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EVALUATION OF THE RESOLUTION OF CTC AND SVX

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Abstract

We summarize the status of understanding of the resolution of the CTC and the SVX tracking systems at this point. The CTC track parameter errors as obtained in V6.01 tracking are underestimated by factors which range from 1.6, from studies of residual distributions, to about 2.7, from studies of mass and χ^2 distributions. These scale factors, however, do not seem to affect the uncertainty on the impact parameter or decay length measurements in the SVX, as the dominant factor in this measurement is the uncertainty due to the beam size. For V6.1 tracking, preliminary analyses show that the CTC errors scale factor obtained from residuals is 1.2, whereas the factor obtained from track parameters is 1.7. The tuning of the SVX errors for each cluster size is also discussed.

1 Introduction

Understanding the errors on the track parameters as returned from the fits in the CTC, SVX or the combination of the two is important for a variety of reasons:

- verify our understanding of the CTC and SVX tracking systems, which can be somehow quantified by our ability to describe the observed track parameter errors;
- weigh properly the CTC and SVX part of a track, when we combine the two;
- have the right values of the predicted track parameter errors in physics analysis where these errors are relevant. In particular this is very important for B tagging algorithms based on the significance of some track related variable.

In the following we shall not attempt to improve the resolution of the tracking detectors (eg. by improving their alignments or calibrations), but will just take a snapshot of the current status of their resolution.

In section 2 we will study the CTC errors, in section 3 the SVX errors, and in section 4 the combination of the two.

2 CTC errors

This study is based on production 6.01 tracking. Major improvements in the space-time relation have been made lately and will become part of version 6.1. This makes the CTC performance better than it is described here. We will quote the few results available with V7.07 tracking using better CTC calibration and V6.1 tracking which has still better calibrations. The tracking code has changed very little for these different tracking versions, whereas the CTC calibration has changed substantially. To illustrate this point, Fig. 1 shows the CTC t_0 as a function of run number for V6.01 and V7.07 (data base as of January 15). This illustrates the famous t_0 problem for runs 42600 to 42744 when the STAGE 0 code was not working properly.

Fig. 2 shows the impact parameter resolution as a function of the run number for the electrons of the W sample processed with V6.01 (top) and with V7.07 (bottom) using the CTC calibrations as of January 15. The retracking has been done at Chicago by Sarah Eno. The CTC impact parameter resolution with V6.01 is on the average 660 μm and follows clearly the pattern of the t_0 used. The new CTC constants show a remarkable improvement of the impact parameter resolution, as seen in Fig. 2b. In both cases the beam position as determined by the SVX was used.

In order to assess what is the situation with the CTC errors, first we look at the residual distributions. Fig.3 shows the residuals for tracks in a sample of 100 random events tracked with V6.01. In the table below we display the resolutions currently used by the CTC reconstruction code and compare them with the width measured from these data. Fig. 4 shows the residual obtained in min-bias events with V6.1 and calibration data base as of February 8. The values of these residuals and their ratio to the resolutions used in the tracking code, are also shown.

Superlayer	0	1	2	3	4	5	6	7	8
Resolution used	200	163	162	161	151	192	152	146	146
Width (μm) V6.01	332	193	276	225	220	297	239	231	238
Ratio V6.01	1.66	1.18	1.70	1.40	1.46	1.55	1.93	1.58	1.63
Width (μm) V6.1	217	192	200	193	179	235	195	188	189
Ratio V6.1	1.04	1.18	1.23	1.20	1.19	1.22	1.28	1.28	1.29

We observe that the width of these distributions is systematically wider than the resolutions used in the CTC reconstruction code. The ratio between the observed widths and the resolutions used in V6.01 tracking ranges between 1.2 and 1.9, depending on the superlayer number, with an average value of 1.6. For V6.1 tracking, the situation is clearly improved, the average factor being 1.2.

For V6.01 the average factor 1.6 naively should reflect directly into the track parameter errors returned by the CTC. In practice, however, this is just a lower bound on the actual rescaling factor for the track parameter errors. Indeed there are various factors which make the widths of the residual distribution narrower than the actual CTC resolution. The major one is that it is believed that the systematics of the CTC are highly correlated (e.g. common errors in the space-time relation), the helix fit then adjusts the track parameters to compensate for these correlated errors. It is therefore necessary to add additional information in order to extract more realistic rescaling factors.

Two approaches have so far given results: one is the study of the width of the invariant mass distribution of various resonances (J/ψ, Υ, Z_0), the other is the study of the behaviour of the χ^2 of the 2 μ vertex for the J/ψ sample.

2.1 Invariant mass study

Let P_{t1}, ϕ_1, s_1 and P_{t2}, ϕ_2, s_2 be the transverse momentum, direction in the transverse plane and $cot(\theta)$ of 2 tracks into which a resonance decays. Their invariant mass can be written, in the limit of massless decay products, as:

$$M^2 = 2P_{t1}P_{t2}\left(\sqrt{1+s_1^2}\sqrt{1+s_2^2} - cos(\phi_1-\phi_2) - s_1 s_2\right) + m_1^2 + m_2^2$$

If s_1 and s_2 are small relative to 1, which amounts to select fairly central tracks, then the above formula can be expanded to first order in s^2 as:

$$M^2 = 2P_{t1}P_{t2}(1 - cos(\Delta\phi) + \frac{1}{2}\Delta s^2) + m_1^2 + m_2^2$$

Applying error propagation on the above formula and disregarding the correlation terms, we obtain:

$$\frac{\sigma_M^2}{M^2} = (\frac{\sigma_{P_{t1}^2}}{P_{t1}})^2 + (\frac{\sigma_{P_{t2}^2}}{P_{t2}})^2 + \left(sin^2\Delta\phi(\sigma_{\phi_1}^2 + \sigma_{\phi_2}^2) + (s_1 - s_2)^2(\sigma_{s_1}^2 + \sigma_{s_1}^2)\right)\left(\frac{2P_{t1}P_{t2}}{M^2}\right)$$

If M is large we can make the additional approximation that $P_{t1} \sim P_{t2} \sim M/2$, and remembering that the fractional P_t resolution is proportional to P_t, we can further simplify the above formula and obtain:

$$\frac{\sigma_M^2}{M^2} = k^2M^2/2 + \left(sin^2\Delta\phi(\sigma_{\phi_1}^2 + \sigma_{\phi_2}^2) + (s_1 - s_2)^2(\sigma_{s_1}^2 + \sigma_{s_1}^2)\right)/4$$

which reduces to just $k^2M^2/2$ for large masses.

Applying this equation to the study of the mass width of the Z_0 into muons (fig. 5a), we can extract a scale factor for the curvature measurement. The value actually obtained is 3.2. A similar analysis of the Υ peak (fig. 5b) gives a lower factor, 2.6. The discrepancy can probably be understood in terms of the relative effect of the systematics in the two cases. For the Z_0 the curvature measurement is at the limit of the CTC resolution, so systematic effects can have a larger influence on the curvature measurement than in the case of the Υ. The low statistics and the non gaussian tails in the mass distribution make in any case this measurement quite difficult.

The situation is better in the J/ψ case, since we have a much larger statistics available. We can therefore split the data into different kinematical regions to make alternatively dominant the contribution of the P_t, ϕ and s resolutions. The results of this analysis (fig. 6) are consistent with scale factors on the calculated errors of 2.7 ± 0.2 for the kinematic regions dominated by p_t and ϕ_0 errors and 2.1 ± 0.4 for the region dominated by $cot(\theta)$ errors.

In summary the scale factors obtained with this analysis are as follows:

Sample	Parameter	Scale Factor
Z_0	C	3.2
Υ	C	2.6
J/ψ	C	2.7
J/ψ	ϕ_0	2.7
J/ψ	$cot(\theta)$	2.1

2.2 Vertex χ^2 study

In this study we analyze the χ^2 distribution of the fit to a common vertex of the 2 μ which make the J/ψ. What we expect is a 1 DoF distribution, which we know to be dominated by the longitudinal track parameters, since 2 circles almost always cross in the transverse plane. We then fit the χ^2 distribution to the expected statistical distribution applying a variable scale factor to the observed χ^2. The square root of this scale factor is the rescaling factor on the CTC track parameter errors.

The data is consistent with a rescaling factor of 2.1 (fig. 7a). After retracking with the CTC errors scaled by a factor 2.0 this scale factor becomes 1.18 (fig. 7b). Unfortunately in this study we have used SVX+CTC tracks. Since the SVX measurement is improving the Z_0, $cot(\theta)$ measurements by about 20% (these errors are indeed correlated to the transverse parameter errors, so even if the SVX does not measure directly these parameters, an indirect improvement is induced by the improvement in the transverse parameter resolution through these correlations), this could mean that the actual rescaling factor for the CTC errors is 20% higher or 2.5, in better agreement with other results.

In conclusion the CTC errors are clearly underestimated. Also it is clear that the residual distributions are showing a very incomplete side of the issue. The scale factor to apply to the CTC track errors appears to be about 1.6 from the residuals and in the vicinity of 2.7 when additional CTC independent constraints are applied like a mass or a 2 track vertex. Factors in the 2.7 range are confirmed from other independent analyses which involve a comparison with SVX, as will be shown later.

3 SVX errors

Given the working mechanisms of a microstrip detector we expect the position resolution of the SVX to be dependent on the size of the cluster of strips used to calculate the hit position. We have therefore determined the SVX resolution for clusters of size 1, 2, 3. Higher cluster sizes are known to be usually due to track overlap or noise, in this case therefore the SVX tracking code does not trust the charge sharing algorithm and defaults an overestimated resolution of pitch × cluster size/$\sqrt{12}$.

The method used consists in scanning a reasonable range of possible errors for each cluster size and studying the normalized residual distribution (pulls) for each of these values. For each value chosen for the cluster resolution, σ_C, we measure the corresponding σ_{NR} of the normalized residual distribution. The calculation of the normalization factor follows these lines: we first calculate the dependence of the residuals, $\vec{R}$, on the difference between the measured and true cluster position, $\Delta\vec{x}$:

$$\vec{R} = (ACA^tS^{-1} - I)\Delta\vec{x}$$

where:

- A is the derivative of the predicted hit position with respect to the track parameters;
- C is the covariance matrix of the parameters;
- S is the covariance matrix of the measured points.

Since by definition we have that $< \Delta\vec{x}\Delta\vec{x}^t >= S$, we can derive easily the covariance matrix of the residuals:

$$< \vec{R}\vec{R}^t >= S - ACA^t$$

the diagonal terms of this matrix give us, on a track by track basis, the normalization factor to apply to each residual. A plot of σ_{NR} versus σ_C (fig.8) shows a roughly linear dependence, allowing to extrapolate the value of σ_C which makes $\sigma_{NR} = 1$.

The results obtained are shown in the following table:

Cluster Length	1	2	3
σ_C	15 μm	13 μm	25 μm

In this study the track helix fit was performed only in the transverse plane holding the curvature constant and equal to the value determined by the CTC, and using only SVX hits. This measurement is therefore free of CTC systematics, but could still be sensitive to SVX misalignments which could introduce unaccounted correlations. These, however possible effects, should be mitigated by the fact that we have averaged over all barrels and wedges, when we performed the above analysis.

4 Combining CTC and SVX

We can use the SVX information to get a better measurement of the CTC error rescaling factors. The technique is quite simple. One has to assume that the errors of the CTC+SVX track are basically correct, or that in any case the amount by which they are mistaken is small compared to the difference between the CTC and CTC+SVX errors, which is in general the case. Let $\vec{\alpha_C}$ be the set of track parameters returned by the CTC fit and $\vec{\alpha_S}$ the set of track parameters returned by the CTC+SVX fit, and C_C and C_S their covariance matrices. It is fairly straightforward to prove that the covariance matrix of the parameter variation after the addition of the SVX information is given by the difference $C_C - C_S$. We can therefore plot the pulls of the parameter variations and see how much they differ from 1.

We have done this with the sample of J/ψ muons and obtained rescaling factors in the range from 1.7 to 2 depending on the parameter under consideration. We expect however that some of the effect can be washed out by the effect of multiple scattering in the CTC can, the VTX region and the SVX. To cope with this we have applied a P_t of 3 GeV on all tracks, not completely out of multiple scattering effect, but better. Indeed the rescaling factors now range from 1.9 to 2.1. To make sure to be free of SVX dependent systematics we have added then a J/ψ mass constraint and a common vertex constraint and recalculated the pulls. This measurement is somewhat more difficult since it introduces some background from "fake" J/ψ for which the mass constraint cannot be applied. In this case we fit a gaussian to the resulting distributions to remove tails due to this problem. The result is quite consistent with the previous approach, with scaling factor again in the range of 1.9 to 2.1. All these results are summarized in the following table:

Parameter	$cot(\theta)$	C	Z_0	D	ϕ_0
Widths (All p_t)	1.79	1.99	2.04	1.92	1.74
Widths ($p_t > 3$)	2.04	2.13	2.14	2.10	1.87
Widths (with constraints)	2.11	2.09	1.85	1.94	1.88

A similar approach applied to W electrons, where multiple scattering is totally negligible, yields the following rescaling factors (for V6.01 tracking):

Parameter	Before rescaling		After rescaling	
V6.01	Positive	Negative	Positive	Negative
$Cot(\theta)$	2.55	2.82	1.12	1.19
C	2.60	3.11	1.10	1.13
Z_0	2.58	2.80	1.14	1.16
D	2.52	3.01	1.16	1.18
ϕ_0	2.49	2.74	1.13	1.16

In the above table we have also shown the effect of repeating the study after rescaling the CTC errors by a factor 2.7. This has not only the effect of reducing the σ of the pull distribution, but also it makes much more symmetric the differences between positive and negative charges. Fig. 9 shows the D pull distributions before and after the rescaling.

We have repeated the study with V7.07 tracking that used the new CTC constants, after the famous t_0 problem discussed earlier. Sarah Eno reprocessed the data during the period Jan 12-15. We obtain the following factors:

Parameter	Before rescaling		After rescaling	
V7.07	Positive	Negative	Positive	Negative
$Cot(\theta)$	1.98	2.05	1.01	1.08
C	2.03	1.96	1.04	1.05
Z_0	1.94	1.97	1.01	1.06
D	1.97	2.02	1.02	1.08
ϕ_0	2.03	2.07	1.04	1.07

The factors in the last two columns were obtained after the CTC errors were multiplied by a factor 2. There is a clear improvement when we use the new CTC constants. Still better constants are already available and we have rerun the W sample with V6.1 using the data base available as of Feb.12. We obtain the scale factors shown in the following table.

Parameter	Before rescaling	
V6.1	Positive	Negative
$Cot(\theta)$	1.68	1.73
C	1.66	1.72
Z_0	1.67	1.71
D	1.69	1.74
ϕ_0	1.70	1.77

The situation has clearly improved, with an average scale factor of 1.7, but it is not optimal as yet. As already observed, the residuals for the same V6.1 run on min-bias events still on the average a factor 1.2 larger then expected.

While the CTC errors appear to be underestimated, the status of the ϕ_0 and impact parameter errors for CTC+SVX tracks is much more under control. Indeed it even seems to be very weakly correlated with the CTC error problem. We have approached the problem from 2 sides: (i) study the SVX component of the track χ^2, (ii) study of the L_{xy} distribution for J/ψ and Υ.

4.1 SVX χ^2

The SVX χ^2 is made up of 2 components: one is related to the SVX residuals, the other to the variation of the CTC parameters. This χ^2 should be distributed as a 4 DoF.

Fig. 10 shows the χ^2 distribution for muon tracks from the J/ψ sample. If we fit the shape of this distribution applying an overall scale factor, we find a 1.4 scale factor on the errors and a rather poor fit to the distribution (Fig. 10a). We repeat this exercize after multiplying by a factor of 2 the CTC errors, and obtain a scale factor of 1.18, with a better fit to the distribution. It is possible that this remaining 18% is still due to the CTC, since most measurements tend to point to rescaling factors bigger than 2.

Fig. 11a shows the χ^2 distribution for the electrons in the W sample ($P_t > 21$ GeV/c). The overall scale factor for this distribution is again 1.39 and the fit appears to be very poor. When we multiply the CTC errors by a factor 2.7, as previously determined for this sample, we obtain a scale factor of 1.02 and a much better fit as shown in Fig. 11b. The factor 2.7 seems to be a better estimate of the degradation of the CTC errors over the errors assumed in the tracking code.

4.2 L_{xy} distributions

The J/ψ L_{xy} distribution (distance of the J/ψ vertex from the primary vertex projected along the ψ direction) has a large gaussian component since about 80% of our J/ψ seem to originate from the primary vertex. Fitting a gaussian to the core of the distribution of L_{xy}/σ is a very effective way to verify the quality of the ϕ_0 and impact parameter errors. The result of the gaussian fit is 1 with a very small statistical error. A similar approach using Υ's gives a comparable result though with lower statistics. Both distributions are shown in fig.12. It is worth pointing out that in the calculation of this error we need to include the width of the beam, which turns out to be the major contribution in the calculation of the errors. Indeed, we have verified that the width of the L_{xy} distribution is very weakly dependent on the value assigned to the CTC errors. We keep in fact getting a width of about 1 for these distributions whether we rescale or not the CTC errors.

5 Conclusions

In conclusion, there are many indications that the CTC errors are underestimated. The exact amount of this underestimation appears to be hard to explain in full using only the CTC residual distributions. Additional studies involving external constraints (SVX matching or a physical mass) indicate that the CTC errors are underestimated by a most likely factor of about 2.7 for V6.01 tracking. This factor is about twice as large as one would estimate from an inspection of the CTC residual distributions.

For V6.1 tracking, using the CTC calibrations as of Feb 12, we find a scale factor of 1.7, whereas the residuals suggest a scale factor of 1.2.

Finally, the SVX errors appear to be fairly well understood from an analysis of the combined CTC/SVX χ^2's and of the L_{xy} error. In particular the latter does not seem to be affected by the underestimation of the CTC errors.

Figure captions

Fig. 1. a.) CTC t_0 used for V6.01 tracking versus run number. The region of values between 4.5 ns and 8 ns corresponds to runs 42146-42356 when the trigger t_0 was delayed by 3 ns. b.) t_0 versus run number for V7.07 tracking with data base of January 15.

Fig. 2. The resolution of the impact parameter as measured by the CTC, for a.) V6.01 tracking and b.) V7.07 tracking. Electrons from W's were used in both cases.

Fig. 3. CTC residual for the 9 superlayers for V6.01 tracking . All tracks were used and all hits were plotted whether or not they were used in the fit.

Fig. 4 CTC residual for V6.1 tracking. Only hits used by the fit are plotted.

Fig. 5 Distribution of the square of the (measured mass - nominal value) divided by its error, for a.) Z_0 events and b.) Υ events.

Fig. 6 Same as Fig. 5, but for the J/ψ. Distributions for three different kinematic regions are shown.

Fig. 7 χ^2 distribution for the fit to a common vertex of the two muons from the J/ψ. a.)For V6.01 tracking, b.) After the CTC errors have been multiplied by a factor 2.

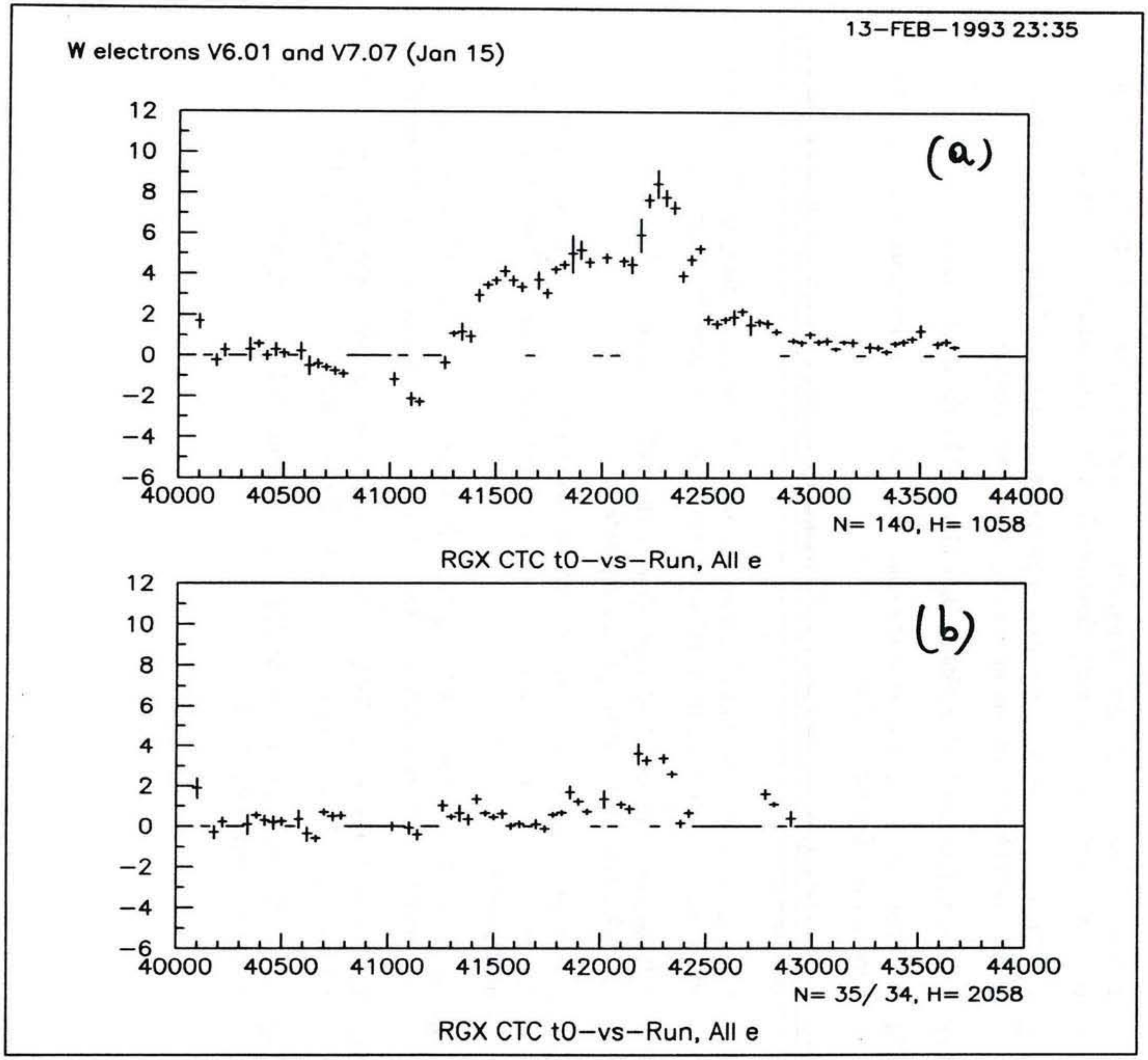
Fig. 8 Dependence of normalized residuals upon the SVX resolution for clusters of different size. The different layers are plotted separately.

Fig. 9 Pulls of the impact parameter for W electrons for V6.01 tracking for a.) positrons, b.) electrons. c.) and d.) show the same distributions after the CTC errors have been multiplied by a factor 2.7.

Fig. 10 χ^2 distribution for the SVX tracks of the two muons from the J/ψ for V6.01, a.) for standard CTC errors and b.) after refit using CTC errors multiplied by a factor 2.

Fig. 11 χ^2 distribution for the SVX tracks of the electron from W's using V6.01 tracking. a.) for standard CTC errors and b.) for CTC errors multiplied by a factor 2.7.

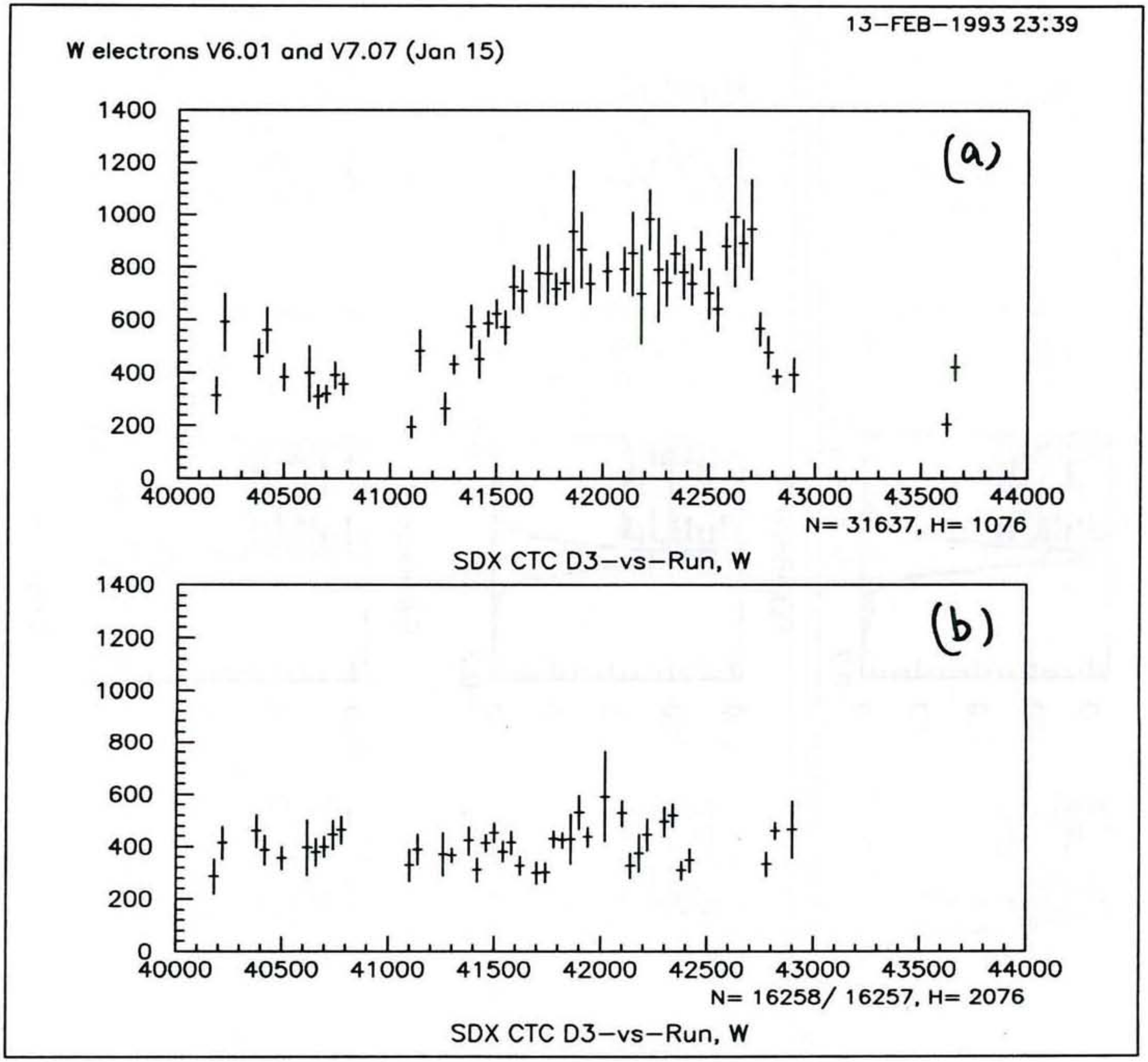
Fig. 12 a.) The flight distance of the J/ψ in the xy plane divided by its error, and b.) the same distribution for the Υ.



13-FEB-1993 23:35
W electrons V6.01 and V7.07 (Jan 15)
(a)
N= 140, H= 1058
RGX CTC t0-vs-Run, All e
(b)
N= 35/ 34, H= 2058
RGX CTC t0-vs-Run, All e

Fig. 1

Fig. 2


13-FEB-1993 23:39
W electrons V6.01 and V7.07 (Jan 15)
(a)
1400
1200
1000
800
600
400
200
0
40000
40500
41000
41500
42000
42500
43000
43500
44000
N= 31637, H= 1076
SDX CTC D3-vs-Run, W
(b)
1400
1200
1000
800
600
400
200
0
40000
40500
41000
41500
42000
42500
43000
43500
44000
N= 16258/ 16257, H= 2076
SDX CTC D3-vs-Run, W

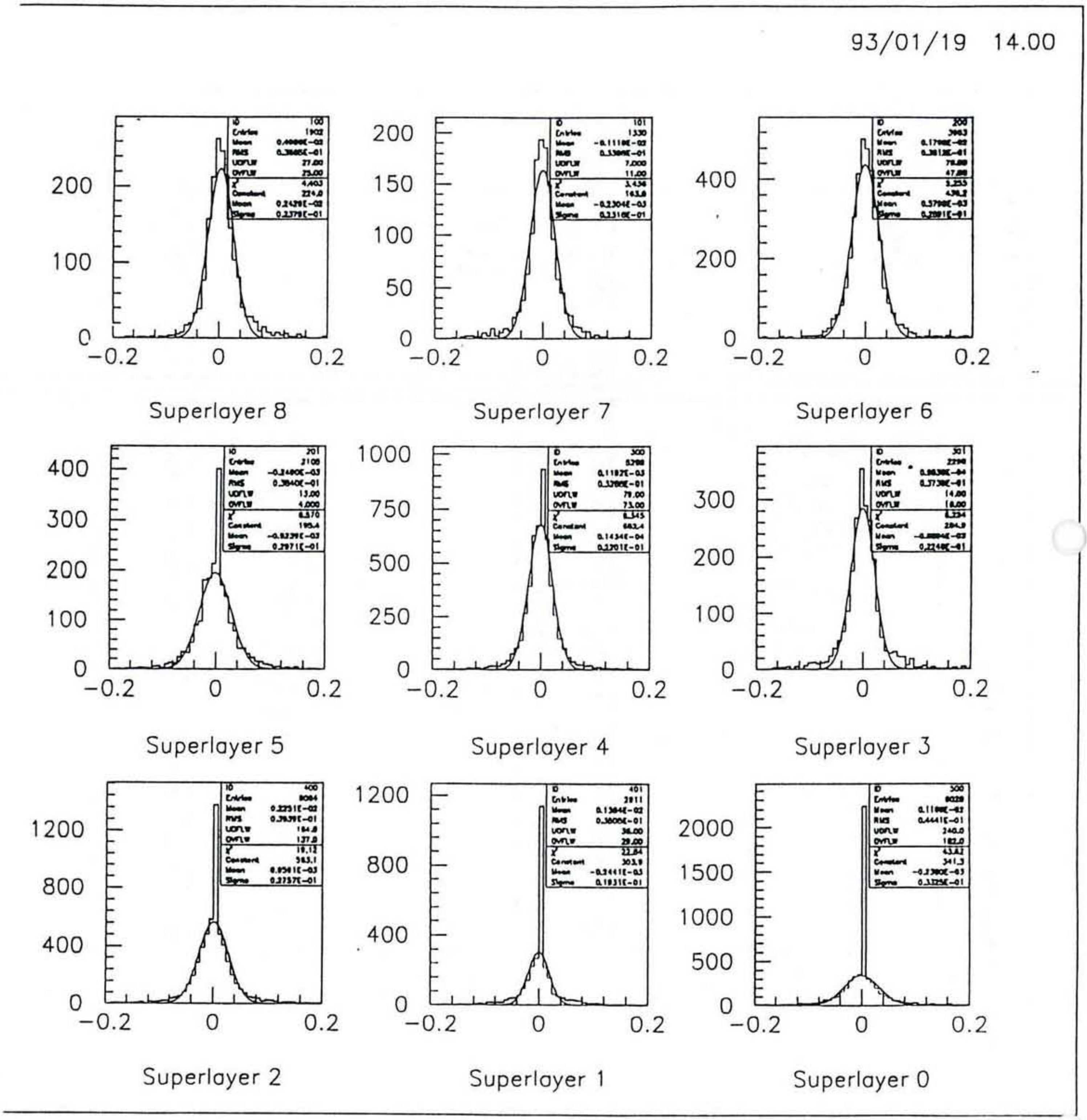
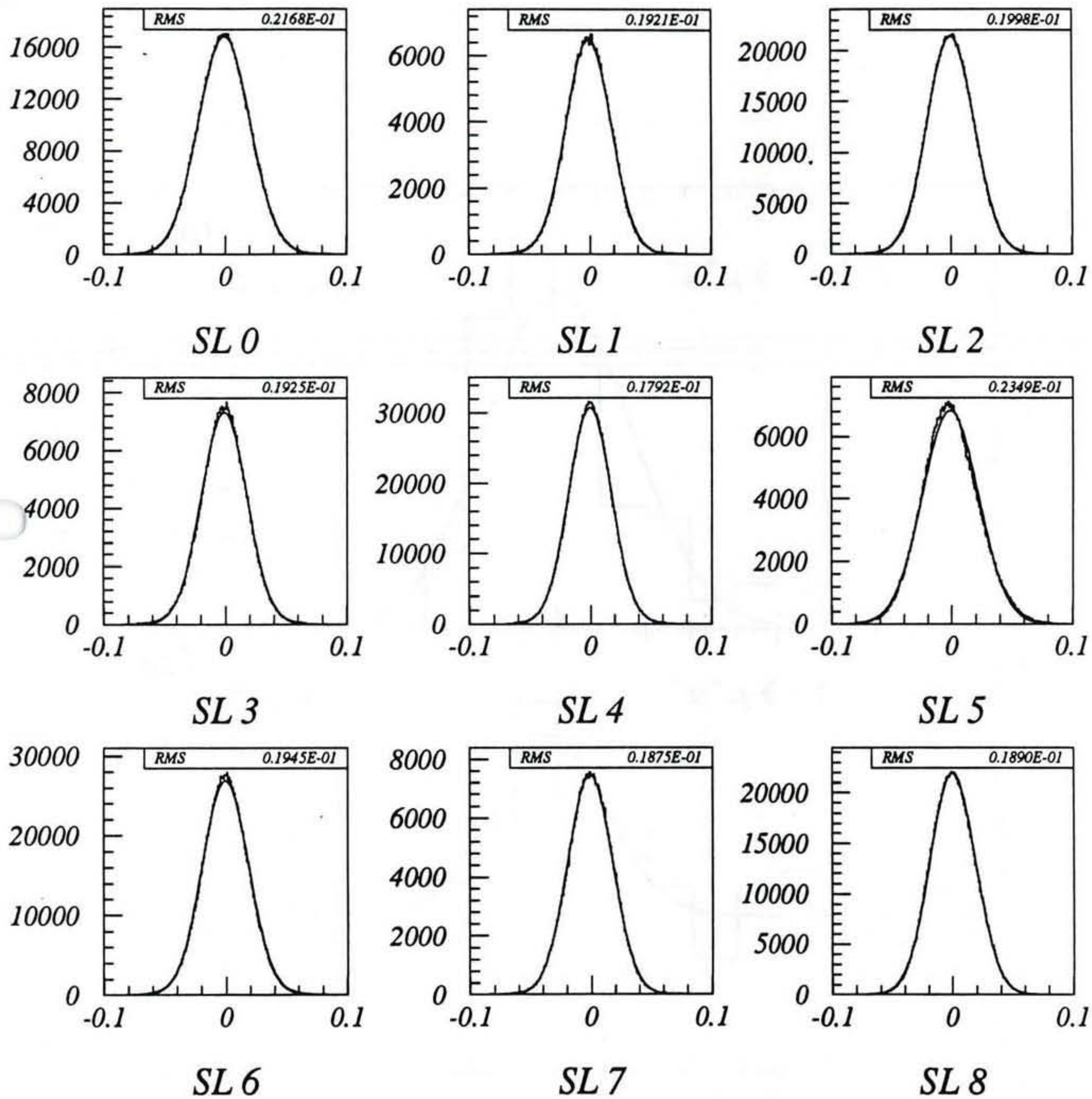

93/01/19 14.00
Superlayer 8
Superlayer 7
Superlayer 6
Superlayer 5
Superlayer 4
Superlayer 3
Superlayer 2
Superlayer 1
Superlayer 0
−0.2
0
0.2

fig. 3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SL 0
RMS 0.1921E-01
SL 1
RMS 0.1998E-01
SL 2
RMS 0.1925E-01
SL 3
RMS 0.1792E-01
SL 4
RMS 0.2349E-01
SL 5
RMS 0.1945E-01
SL 6
RMS 0.1875E-01
SL 7
RMS 0.1890E-01
SL 8

Fig. 4

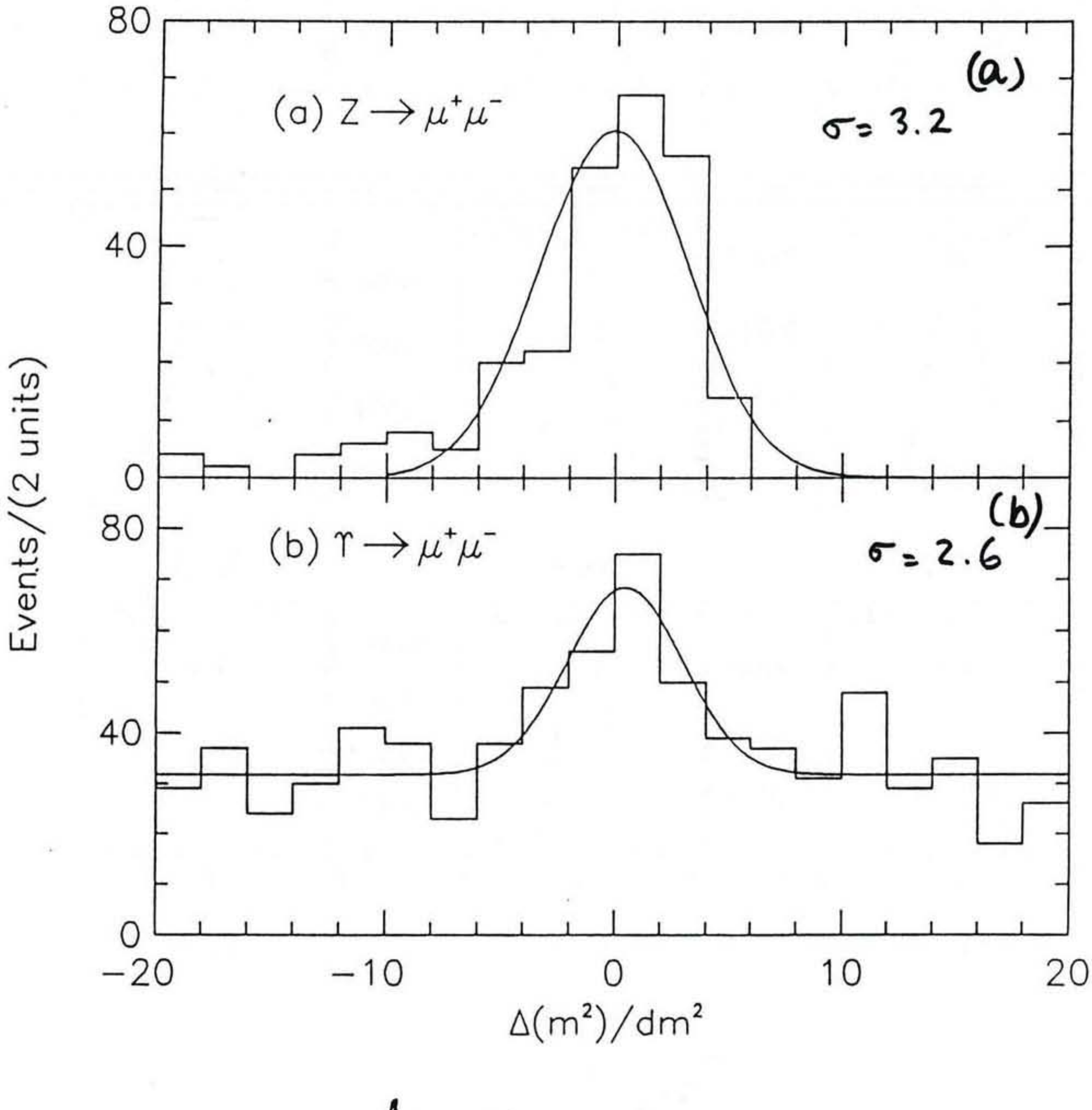
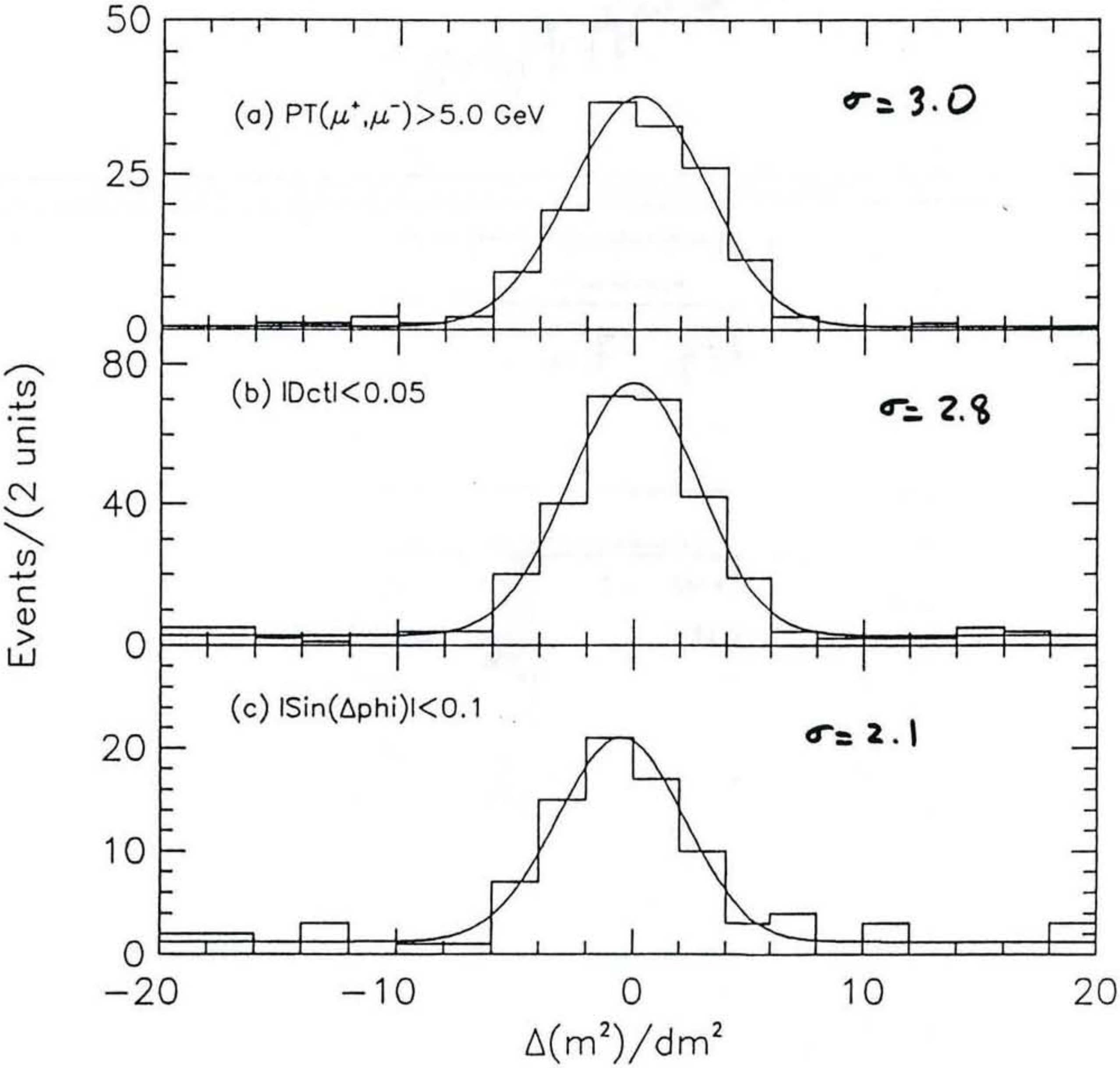


fig. 5.



(a) PT(μ⁺,μ⁻)>5.0 GeV
σ= 3.0
(b) |Dct|<0.05
σ= 2.8
(c) |Sin(Δphi)|<0.1
σ= 2.1
Events/(2 units)
Δ(m²)/dm²
50
25
0
80
40
0
20
10
0
−20
−10
0
10
20

fig. 6

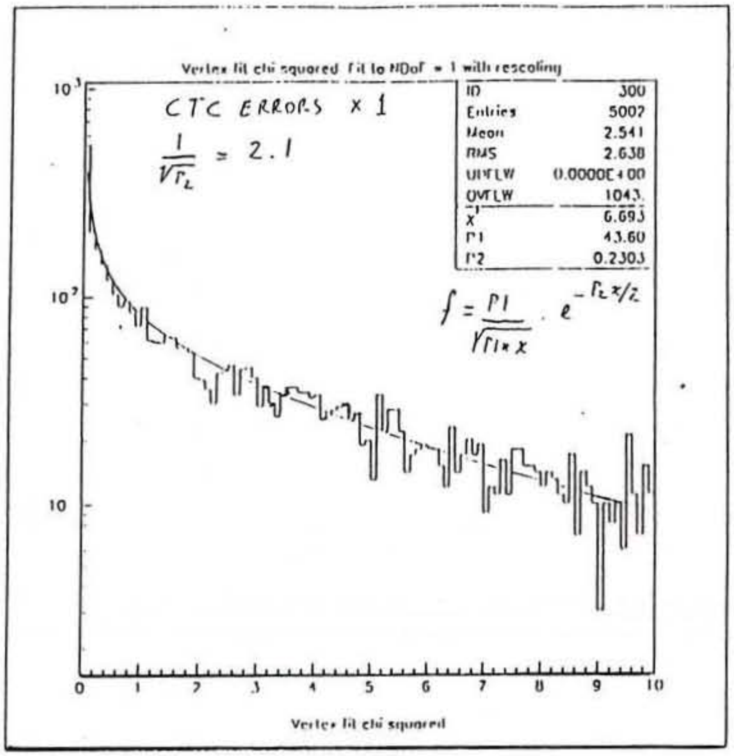

Vertex fit chi squared fit to NDoF = 1 with rescaling
CTC ERRORS x 1
$\frac{1}{\sqrt{P_2}} = 2.1$
ID 300
Entries 5007
Mean 2.541
RMS 2.638
UDFLW 0.0000E+00
OVFLW 1043.
χ^2 6.693
P1 43.60
P2 0.2303
$f = \frac{P1}{\sqrt{P1 * x}} \cdot e^{-P_2 x/2}$
10³
10²
10
0 1 2 3 4 5 6 7 8 9 10
Vertex fit chi squared

fig. 7.a

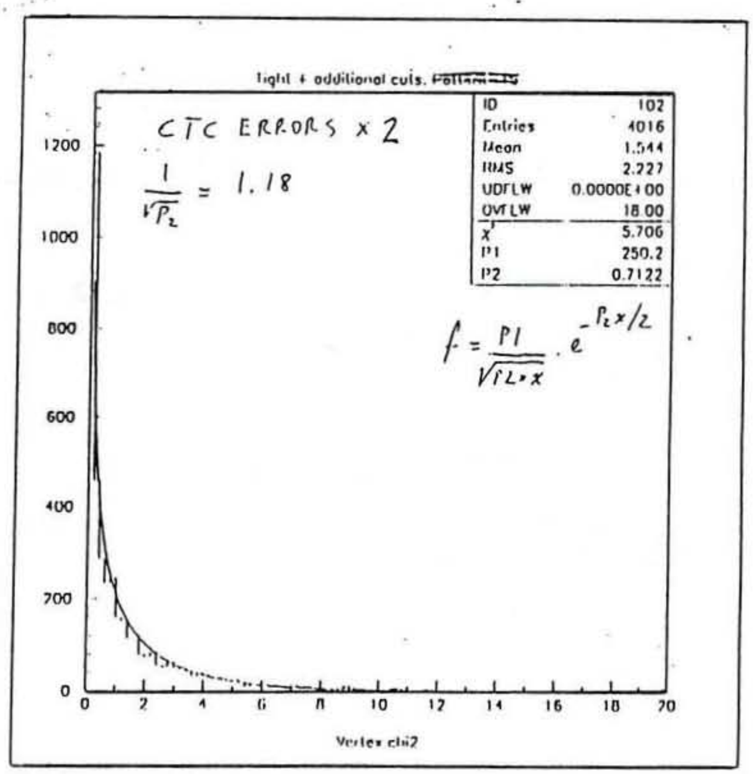

tight + additional cuts.
CTC ERRORS x 2
$\frac{1}{\sqrt{P_2}} = 1.18$
ID 102
Entries 4016
Mean 1.544
RMS 2.227
UDFLW 0.0000E+00
OVFLW 18.00
χ^2 5.706
P1 250.2
P2 0.7122
$f = \frac{P1}{\sqrt{P2 * x}} \cdot e^{-P_2 x/2}$
1200
1000
800
600
400
200
0
0 2 4 6 8 10 12 14 16 18 20
Vertex chi2

fig. 7.b

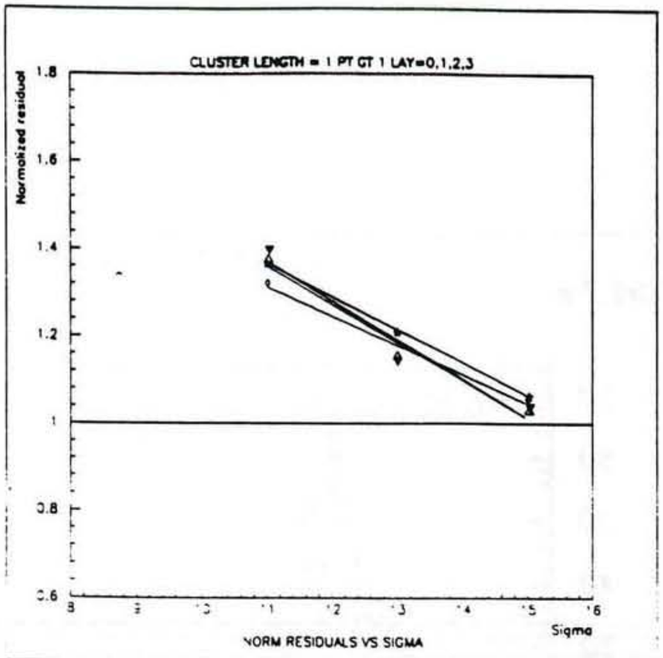


Fig 8a

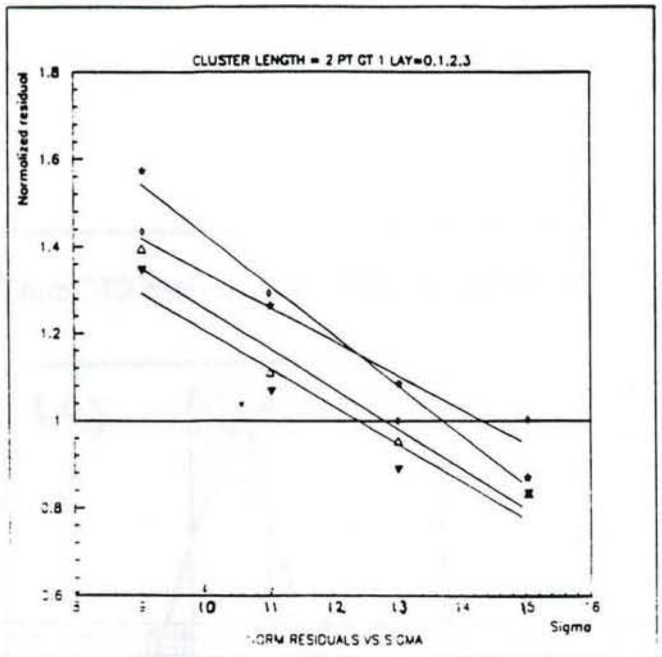


Fig 8b

Different symbols correspond to different layers.

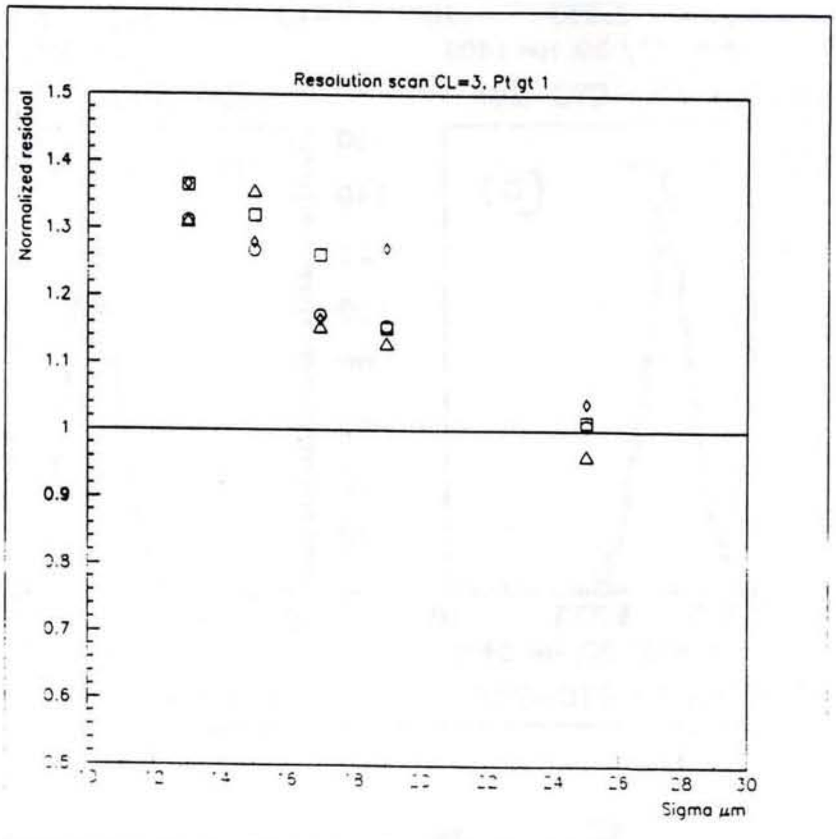


Fig. 8c

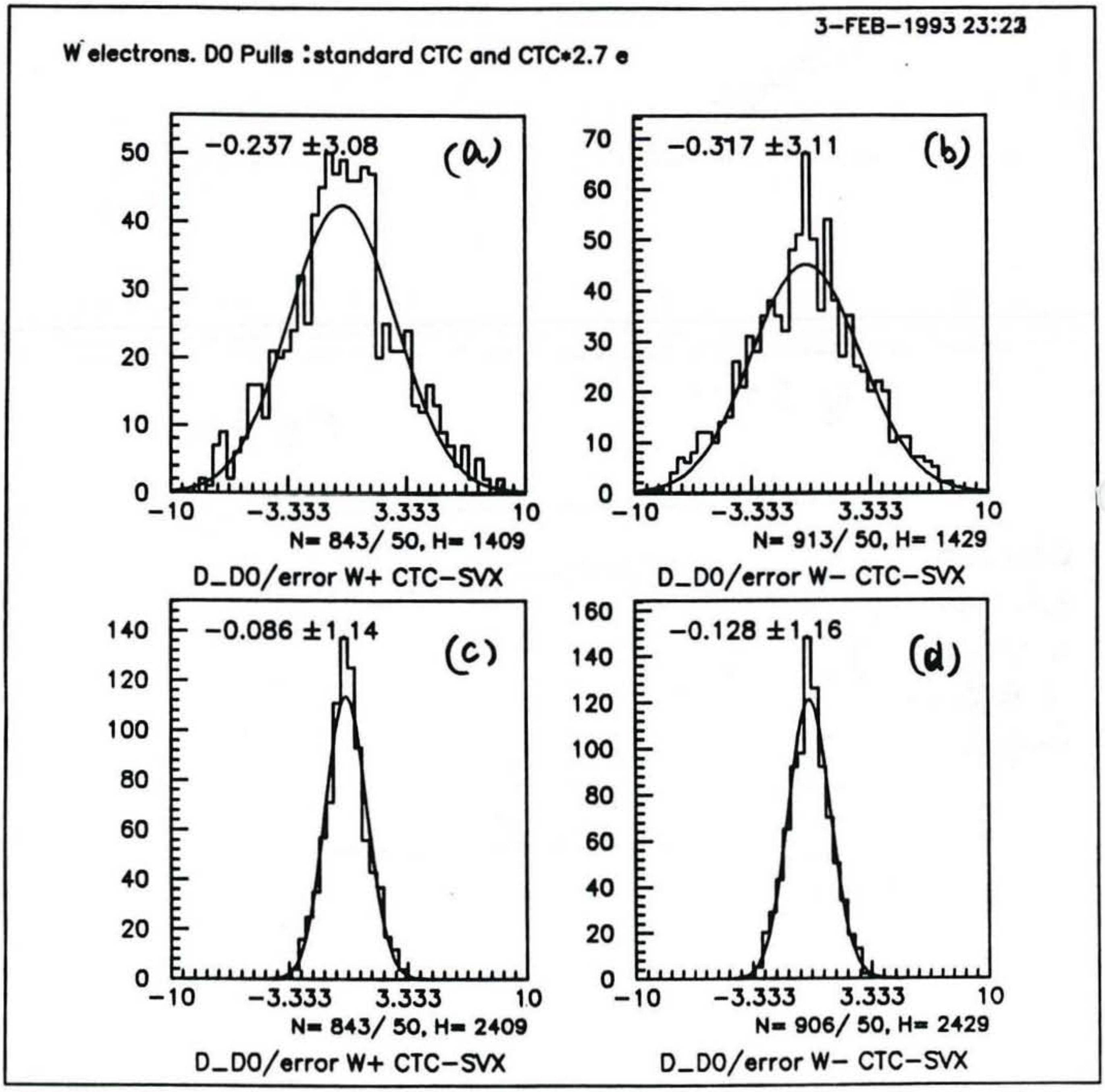

3-FEB-1993 23:23
W electrons. D0 Pulls :standard CTC and CTC*2.7 e
-0.237 ±3.08
(a)
N= 843/ 50, H= 1409
D_D0/error W+ CTC-SVX
-0.317 ±3.11
(b)
N= 913/ 50, H= 1429
D_D0/error W- CTC-SVX
-0.086 ±1.14
(c)
N= 843/ 50, H= 2409
D_D0/error W+ CTC-SVX
-0.128 ±1.16
(d)
N= 906/ 50, H= 2429
D_D0/error W- CTC-SVX

Fig. 9

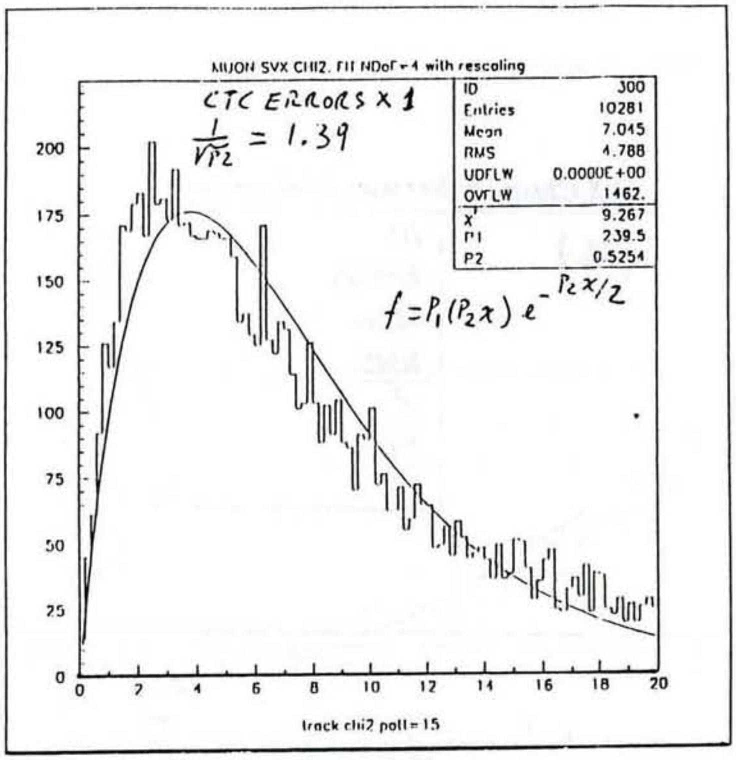


fig. 10a

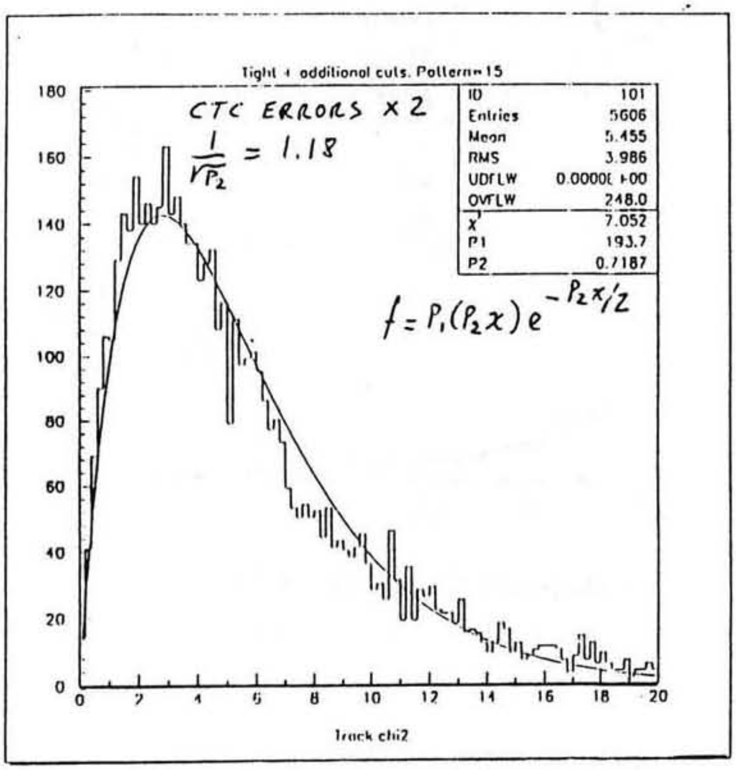


fig. 10b

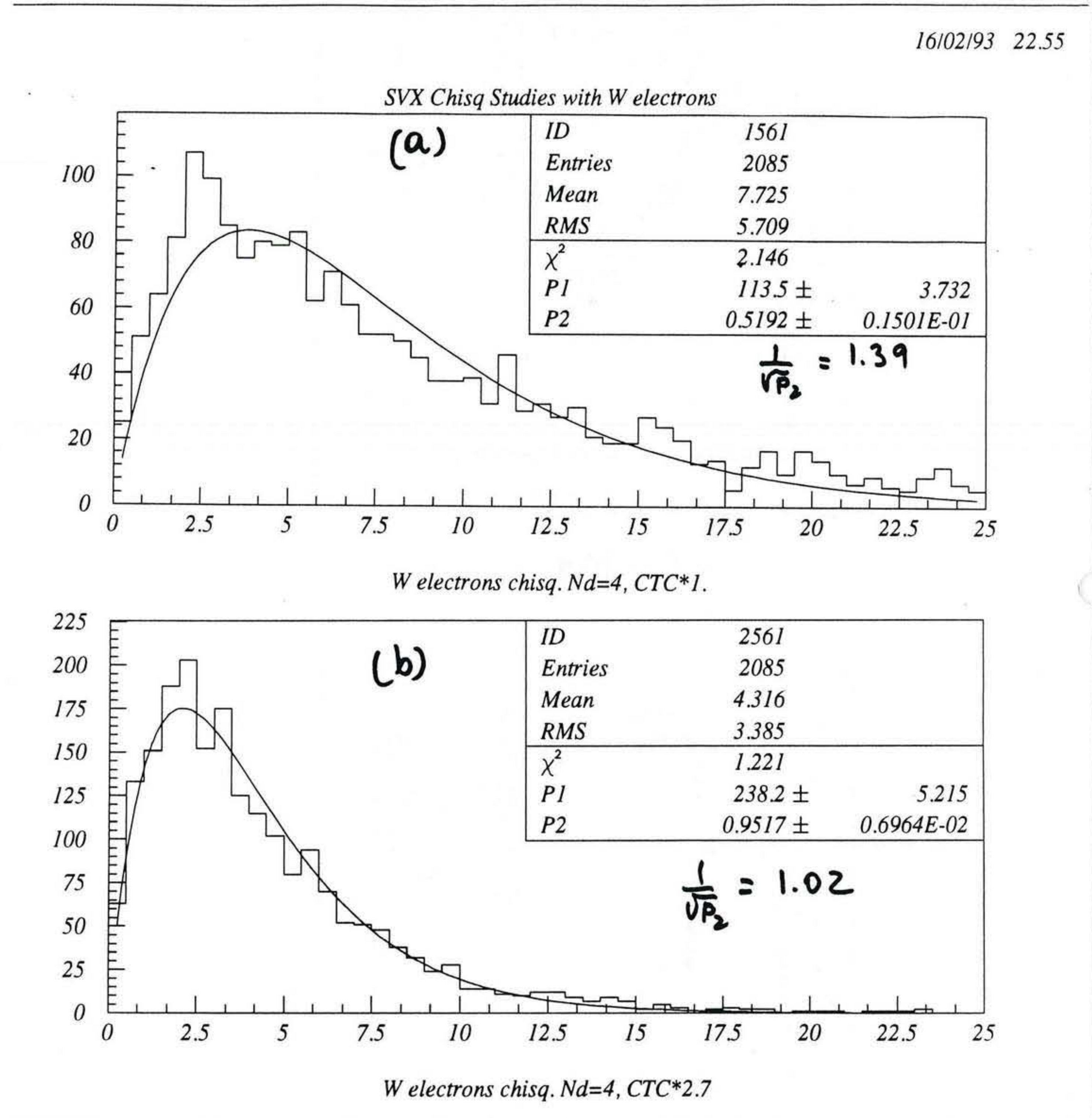

16/02/93 22.55
SVX Chisq Studies with W electrons
(a)
ID 1561
Entries 2085
Mean 7.725
RMS 5.709
χ² 2.146
P1 113.5 ± 3.732
P2 0.5192 ± 0.1501E-01
1/√P2 = 1.39
W electrons chisq. Nd=4, CTC*1.
(b)
ID 2561
Entries 2085
Mean 4.316
RMS 3.385
χ² 1.221
P1 238.2 ± 5.215
P2 0.9517 ± 0.6964E-02
1/√P2 = 1.02
W electrons chisq. Nd=4, CTC*2.7

Fig. 11

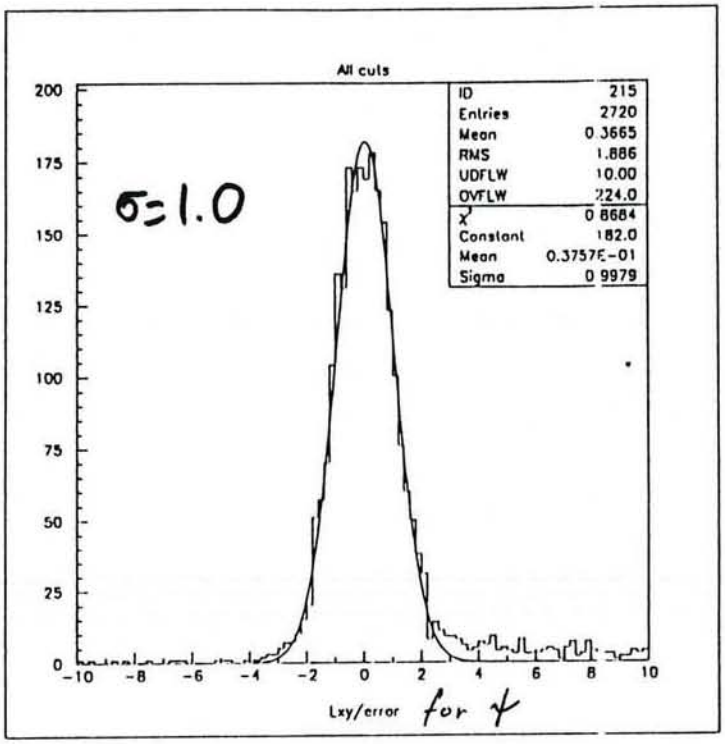


fig. 12a

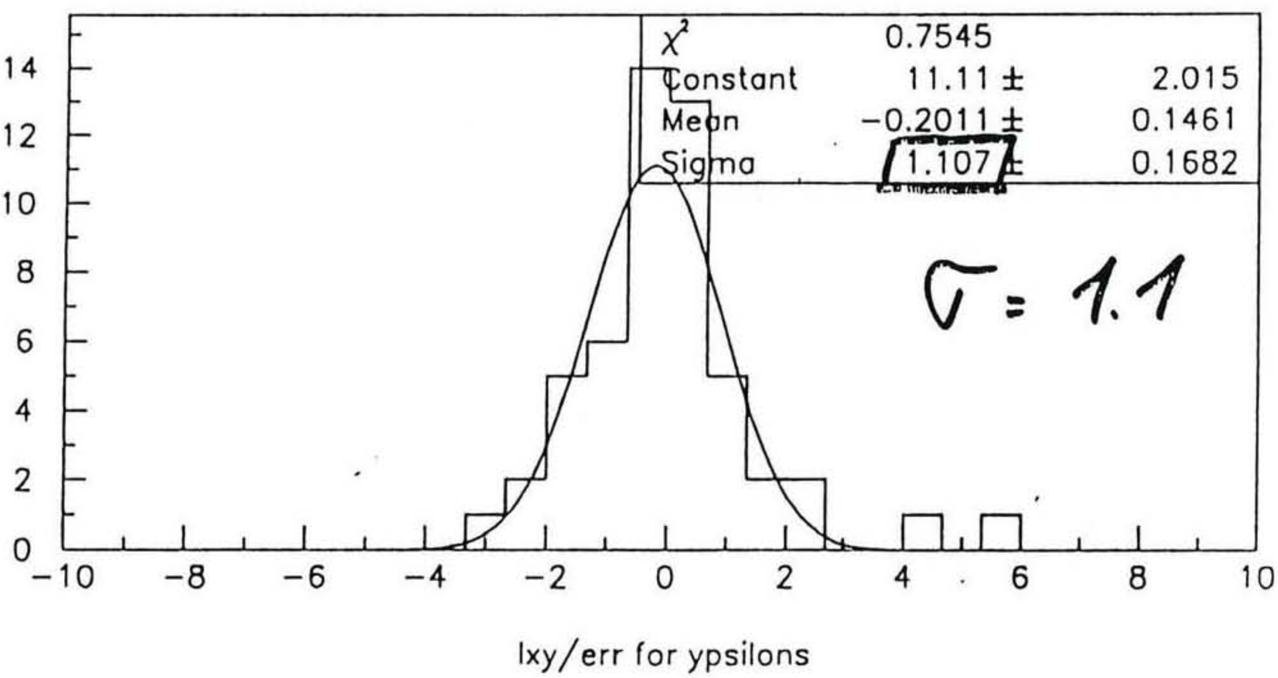


fig. 12b