

Pitch-angle distribution of TeV cosmic rays in the local interstellar medium

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The electric and magnetic fields encountered by Galactic TeV cosmic rays (CRs) as they propagate deep into the heliosphere can alter their energy and arrival direction upon reaching Earth. This perturbation of trajectories in phase space can distort the angular distribution of particle flux, also known as “anisotropy”. Consequently, the maps of TeV CR anisotropy obtained by shower experiments appear quite complex. To undo heliospheric distortions, we developed a theory of flux mapping based upon applying Liouville’s theorem to the CR trajectories obtained via a multi-fluid magnetohydrodynamic model of the heliosphere. With this technique, we determined the original CR distribution function in the pristine local interstellar medium. In this paper, we focus on the dependence of TeV CR intensity on the particle pitch-angle relative to the direction of the local interstellar magnetic field. The pitch-angle distribution has two significant features: it is dominated by a dipole anisotropy, and it displays a notable increase in particle intensity towards the direction aligning with the magnetic field, which corresponds to a pitch-angle of zero. The dipole anisotropy suggests that TeV CRs are scattered nearly isotropically by interstellar magnetic field turbulence instead of expected behavior resulting from resonant scattering by turbulence with a Kolmogorov power spectrum. The increase of particles towards zero pitch-angle implies particle focusing by an inhomogeneous interstellar magnetic field with a gradient length approximately equal to 6 times the CR mean free path.

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1. Introduction

Galactic cosmic rays (CRs) below the knee energy (~ 3000 TeV) are most likely accelerated by supernovae. Below this energy, charged particles are confined by interstellar magnetic fields through gyration along the mean magnetic field, and through pitch angle scattering which prevents them from streaming away along the magnetic field lines. As a result, charged particles below the knee can spend tens of millions of years in the Galaxy before some of them reach the Earth and are detected by CR experiments. The speed at which CRs can propagate from their sources to reach our planet is largely affected by the properties of the magnetic field, such as its strength, inhomogeneity, motional velocity, and fluctuation or turbulence. Therefore, while measuring the CR spectrum and its composition can provide some insight into interstellar CR propagation, the study of the anisotropy of CR intensity as a function of arrival direction on Earth is more likely to provide us a better understanding of CR propagation. For instance, if there is a relatively recent source located within a few particle mean free paths from us, there may exist some signature of the point source in anisotropy maps. This prospect has motivated teams of air shower experimentalists to study CR anisotropy. So far, the observed sky maps show that the anisotropy amplitude of TeV CRs is rather small, about $10^{-4} - 10^{-3}$ in relative intensity [e.g. 1–3]. The anisotropy patterns appear broad and complex, puzzling many of us trying to reconstruct the physical processes responsible for the observations.

Earth resides near the center of the heliosphere carved by the solar wind plasma, within which the electric and magnetic fields are drastically different from those in the local interstellar medium. The heliosphere affects the distribution of local interstellar quantities at distances exceeding 10^4 AU. The radial distance to the heliopause in all directions is larger than the gyroradius of TeV CRs in a typical interstellar magnetic field of $\sim 3 \mu\text{G}$. The electric and fields within the heliosphere can severely alter the trajectories of these CRs on their way to Earth, changing their arrival direction and energy, and distorting the CR anisotropy patterns as we observe. Thus, in order to use the anisotropy measurements to understand CR propagation in the interstellar medium, we must undo heliospheric distortion. We developed a theory founded upon using Liouville's theorem to map the CR distribution function from the interstellar medium to Earth [18, 19]. From measurements by Tibet ASy, we reconstructed the particle distribution as a function of the particle pitch-angle relative to the local interstellar magnetic field, or the interstellar pitch-angle distribution, by removing several other contributions to the observations. The result sheds more light on the physics of CR transport mechanisms in the local interstellar medium.

2. Method

CR flux is proportional to the particle distribution function in the observer's reference frame [18]. Due to Liouville's theorem and to Lorentz invariance, one can map the particle distribution from interstellar space to Earth along particle trajectories in the phase space. Moreover, one can predict the motion of charged CRs through the heliosphere using the Lorentz force and a model of the heliospheric electric and magnetic fields. We used the magnetohydrodynamic (MHD) heliosphere model produced by the Multi-scale Fluid-Kinetic Simulation Suite of the University of Alabama in Huntsville [9? , 10]. This simulation package solves the multi-fluid MHD equations for

plasmas coupled with the kinetic (or multi-fluid) transport models for neutral atoms, and takes into account the physics of space plasma/magnetic field and neutral gas interaction. The inputs, the SW parameters, and the LISM velocity were estimated as accurately as possible under the constraints set by *in-situ* observations of the solar wind and of solar magnetic field in the inner heliosphere, and in the outer heliosphere. These were obtained respectively by many heliospheric missions and by the Voyagers and remote-sensing observations of the IBEX mission. In particular, with the help of the heliosphere model, it was possible to use the Voyager measurements in the local interstellar medium beyond the heliopause [12–14] to greatly constrain the interstellar magnetic field and the gas parameters, even though the spacecraft has not reached the unperturbed interstellar medium. The model has also helped greatly in interpreting NASA IBEX observations of energetic neutral atom fluxes originating from the heliospheric boundary region [e.g., 5, 8, 20]. Moreover, the model was validated against numerous *in-situ* and remote observations, e.g., (SOHO Ly α backscattered emission, Ly α absorption profiles in the direction of nearby stars, New Horizons observations in the distant solar wind [see, e.g., 6, 7]). Although the local interstellar conditions are not expected to change, our model takes into account solar cycle effects by taking as inputs remote measurements of the photospheric magnetic field [15–17]. The output of the MHD heliosphere simulation includes solutions to the distribution of magnetic field vector \mathbf{B} and plasma velocity \mathbf{V} . Since the solar wind and interstellar plasmas are highly conductive, we can calculate the electric field distribution using the ideal MHD induction equation $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$.

The electric and magnetic fields contained in the heliosphere model are average fields without fluctuations or turbulence, as these are unpredictable. As a result, the calculations of CR trajectories used to map CR flux suffer some inaccuracy. However, because heliospheric field fluctuations typically have scales much smaller than the gyroradius of TeV CRs, the perturbations caused by the unknown fluctuating fields are negligible compared to the trajectory curvature caused by the large-scale heliospheric magnetic field. Similarly, fluctuating interstellar magnetic fields can only cause significant scattering over a time-scale of a few years, as estimated from the typical length of particle mean free paths. During propagation through the heliosphere, which typically only lasts a few days, the effect of fluctuating interstellar magnetic fields is also negligible. Therefore we can safely use the Lorentz force from the fields generated by the MHD heliosphere model to calculate CR trajectories and map CR flux.

The mapping of the CR distribution function requires us to know it either on Earth or on the interstellar side. Measurements on Earth make it possible to quantify the momentum dependence, but not the spatial dependence of the particle distribution. Thus, we cannot directly map out the measurements to the interstellar medium. Instead, we assume a certain form of particle distribution in the interstellar medium, map it to Earth, and verify if the mapped distribution matches observations on Earth. Because the observed CR anisotropy is small and its energy dependence must be very close to the observed CR energy spectrum, we take a perturbative approach. We expand the dependence of particle distribution function on interstellar pitch-angle cosine μ and particle guiding center location ($\mathbf{R}_g = \mathbf{r} - \boldsymbol{\rho}_g$), which is displaced by a gyroradius $\boldsymbol{\rho}_g = \frac{\mathbf{B}_{ism} \times \mathbf{p}}{qB_{ism}^2}$ of the particle with charge q and momentum \mathbf{p} in interstellar magnetic field \mathbf{B}_{ism} .

$$f(\mathbf{r}, \mathbf{p}) = f_0 p^{-\gamma} [1 + \mathbf{G}_\perp \cdot \mathbf{R}_g + PAD(\mu)] \quad (1)$$

Here p is the magnitude of particle momentum, $\gamma \approx 4.75$ is the slope of the CR power-law

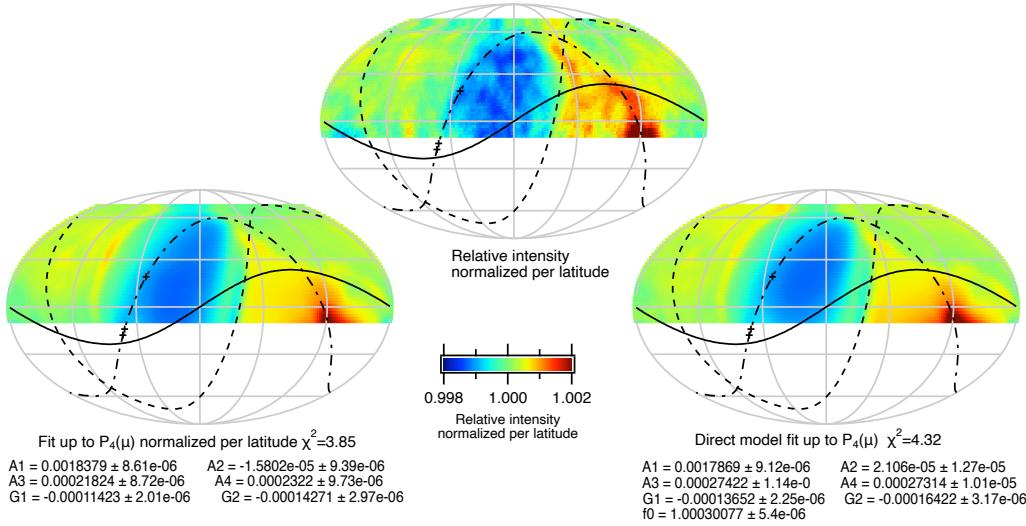


Figure 1: Anisotropy map of 4 TeV CR as measured in the Tibet Air Shower experiment (middle), the Liouville mapping model calculation with least- χ^2 linear fit (right) and nonlinear fit (left). The solid, dashed, and dash-dotted curves show the ecliptic plane, the plane perpendicular to LISM magnetic field and passing through the Sun, and the hydrogen deflection plane, respectively.

momentum distribution, f_0 is a reference constant, $\mathbf{G}_\perp = \nabla_\perp \ln f$ is the spatial gradient of particle intensity perpendicular to the local interstellar medium and pitch-angle distribution $PAD(\mu)$ is the expanded into a series of Legendre polynomials up to an order of N

$$PAD(\mu) = \sum_{i=1}^N A_n P_n(\mu) \quad (2)$$

where the linear coefficient A_n is commonly referred to as the amplitudes of pitch-angle anisotropy of N -th order. Once the interstellar distribution in Equation (1) is mapped to Earth, we can compare its output with the observed angular map of CR anisotropy of relative intensity to determine the parameters A_n and \mathbf{G}_\perp .

While air shower experiments can accurately measure the arrival direction of individual CRs, they cannot determine particle flux accurately enough to distinguish the maximum variation of CR anisotropy, i.e., 10^{-4} to 10^{-3} relative intensity. The determination of flux variation in longitude (right ascension) can rely on the Earth's spin, but the sensitivity to latitudinal (declination) variation is not obtainable. Thus generally, in anisotropy measurements presented by experimental teams, the so-called relative intensity as a function of declination δ and right ascension α is defined as:

$$I(\delta, \alpha) = \frac{f(\mathbf{r}((\delta, \alpha), \mathbf{p})}{\int_0^{2\pi} f(\mathbf{r}((\delta, \alpha), \mathbf{p}) d\alpha} \quad (3)$$

We fit Equation (3) to the 4 TeV anisotropy data provided by Tibet AS γ using a nonlinear optimization procedure. The nonlinear fit has improved the reduced χ^2 value of fitting from 4.32 to 3.85, even though the degree of freedom is reduced by 1. Figure 1 shows a comparison between the fits produced by Equation (3) and (1), and the experimental skymap for comparison.

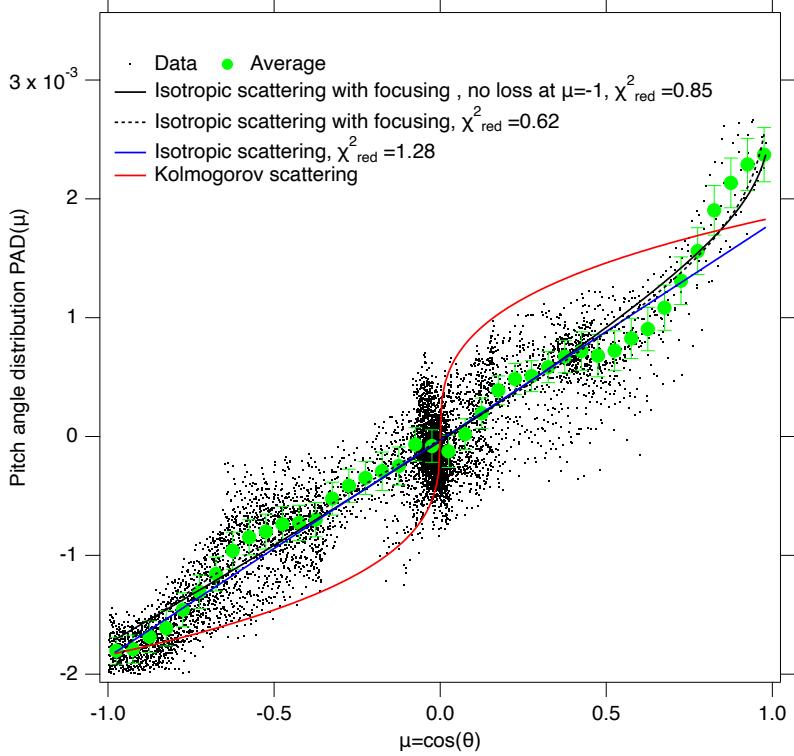


Figure 2: Relative pitch-angle distribution as a function of particle pitch-angle in the local interstellar medium after the other anisotropies are removed and the arrival direction is corrected. The colored curves are model calculations with a $D_{\mu\mu} \propto (1 - \mu^2)$ (red) and a $D_{\mu\mu} \propto (1 - \mu^2)|\mu|^{2/3}$ (blue) caused by the resonant scattering in the presence of Alfvénic turbulence with the Kolmogorov spectrum. The black curves are model fit with magnetic focusing with (solid) and without (dashed) constraint of nondivergence at $\mu = -1$.

If our anisotropy model is correct, we can reconstruct the true relative intensity of interstellar CR distribution by inverting Equation (3).

$$\frac{f}{f_0} = I(\delta, \alpha) \int_0^{2\pi} \frac{f(\mathbf{r}((\delta, \alpha), \mathbf{p}))}{f_0} d\alpha \quad (4)$$

Furthermore, if our calculations of particle momentum change, of guiding center drift during heliospheric propagation, and of the contributions of the aforementioned quantities to the observed anisotropy are correct, then we can derive the pitch angle distribution in the local interstellar medium by inverting Equation (1):

$$PAD(\mu) = \frac{f}{f_0 p^{-\gamma}} - \mathbf{G}_\perp \cdot \mathbf{R}_g \quad (5)$$

3. Result

Figure 2 shows a scatterplot of the PAD of 4 TeV CRs as a function of particle pitch-angle cosine $\mu = \cos(\theta)$. We divided the plot into equal bins of pitch-angle and took the average PAD value within each bin. The result is shown by the green points in Figure 2. This makes the fitting process less biased towards the center of the scatterplot, where the data points are more densely

concentrated. The pitch-angle distribution represents the particle distribution function as a function of the pitch-angle defined relative to the pristine interstellar magnetic field, without the presence of the heliosphere at fixed energy and location. Other contributions to the production of particle flux anisotropy have been removed.

At a glance, the pitch-angle distribution appears to be a single-value function, though some data points carry sizable error bars. Normally, a two-dimensional angular sky map should have two angular coordinates. The fact that our distribution depends only on the pitch-angle confirms that CR distribution in the interstellar reference frame is gyrotrropic. In other words, the time-scale of gyration around magnetic field lines is very small compared to the time-scales of all other particle transport mechanisms, including scattering and diffusion.

The pitch-angle distribution sheds light on the physics of particle scattering by the interstellar magnetic field and turbulence. The distribution is almost linearly proportional to μ . According to the CR transport theory in the diffusive approximation [e.g. 11] the following relation holds:

$$f(\mu) = f_0 - \frac{v}{2} \nabla_{||} f_0 \int_0^\mu d\mu \frac{1 - \mu^2}{D_{\mu\mu}}, \quad (6)$$

Here v is the particle speed, $\nabla_{||} f_0$ is the CR density gradient parallel to the magnetic field, and $D_{\mu\mu}$ is the pitch angle diffusion coefficient. The linear dependence indicates isotropic pitch-angle scattering with a $D_{\mu\mu} \propto (1 - \mu^2)$, as shown by the blue line. If TeV CRs were resonantly scattered by an incompressible or Alfvénic interstellar magnetic field turbulence, with a Kolmogorov spectrum characterized by a power slope of $-5/3$, $D_{\mu\mu}$ would be proportional to $(1 - \mu^2)|\mu|^{2/3}$, which would yield a pitch angle distribution that follows the red curve. Clearly, the isotropic scattering model is in better agreement with the data than the Kolmogorov turbulence model. We have three hypotheses to explain the nearly isotropic pitch angle scattering we observe: (1) Interstellar turbulence on the scales relevant to TeV CRs (roughly tens to thousands of AU) is dynamic and can disrupt resonance with particle gyration, (2) Interstellar turbulence is compressible rather than Alfvénic, or (3) The slope of the Kolmogorov power spectrum deviates significantly from $-5/3$ to -1 .

We notice that the data demonstrates a significant deviation from the linear fit at high μ . The fact that most data points in that range lie above the line suggests that particles become increasingly field-aligned towards $\mu = 1$. In other words, the data indicates focusing by an inhomogeneous large-scale magnetic field. The equation governing particle pitch diffusion with magnetic focusing is the following:

$$\frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial f}{\partial \mu} + \frac{\partial}{\partial \mu} D_{\mu p} \frac{\partial f}{\partial p} - B_\mu \frac{\partial f}{\partial \mu} = \frac{\partial f}{\partial t} + v \mu \nabla_{||} f \quad (7)$$

Here, we have a focusing rate $B_\mu = v(1 - \mu^2)/(2L_B)$ with magnetic field strength gradient $L_B^{-1} = -\nabla_{||} \ln B_{ism}$. If pitch-angle scattering is fast compared to the rate due to the time and spatial variation on the right-hand side of Equation (7), we can use a quasilinear approximation similar to [11]. Using an isotropic pitch angle scattering, i.e., $D_{\mu\mu} = D_{\mu 0}(1 - \mu^2)$ and $D_{\mu p} = D_{p0}(1 - \mu^2)$

with a constant $D_{\mu 0}$ and $D_{p 0}$, we can solve Equation (7):

$$\begin{aligned} f(\mu) = f_0 &+ C_1 \{ \exp(B_0) [\text{ei}(-B_0 - B_0\mu) - \text{ei}(-B_0)] \exp(-B_0) [\text{ei}(B_0 - B_0\mu) - \text{ei}(B_0)] \} \\ &- \frac{1}{2B_0^2 D_0} \left(v \nabla_{||} f_0 + D_{p 0} \frac{\partial f_0}{\partial p} \right) [(1 + B_0) \ln(1 + \mu) + (1 - B_0) \ln(1 - \mu)] \\ &- \frac{1}{2B_0 D_0} \frac{\partial f_0}{\partial t} [\ln(1 - \mu) + \ln(1 + \mu)] \end{aligned} \quad (8)$$

Here, $B_0 = v/(2L_b D_0) = \lambda_{||}/L_B$, $\text{ei}()$ is the exponential integral, and C_1 is an integration constant to be fixed by the boundary condition at the nodes $\mu = \pm 1$. We have assumed that the diffusive condition is valid, such that the terms on the right-hand side of Equation (7) are small and the derivatives $\partial f_0/\partial t$, $\nabla_{||} f_0$ and $\partial f_0/\partial p$ do not depend on μ . Normally, without the focusing term or $B_0 = 0$, C_1 can be set so that $f(\mu = \pm 1)$ is finite. However, this cannot be guaranteed in our case. The pitch angle distribution scatterplot in Figure 2 displays a trend of divergence at $\mu = 1$. We fit Equation (8) to the measured data in Figure 2. The black lines represent two different fits. The solid line is constrained, whereas the dashed line is allowed to diverge also at $\mu = -1$. Both fits are very close, and yield roughly the same parameters:

$$\frac{1}{B_0 D_0} \frac{\partial \ln f_0}{\partial t} < 0.002 \quad (9)$$

$$B_0 = \frac{\lambda_{||}}{L_b} = 0.16 \pm 0.05 \quad (10)$$

$$\lambda_{||} \nabla_{||} \ln f_0 = -0.0024 \pm 0.0004 \quad (11)$$

We have neglected the the contribution from the $\partial f_0/\partial p$ term, because $D_{p 0}/(p D_{\mu 0}) \sim V_A/c$ where $V_A \approx 30$ km/s is the Alfvén speed in the local interstellar medium.

4. Summary

We corrected the arrival direction of CRs by taking into account the bending of particle trajectories by the heliospheric magnetic field. Then, we amended the measured anisotropy map by undoing the latitudinal normalization caused by the lack of sensitivity in the declination direction of air shower experiments. Finally, we removed the contributions to the observed anisotropy from energy change—due to the Compton-Getting effect—and from both heliospheric acceleration and the particle density gradient. This allowed us to derive a pitch angle distribution of TeV CRs in the original local interstellar medium. The resulting distribution is almost a linear function of μ , which indicates nearly isotropic pitch angle scattering by interstellar magnetic field turbulence. We observed an excess of particle flux towards $\mu = 1$ compared to the prediction made by a linear fit. This is likely a result of focusing by an inhomogeneous interstellar magnetic field, which becomes weaker along the field line into the northern Galactic halo. The gradient length of magnetic field strength is about 6 times the particle mean free path. The gradient scale of particle flux along the field line is about 416 times the particle mean free path.

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