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Emittance growth and beam lifetime limitations due to beam–beam effects in e^+e^- storage ring colliders

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Abstract

In this paper we give analytical expressions for the maximum beam–beam parameter and related beam–beam limited beam lifetime in e^+e^- storage ring colliders. The performances of some existing or existed machines are analyzed.

1. Introduction

For about four decades, beam–beam effect has been a subject of scientific research for its limiting nature on the performance of storage ring colliders, and countless publications have been dedicated to it. As a very comprehensive and classical review on beam–beam effect, readers are directed to Ref. [1] for detailed information. In this paper we treat the beam–beam limitations from two directions, firstly, from emittance blow-up point of view (see Ref. [2], which is modified in this paper), secondly, from the point of view of beam–beam limited beam lifetime (see Ref. [3]), and finally, we combine them to a unified theory. Since there are some modifications to Ref. [2], we investigate further in Section 2 to clarify emittance blow-up mechanism, and Section 3 is devoted to the unified beam–beam effect theory, and finally, in Section 4 experimental results obtained in different machines are compared with analytical ones.

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2. Beam–beam parameter limit coming from beam emittance blow-up

In e^+e^- storage ring colliders, due to strong quantum excitation and synchrotron damping

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where analytical expressions of τ_y and $K_{\mathrm{bb},y}$ have been inserted into Eq. (11). Defining

$$\mathscr{H} = \frac{\sigma_{x,*,+}\sigma_{y,*,+}}{\sigma_{x,*}\sigma_{y,*}} \tag{13}$$

where $\mathscr{H}$ is a measure of the plasma pinch effect, assuming that H can be expressed as follows:

$$\mathscr{H} = \frac{\mathscr{H}_0}{\sqrt{\gamma}} \tag{14}$$

and recalling the beam–beam parameter definition

$$\xi_y = \frac{N_e r_e \beta_{y,*}}{2\pi\gamma\sigma_{y,*}(\sigma_{x,*}+\sigma_{y,*})} \tag{15}$$

where β_y^* is the beta function value at the interaction point, σ_x^* and σ_y^* are the bunch transverse dimensions after the plasma pinch effect, respectively, and finally, by combining Eqs. (12), (14) and (15) one gets

$$\xi_y \leqslant \xi_{y,\mathrm{max,em,flat}} = \frac{h\mathscr{H}_0\gamma}{F}\sqrt{\frac{r_e}{6\pi R N_{\mathrm{IP}}}} \tag{16}$$

or, in general case, one has

$$\xi_y \leqslant \xi_{y,\mathrm{max,em,flat}} = \frac{h\mathscr{H}_0}{2\pi F}\sqrt{\frac{T_0}{\tau_y\gamma N_{\mathrm{IP}}}} \tag{17}$$

where h is a constant used to quantify how the denominator in Eq. (11) is approaching to zero, defining $H_0 = h\mathscr{H}_0$, one has $H_0 \approx 2845$, which is not a derived value, but obtained by comparing with experimental results, R is the local dipole bending radius, and F is expressed as follows:

$$F = \frac{\sigma_s}{\sqrt{2}\beta_{y,*}}\left(1+\left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right)^{1/2}. \tag{18}$$

The subscript em in Eqs. (16) and (17) denotes the emittance blow-up limited beam–beam parameter. When $\sigma_s = \beta_{y,*}$ one has $F = 1$.

3. Beam–beam parameter limit coming from beam–beam-induced beam lifetime

In Ref. [3] we have derived beam–beam effect limited beam lifetimes for a rigid flat beam

$$\tau_{\mathrm{bb},y,\mathrm{flat}} = \frac{\tau_y}{2}\left(\frac{3}{\sqrt{2}\pi\xi_y N_{\mathrm{IP}}}\right)^{-1}\exp\left(\frac{3}{\sqrt{2}\pi\xi_y N_{\mathrm{IP}}}\right) \tag{19}$$

$$\tau_{\mathrm{bb},x,\mathrm{flat}} = \frac{\tau_x}{2}\left(\frac{3}{\pi\xi_x N_{\mathrm{IP}}}\right)^{-1}\exp\left(\frac{3}{\pi\xi_x N_{\mathrm{IP}}}\right) \tag{20}$$

and a rigid round beam

$$\tau_{\mathrm{bb},y,\mathrm{round}} = \frac{\tau_y}{2}\left(\frac{4}{\pi\xi_y N_{\mathrm{IP}}}\right)^{-1}\exp\left(\frac{4}{\pi\xi_y N_{\mathrm{IP}}}\right). \tag{21}$$

From Eqs. (19) and (20) one finds that for the same $\tau_{y,\mathrm{bb,flat}}/\tau_y$, $\tau_{x,\mathrm{bb,flat}}/\tau_x$, and $\tau_{y,\mathrm{bb,round}}/\tau_y$, one has $\xi_{x,\mathrm{flat}} = \sqrt{2}\xi_{y,\mathrm{flat}}$, and $\xi_{y,\mathrm{round}} = \frac{4\sqrt{2}}{3}\xi_{y,\mathrm{flat}} = 1.89\xi_{y,\mathrm{flat}}$.

Eqs. (20) and (21) are derived for the case of rigid colliding bunches, where one gets the beam–beam parameter limitations from beam-beam interaction limited beam lifetimes' point of view. To combine the two physical processes discussed above, in a heuristic way, we propose the analytical expressions for the beam–beam interaction limited beam lifetimes for colliding bunches undergoing emittance blow-ups

$$\tau_{\mathrm{bb},y,\mathrm{flat}} = \frac{\tau_y}{2}\left(\frac{3\xi_{y,\mathrm{max,em,flat}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right)^{-1} \times \exp\left(\frac{3\xi_{y,\mathrm{max,em,flat}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right) \tag{22}$$

and

$$\tau_{\mathrm{bb},y,\mathrm{round}} = \frac{\tau_y}{2}\left(\frac{3\xi_{y,\mathrm{max,em,round}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right)^{-1} \times \exp\left(\frac{3\xi_{y,\mathrm{max,em,round}}}{\sqrt{2}\pi\xi_{y,\mathrm{max},0}\xi_y N_{\mathrm{IP}}}\right) \tag{23}$$

with

$$\xi_{y,\mathrm{max,em,round}} = 1.89\xi_{y,\mathrm{max,em,flat}} \tag{24}$$

effects, the particles are confined inside a bunch. The state of the particles can be regarded as a gas, where the positions of the particles follow statistic laws. When two bunches undergo collision at an interaction point (IP, denoted by "*") the particles in each bunch will suffer from additional heatings. Taking the vertical plane for example, one has beam–beam induced kicks in y and $y' = \mathrm{d}y/\mathrm{d}s$ expressed as

$$\delta y = -\frac{\sigma_s}{f_y} y \tag{1}$$

$$\delta y' = -\frac{1}{f_y} y \tag{2}$$

$$\frac{1}{f_y} = \frac{2N_e r_e}{\gamma \sigma_{y,*,+}(\sigma_{x,*,+} + \sigma_{y,*,+})} \tag{3}$$

where σ_s is the bunch length, N_e is the particle number inside the bunch, r_e is the electron classical radius, $\sigma_{x,*,+}$ and $\sigma_{y,*,+}$ are bunch transverse dimensions just before the two colliding bunches overlapping each other, and $\sigma_{x,*}$ and $\sigma_{y,*}$ are defined as the transverse dimensions when the two bunches are fully overlapped at IP. The invariant of vertical betatron motion can be expressed as [4]

$$a_y^2 = \frac{1}{\beta_y^*}\left(y_*^2 + \left(\beta_{y,*} y'_* - \frac{1}{2}\beta'_{y,*} y_*\right)^2\right). \tag{4}$$

From Eqs. (1) and (2) one finds that

$$\delta a_y^2 = \frac{1}{\beta_{y,*}}\left(\frac{\sigma_s}{f_y}\right)^2 y_*^2 \left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right) \tag{5}$$

where y_* is the vertical displacement of the test particle with respect to the center of the colliding bunch. Due to the gaseous nature of the particles, one has to take an average of all possible values of y_* according to its statistical distribution function, and from Eq. (5) one obtains

$$\langle \delta a^2 \rangle = \frac{1}{\beta_{y,*}}\left(\frac{\sigma_s \sigma_{y,*}}{f_y}\right)^2 \left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right). \tag{6}$$

The resultant particles' vertical dimension combining the synchrotron radiation and beam–beam effects can be expressed as follows:

$$\sigma_{y,*}^2 = \frac{1}{4}\tau_y \beta_{y,*}\left(Q_y + \frac{1}{T_0 \beta_{y,*}}\left(\frac{\sigma_s \sigma_{y,*}}{f_y}\right)^2 \times \left(1 + \left(\frac{\beta_{y,*}}{\sigma_s}\right)^2\right)\right) \tag{7}$$

where T_0 is the revolution time, τ_y is the radiation damping time, and Q_y is defined according to Ref. [4] as $\sigma_{y,*,0}^2 = \frac{1}{4}\tau_y \beta_{y,*} Q_y$ with $\sigma_{y,*,0}$ being bunch natural vertical dimension at IP. Solving Eq. (7), one finds

$$\sigma_{y,*}^2 = \frac{\sigma_{y,*,0}^2}{(1 - \frac{\tau_y}{4T_0}(\frac{e^2 N_e K_{\mathrm{bb},y}}{E_0})^2)} \tag{8}$$

where E_0 is particles' energy, and

$$K_{\mathrm{bb},y} = \frac{\sigma_s}{2\pi\varepsilon_0 \sigma_{y,*,+}(\sigma_{x,*,+} + \sigma_{y,*,+})} \times \left(1 + \left(\frac{\beta_{y,*,+}}{\sigma_s}\right)^2\right)^{1/2}. \tag{9}$$

Since $\sigma_y(s) = \sqrt{\varepsilon_y \beta_y(s)}$, from Eq. (8) one gets

$$\varepsilon_y = \frac{\varepsilon_{y,0}}{(1 - \frac{\tau_y}{4T_0}(\frac{e^2 N_e K_{\mathrm{bb},y}}{E_0})^2)} \tag{10}$$

where $\varepsilon_{y,0}$ is the natural transverse emittance. When there are N_{IP} interaction points, the independent heating effects have to be added in a statistical way, and Eq. (10) becomes

$$\varepsilon_y = \frac{\varepsilon_{y,0}}{(1 - \frac{\tau_y N_{\mathrm{IP}}}{4T_0}(\frac{e^2 N_e K_{\mathrm{bb},y}}{E_0})^2)}. \tag{11}$$

From Eq. (11) one knows that the bunch particle population, N_e, cannot be increased beyond a limit that the denominator on the right hand of equation is approaching zero. Using this limiting condition, for an isomagnetic ring, and for a flat bunch ($\sigma_{y,*,+} \ll \sigma_{x,*,+}$), from Eq. (11) one knows that

$$\sigma_{x,*,+}\sigma_{y,*,+} > \left(\frac{3RN_{\mathrm{IP}}(e^2 f N_e \beta_{y,*})^2}{8\pi^2 \varepsilon_0 m_0 c^2 \gamma^5}\right)^{1/2} \tag{12}$$

where $\xi_{y,\max,0}$ is rigid beam case limiting value. Taking $\xi_{y,\max,0} = 0.0447$ means that we quantify the term "beam–beam limit" for the beam–beam limited beam lifetime being 1 h at $\tau_y = 30$ ms.

4. Comparison of some machine performances with respect to theoretical estimations

We start with the machine parameters [5] shown in Table 1 where the beam energy ranges from half GeV (DAFNE) up to almost hundred GeV, LEP-200, among which there are two machines make the collisions with non zero crossing angle, i.e., DAFNE and KEK-B. Using Table 1 and Eq. (17) and assuming $F = 1$, the theoretical head-on collision beam–beam parameter limits are given in Table 2. The experimentally achieved maximum beam–beam parameters [5] are shown in Table 2 also with or without crossing angle. The agreement between the two sets of values is quite well. As for PEP-II beam–beam experimental values reported in Ref. [6] where $\xi_{y,\mathrm{Low}}$ and $\xi_{y,\mathrm{High}}$ are 0.084 and 0.04, respectively, it should be noted that the vertical beam–beam parameter in the high-energy ring has not reached the limit, and therefore, the apparent large vertical beam–beam parameter in the low energy ring does not correspond to the case studied in Table 2 where both rings reach beam–beam limits. Two machines, KEK-B factory and DAFNE, which have finite crossing angles, deserve further analyses. From Table 2 one finds that with Piwinski crossing angle $\Phi = 0.69$ the experimentally achieved KEK-B low-energy ring's (positron) maximum vertical beam–beam parameter is 20% lower than that of head-on collision. On the contrary, the maximum achieved vertical beam–beam parameter of high energy ring seems not have been affected by the large crossing angle. As for DAFNE, according the theoretical analysis method described in Ref. [7], it seems that the experimentally achieved rather low vertical beam-beam parameter (0.02) should not be due to Piwinski angle of $\Phi = 0.22$, but might be due to other physical causes.

Table 1
Machine parameters

Machine	N_{IP}	Energy (GeV)	γ	τ_y (ms)	T_0 (μs)	Φ_{Piwin}
DAFNE	1	0.51	10^3	36	0.325	0.22
BEPC	1	1.89	3.7×10^3	28	0.8	0
PEP-II(L)	1	3.12	6.12×10^3	62	7.33	0
KEKB(L)	1	3.5	6.86×10^3	43	10.05	0.69
KEKB(H)	1	8	1.57×10^4	46	10.05	0.69
PEP-II(H)	1	8.99	1.76×10^4	37	7.33	0
LEP-100	4	45	8.82×10^4	38	88.9	0
LEP-200	4	80.5	1.58×10^5	5	88.9	0

Table 2
Theoretical maximum and experimentally achieved beam–beam parameters

Machine	$\xi_{y,\max,\mathrm{theory}}$	$\xi_{y,\max,\exp}$
DAFNE	0.043	0.02
BEPC	0.04	0.04
PEP-II(L)	0.063	0.06
KEKB(L)	0.084	0.069
KEKB(H)	0.053	0.052
PEP-II(H)	0.048	0.048
LEP-I	0.037	0.033
LEP-II	0.076	0.079

5. Conclusions

In this paper, we have presented analytical expressions for the maximum beam–beam parameters and the corresponding beam–beam limited beam lifetimes for flat and round colliding beam cases. It should be stressed that this theory aims at providing an analytical framework to describe the sophisticated physical process of beam–beam interactions in electron–positron storage ring colliders, from which one could draw useful scaling laws or insights for designs and experiments.

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