\bar{g}} \ \overline{G}_{\mu\nu} \ \kappa h^{\mu\nu}, \tag{C.12}$$

$$\underline{\int d^4x \ \sqrt{-g} \ R^2} = \int d^4x \sqrt{-\bar{g}} \ \kappa h^{\mu\nu} \left[-2\overline{R} \ \overline{R}_{\mu\nu} + 2 \left(\nabla_{\mu\nu} - \eta_{\mu\nu}\Box \right) \overline{R} + \frac{1}{2}\eta_{\mu\nu}\overline{R}^2 \right],$$

$$\underline{\int d^4x \ \sqrt{-g} \ R_{\mu\nu}R^{\mu\nu}} = \int d^4x \sqrt{-\bar{g}} \ \kappa h_{\mu\nu} \left[\nabla^\mu\nabla_\sigma\overline{R}^{\sigma\nu} + \nabla^\nu\nabla_\sigma\overline{R}^{\sigma\mu} \right.$$

$$\left. -\eta^{\mu\nu}\nabla_{\rho\sigma}\overline{R}^{\rho\sigma} - \Box\overline{R}^{\mu\nu} + \frac{1}{2} \eta^{\mu\nu} \ \overline{R}_{\rho\sigma}\overline{R}^{\rho\sigma} \right].$$

Correspondent contributions quadratic in perturbations are given by the following expressions (in terms of Nieuwenhuizen operators) over the flat spacetime:

$$-\frac{2}{\kappa^2} \underline{\underline{\int d^4x \sqrt{-g}R}} = \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \left(P^2_{\mu\nu\alpha\beta} - 2P^0_{\mu\nu\alpha\beta} \right) \Box h^{\alpha\beta} \right],$$

$$\underline{\underline{\int d^4x \sqrt{-g}R^2}} = \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \left(6P^0_{\mu\nu\alpha\beta} \right) \ \kappa^2\Box^2 h^{\alpha\beta} \right], \tag{C.13}$$

$$\underline{\underline{\int d^4x \sqrt{-g}R^2_{\mu\nu}}} = \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \left(-\frac{1}{2} P^2_{\mu\nu\alpha\beta} - 2P^0_{\mu\nu\alpha\beta} \right) \kappa^2\Box^2 h^{\alpha\beta} \right].$$

Appendix D

Feynman Rules for Quantum General Relativity

Feynman rules for quantum GR are often used in this thesis. They are derived with the use of external field quantisation technique within functional integral formalism [253, 257].

The gravitational field $g_{\mu\nu}$ consists of the background metric $\overline{g}_{\mu\nu}$ and small perturbations $h_{\mu\nu}$. The generating functional describing such a system reads:

$$Z = \int \mathcal{D}[h] \exp\left[i\mathcal{A}_{\mathrm{GR}}[\overline{g}_{\mu\nu} + \kappa h_{\mu\nu}]\right]. \tag{D.1}$$

We assume that the background field satisfy vacuum Einstein equations and separate the part quadratic in perturbations:

$$Z = \int \mathcal{D}[h] \exp\left[-\frac{i}{2}\int d^4x \; h^{\mu\nu} \cdot (\mathcal{O}_{\mathrm{GR}})_{\mu\nu\alpha\beta} \;\Box h^{\alpha\beta} + i\mathcal{A}_{\mathrm{int}}[h]\right]. \tag{D.2}$$

Here $\mathcal{A}_{\mathrm{int}}$ denotes all $O(h^2)$ terms that correspond to gravitons self-interaction. The following brief notations are used:

$$h^{\mu\nu} \cdot (\mathcal{O}_{\mathrm{GR}})_{\mu\nu\alpha\beta}\Box h^{\alpha\beta} \overset{\mathrm{notes}}{=} \int d^4x \left[h^{\mu\nu}(x)(\mathcal{O}_{\mathrm{GR}})_{\mu\nu\alpha\beta}\Box h^{\alpha\beta}(x)\right]. \tag{D.3}$$

For the Einstein-Hilber action operator $\mathcal{O}_{\mu\nu\alpha\beta}$ was evaluated in Appendix C in terms of Nieuwenhuizen operators reads:

$$(\mathcal{O}_{\mathrm{GR}})_{\mu\nu\alpha\beta} = P^2_{\mu\nu\alpha\beta} - 2P^0_{\mu\nu\alpha\beta} \;. \tag{D.4}$$

To evaluate this formula the following finite-dimensional Gauss integrals should be

used:

$$\frac{\int d^N x \; f(x) \; \exp\left[-\frac{i}{2}\, x \cdot A \cdot x\right]}{\int d^N x \; \exp\left[-\frac{i}{2}\, x \cdot A \cdot x\right]} = f\left(\frac{1}{i}\frac{\partial}{\partial J}\right) \exp\left[\frac{i}{2}\, J \cdot A^{-1} \cdot J\right]\Bigg|_{J=0} \; . \tag{D.5}$$

Because of the gauge symmetry $\mathcal{O}_{\mu\nu\alpha\beta}k^\beta = 0$, so operator $\mathcal{O}_{\mu\nu\alpha\beta}$ is irreversible. In full analogy with other gauge models a gauge-fixing term should be introduced via Faddeev-Popov technique [258]. The gauge-fixing technique is discussed in Appendix E. For the sake of convenience I use Feynman gauge, as it allows one to decoupled ghost states completely.

I use the following gauge-fixing term:

$$\mathcal{A}_{\mathrm{gf}} = \int d^4x \; \left(\partial_\sigma h^{\sigma\mu} - \frac{1}{2}\partial^\mu h\right)\left(\partial^\rho h_{\rho\mu} - \frac{1}{2}\partial_\rho h\right) =$$
$$= \int d^4x \; -\frac{1}{2}h^{\mu\nu}(\mathcal{O}_{gf})_{\mu\nu\alpha\beta} \, \Box h^{\alpha\beta} \; . \tag{D.6}$$

Operator $\mathcal{O}_{\mathrm{gf}}$ is given by the following:

$$(\mathcal{O}_{\mathrm{gf}})_{\mu\nu\alpha\beta} = P^1_{\mu\nu\alpha\beta} + \frac{3}{2}\, P^0_{\mu\nu\alpha\beta} + \frac{1}{2}\, \overline{P}^0_{\mu\nu\alpha\beta} - \frac{1}{2}\, \overline{\overline{P}}^0_{\mu\nu\alpha\beta} \; . \tag{D.7}$$

The generating functional with such a gauge-fixing term reads:

$$Z = \int \mathcal{D}[h] \exp\left[-\frac{i}{2}\, h^{\mu\nu}(\mathcal{O}_{\mathrm{GR+gf}})_{\mu\nu\alpha\beta}\Box h^{\mu\nu} + i\mathcal{A}_{\mathrm{int}}[h]\right] \; . \tag{D.8}$$

Operator $\mathcal{O}_{\mathrm{GR+gf}}$ can be reversed and is given by the following:

$$(\mathcal{O}_{\mathrm{GR+gf}})_{\mu\nu\alpha\beta} = P^1_{\mu\nu\alpha\beta} + P^2_{\mu\nu\alpha\beta} - \frac{1}{2}\, P^0_{\mu\nu\alpha\beta} + \frac{1}{2}\, \overline{P}^0_{\mu\nu\alpha\beta} - \frac{1}{2}\, \overline{\overline{P}}^0_{\mu\nu\alpha\beta} =$$
$$= \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right) = \frac{C_{\mu\nu\alpha\beta}}{2} \; . \tag{D.9}$$

In Appendix B we presented the set of Nieuwenhuizen operators which form the complete basis of projection operators. Arbitrary projection operator $\mathcal{O}_{\mu\nu\alpha\beta}$ can be presented in terms of these operators:

$$\mathcal{O}_{\mu\nu\alpha\beta} = x_1 P^1_{\mu\nu\alpha\beta} + x_2 P^2_{\mu\nu\alpha\beta} + x_0 P^0_{\mu\nu\alpha\beta} + \overline{x}_0 \overline{P}^0_{\mu\nu\alpha\beta} + \overline{\overline{x}}_0 \overline{\overline{P}}^0_{\mu\nu\alpha\beta} \; . \tag{D.10}$$

The operator can be inverted if and only if $x_1 \neq 0$, $x_2 \neq 0$, and $x_0\overline{x}_0 - 3\,\overline{\overline{x}}_2 \neq 0$. In that case the inverted operator is given by the following formula:

$$(\mathcal{O}_{\mu\nu\alpha\beta})^{-1} = \frac{1}{x_1}P^1_{\mu\nu\alpha\beta} + \frac{1}{x_2}P^2_{\mu\nu\alpha\beta} + \frac{\overline{x}_0 P^0_{\mu\nu\alpha\beta} + x_0 \overline{P}^0_{\mu\nu\alpha\beta} - \overline{\overline{x}}_0 \overline{\overline{P}}^0_{\mu\nu\alpha\beta}}{x_0\overline{x}_0 - 3\,\overline{\overline{x}}_0^2} \; . \tag{D.11}$$

In accordance with this formula the inverse operator reads:

$$(\mathcal{O}_{\text{GR+gf}})^{-1}_{\mu\nu\alpha\beta} = (\mathcal{O}_{\text{GR+gf}})_{\mu\nu\alpha\beta} = \frac{C_{\mu\nu\alpha\beta}}{2} \; . \tag{D.12}$$

To complete the derivation I introduce the external current $J^{\mu\nu}$ and evaluate the generating functional:

$$Z[J] = \int \mathcal{D}[h] \exp\left[-\frac{i}{2} \, h^{\mu\nu} \cdot \left(\frac{C_{\mu\nu\alpha\beta}}{2} \right) \Box h^{\alpha\beta} + i\mathcal{A}_{\text{int}} + iJ^{\mu\nu}h_{\mu\nu} \right] =$$

$$= \exp\left[\mathcal{A}_{\text{int}} \left[\frac{1}{i} \frac{\delta}{\delta J^{\mu\nu}} \right] \right] \exp\left[\frac{i}{2} \, J^{\mu\nu} \cdot \left(\frac{C_{\mu\nu\alpha\beta}}{2} \right) \Box^{-1} J^{\alpha\beta} \right] \; . \tag{D.13}$$

This expression allows one to use the standard technique to derive interaction rules to any order in perturbation theory [143].

Appendix E

Gauge Fixing

In this thesis the standard technique of gauge fixing within Feynman integral formalism is used. The metric admits a gauge symmetry similar to the gauge symmetry of the electromagnetic field, because of this the standard gauge fixing technique can be used [257, 258]. For the sake of consistency we briefly discuss it here.

Gravity is a gauge theory and it is invariant with respect to coordinate transformations. At the level of the metric a change of coordinates induces the following gauge transformations:

$$g_{\mu\nu} \to g_{\mu\nu} + \partial_\mu\, \zeta_\nu + \partial_\nu\, \zeta_\mu \ . \tag{E.1}$$

Here ζ_μ is an arbitrary four-vector.

The action of a gravity model that respects the coordinate transformation invariance should be invariant with respect to these metric transformations. The part of the action quadratic in the small metric perturbations should also respect the symmetry. If the quadratic part is given by the following expression

$$h^{\mu\nu}\, (\mathcal{O}_{\text{quadratic}})_{\mu\nu\alpha\beta}\ \Box h^{\alpha\beta} \ , \tag{E.2}$$

then operator $\mathcal{O}_{\text{quadratic}}$ should have the following symmetry features:

$$(\mathcal{O}_{\text{quadratic}})_{\mu\nu\alpha\beta} = (\mathcal{O}_{\text{quadratic}})_{\nu\mu\alpha\beta} = (\mathcal{O}_{\text{quadratic}})_{\alpha\beta\mu\nu} \tag{E.3}$$

and should satisfy the following gauge condition:

$$\partial^\mu\, (\mathcal{O}_{\text{quadratic}})_{\mu\nu\alpha\beta} = 0 \ . \tag{E.4}$$

Only operators P^2 and P^0 from the set of Nieuwenhuizen operators (presented in Appendix B) satisfy these conditions. However, these operators are projectors, thus the quadratic part of the action cannot be inverted.

This feature of the quadratic part of an action is typical for gauge theories [173]. At the physical level it corresponds to the following feature for the theory. Because the metric is invariant with respect to the gauge transformation (E.1), a family of different metric can describe the same physical spacetime. At the same time within the Feynman integral formalism the integration should be performed over all conceivable metric $g_{\mu\nu}$. Thus within the Feynman integral the integration measure is redundant, as each particular configuration of the physical spacetime has multiple contributions to the integral, in full analogy with the other gauge theories [173].

To eliminate this redundancy the following steps should be done. First of all, the integration over all conceivable metrics should be transformed to an integration over classes of metrics that describe different configurations of the physical spacetime. At the level of Feynman integral it should be done as follows:

$$\int \mathcal{D}\left[g_{\mu\nu}\right] = \int \mathcal{D}[\mathfrak{g}_{\mu\nu}] \int \mathcal{D}[\zeta_\mu] \, \delta\left(\mathcal{G}\right) \, \det\left|\frac{\delta\mathcal{G}}{\delta\zeta_\mu}\right| . \tag{E.5}$$

Here on the left hand side the integration is performed over all conceivable metrics $g_{\mu\nu}$. On the right hand side the integration over $\mathfrak{g}_{\mu\nu}$ is also performed over all conceivable metrics while the integration over ζ_μ corresponds to all possible gauge parameters. Because of the δ-function on the right hand side the integration over $\mathfrak{g}_{\mu\nu}$ accounts for a single representative from every class of physically equivalent metric which is chosen by the gauge conditions $\mathcal{G}$. Finally, the determinant $\delta\mathcal{G}/\delta\zeta_\mu$ is required to preserve the invariance of the integration measure.

Secondly, a gauge fixing term should be introduced in the action. This is required, as the quadratic part of the action is still gauge-invariant and cannot be inverted. The following standard expression for the Gaussian integral should be used:

$$\mathcal{N}^{-1} \int \mathcal{D}[\omega] \exp\left[-i\,\omega^2\right] = 1 . \tag{E.6}$$

Here $\mathcal{N}$ is the infinite normalisation factor that is to be omitted for the sake of simplicity. Consequently, the Feynman integral can be rewritten in the following form (presented here up to the infinite normalisation factor):

$$\int \mathcal{D}\left[g_{\mu\nu}\right] \exp\left[i\mathcal{A}[g]\right] =$$

$$= \int \mathcal{D}\left[\mathfrak{g}_{\mu\nu}\right] \int \mathcal{D}[\zeta_\mu] \int \mathcal{D}\left[\omega\right] \, \delta\left(\mathcal{G}\right) \, \det\left|\frac{\delta\mathcal{G}}{\delta\zeta_\mu}\right| \exp\left[i\mathcal{A}[g] - i\omega^2\right] . \tag{E.7}$$

Finally, it is required to specify the gauge condition $\mathcal{G}$. There are multiple ways to define the gauge conditions, but for the sake of simplicity the Feynman gauge is used in this thesis:

$$\mathcal{G} = \partial_\mu h^{\mu\nu} - \frac{1}{2}\,\partial^\nu h + \omega \ . \tag{E.8}$$

In an arbitrary gauge the determinant presenting in the integral may contain metric $g_{\mu\nu}$. Because of this it is required to introduce Faddeev-Popov ghost to evaluate the integral and to obtain the Feynman rules [257, 258]. However in the Feynman gauge the determinant is reduced to the following expression:

$$\det\left|\frac{\delta\mathcal{G}}{\delta\zeta_\mu}\right| \rightarrow \det\left|\frac{\partial_\mu h^{\mu\nu} - \frac{1}{2}\delta^\nu h + \omega}{\delta\zeta_\sigma}\right| = \det\left(\Box\,\delta_\sigma^\nu\right) \ . \tag{E.9}$$

The expression is free from the spacetime metric $g_{\mu\nu}$, therefore it can be included in the infinite normalisation faction of the integral.

In such a way the gauge fixing procedure presented here results in the following expression for the Feynman integral (presented up to the infinite normalisation factor):

$$\int \mathcal{D}\left[g_{\mu\nu}\right] \exp\left[i\mathcal{A}[g]\right] = \int \mathcal{D}\left[\mathfrak{g}_{\mu\nu}\right]\, \exp\left[i\mathcal{A}[g] - i\left(\partial_\mu h^{\mu\nu} - \frac{1}{2}\partial^\mu h\right)^2\right] \ . \tag{E.10}$$

This procedure is used throughout the thesis to obtain various sets of Feynman rules.

Appendix F

Feynman Rules for Quadratic Gravity

This thesis refers to quadratic gravity first studied by Stelle [53, 54] multiple times. This appendix contains derivations of Feynman rules and a brief discussion of its perturbation spectrum [253]. For the sake of simplicity only flat background is concerned.

Functional integral formalism allows one to obtain Feynman rules for quadratic gravity via the following generating functional:

$$Z = \int \mathcal{D}[h] \, \exp\left[i \, \mathcal{A}_{\text{Stelle}}[\eta_{\mu\nu} + \kappa h_{\mu\nu}]\right] \ . \tag{F.1}$$

Here $\mathcal{A}_{\text{Stelle}}$ notes Stelle action in the following parametrisation:

$$\mathcal{A}_{\text{Stelle}}[g_{\mu\nu}] \stackrel{\text{def}}{=} \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} \, R + c_1 \, R^2 + c_2 \, R_{\mu\nu}R^{\mu\nu} \right] \ . \tag{F.2}$$

In full analogy with GR one can separate the following kinetic part of the action

$$\mathcal{A}_{\text{Stelle}} = \int d^4x \left[-\frac{1}{2} \, h^{\mu\nu} (\mathcal{O}_{\text{Stelle}})_{\mu\nu\alpha\beta} \, \Box h^{\alpha\beta} + O(h^3) \right] . \tag{F.3}$$

Terms $O(h^3)$ describe particle interaction and lie beyond the scope of our interest. Oper-

ator $\mathcal{O}_{\text{Stelle}}$ has the following structure in coordinate representation:

$$(\mathcal{O}_{\text{Stelle}})_{\mu\nu\alpha\beta} = I_{\mu\nu\alpha\beta} - \eta_{\mu\nu}\eta_{\alpha\beta} + \left(\frac{\partial_\mu \partial_\nu}{\Box}\eta_{\alpha\beta} + \frac{\partial_\alpha \partial_\beta}{\Box}\eta_{\mu\nu} \right) \tag{F.4}$$

$$-\frac{1}{2}\left(\frac{\partial_\mu \partial_\alpha}{\Box}\eta_{\nu\beta} + \frac{\partial_\mu \partial_\beta}{\Box}\eta_{\nu\alpha} + \frac{\partial_\nu \partial_\alpha}{\Box}\eta_{\mu\beta} + \frac{\partial_\nu \partial_\beta}{\Box}\eta_{\mu\alpha} \right)$$

$$-\frac{c_2 \kappa^2}{2}\left[I_{\mu\nu\alpha\beta} - \frac{1}{2}\left(\frac{\partial_\mu \partial_\alpha}{\Box}\eta_{\nu\beta} + \frac{\partial_\mu \partial_\beta}{\Box}\eta_{\nu\alpha} + \frac{\partial_\nu \partial_\alpha}{\Box}\eta_{\mu\beta} + \frac{\partial_\nu \partial_\beta}{\Box}\eta_{\mu\alpha} \right) \right.$$

$$\left. +2\left(1 + 2\frac{c_1}{c_2}\right) \frac{\partial_\mu \partial_\nu}{\Box} \frac{\partial_\alpha \partial_\beta}{\Box} + \left(1 + 4\frac{c_1}{c_2}\right)\eta_{\mu\nu}\eta_{\alpha\beta} - \left(1 + 4\frac{c_1}{c_2}\right)\left(\frac{\partial_\mu \partial_\nu}{\Box}\eta_{\alpha\beta} + \frac{\partial_\alpha \partial_\beta}{\Box}\eta_{\mu\nu} \right) \right]\Box \ .$$

In momentum representation operator $\mathcal{O}_{\mu\nu\alpha\beta}$ is given in terms of Nieuwenhuizen operators as follows:

$$(\mathcal{O}_{\text{Stelle}})_{\mu\nu\alpha\beta} = \left(1 + \frac{c_2 \kappa^2}{2}k^2\right)P^2_{\mu\nu\alpha\beta} - 2\left(1 - (3c_1 + c_2)\kappa^2 k^2\right)P^0_{\mu\nu\alpha\beta} \ . \tag{F.5}$$

Operator $\mathcal{O}_{\text{Stelle}}$ is composed from P^2 and P^0 Nieuwenhuizen operators which are gauge-invariant, so the operator is irreversible. In full analogy with the standard GR case one should introduce a gauge-fixing term and use the standard technique discussed in Appendix E. In full analogy with GR gauge-fixing term of the following form allows one to decouple Faddeev-Popov ghosts completely:

$$\mathcal{A}_{\text{gf}} = \int d^4x \left(\partial_\mu h^{\mu\nu} - \frac{1}{2}\partial^\nu h \right)^2 = \int d^4x \left[-\frac{1}{2} h^{\mu\nu}(\mathcal{O}_{\text{gf}})_{\mu\nu\alpha\beta}\Box h^{\alpha\beta} \right] \ . \tag{F.6}$$

In terms of Nieuwenhuizen operators $\mathcal{O}_{\text{gf}}$ reads

$$(\mathcal{O}_{\text{gf}})_{\mu\nu\alpha\beta} = P^1_{\mu\nu\alpha\beta} + \frac{3}{2}P^0_{\mu\nu\alpha\beta} + \frac{1}{2}\overline{P}^0_{\mu\nu\alpha\beta} - \frac{1}{2}\overline{\overline{P}}^0_{\mu\nu\alpha\beta} \ . \tag{F.7}$$

Combined $\mathcal{O}_{\text{Stelle}}$ and $\mathcal{O}_{\text{gf}}$ operator is not gauge-invariant and can be inverted:

$$(\mathcal{O}_{\text{Stelle+gf}})_{\mu\nu\alpha\beta} = \left(1 + \frac{c_2 \kappa^2}{2}k^2\right)P^2_{\mu\nu\alpha\beta} + \left(-\frac{1}{2} + 2\kappa^2 k^2(3c_1 + c_2)\right)P^0_{\mu\nu\alpha\beta}$$

$$+ P^1_{\mu\nu\alpha\beta} + \frac{1}{2}\overline{P}^0_{\mu\nu\alpha\beta} - \frac{1}{2}\overline{\overline{P}}^0_{\mu\nu\alpha\beta} \ . \tag{F.8}$$

$$(\mathcal{O}^{-1}_{\text{Stelle+gf}})_{\mu\nu\alpha\beta} = \frac{P^2_{\mu\nu\alpha\beta}}{1 + \dfrac{c_2\kappa^2}{2}k^2} - \frac{1}{2}\frac{P^0_{\mu\nu\alpha\beta}}{1 - k^2\kappa^2(3c_1 + c_2)} \tag{F.9}$$

$$P^1_{\mu\nu\alpha\beta} - \frac{1}{2}\frac{\overline{\overline{P}}^0_{\mu\nu\alpha\beta} + [3 - 4\{1 - k^2\kappa^2(3c_1 + c_2)\}]\overline{P}^0_{\mu\nu\alpha\beta}}{1 - k^2\ \kappa^2(3c_1 + c_2)} \ .$$

This expression allows one to define two constants m_0 and m_2 with dimension of mass

$$\begin{cases} m_2^2 = -\dfrac{2}{c_2\kappa^2} \ , \\[4mm] m_0^2 = \dfrac{1}{\kappa^2(3c_1 + c_2)} \ . \end{cases} \tag{F.10}$$

In terms of this constants quadratic gravity action reads [130]:

$$\mathcal{A}_{\text{Stelle}} = -\frac{2}{\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{6m_0^2}\,R^2 + \frac{1}{2m_2^2}\,\mathcal{C}^2_{\mu\nu\alpha\beta}\right] \ . \tag{F.11}$$

In terms of small perturbations $h_{\mu\nu}$ with a fixed gauge the action takes the following form [253]:

$$\mathcal{A}_{\text{Stelle+gf}} = \int\frac{d^4k}{(2\pi)^4}\left\{\frac{1}{2}\,h^{\mu\nu}\left[\frac{1}{2}\frac{C_{\mu\nu\alpha\beta}}{k^2} - \frac{P^2_{\mu\nu\alpha\beta}}{k^2 - m_2^2} + \frac{1}{2}\frac{P^0_{\mu\nu\alpha\beta} + 3\bar{P}^0_{\mu\nu\alpha\beta} + \overline{\overline{P}}^0_{\mu\nu\alpha\beta}}{k^2 - m_0^2}\right]^{-1}h^{\alpha\beta}\right\}$$

$$+ \mathcal{A}_{\text{interaction}} \ . \tag{F.12}$$

The standard functional integral technique results in the following expression for the gauge-invariant parto of the graviton propagator:

$$-i\langle 0|\,h_{\mu\nu}h_{\alpha\beta}\,|0\rangle = \tag{F.13}$$

$$= \mu\nu\ \text{〰〰〰}\ \alpha\beta\,\Bigg|_{\text{Stelle}} \quad = \frac{1}{2}\frac{C_{\mu\nu\alpha\beta}}{k^2} - \frac{P^2_{\mu\nu\alpha\beta}}{k^2 - m_2^2} + \frac{1}{2}\frac{P^0_{\mu\nu\alpha\beta}}{k^2 - m_0^2} \ .$$

This propagator consists of three parts that carry perturbations with different spin and masses:

$$\text{〰〰}\,\Bigg|_{\text{Stelle}} \rightarrow \text{〰〰}\,\Bigg|_{s=2,m=0} - \text{〰〰}\,\Bigg|_{s=2,m=m_2} + \text{----}\,\Bigg|_{s=0,m=m_0} \ . \tag{F.14}$$

First term corresponds to propagation of massless particle with spin 2, i.e. the standard graviton. The second term has an opposite sign with respect to the former one. Because of this correspondent perturbation carry negative kinetic energy, has spin 2 and mass m_2,

i.e. it is massive spin-2 ghost. Finally, the last term has the same sign as the first term, so it carry positive kinetic energy. It also carry zero spin and has non-vanishing mass m_0. In other words, the last term describes propagation of massive scalar particle. Therefore quadratic gravity perturbation spectrum contains the standard graviton, massive spin-2 ghost and massive scalar, which was first proven in [53, 54].

One can introduce the following variables corresponding to these components:

- $k_{\mu\nu}$ desribes $s = 2$, $m = 0$ perturbations;

- $\psi_{\mu\nu}$ describes $s = 2$, $m = m_2$ perturbations;

- χ describes $s = 0$, $m = m_0$ perturbations.

The following relation holds for their propagators:

$$\langle 0| h_{\mu\nu}(x)h_{\alpha\beta}(y) |0\rangle =$$
$$= \langle 0| k_{\mu\nu}(x)k_{\alpha\beta}(y) |0\rangle + \langle 0| \psi_{\mu\nu}(x)\psi_{\alpha\beta}(y) |0\rangle + \eta_{\mu\nu}\eta_{\alpha\beta} \langle 0| \chi(x)\chi(y) |0\rangle . \tag{F.15}$$

Because of this modes $k_{\mu\nu}$, $\psi_{\mu\nu}$, χ, and $h_{\mu\nu}$ are related as follows:

$$h_{\mu\nu}(x) = k_{\mu\nu}(x) + h_{\mu\nu}(x) + \eta_{\mu\nu} \chi(x) . \tag{F.16}$$

This allows one to use quadratic gravity action (given up to interaction terms) in terms of $k_{\mu\nu}$, $\psi_{\mu\nu}$, χ modes reads:

$$S_{\text{Stelle}} \rightarrow \int d^4x \left[-\frac{1}{2} k^{\mu\nu} \left(P^2_{\mu\nu\alpha\beta} - 2P^0_{\mu\nu\alpha\beta} \right) \Box k^{\alpha\beta} + \frac{1}{2} \psi^{\mu\nu} (P^2_{\mu\nu\alpha\beta} - 2P^0_{\mu\nu\alpha\beta}) \Box \psi^{\alpha\beta} \right.$$
$$\left. -\frac{m_2^2}{2} (h_{\mu\nu}^2 - h^2) - \frac{1}{2} \chi(\Box + m_0^2)\chi + \kappa(k_{\mu\nu} + \psi_{\mu\nu} - \eta_{\mu\nu}\chi)T^{\mu\nu} \right] . \tag{F.17}$$