Gauge coupling unification in correlation study of proton decay

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Abstract. Proton decay has been studied for decades now as one of the consequences of grand unified theories. Among those theories exists $SU(5)$ theory, firstly postulated by H. Georgi and S. Glashow [1]. However, there were some problems with this theory such as mass degeneration and coupling unification [1-3]. This created a need for an extension of an original $SU(5)$ model – a specific minimal $SU(5)$ [4-5]. In this minimal $SU(5)$ there is a viable parameter space with achievable gauge coupling unification. In this article, we present the process of gauge coupling unification for three mass scales of new physics states in this model, namely for 1 TeV, 10 TeV, and 100 TeV.

1. Introduction

Unification as a term refers to unification of fundamental forces. This process started with James Clerk Maxwell in 1865, when his paper [6] on the dynamical theory of electromagnetism was published. Some would argue that it started even earlier, with Isaac Newton and his unification of gravity and astronomy. Nevertheless, when it comes to fundamental forces, it was the theory of electromagnetism that unified electricity and magnetism. The electromagnetism and the weak nuclear force were later in the 20th century unified in electroweak force [7].

Today we have the unification of three forces in the Standard Model. However, there are some problems that remain unsolved within this theory, such as neutrino masses, gauge coupling unification etc. This theory does not provide a unification at any energy scale. Since the Standard Model is a gauge theory based on the $SU(3) \times SU(2) \times U(1)$ gauge group, the search for unification overpasses this gauge group and goes to some other theories and models in what is called today – theories Beyond the Standard Model (BSM).

There is a plethora of Grand Unified Theories trying to provide a theoretical framework for solving issues present today in the Standard Model. One of the features or a result of those theories is proton decay. In Figure 1, we present the proton decay mediation options – process induced via gauge boson or via scalar leptoquark.

2. Proton decay

Nucleon decay, in general, has become something like a probing tool for unification theories since, more specifically, proton decay is an inevitable consequence of unification at some larger scale. This happens due to a unification to some greater symmetry groups such as $SU(5), SO(10)$ etc. With unification, new bosonic states arise. As an example, one can take $SU(2)$ doublet and its

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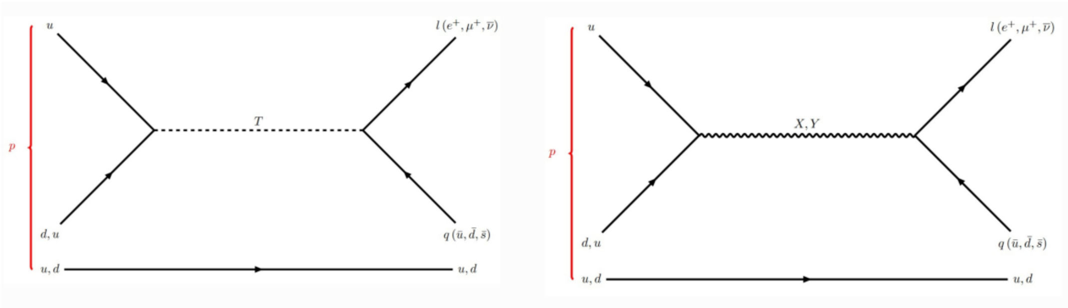

Figure 1. Proton decay processes: proton decay via scalar leptoquark (left) and proton decay via gauge bosons (right)

states of charged lepton and accompanying neutrino. Gauge bosons $W^{\pm}$ enable the transitions between the two states. The same process occurs within the greater group of unification where new gauge bosons enable the communication between quarks and leptons leading to baryon number violation and, therefore, nucleon decay.

Precisely, these new gauge bosons, serving as a communication channel between quarks and leptons and portaiting the connection between nucleon decay and unification process, determine the energy scale of unification. For initial consideration of gauge coupling unification, one must have a range of values for mass scale. Depending on the mediator of proton decay, different mass scales appear. As a consequence of unification, new gauge bosons have a mass approximate to the unification mass scale, while scalar leptoquarks are at least 1000 GeV lighter.

Two possible mediators for proton decay are gauge bosons or scalar leptoquarks (Figure 1). Each of the two posits as a possibility, since there are overall eight possible channels for proton decay regardless of mediator: $p \to \pi^0 e^+$, $p \to \pi^0 \mu^+$, $p \to \pi^+ \bar{\nu}$, $p \to \eta^0 e^+$, $p \to \eta^0 \mu^+$, $p \to K^0 e^+$, $p \to K^0 \mu^+$, and $p \to K^+ \bar{\nu}$.

In this correlation study, we have examined and found all eight partial proton lifetimes for each mediator that correspond to the experimental data and proton decay predictions [8-13].

3. Specific $SU(5)$ model

3.1 Description of the model

$SU(5)$ as a unification group is known since the Georgi-Glashow model [1]. However, this model had some flaws and could not answer to questions of neutrinos masses, mass degeneracy in some sectors, and gauge coupling unification. Therefore, there is a vast space around this original $SU(5)$ for improvement and amelioration. For example, one of the models [14] offers an extension via 45_H, a new scalar representation with seven new states within that extension representation. However, this and other models do not provide minimalistic extension but rather more complex with more parameters coming from new states.

The specific SU(5) we have been working in, firstly emerged in the field in 2020 [4]. The complete model counts overall eight representations, where we distinguish representations with gauge bosons (24_V), fermions ($\overline{5}_{Fi}, 10_{Fi}, and\ 15_F + \overline{15}_F$), and scalars ($5_H, 24_H,$ and 35_H). In comparison with the original Georgi-Glashow model, one can observe that the extensions are in scalar and fermion fields. Namely, there are overall seven new states in these representations needed for creating a mismatch between the masses of the down-type quarks and charged leptons that is experimentally observed and confirmed, as well as for gauge coupling unification and neutrino mass sector.

3.2 Lagrangian of the model

Lagrangian for this specific SU(5) model is of the following form[5, 15]:

$$\begin{aligned}\mathcal{L} \supset \Big\{&+Y_{ij}^{u}T_{i}^{\alpha\beta}T_{j}^{\gamma\delta}\Lambda^{\rho}\epsilon_{\alpha\beta\gamma\delta\rho}+Y_{ij}^{d}T_{i}^{\alpha\beta}F_{\alpha j}\Lambda_{\beta}^{*}+Y_{i}^{a}\Sigma^{\alpha\beta}F_{\alpha i}\Lambda_{\beta}^{*}+Y_{i}^{b}\overline{\Sigma}_{\beta\gamma}F_{\alpha i}\Phi^{*\alpha\beta\gamma}+Y_{i}^{c}T_{i}^{\alpha\beta}\overline{\Sigma}_{\beta\gamma}\phi_{\alpha}^{\gamma}\\ &+\mathrm{h.c.}\Big\}+M_{\Sigma}\overline{\Sigma}_{\alpha\beta}\Sigma^{\alpha\beta}+y\overline{\Sigma}_{\alpha\beta}\Sigma^{\beta\gamma}\phi_{\gamma}^{\alpha}-\mu_{\Lambda}^{2}(\Lambda_{\alpha}^{*}\Lambda^{\alpha})+\lambda_{0}^{\Lambda}(\Lambda_{\alpha}^{*}\Lambda^{\alpha})^{2}+\mu_{1}\Lambda_{\alpha}^{*}\Lambda^{\beta}\phi_{\beta}^{\alpha}\\ &+\lambda_{1}^{\Lambda}(\Lambda_{\alpha}^{*}\Lambda^{\alpha})\left(\phi_{\gamma}^{\beta}\phi_{\beta}^{\gamma}\right)+\lambda_{2}^{\Lambda}\Lambda_{\alpha}^{*}\Lambda^{\beta}\phi_{\beta}^{\gamma}\phi_{\gamma}^{\alpha}-\mu_{\phi}^{2}\left(\phi_{\gamma}^{\beta}\phi_{\beta}^{\gamma}\right)+\mu_{2}\phi_{\beta}^{\alpha}\phi_{\gamma}^{\beta}\phi_{\alpha}^{\gamma}+\lambda_{0}^{\phi}\left(\phi_{\gamma}^{\beta}\phi_{\beta}^{\gamma}\right)^{2}\\ &+\lambda_{1}^{\phi}\phi_{\beta}^{\alpha}\phi_{\gamma}^{\beta}\phi_{\delta}^{\gamma}\phi_{\alpha}^{\delta}+\mu_{\Phi}^{2}\left(\Phi^{*\alpha\beta\gamma}\Phi_{\alpha\beta\gamma}\right)+\lambda_{0}^{\Phi}\Phi^{*\alpha\beta\gamma}\Phi_{\alpha\beta\gamma}\left(\Lambda_{\rho}^{*}\Lambda^{\rho}\right)\\ &+\lambda_{0}^{\prime\prime}\Phi^{*\alpha\beta\gamma}\Phi_{\beta\gamma\delta}\Lambda^{\delta}\Lambda_{\alpha}^{*}+\mu_{3}\Phi^{*\alpha\beta\gamma}\Phi_{\beta\gamma\delta}\phi_{\alpha}^{\delta}+\lambda_{1}\Phi^{*\alpha\beta\gamma}\Phi_{\alpha\delta\rho}\phi_{\beta}^{\delta}\phi_{\gamma}^{\rho}\\ &+\lambda_{2}\Phi^{*\alpha\beta\rho}\Phi_{\alpha\beta\delta}\phi_{\rho}^{\gamma}\phi_{\gamma}^{\delta}+\left\{\lambda^{\prime}\Lambda^{\alpha}\Lambda^{\beta}\Lambda^{\gamma}\Phi_{\alpha\beta\gamma}+\mathrm{h.c.}\right\}\end{aligned} \tag{1}$$

where states in the Lagrangian itself are presented in Table 1 and Table 2. This Lagrangian does not represent kinetic terms, because presented part aims to show all interactions between the states in order to demonstrate where do masses of neutrinos come from. Since neutrinos reside in fermion representations $\overline{5}_{Fi}$, the states responsible for their masses are Σ_1 and Φ_1.

Table 1. Particle content of the specific $SU(5)$ model – fermionic representations

Representation	$SU(5)$ field	$SU(3)\times SU(2)\times U(1)$ state (from Standard Model)
Fermion	$\overline{5}_{Fi}$	$\mathrm{L}_i = \left(1,2,-\frac{1}{2}\right)$
		$d_i^C = \left(\overline{3},1,\frac{1}{3}\right)$
	10_{Fi}	$Q_i = \left(3,2,\frac{1}{6}\right)$
		$u_i^C = \left(\overline{3},1,-\frac{2}{3}\right)$
		$e_i^C = (1,1,1)$
	15_F	$\Sigma_1 = (1,3,1)$
		$\Sigma_3 = \left(3,2,\frac{1}{6}\right)$
		$\Sigma_6 = \left(6,1,-\frac{2}{3}\right)$
	$\overline{15}_F$	$\overline{\Sigma}_1 = (1,3,-1)$
		$\overline{\Sigma}_3 = \left(3,2,-\frac{1}{6}\right)$
		$\overline{\Sigma}_6 = \left(6,1,\frac{2}{3}\right)$

Table 2. Particle content of the specific $SU(5)$ model – scalar representations

Representation	$SU(5)$ field	$SU(3)\times SU(2)\times U(1)$ state (from Standard Model)
Scalar	5_H	$\Lambda_1 = \left(1,2,\frac{1}{2}\right)$
		$\Lambda_3 = \left(3,1,-\frac{1}{3}\right)$
	24_H	$\phi_0 = (0,0,0)$
		$\phi_1 = (1,3,0)$
		$\phi_3 = \left(3,2,-\frac{5}{6}\right)$
		$\phi_{\bar{3}} = \left(\bar{3},2,\frac{5}{6}\right)$
		$\phi_8 = (8,1,0)$
	35_H	$\Phi_1 = \left(1,4,-\frac{3}{2}\right)$
		$\Phi_3 = \left(\bar{3},3,-\frac{2}{3}\right)$
		$\Phi_6 = \left(\bar{6},2,\frac{1}{6}\right)$
		$\Phi_{10} = (\overline{10},1,1)$

Higgs state that participates in neutrino masses generation process is $\Lambda_1 = \left(1,2,\frac{1}{2}\right)$ and the relevant contraction term is the term $\lambda' \Lambda^\alpha \Lambda^\beta \Lambda^\gamma \Phi_{\alpha\beta\gamma}$ from Lagrangian of the model.

3.3 Unification and symmetry breaking

Unification is a term that refers to a unification of gauge coupling constants or parameters determining the strength of the forces of unification. In Standard Model framework, there are three gauge coupling constants that do not meet at any energy scale. With notation of gauge coupling constants for Standard Model g_i where $i = 1,2,3$ for three forces (electromagnetic, weak, and strong), we can write two-loop renormalization group equations in the following form[16-18]:

$$(4\pi)^2 \frac{d}{dt} g_i = A_i g_i^3 + \frac{g_i^3}{(4\pi)^2}\left[\sum_{j=1}^{3} B_{ij} g_j^2 - \sum_{x=u,d,e} C_{i,x} \mathrm{Tr}\left(Y_x^\dagger Y_x\right)\right] \tag{2}$$

where Y_x are Yukawa coupling constants, t is the logarithm of the renormalization scale, and A_i, B_{ij}, and $C_{i,x}$ are the following coefficients [19]:

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$$A_i = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}$$

$$B_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}$$

$$C_{i,x} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} \tag{3}$$

With reformulation $\omega_i = \alpha_i^{-1} = \frac{4\pi}{g_i^2}$, Equation (2) can be rewritten:

$$\mu \frac{d\omega_i\,(\mu)}{d\mu} = -\frac{\alpha_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2 \omega_i\,(\mu)} \tag{4}$$

where μ represents the energy scale.

The symmetry of the group SU(5) is broken at two stages. It can be formulated in the following way:

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)_{\text{em}} \tag{5}$$

The first stage of symmetry breaking happens at M_{GUT} energy scale with a vacuum expectation value of 24_H:

$$\langle 24_H \rangle = \frac{v_{24}}{\sqrt{15}} \text{diag}\,(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}) \tag{6}$$

from where arise 12 new gauge bosons that appear to be proton decay mediators. Twelve remaining undergo a second stage of symmetry breaking via vacuum expectation value:

$$\langle 5_H \rangle = \text{diag}\,(0, 0, 0, 0, v_5) \tag{7}$$

where $v_5 = 174.104$ GeV (Standard Model VEV). This second stage of symmetry breaking gives eight gluons, photon, Z, and $W^{\pm}$ bosons.

4. Gauge coupling unification

4.1 New states and constraints

In a search for a greater unification gauge group than the current one of the Standard Model, new physics states appear at higher energy scales. The specific $SU(5)$ model explained in section 3 of this article is an extension of the original Georgi-Glashow and, therefore, brings some new physics states with masses of unknown range value at the beginning. Those states reside in the following representations: 35_H, 15_F, 24_H, and 5_H. There are three states in 15_F related as:

$$M_{\Sigma_6} = 2M_{\Sigma_3} - M_{\Sigma_1} \tag{8}$$

These states are degenerate in their masses since we are opting for positive masses. Consequently, we conclude that these three states live at one same energy scale.

A similar conclusion can be drawn for states in 35_H from their mass relation:

$$M_{\Phi_{10}}^2 = M_{\Phi_1}^2 - 3M_{\Phi_3}^2 + 3M_{\Phi_6}^2 \tag{9}$$

Since the idea is to maximize GUT scale, one needs to search for those states with beta function coefficients that would increase GUT scale. In 35_H, those are Φ_3 and Φ_6, which means that they need to be as light as possible, implying that reaming two are very heavy.

With previous two constraints on mass scales of new states from 15_F and 35_H, third one arises from a state in 5_H that requires $\Lambda_3 = \left(3, 1, -\frac{1}{3}\right)$ to be greater than 10^{11} GeV.

4.2 Numerical analysis

What frames and constrains the parameter space is proton decay with its experimental limits on partial decay lifetimes. These data are used to portray the parameter space in order to maximize GUT scale. The complete numerical analysis is conducted in Wolfram System Modeler, Mathematica [20].

The condition of maximization of GUT scale and one-loop level gauge coupling analysis yields the order of magnitude for fields Φ_{10} and Φ_1 (10^{11} GeV) and the order of magnitude for fields Φ_3 and Φ_6 (10 TeV). After fixing mass values for fields Φ_1 and Σ_1, and knowing parameters from Standard Model such as $M_Z = 91.1876$ GeV, $\alpha_S\,(M_Z) = 0.1193 \pm 0.0016, \alpha^{-1}(M_Z) = 127.906 \pm 0.019$, and $(\sin\theta_W)^2(M_Z) = 0.23126 \pm 0.00005$ [21], we vary the masses of the remaining states from 35_H, 15_F, 24_H, and 5_H.

Variation of the masses for states Φ_1, Φ_3, Φ_6, $\Phi_{10}, \Sigma_1, \Sigma_3, \Sigma_6, \phi_1, \phi_8, \Lambda_3$ happens at three energy scales: 1, 10 and 100 TeV. After setting the mass scale for the aforementioned states, next step is to set a range of masses for Φ_1 and Σ_1 that would provide a discreet set of values spanning

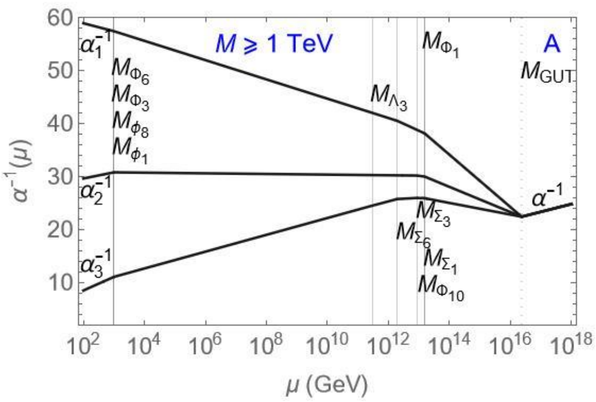

Figure 2. Gauge coupling unification at mass scale greater than 1 TeV

the parameter space. For the first, the range is from 10^{10} to 10^{13} GeV, and for the latter from 10^8

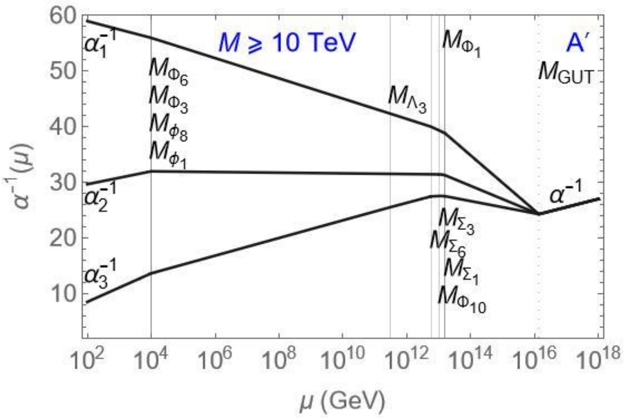

Figure 3. Gauge coupling unification at a mass scale greater than 10 TeV

to 10^{14} GeV. Now, for each of these values, a mass spectrum for new physics states is created. Therefore, for three variations or three mass spectra, gauge coupling unification provides three scenarios presented in section 5. Results.

5. Results

For all three scenarios (1, 10, and 100 TeV) gauge coupling unification is achieved at a point corresponding approximately to 10^{16} GeV, presented in Figures 2, 3 and 4.

The distinction between the three graphs can be made regarding the new physics states, namely Λ_3, Φ_1, Φ_{10}, Σ_1, Σ_3, and Σ_6. For first two scenarios, state Λ_3 stays below 10^{12} GeV due to the proton decay bound. As mass spectrum increses, other states tend to be at the same scale,

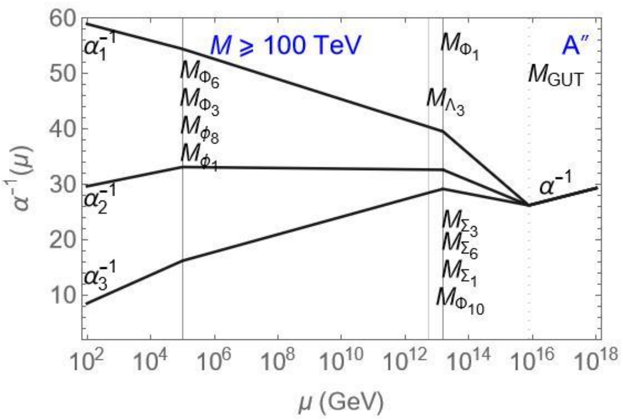

Figure 4. Gauge coupling unification at a mass scale greater than 100 TeV

around 10^{13} GeV, where all three gauge coupling constants bend to reach a unification. From Figures 2 and 3, it is observable that states Σ_3 and Σ_6 bend gauge coupling constants α_2^{-1} and α_3^{-1}, while α_1^{-1} bends at the mass scale of Φ_1 and Φ_{10}. Also, it is precisely these gauge coupling unification graphs that show constraints on the mass scales spectrum for new physics state introduced in section 4.

Another interesting observation is a raising line after the unification scale for common or joint gauge coupling constant for $SU(5)$ in this specific case. It shows the stability of the theory and the specific model presented.

6. Conclusion

Gauge coupling analysis is an important tool for probing the unification model and accompanying features. Since the current unification of forces within the Standard Model framework does not provide gauge coupling unification at any energy level, the vast field of theories Beyond the Standard Model opens up, among which one of the simplest and most elegant ones is the $SU(5)$ model.

Gauge coupling unification described in this article enables a viable parameter space in which neutrino masses can be generated, proton can undergo a decay process in a time frame in accordance with experimental results and predictions, and GUT scale corresponds to theoretical and experimental postulations. Legit proof for this model and therefore a unification to be testable, would be an observation of a proton decay process at best, or an observation of a new physics state such as leptoquark as a potential mediator of a proton decay, at least. In the past 30 years, SuperKamiokande has been the most promising collaboration and experiment for testing these theories. News that brings even greater hope is about HyperKamiokande as a successor of SuperKamiokande, expected to start in 2027 with its operations [22-23].

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