GAMMA-TRANSITION-JUMP SCHEME OF THE CPS

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Summary

In 1973, the former Q-jump system has been replaced by 8 fast and 6 slowly pulsed quadrupoles forming 4 doublets and 2 triplets respectively. They allow presently a change of γ_t by $\simeq$ 2.3 (instead of $\simeq$ 0.23), limited only by the power supply within 0.8 ms (< 0.4 ms with a new supply under construction). Transition is crossed 50 times (instead of 5 times) faster than without jump. More than 5 10^{12} p/p have been accelerated operationally and brought through transition within a bunch area of $\simeq$ 10 mrad. In the future, it should be possible to handle 10^{13} p/p within the same bunch area. The scaling laws for achievable longitudinal density Ψ (apart from logarithmic terms $\Psi \propto N^{1/3} |\Delta\gamma_t|^{\frac{1}{2}}$ for bunch matching and $\Psi \propto N^{1/5} |\dot{\gamma}_t|^{2/5} |\Delta\gamma_t|^{-1/10}$ for negative mass instability) are in good agreement with experiments.

1. Introduction

High longitudinal phase space density is of particular importance in the CPS as it serves as injector for the ISR and, in the future for the SPS. The most severe limitation occurs at transition where a blow-up due to negative mass instability must be avoided. As shown in chapter "The negative mass instability", the only efficient method to achieve this at high intensity is to cross transition much faster than it would be crossed without special precautions. The method used to manipulate the transition energy is the subject of chapters 2 to 5. Chapter 6 deals with bunch matching, chapter 7 with the negative mass instability leading to a simple criterion for avoiding blow-up trouble. Chapter 8 summarizes the practical experience with the new scheme.

2. The Lens Configuration

A convenient way to influence γ_t without affecting the betatron tune is to form doublets : two lenses of opposite polarity are placed half a betatron wavelength apart. The phase advance per cell of the CPS being $\pi/4$ and the structure being FOFDOD, the doublet lenses can all be placed in mid-F straight sections. A doublet leaves the amplitude function β_H unperturbed outside but not the dispersion function x_p. Two consecutive doublets of opposite polarity form a triplet whose center lens is twice as strong as the outer lenses.

The lens configuration consists of two superperiods, each containing : 1 triplet, 2 cells, 1 doublet, 1 cell, 1 doublet, 6 cells. In the following three chapters, expressionsfor β, x_p and γ_t are derived for a machine assuming :

i) all unperturbed β and x_p values at the lenses are equal to β_F and x_{pF} respectively;

ii) the phase advances are precisely as required.

3. The Excursion of the Amplitude Function

The knowledge of β is not really needed for obtaining γ_t but we want an expression for $\hat{\beta}$ and we shall prove that the Q-value remains constant. As a measure for the lens strength we introduce the dimensionless parameter :

$$q = \beta_F / f \qquad \text{(f : focusing length)}$$

With $\psi = \int \frac{ds}{\beta}$ the phase advance of the unperturbed machine counted from the first quadrupole, we have in short lens approximation with initial conditions

$$\Delta\beta = 0 \quad \text{and} \quad \frac{d}{d\psi}\left(\frac{\Delta\beta}{\beta}\right) = 0$$

just before the lens :

$$\Delta\beta = - \beta \, \Delta q \sin 2\psi \quad \text{for small } \Delta q.$$

For large q, we have :

$$\frac{\partial \tilde{\beta}}{\partial q} = - \tilde{\beta} \sin 2\tilde{\psi} \tag{3.1}$$

where $\tilde{\beta}$ and $\tilde{\psi}$ are the new values such that

$$ds = \tilde{\beta} \, d\tilde{\psi} = \beta \, d\psi$$

It follows together with (3.1)

$$\frac{d^2 \tilde{\psi}}{d q^2} = \frac{d\tilde{\psi}}{dq} \sin 2\tilde{\psi} \tag{3.2}$$

The system (3.1, 3.2) has a solution which can be written :

$$\left.\begin{aligned}
& q = \text{ctg}\, \psi - \text{ctg}\, \tilde{\psi} \qquad \text{from which} \\
& \quad \tilde{\psi} = \pi \text{ for } \psi = \pi \qquad \text{q.e.d.} \\
& \frac{\tilde{\beta}}{\beta} = \cosh \chi - \sinh \chi \sin(2\psi + \text{arctg}\, \frac{q}{2}) \\
& \frac{\beta}{\tilde{\beta}} = \cosh \chi + \sinh \chi \sin(2\tilde{\psi} - \text{arctg}\, \frac{q}{2}) \\
& \frac{q}{2} = \sinh (\frac{\chi}{2})
\end{aligned}\right\} \tag{3.3}$$

It follows :

$$\left(\frac{\tilde{\beta}}{\beta}\right)_{\substack{max \\ min}} = e^{\pm\chi} = 1 + \frac{q^2}{2} \pm q \sqrt{1 + \frac{q^2}{4}}$$

4. The Excursion of the Dispersion Function

A simple way of treating x_p is to consider $\Delta x_p = x_p - x_{po}$ as a betatron oscillation excited by kicks at the lenses, the magnitude of the kick being proportional to x_p. A convenient measure for Δx_p is

$$\xi = \sqrt{\frac{\beta_F}{\beta(\psi)}} \; \frac{\Delta x_p(\psi)}{x_{pF}}$$

ξ must obey the differential equation

$$\frac{d^2\xi}{d\psi^2} + \xi = -q_k(1+\xi_k)\,\delta(\psi_k)$$

With complex notation $z = \xi + i\frac{d\xi}{d\psi}$ and the initial conditions $z(\psi_1) = z_1$ at $\psi_1 = 0$, one finds after a lens of strength q_1 at $\psi_1 = 0$

$$z = \left(z_1 - iq_1(1+\xi_1)\right)\exp(-i\psi)\ ;\ 0 < \psi < \pi \qquad (4.1)$$

and after the second doublet lens of strength $-q_1$ at $\psi = \pi$:

$$z = (z_1 - 2iq_1)\exp(-i\psi) \qquad ;\ \pi < \psi \qquad (4.2)$$

The term $-q_1\xi_1\sin\psi$ which occurs inside the doublet deserves particular attention. As we shall see in chapter 5, this term causes the γ_t change. With $\varphi_k = \psi_{k+1} - \psi_k$, the tracing law is

$$z_{k+1} = (z_k - 2iq_k)\exp(-i\varphi_k) \qquad (4.3)$$

For n doublets within a superperiod, the periodicity condition leads to

$$z_1 = \left(-\,\mathrm{ctg}\,\frac{\phi}{2} + i\right)\sum_{k=1}^{n} q_k \exp\left(i\sum_{\ell=1}^{k-1}\varphi_\ell\right) \qquad (4.4)$$

where $\phi = \sum_{k=1}^{n}\varphi_k$ is the phase advance over the superperiod.

In the CPS, there are two sets of lenses powered differently. Let $q_1 = -q_2 = T$; $-q_3 = q_4 = D$. The choice $\varphi_3 = \phi$ eliminates the term containing D^2 in the characteristic polynominal P_2 (see 5.2) and simultaneously maximizes the term containing DT if φ_2 is put $3\pi/2$ (mod 2π) and $\varphi_4 = 5\pi/2$ (mod 2π). Modifications arise from shifting doublets by π and reversing the polarity. The scheme

$$(T)\ \pi\ (-T)\ \frac{3\pi}{2}\ (-D)\ \phi\ (D)\ \frac{5\pi}{2}$$

yields with (4.3) and (4.4)

$$\left\{\begin{array}{l} \frac{z_1}{2} = \frac{z_T}{2} = \left(-\,\mathrm{ctg}\,\frac{\phi}{2} + i\right)T{+}D\ ;\ \frac{z_2}{2} = \frac{z_{-T}}{2} = \mathrm{ctg}\,\frac{\phi}{2}\,T{-}D \\ \\ \frac{z_3}{2} = \frac{z_{-D}}{2} = \left(-1 + i\,\mathrm{ctg}\,\frac{\phi}{2}\right)T - iD\ ;\ \frac{z_4}{2} = \frac{z_D}{2} = \left(1 + i\,\mathrm{ctg}\frac{\phi}{2}\right)T \end{array}\right. \qquad (4.5)$$

from which results

$$\sum_{k=1}^{4} q_k\,\xi_k = 4T\left(2D - \mathrm{ctg}(\frac{\phi}{2})\,T\right)$$

In the CPS, $|-\,\mathrm{ctg}\,\phi/2|$ could be kept small by going to the antisymmetric disposition which makes

$$\phi = \frac{6.25 - 1}{2}\,2\pi$$

and

$$-\,\mathrm{ctg}\,\frac{\phi}{2} = \mathrm{tg}\,\frac{\pi}{8} = \sqrt{2} - 1$$

(a corresponding scheme for the AGS is feasible with $\phi = 8.75/2\cdot 2\pi$ or $-\mathrm{ctg}\,\phi/2 = -(\sqrt{2} - 1)$ with lenses placed in FD or DF straights for which the desired phase distances can be approximated rather closely i.e. : $\pi \simeq 3\frac{1}{2}$ cells; $3\pi/2 \approx 5$ cells; $7\pi/4 \approx 6$ cells).

5. The new Transition Energy

As γ_t is defined by $1/\gamma_t^2 = \Delta C/C$ where ΔC is the additional circumference of a particle with unit momentum displacement $\Delta p/p = 1$ we have to sum over the contributions of all elements $\Delta C = \Sigma\,\delta C_k$. The kth doublet contributes to this sum

$$\delta C_k = \int \frac{\Delta x_p}{\rho}\,ds \approx \frac{x_{pF}}{R\sqrt{\beta_F}} \int_0^{\pi} \beta^{3/2}\left[\xi_k\cos\psi + \left(\left(\frac{d\xi}{d\psi}\right)_k - q_k(1+\xi_k)\right)\sin\psi\right]d\psi$$

$$= \frac{x_{pF}\ \overline{\beta^{3/2}}^{\psi}}{R\sqrt{\beta_F}}\ 2\left(\left(\frac{d\xi}{d\psi}\right)_k - q_k(1+\xi_k)\right) \qquad (5.1)$$

When summing over the whole circumference the only remaining terms involve the combinations $q_k\xi_k$ leading to

$$\left.\begin{array}{l} \left(\frac{\gamma_{to}}{\gamma_t}\right)^2 = 1 - c_p\,P_2 \\ \\ c_p = 8\,\frac{x_{pF}\ \overline{\beta^{3/2}}^{\psi}\ \gamma_{to}^2}{\pi\ R^2\sqrt{\beta_F}} \approx 0.4 \text{ for the CPS} \\ \\ P_2 = T\left(2D + (\sqrt{2} - 1)\,T\right) \end{array}\right\} \qquad (5.2)$$

With this P_2, γ_t can be changed to either side as with a first order perturbation by keeping one set (triplets T) fixed and pulsing the other set (doublets D). Applying reasonable criteria, this scheme is superior to those studied previously[1,2].

An additional application has arisen which helps to ease debunching the beam prior to the continuous transfer into the SPS [3]. The combination $|Z|\ I\ \eta^{-1}\ (\Delta p/p)^{-2}$ should be small (Z : coupling impedance; I : beam current). As long as $|Z|$ is rather high as at present, it is helpful to increase η via a decrease of γ_t. The phase advance over the superperiod ϕ can be made $6.25\ \pi$ by reversing the polarity of one triplet and of one doublet within each of the two superperiods so that P_2 becomes

$$P_2 = -\,T\left(2D + (\sqrt{2}{+}1)\,T\right)$$

The quadratic term is so efficient now that with the (water-cooled) triplet lenses alone, a gain of 2.5 in η is obtained with $T \approx 1$; $\gamma_t \approx 4.32$; $\hat{\beta} \approx 60$ m; $\hat{x}_p = 23$ m. The peak excursion of x_p is high, but as for this application $\Delta p/p$ is small, the synchrotron width of the beam is not excessive. Also betatron width and synchrotron width add in quadrature to the total beam width; this is rigorous for the rms values of symmetric and uncorrelated distributions, and 4σ are a good measure for the total beam width.

The scheme is fairly flexible and allows for other modes of operation if the criteria (e.g. permissible $\hat{\beta}$, $\hat{x}_p$) are altered.

6. Bunch Matching

Since we shall describe the features of the negative mass instability also in terms of the matching theory, it is justified to recall briefly the most important facts of the matching theory[4]. They are

i) RF forces are linearized, so trajectories are elliptic ;

ii) Space-charge forces are also linearized by assuming a parabolic shape for the linear density which results from phase space density of semi-circle shape across trajectories;

iii) Time is measured in units of the non-adiabatic time τ for ordinary transition crossing

$$\tau = \left(\frac{\gamma_{cr}^4 \,|tg\ \phi_s|}{4\ \pi\ \nu_{RF}\ \dot{\gamma}_s^{\ 2}}\right)^{1/3}$$

and the new time given by $x = t/\tau$; $d/dx = \ '$; γ_{cr} is the γ at which transition crossing occurs (i.e. γ_t equals γ_s the value of the particles), ϕ_s the synchronous phase angle and ν_{RF} the RF frequency.

iv) Manipulations on γ_t are described by a function $f(x) = \eta/\eta_s'$ where

$$\eta = \frac{1}{\gamma_t^{\ 2}} - \frac{1}{\gamma_s^{\ 2}} \quad \text{and} \quad \eta_s' = -\frac{d}{dx}(\gamma_s^{\ -2}) \simeq \frac{2\gamma_s'}{\gamma_{cr}^{\ 3}}$$

Note that $f(x) \simeq x$ for γ_t = const.

v) A longitudinal space-charge parameter n_o is introduced by

$$n_o = \frac{3\pi^2}{2}\frac{r_p\ N\ g_o}{R\ A^{3/2}}\left(\frac{\nu_{RF}}{\dot{\gamma}_s}\right)^{\frac{1}{2}}$$

$$n_o = 0.772334\ n_o\ (0)\ ;\ n_o(0) \text{ of ref. 4.}$$

r_p = classical proton radius; $2\pi R$ = machine circumference; N = number of particles; A = bunch area in units of $\Delta(\beta\gamma)$·RF-angle (cut-off value); $g_o \simeq 1 + 2\ \ln\ b/a$ (see 7.i).

vi) A variable θ is introduced denoting the half length of a normalized bunch of area π. θ is related to the physical bunch half length $\hat{\phi}$ by

$$\hat{\phi} = \frac{2\sqrt{A\ \nu_{RF}\ \gamma_s'\ \tau}}{\gamma_s^2}\ \theta$$

θ obeys a second order differential equation. Far away from transition, the equilibrium values of θ are below (-) and above (+) transition given by

$$|f_\pm| = \theta_\pm^4 \pm n_o\ \theta_\pm \qquad (6.1)$$

A large and fast jump is aimed at such that θ_- is reestablished $\theta_+ = \theta_- = \theta$. For this case, summation and subtraction of (6.1) yields

$$\theta = \left(\frac{\Sigma f}{2}\right)^{\frac{1}{4}} \quad \text{and} \quad n_o = \left(\frac{\Delta f}{2\theta}\right) \qquad (6.2)$$

with $\Sigma f = f_+ + |f_-|$ and $\Delta f = f_+ - |f_-|$. For $f_- = 0$, the minimum jump size[5] is $\Sigma f = f_+ = 2(n_o)^{4/3}$.

The system installed now in the CPS allows at least twice this jump.

The advantages of such a "superjump" are :

i) The mismatch is little sensitive against space-charge parameter variations. This is important at the CPS since each of the four Booster rings has some individuality. Roughly one has :

$$\frac{\partial}{\partial\ n_o}\left(\frac{\Delta\theta}{\theta}\right) \propto \left(\Sigma f\right)^{-3/4}$$

ii) For systematic intensity modulations from cycle to cycle, as planned, it is sufficient to change only the timing. This is much simpler than to change current amplitudes too.

iii) The negative mass unstable region is crossed near the peak of $|\dot{\gamma}_t|$, a very important aspect which leads us to the next chapter.

7. The Negative Mass Instability

The linear region of negative mass (n.m.) instability[6] is characterized by the fact that (small) modulations of the linear density grow exponentially with a growth rate which is a function of both the mode number and, in our case, of time, since we change η rapidly with the aim of reaching the threshold before the modulations have grown too much. In order to treat the phenomenon conveniently, we shall make a few assumptions and simplifications none of which are unrealistic or too rough.

i) The dependence of the geometrical factor g on the mode number is given for a beam of radius a, within a perfectly conducting pipe of radius b, at the axis by

$$g = \frac{4}{w}\left(\frac{1}{w} - K_1(w) - I_1(w)\ \frac{K_o\left(\frac{wb}{a}\right)}{I_o\left(\frac{wb}{a}\right)}\right)$$

with $$w = \frac{a\ k_c}{\gamma_{cr}R} = \frac{\pi\ a\ h\ k_b}{\gamma_{cr}R\ \hat{\phi}}$$

k_c, k_b : mode number per circumference or per bunch length respectively.

I_o, I_1, K_o, K_1 : modified Bessel functions.

The space-charge parameter n_o contains :

$$g_o = \lim_{w\to 0} g = 1 + 2\ \ln\frac{b}{a}$$

For larger arguments w and b/a not too big, g is approximated by

$$g = \frac{g_o}{1 + \left(\frac{w}{1.6\ a/b + 0.52}\right)^2} \qquad (7.1)$$

$$g = \frac{g_o}{2} \quad \text{at} \quad k_{c\frac{1}{2}} = \gamma_{cr}\ R\left(\frac{1.6}{b} + \frac{0.52}{a}\right)$$

using the simplified kernel[7] for the space-charge potential, together with a numerical fit.

ii) From inspecting the Vlasov equation, it can be concluded that for $k_b >> 1$, the coasting beam model is a good approximation as proposed earlier by Pease[8].

iii) For the unperturbed density, the same distribution is assumed as in chapter 6.

iv) The initial modulation is assumed to be given entirely by statistical fluctuations due to the finite number of particles within the bunch $N_b = N/h$. The small amplitude approximation being based on Fourier decomposition, we can calculate the expectation value of the modes when referred to the average linear density $\lambda = N/2\hat{\phi}$ and find them independent of the mode number and the distribution function to be (see the Appendix)

$$|c_k(0)| = \frac{2}{\sqrt{N_b}}$$

A detailed study of the initial value problem shows that the behaviour after transition crossing as function of the growth rate $G(x,k)$ is

$$|c_k(x)| \simeq |c_k(0)| \cosh \int G(x,k)\, dx$$

so that after a short time

$$|c_k(x)| \simeq \frac{\exp \int G(x,k)\, dx}{\sqrt{N_b}} \qquad (7.2)$$

With these assumptions, the growth rate and the threshold can be found via the Vlasov equation and the dispersion relation. The threshold can be expressed conveniently by the parameters of chapter 6 except that

$$n = \frac{g}{g_o} n_o \quad \text{is to be used (7.1)} : \frac{n}{n_{th}} = \frac{f}{n\theta} \qquad (7.3)$$

This relation is valid along the whole bunch and allows to define a threshold value $f_{th} = n\theta$ which is always smaller than f_+ the value to be reached for matching. The growth rate varies along the bunch as the square root of the linear density and involves additional parameters. At the bunch center, the growth rate is

$$G(x,k_c) = \frac{k_c n}{h} \sqrt{\frac{A\,|tg\,\phi_s|}{\pi\,\gamma_s'}}\; \frac{1 - \left(\frac{f}{n\theta}\right)}{\sqrt{2\frac{n\theta}{f} - 1}} \qquad (7.4)$$

For θ = const. and $f = f'x$, the integral throughout the unstable region which measures the accumulated e-folding times of the n.m. instability is

$$E_{acc}(k_c) = \int G(x,k_c)\, dx = \frac{k_c\, n^2 \theta}{h\, f'} \left(1 - \frac{\pi}{4}\right) \sqrt{\frac{A\,|tg\,\phi_s|}{\pi\,\gamma_s'}} \qquad (7.5)$$

E_{acc} has a maximum $\hat{E}_{acc}$ for $k_c = k_{c\frac{1}{2}}/\sqrt{3}$, yielding

$$k_c n^2 = \frac{3\sqrt{3}}{16} k_{c\frac{1}{2}}\; n_o^2$$

$\hat{E}_{acc}$ must remain smaller than some critical value E_{crit} in order to avoid a blow-up which happens when

$$(\Sigma|c_k|^2)^{\frac{1}{2}}$$

of equation (7.2) approaches unity.

The implied summation

$$\sum_k \exp\left(2E_{acc}(k)\right) = N_b$$

can be carried out by second order expansion of $E_{acc}(k)$ around E_{crit}

$$E_{acc}(k_b) \approx E_{crit}\left(1 - \left(\frac{3}{4}\frac{\Delta k_b}{k_{b\frac{1}{2}}}\right)^2\right)$$

$k_{b\frac{1}{2}}$ has to be taken in order to achieve the proper mode spacing within a bunch. The sum may then be approximated by the error integral yielding the transcendental equation for E_{crit}

$$\frac{k_{b\frac{1}{2}}}{3}\sqrt{\frac{8\pi}{E_{crit}}}\, \exp(2E_{crit})\; \underbrace{\mathrm{erf}\left[3\sqrt{\frac{E_{crit}}{8}}\right]}_{\simeq 1} = N_b$$

An approximate solution is

$$E_{crit} = \tfrac{1}{2}\left[\ln N_b - \ln\left(\frac{k_{b\frac{1}{2}}\sqrt{8\pi}}{3\sqrt{\frac{1}{2}\ln N_b}}\right)\right] \simeq 10$$

($\frac{1}{2}\ln N_b$ is E_{crit} for only one mode; the second term arises from the presence of many modes).

The criterion for no blow-up due to n.m. instability is then

$$\boxed{\frac{k_{eff}\, n_o^2\, \theta}{h\, f'}\left(1 - \frac{\pi}{4}\right)\sqrt{\frac{A\,|tg\,\phi_s|}{\pi\,\gamma_s'}} < E_{crit}} \qquad (7.6)$$

with $k_{eff} \simeq \frac{\gamma_{cr} R}{3}\left(\frac{1.6}{b} + \frac{0.52}{a}\right)$; $k_{eff}(\mathrm{CPS}) \simeq 2\; 10^4$

This should be considered as a condition for f' indicating how much faster transition energy is to be crossed by the jump. If f' prooves to be insufficient, the magnitude of the blow-up might be estimated by applying Dory's law[9] together with (7.4).

The inequality (7.6) allows also to study other methods, for example, playing with ϕ_s. Since they only provide factors of the order unity, at least in the good direction, we conclude that a fast jump is indispensible*.

If we put $\theta = (\frac{\Sigma f}{2})^{\frac{1}{4}}$ and note that n_o^2 contains A^{-3}, we see that the longitudinal phase-space density scales as $N/A \propto N^{1/5} (f')^{2/5} (\Sigma f)^{-1/10}$. The scaling law for bunch matching taken from (6.2) goes as $N/A \propto N^{1/3} (\Delta f)^{2/3} (\Sigma f)^{-1/6}$ and simplifies to $N/A \propto N^{1/3}(\Sigma f)^{\frac{1}{2}}$ if $\Sigma f/\Delta f$ remains constant.

* A promising method at a first glance seems to be passive compensation[10], i.e. reducing n_o via g_o. As we need really to reduce n(k) via g(k), a critical frequency characteristic of the wall impedance over a wide range is required to achieve this without over-compensation[11]. Since that is also sensitive to the density distribution, an awkward problem would have to be mastered.

8. Experimental observations

All quadrupoles are compact in length (< 0.25 m), the doublet quadrupoles (except one) are also compact in cross-section. The present power supplies allow for $|D| \approx 0.45$ and $T \approx 0.82$ corresponding to $\hat{\gamma}_t \approx 7.9$ and $\check{\gamma}_t \approx 5.6$, the jump is performed within 0.8 ms corresponding to $f' \approx 50.$ ($\gamma_s = 50\ s^{-1}$; $\gamma_s' \approx 0.09$).

The scheme came into operation smoothly. Some initial beam loss due to closed orbit perturbation could be cured by centering carefully the mean radial position of the beam. The pulse form of the triplet current is rather uncritical, the peak should approximately coincide with the jump. The doublet current should not rise too quickly after the jump (see Fig. 1). The triple switch on the RF phase gives perfect bunch matching but a single switch is sufficient (see Fig.2). The bunch size of the booster seems to decrease with increasing intensity. Above 6 10^{12} p/p in the CPS, a negative-mass instability blow-up was sometimes observed at $A \approx 8$ mrad (in quantitative agreement with (7.6)). In order to improve the situation, more powerful supplies are under design for a larger jump and more than doubled speed aiming at $|D| \geq 0.55$, $T \geq 1.0$, $f' \approx 200\text{-}400$. 10^{13} p/p could then be handled within a bunch area of 10 mrad or less. The betatron tunes Q_H and Q_V change only by $\Delta Q =$ a few 10^{-3} in agreement with computations by a lattice programme which also show that the real CPS yields the analytical results of chapters 3-5 very closely, in particular for γ_t.

Appendix

Let N_b = number of particles per bunch, $2\hat{\phi}$ = bunch length, $F(\phi)$ = bunch distribution function normalized by

$$\int_{-\hat{\phi}}^{\hat{\phi}} F(\phi)\, d\phi = 2\hat{\phi}$$

Subdivide the bunch into M bins indexed by m (or n). Each bin contains $F(\phi)\ \Delta N$ particles where $\Delta N = N_b/M$ is the average number of particles per bin. Random fluctuations lead to particle numbers per bin different by δN from that determined by the distribution function. The expectation value of that difference is assumed to be given by

$$E\left\{\frac{\delta N}{\Delta N}(m)\ \frac{\delta N}{\Delta N}(n)\right\} = \frac{F}{\Delta N}\ \delta_{m\,n}$$

The implied step function $f(\phi) = (\delta N/\Delta N)(m)$ for $(m-1)/M < (\phi+\hat{\phi})/2\hat{\phi} < m/M$ represented by Fourier expansion is to be

$$f(\phi) = \sum_{k=-\infty}^{\infty} c_k \exp\left(\frac{i2\pi k}{2\hat{\phi}}\ \phi\right) \rightarrow c_k = \frac{1}{2\hat{\phi}} \int_{-\hat{\phi}}^{\hat{\phi}} f(\phi)$$

$$\exp\left(-\frac{i2\pi k}{2\hat{\phi}}\ \phi\right) d\phi = \frac{1}{2\hat{\phi}} \sum_{m=1}^{M} \int_{\frac{m-1}{M}2\hat{\phi}}^{\frac{m}{M}2\hat{\phi}} \frac{\delta N}{\Delta N}(m)$$

$$\exp\left(-\frac{i2\pi k}{2\hat{\phi}}\ (\phi+\hat{\phi})\right)\ d\phi$$

$$= \frac{1}{M} \sum_{m=1}^{M} \frac{\delta N}{\Delta N}(m)\ \exp\left(-\frac{i2\pi km}{M}\right) \quad \rightarrow \quad E\left\{c_k\ \bar{c}_k\right\}$$

$$= \frac{1}{M^2} \sum_{m=1}^{M} \sum_{n=1}^{M} E\left\{\frac{\delta N}{\Delta N}(m)\ \frac{\delta N}{\Delta N}(n)\right\} \exp\left(-\frac{i2\pi k}{M}(m-n)\right)$$

$$= \frac{1}{M^2} \sum_{m=1}^{M} \frac{F}{\Delta N} = \frac{1}{2\hat{\phi}} \int_{-\hat{\phi}}^{\phi} \frac{F}{N_b}\ d\phi = N_b$$

The modulus of the Fourier coefficient c_k is composed of c_k and c_{-k} so that $|c_k| = 2/\sqrt{N_b}$ q.e.d.

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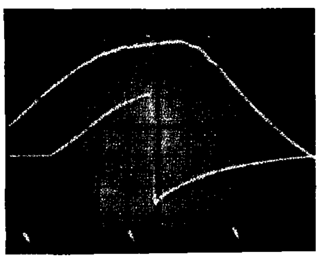

Figure 1
Triplet current (upper trace). Doublet current (lower trace). Sweep : 5 ms/div.

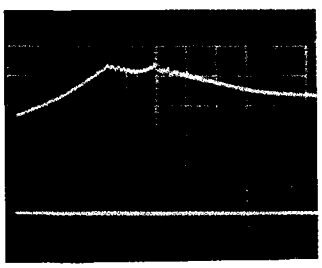

Figure 2
Wide-band pick-up station signal, inversely proportional to bunch length. First peak : doublet current starts rising. Second peak : Jump. N = 5.5 10^{12} p/p. Sweep : 10 ms/div.