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Topological defect solutions for a system of three scalar fields

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Abstract. In this paper, we studied on the defect structures as topological by non-linear three scalar fields. By using modified Adomian decomposition method (MADM), and Adomian decomposition method (ADM) we have found the solutions of three scalar fields. Then we compared the obtained results each other by numerical solution. Also, we consider the static case and draw $\phi(x)$, $\chi(x)$, and $\rho(x)$ with the choice of different values for parameter r.

1. Introduction

Recently, a lot of literatures have been considered nonlinear equations as ordinary differential equation (ODE) and partial differential equation (PDE). These nonlinear equations are used in the different branches of physics, engineering and the other sciences. One of applications is defect structures that plays an important role in cosmology and high energy physics and can be of topological or non-topological nature [1, 2, 3]. Kink-like and lump-like are topological and non-topological structures respectively, which can be described by real scalar fields in $1+1$ space-time dimensions under the action of nonlinear interactions. One of main applications of topological defects is in cosmology especially for formation of structure in the early Universe, because topological defects are as carriers of attractive gravitational force [4, 5].
So far, the defect structures solutions have investigated by orbit method [6, 7]. In the present work, we focus attention on three coupled real scalar fields which are topological or kink-like defects [8]. For example the single real scalar field support just single defect as tanh-like kink, and the double sin-Gordon model may support two different defects, large and small kink [8]. In other words, a system containing two or more real scalar fields give rise to at least two other classes of systems those that support defects that engender internal structure and those that support junctions of defects [6, 7, 9, 10]. Also, we know that the regular hexagonal network of defects described by two and three real scalar fields [11, 12, 13, 14].

Also the three fields solutions in the Einstein equations for describing black holes with the cosmic string discussed by [15]. In general, these give us motivation to study three scalar fields. However, to make the present investigation as general as possible.

In most of the natural problems and modelings we encounter the nonlinear equations which could be solved by using different methods, such as Variational Iteration Method (VIM) [16], Modified Variational Iteration Method (MVIM) [17, 18], Homotopy Perturbation Method (HPM) [19, 20, 21, 22, 23], Adomian Decomposition Method (ADM) [24, 25], Modified Adomian

Decomposition Method (MADM) [26] and so on. Here we investigate the three last ones with comparing the results to show the accuracy of these methods and obtain a reliable solution for this physical problem.

The present work are organized as follows:
In the section 2 we will study topological defect by three scalar fields. Afterward, we will present the general form of ADM and MADM methods in section 3. Then, we will apply the ADM and MADM methods to three coupled scalar fields and obtain the corresponding solutions for the fields $\phi(x)$, $\chi(x)$ and $\rho(x)$ in section 4. In section 5, the result is showed in a table and the fields $\phi(x)$, $\chi(x)$ and $\rho(x)$ are drew in terms of position for three different values of parameter r. Finally, we conclude in section 6 that this solutions is capable to describe the topological defect.

2. Three scalar fields system

We start with the following Lagrangian density [1],

$$\mathcal{L} = \frac{1}{2}\frac{\partial\phi}{\partial x_\alpha}\frac{\partial\phi}{\partial x^\alpha} + \frac{1}{2}\frac{\partial\chi}{\partial x_\alpha}\frac{\partial\chi}{\partial x^\alpha} + \frac{1}{2}\frac{\partial\rho}{\partial x_\alpha}\frac{\partial\rho}{\partial x^\alpha} + U(\phi,\chi,\rho), \tag{1}$$

where x^α and x_α are as $x^0 = x_0 = t,\ x^1 = -x_1 = x$ and $U(\phi,\chi,\rho)$ is the potential which is a linear function of the three fields. The variation of Lagrangian density with respect to fields lead to equations of Euler-Lagrange in the following form

$$\frac{\partial}{\partial t}(\frac{\partial\mathcal{L}}{\partial\dot{\phi}}) = \frac{\partial\mathcal{L}}{\partial\phi}, \quad \frac{\partial}{\partial t}(\frac{\partial\mathcal{L}}{\partial\dot{\chi}}) = \frac{\partial\mathcal{L}}{\partial\chi}, \quad \frac{\partial}{\partial t}(\frac{\partial\mathcal{L}}{\partial\dot{\rho}}) = \frac{\partial\mathcal{L}}{\partial\rho} \tag{2}$$

where point symbol demonstrates derivative with respect to time. So, the above model we have

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \frac{\partial U}{\partial\phi} = 0, \quad \frac{\partial^2\chi}{\partial t^2} - \frac{\partial^2\chi}{\partial x^2} + \frac{\partial U}{\partial\chi} = 0, \quad \frac{\partial^2\rho}{\partial t^2} - \frac{\partial^2\rho}{\partial x^2} + \frac{\partial U}{\partial\rho} = 0. \tag{3}$$

If the field is constant we get $\frac{\partial U}{\partial\phi} = \frac{\partial U}{\partial\chi} = \frac{\partial U}{\partial\rho} = 0$. However, for static fields configurations (i.e. $\phi = \phi(x),\ \chi = \chi(x),\ \rho = \rho(x)$) we have

$$\frac{\partial^2\phi}{\partial x^2} = \frac{\partial U}{\partial\phi}, \quad \frac{\partial^2\chi}{\partial x^2} = \frac{\partial U}{\partial\chi}, \quad \frac{\partial^2\rho}{\partial x^2} = \frac{\partial U}{\partial\rho}. \tag{4}$$

So the above equations of motion are a system of three non-linear differential equations. The energy density associated with these configurations could be written as

$$\epsilon = \frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 + \frac{1}{2}\left(\frac{d\chi}{dx}\right)^2 + \frac{1}{2}\left(\frac{d\rho}{dx}\right)^2 + U(\phi,\chi,\rho) \tag{5}$$

We note, for existence of soliton solutions the following restrictions are require

$$U(\phi,\chi,\rho) = U(-\phi,\chi,\rho), \quad U(\phi,\chi,\rho) = U(\phi,-\chi,\rho), \quad U(\phi,\chi,\rho) = U(\phi,\chi,-\rho).$$

Let us now consider models described by three scalar fields, given by [6, 7, 8, 9]

$$U(\phi,\chi,\rho) = \frac{1}{2}(1-\phi^2-r(\chi^2+\rho^2))^2 + 2r^2\phi^2(\chi^2+\rho^2) \tag{6}$$

by inserting (6) into (4), the equations of motion for above system be in the form

$$\begin{aligned} \frac{d^2\phi}{dx^2} &= -2\phi(1-\phi^2-r(\chi^2+\rho^2)) + 4r^2\phi(\chi^2+\rho^2), \\ \frac{d^2\chi}{dx^2} &= -2r\chi(1-\phi^2-r(\chi^2+\rho^2)) + 4r^2\phi^2\chi, \\ \frac{d^2\rho}{dx^2} &= -2r\rho(1-\phi^2-r(\chi^2+\rho^2)) + 4r^2\phi^2\rho. \end{aligned} \tag{7}$$

In next section, we will study a brief review on ADM and MADM methods.

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3. A brief review of ADM, and MADM methods

In here we explain ADM and MADM methods in below separately.

3.1. Fundamentals of Adomian decomposition method

We start with a general nonlinear differential equation in the following form

$$Lu + Ru + Nu = g(t), \tag{8}$$

where linear term is represented by Lu and L is a linear operator and easily invertible. We choose L as the highest-ordered derivative, R is the reminder of the linear operator, namely a term consists only u with a coefficient (constant or variable). The nonlinear term is represented by Nu, L^{-1} is defined as n-fold integration for $L = \frac{d^n}{dt^n}$. For example, we can write $L = \frac{d^2}{dt^2}$ as

$$L^{-1}L = u - u(0) - tu\prime(0). \tag{9}$$

where $L^{-1} = \int_0^t \int_0^t [.]\, dtdt$. From equation Eq. (8) we get

$$Lu = g(t) - Ru - Nu, \quad L^{-1}Lu = L^{-1}g(t) - L^{-1}Ru - L^{-1}Nu, \tag{10}$$

in that case one can obtain u as follows,

$$u = u(0) + tu\prime(0) + L^{-1}g(t) - L^{-1}Ru - L^{-1}Nu. \tag{11}$$

The first three terms are identified as in the assumed decomposition $u = \sum_{n=0}^{\infty} u_n$,

$$\begin{gathered} u_0 = u(0) + tu\prime(0) + L^{-1}g(t), \quad u_1 = -L^{-1}Ru_0 - L^{-1}Nu_0, \\ u_2 = -L^{-1}Ru_1 - L^{-1}Nu_1, \quad ..., \quad u_n = -L^{-1}Ru_{n-1} - L^{-1}Nu_{n-1}, \end{gathered} \tag{12}$$

also we write

$$Nu = \sum_{n=0}^{\infty} A_n(u_0, u_1, ..., u_n),$$

where A_n are called Adomian polynomials and depend only on u components and make a rapidly convergent series (any nonlinearity are written in terms of A_n and Nu need not even be analytic). The Adomian polynomials for one variable function are generated by the following formula :

$$A_n(u_0, u_1, ..., u_n) = \frac{1}{n!}[\frac{d^n}{d\lambda^n} N(\sum_{i=0}^{n} u_i \lambda^i)]_{\lambda=0}, \tag{13}$$

we write here the first five Adomian polynomials for convenience,

$$\begin{gathered} A_0 = f\left(u_0\right), \quad A_1 = u_1 f^{(1)}\left(u_0\right), \quad A_2 = u_2 f^{(1)}\left(u_0\right) + \left(\tfrac{1}{2!}\right) u_1^2 f^{(2)}\left(u_0\right), \\ A_3 = u_3 f^{(1)}\left(u_0\right) + u_1 u_2 f^{(2)}\left(u_0\right) + \left(\tfrac{1}{3!}\right) u_1^3 f^{(3)}\left(u_0\right), \\ A_4 = u_4 f^{(1)}\left(u_0\right) + \left[\left(\tfrac{1}{2!}\right) u_2^2 + u_1 u_3\right] f^{(2)}\left(u_0\right) + \left(\tfrac{1}{2!}\right) u_1^2 u_2 f^{(3)}\left(u_0\right) + \left(\tfrac{1}{4!}\right) u_1^4 f^{(4)}\left(u_0\right), \; ... \end{gathered} \tag{14}$$

The Adomian polynomials for two variables function $f(u(x), v(x))$ are as follow:

$$\begin{gathered} A_0 = f(u_0, v_0), \quad A_1 = u_1 f_{1,0} + v_1 f_{0,1}, \\ A_2 = u_2 f_{1,0} + v_2 f_{0,1} + (\tfrac{u_1^2}{2!}) f_{2,0} + u_1 v_1 f_{1,1} + (\tfrac{v_1^2}{2!}) f_{0,2} \\ A_3 = u_3 f_{1,0} + v_3 f_{0,1} + u_1 u_2 f_{2,0} + [u_1 v_2 + u_2 v_1] f_{1,1} + v_1 v_2 f_{0,2} + (\tfrac{u_1^3}{3!}) f_{3,0} + (\tfrac{u_1^2}{2!}) v_1 f_{2,1} \\ +(\tfrac{u_1 v_1^2}{2!}) f_{1,2} + (\tfrac{v_1^3}{3!}) f_{0,3}, \; ... \end{gathered} \tag{15}$$

where

$$f_{\mu,\nu}(u_0,v_0)=\frac{\partial^{\mu+\nu}}{\partial u^\mu\partial v^\nu}f(u(x),v(x))|_{x=0},$$

We can write the practical solution in n-term approximation [24]

$$\varphi_n=\sum_{i=0}^{n-1}u_i,\quad u=\lim_{n\to\infty}\varphi_n=\sum_{i=0}^{\infty}u_i. \tag{16}$$

where $u_0, u_1, u_2, \ldots$ are determined by the following recursive relation as mentioned above,

$$u_0(x)=f(x),\quad u_n(x)=-L^{-1}Ru_{n-1}-L^{-1}Nu_{n-1},\quad n\geq 1 \tag{17}$$

3.2. Fundamentals of modified Adomian decomposition method

In ADM method the zeroth component $u_0(x)$ is usually identified by the function $f(x)$ defined in Eq. (17). It is obvious that Adomian method changes the differential equations to obtain an easily computable components. The closed form for the solution $u(x)$ if exists can be immediately obtained because of the rapid convergence presented by the method.

The modified decomposition method suggests that the function $f(x)$ defined above in Eq. (17) be decomposed into two parts, namely $f_0(x)$ and $f_1(x)$ such that $f(x)=f_0(x)+f_1(x)$. The proper choice of the parts $f_0(x)$ and $f_1(x)$ depends mainly on trial basis. In view of this decomposition of f(x), a slight variation only on the components $u_0(x)$ and $u_1(x)$ should be introduced. The proposed variation is that only the part $f_0(x)$ be assigned to the zeroth component $u_0(x)$, whereas the remaining part $f_1(x)$ be combined with the other terms given in $u_1(x)$ to define it. In view of this assumption, we formulate the following recursive relation for the modified decomposition method [26],

$$\begin{aligned} u_0(x)=f_0(x),\quad u_1(x)=f_1(x)-L^{-1}(Ru_0(x))-L^{-1}(Nu_0(x)),\\ u_n(x)=-L^{-1}(Ru_{n-1}(x))-L^{-1}(Nu_{n-1}(x)),\quad n\geq 2. \end{aligned} \tag{18}$$

The success of the modified method depends mainly on the proper choice of the parts $f_0(x)$ and $f_1(x)$. We have been unable to establish any criterion to judge what forms of $f_0(x)$ and $f_1(x)$ can be used to yield the acceleration demanded. It appears that trials are the only criteria that can be applied so far.

4. Solving of the three scaler fields

In this section, we are going to solve the equations system of Eqs. (7) by three methods of MADM and ADM as follows:
Now we expand the the system of Eqs. (7) and apply MADM formulism to them. So that we have,

$$\begin{aligned} L\phi &= -2\phi+2\phi^3+(4r^2+2r)\phi\chi^2+(4r^2+2r)\phi\rho^2\\ L\chi &= -2r\chi+2r^2\chi^3+2r^2\chi\rho^2+(4r^2+2r)\chi\phi^2\\ L\rho &= -2r\rho+2r^2\rho^3+2r^2\rho\chi^2+(4r^2+2r)\rho\phi^2, \end{aligned} \tag{19}$$

assuming $\beta=4r^2+2r$, and with the integration of the operator L we can obtain the following expressions

$$\begin{aligned} \phi &= a_0+a_1x-2L^{-1}\phi+2L^{-1}\phi^3+\beta L^{-1}\phi\chi^2+\beta L^{-1}\phi\rho^2\\ \chi &= b_0+b_1x-2rL^{-1}\chi+2r^2L^{-1}\chi^3+2r^2L^{-1}\chi\rho^2+\beta L^{-1}\chi\phi^2\\ \rho &= c_0+c_1x-2rL^{-1}\rho+2r^2L^{-1}\rho^3+2r^2L^{-1}\rho\chi^2+\beta L^{-1}\rho\phi^2, \end{aligned} \tag{20}$$

where, we let (base on modified decomposition)

$$\phi_0 = a_0, \quad \chi_0 = b_0, \quad \rho_0 = c_0. \tag{21}$$

$$\begin{aligned}
\phi_1 &= a_1 x - 2\int_0^x\int_0^x \phi_0(x)dxdx + 2\int_0^x\int_0^x A_0[\phi(x)^3]dxdx + \\
&+ \beta\int_0^x\int_0^x A_0[\phi(x)\chi(x)^2]dxdx + \beta\int_0^x\int_0^x A_0[\phi(x)\rho(x)^2]dxdx \\
\chi_1 &= b_1 x - 2r\int_0^x\int_0^x \chi_0(x)dxdx + 2r^2\int_0^x\int_0^x A_0[\chi(x)^3]dxdx + \\
&+ 2r^2\int_0^x\int_0^x A_0[\chi(x)\rho(x)^2]dxdx + \beta\int_0^x\int_0^x A_0[\chi(x)\phi(x)^2]dxdx \\
\rho_1 &= c_1 x - 2r\int_0^x\int_0^x \rho_0(x)dxdx + 2r^2\int_0^x\int_0^x A_0[\rho(x)^3]dxdx + \\
&+ 2r^2\int_0^x\int_0^x A_0[\rho(x)\chi(x)^2]dxdx + \beta\int_0^x\int_0^x A_0[\rho(x)\phi(x)^2]dxdx
\end{aligned} \tag{22}$$

Finally from equation (9) one can obtain $\phi_n(x)$, $\chi_n(x)$ and $\rho_n(x)$ as follows:

$$\begin{aligned}
\phi_n &= -2\int_0^x\int_0^x \phi_{n-1}(x)dxdx + 2\int_0^x\int_0^x A_{n-1}[\phi(x)^3]dxdx + \\
&+ \beta\int_0^x\int_0^x A_{n-1}[\phi(x)\chi(x)^2]dxdx + \beta\int_0^x\int_0^x A_{n-1}[\phi(x)\rho(x)^2]dxdx \\
\chi_n &= -2r\int_0^x\int_0^x \chi_{n-1}(x)dxdx + 2r^2\int_0^x\int_0^x A_{n-1}[\chi(x)^3]dxdx + \\
&+ 2r^2\int_0^x\int_0^x A_{n-1}[\chi(x)\rho(x)^2]dxdx + \beta\int_0^x\int_0^x A_{n-1}[\chi(x)\phi(x)^2]dxdx \\
\rho_n &= -2r\int_0^x\int_0^x \rho_{n-1}(x)dxdx + 2r^2\int_0^x\int_0^x A_{n-1}[\rho(x)^3]dxdx + \\
&+ 2r^2\int_0^x\int_0^x A_{n-1}[\rho(x)\chi(x)^2]dxdx + \beta\int_0^x\int_0^x A_{n-1}[\rho(x)\phi(x)^2]dxdx
\end{aligned} \tag{23}$$

for $n = 2, 3, 4, \ldots$
where $A_{n-1}[.]$ are $(n-1)$th Adomian polynomials. The exact solution will be

$$\phi = \lim_{n\to\infty}\sum_{i=0}^{n}\phi_i, \quad \chi = \lim_{n\to\infty}\sum_{i=0}^{n}\chi_i, \quad \rho = \lim_{n\to\infty}\sum_{i=0}^{n}\rho_i, \tag{24}$$

with applying initial conditions $\phi(0) = \frac{1}{2}$, $\phi\prime(0) = \frac{1}{2}$, $\chi(0) = \frac{\sqrt{2}}{4}$, $\chi\prime(0) = -\frac{\sqrt{2}}{4}$, $\rho(0) = \frac{\sqrt{2}}{4}$ and $\rho\prime(0) = -\frac{\sqrt{2}}{4}$ the constant coefficient from Eq. (3) will be

$$a_0 = \frac{1}{2}, \qquad a_1 = \frac{1}{2}, \qquad b_0 = \frac{\sqrt{2}}{4}, \qquad b_1 = -\frac{\sqrt{2}}{4}, \qquad c_0 = \frac{\sqrt{2}}{4}, \qquad c_1 = -\frac{\sqrt{2}}{4},$$

so the above information into equation (6) give us,

$$\begin{aligned}
\phi(x) = \frac{1}{2} + \frac{1}{2}x + (\frac{1}{4}r^2 + \frac{1}{8}r - \frac{3}{8})x^2 + (-\frac{1}{12}r^2 - \frac{1}{24}r - \frac{1}{24})x^3 \\
+(\frac{1}{12}r^4 - \frac{1}{96}r^3 - \frac{13}{192}r^2 - \frac{1}{148}r + \frac{1}{64})x^4 + ... + \Re(x),
\end{aligned} \tag{25}$$

$$\chi(x)=\rho(x)=\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4}x+(\frac{3\sqrt{2}}{16}r^2-\frac{3\sqrt{2}}{16}r)x^2+(-\frac{\sqrt{2}}{48}r^2+\frac{5\sqrt{2}}{48}r)x^3$$
$$+(\frac{23\sqrt{2}}{384}r^4-\frac{\sqrt{2}}{24}r^3+\frac{\sqrt{2}}{128}r^2+\frac{5\sqrt{2}}{192}r)x^4+...+\Re(x), \quad (26)$$

where $\Re(x)$ is the reminder.

By using ADM method we obtain the following results

$$\phi(x)=(-\frac{19}{9600}r^5-\frac{73}{1920}+\frac{13}{76800}r^3+\frac{121}{19200}r^4+\frac{403}{6400}r^2+\frac{31}{960}r-\frac{19}{9600}r^6)x^5$$
$$+(\frac{61}{640}-\frac{203}{3840}r^2-\frac{23}{15360}r^3-\frac{5}{192}r+\frac{1}{1920}r^5+\frac{1}{1920}r^6+\frac{1}{3840}r^4)x^4$$
$$+(-\frac{1}{38400}r^3-\frac{13}{320}-\frac{1}{19200}r^4-1/12\,r^2-1/24\,r)x^3+(1/8\,r+\frac{1}{12800}r^3-\frac{121}{320}$$
$$+\frac{1}{6400}r^4+1/4\,r^2)x^2+\frac{67}{120}x+\frac{43}{84}+\ldots+\Re(x). \quad (27)$$

$$\chi(x)=\rho(x)=(-\frac{1169}{38400}r^2\sqrt{2}-\frac{871}{614400}r^6\sqrt{2}-\frac{1}{160}r\sqrt{2}-\frac{931}{76800}r^4\sqrt{2}+\frac{7}{76800}r^5\sqrt{2}$$
$$+\frac{197}{10240}r^3\sqrt{2})x^5+(\frac{43}{3072}r^4\sqrt{2}+\frac{317}{7680}r^2\sqrt{2}-\frac{1}{96}r\sqrt{2}+\frac{21}{40960}r^6\sqrt{2}$$
$$-\frac{1343}{30720}r^3\sqrt{2}+\frac{1}{15360}r^5\sqrt{2})x^4+(-\frac{13}{4800}r^4\sqrt{2}+\frac{5}{48}r\sqrt{2}-\frac{11}{307200}r^6\sqrt{2}$$
$$+\frac{401}{76800}r^3\sqrt{2}-1/48\,r^2\sqrt{2})x^3+(3/16\,r^2\sqrt{2}+\frac{1}{800}r^4\sqrt{2}-\frac{79}{25600}r^3\sqrt{2}-3/16\,r\sqrt{2}$$
$$+\frac{1}{102400}r^6\sqrt{2})x^2+(-\frac{1}{64}r^2\sqrt{2}-1/4\,\sqrt{2}+\frac{1}{960}r^3\sqrt{2})x+1/4\,\sqrt{2}-\frac{1}{6720}r^3\sqrt{2}$$
$$+\frac{1}{320}r^2\sqrt{2}+\ldots+\Re(x). \quad (28)$$

5. Discussions

In this section, we are going to discuss on obtained solutions. As the solutions of nonlinear equations are very sensitive to initial conditions and parameters thus we should choose them carefully, otherwise the result will be very different from our interest. For this purpose, we first solve system of (7) by Runge-Kutta method and the obtained values of $\phi(x)$, $\chi(x)$ and $\rho(x)$ are written in table 1 and 2 for interval $0<x<1$ in $r=0$. Also, the obtained values of $\phi(x)$, $\chi(x)$ and $\rho(x)$ are calculated with MADM and ADM methods by Eqs. (25), (26), (27) and (28) for interval $0<x<1$ in $r=0$ in table 1 and 2. Then, we compare the obtained results of MADM and ADM methods with numerical solution (Runge-Kutta method), and we specify them accuracy with column of error in table 1 and 2.

In what follows, we draw the $\phi(x)$, $\chi(x)$ and $\rho(x)$ with the choice of different values for real parameter $r=0,\frac{1}{6},1$. Also, we add solution of HPM method in figures of 1 and 2 that it has come of Ref. [27]. We can see the diverse results in figures of 1 and 2 in interval $-1<x<1$. It is worth noting that the figures 1 and 2 show kink-like topological solutions.

6. Conclusion

In this paper, we studied on topological defect structure by a system of three scalar fields. For this purpose, we wrote the Lagrangian density in terms of three scalar fields and a potential functional of three fields. We note that the fields are functional of position, and by using of equations of Euler-Lagrange we obtained equations of field as a system of three scalar fields. In

Table 1. The results of field $\phi(x)$ for $r = 0$.

x	$Numerical$	$MADM$	$Error$	ADM	$Error$
0	0.5000000000	0.5000000000	0	0.5119047619	$1.190 * 10^{-2}$
0.2	0.5847777033	0.5846916667	$8.604 * 10^{-5}$	0.6082602709	$2.348 * 10^{-2}$
0.4	0.6388794848	0.6377333333	$1.146 * 10^{-3}$	0.6740968653	$3.522 * 10^{-2}$
0.6	0.66272761418	0.6580250000	$4.703 * 10^{-3}$	0.7104041009	$4.768 * 10^{-2}$
0.8	0.65678514162	0.6450666667	$1.172 * 10^{-2}$	0.7170933419	$6.031 * 10^{-2}$
1	0.62092003909	0.5989583333	$2.196 * 10^{-2}$	0.6902529763	$6.933 * 10^{-2}$

Table 2. The results of fields $\chi(x) = \rho(x)$ for $r = 0$.

x	$Numerical$	$MADM$	$Error$	ADM	$Error$
0	0.3535533905	0.3535533905	0	0.3535533905	0
0.2	0.2828427124	0.2828427124	0	0.2828427124	0
0.4	0.2121320343	0.2121320343	0	0.2121320343	0
0.6	0.1414213562	0.1414213562	0	0.1414213562	0
0.8	0.0707106781	0.0707106780	0	0.0707106780	0
1	0.984455^{-16}	0	$9.84 * 10^{-17}$	0	$9.84 * 10^{-17}$

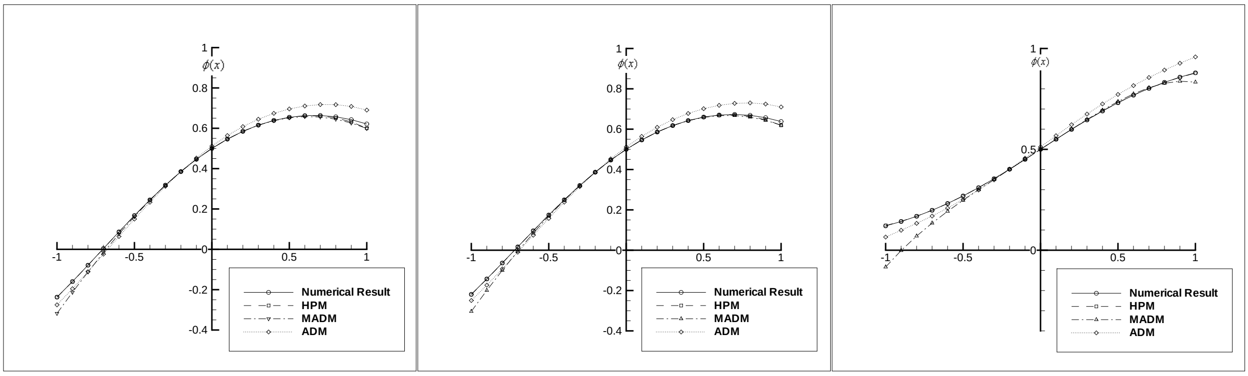

Figure 1. The graph of $\phi(x)$ for cases $r = 0$ (left hand), $r = \frac{1}{6}$ (middle) and $r = 1$ (right hand). The HPM calculated in Ref. [27].

order to solve this system, we used the ADM and MADM methods. In these methods, choice of initial condition is very sensitive, so that we chose them due to find kink-like solutions. Also the parameter r plays an important role in the solution of this system of differential equations. Therefore, we considered its value with $r = 0, \frac{1}{6}, 1$. Afterward, we solve this system as numerical solution by Runge-Kutta method, so that tables 1 and 2 are included by three solutions of numerical, ADM and MADM in interval $0 < x < 1$ for $r = 0$. Finally, we plotted three scalar fields in terms of position, and compared them together. The figures 1 and 2 depicted the variation of three fields against position. Also we entered solution of HPM method (result of Ref. [27]) in figures 1 and 2.

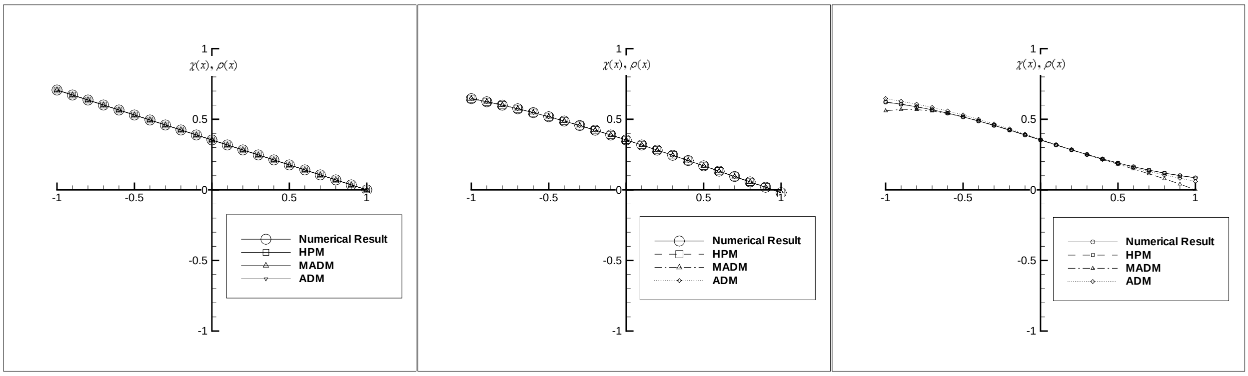

Figure 2. The graph of $\chi(x) = \rho(x)$ for cases $r = 0$ (left hand), $r = \frac{1}{6}$ (middle) and $r = 1$ (right hand). The HPM calculated in Ref. [27]

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