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Shear viscosity to entropy density ratio of nuclear matter by transport model

D. Q. Fang, Y. G. Ma, C. L. Zhou

Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China

E-mail: dqfang@sinap.ac.cn

Abstract. The ratio of shear viscosity to entropy density for an equilibrated nuclear system is investigated at intermediate energy heavy-ion collisions within the framework of the Boltzmann-Uehling-Uhlenbeck (BUU) model. After the collision system almost reaching a local equilibration, the temperature, pressure and energy density are obtained from the phase-space information and shear viscosity is calculated using the Green-Kubo method. The results show that shear viscosity to entropy density ratio decreases with the increase of incident energy and tends toward a constant value around 0.5, which is close the BNL Relativistic Heavy Ion Collider results.

1. Introduction

Heavy ion collisions (HIC) have been extensively studied both experimentally and theoretically for obtaining information about the properties of nuclear matter under a wide range of density and temperature. Due to van der Waals nature of the nucleon-nucleon force, liquid-gas phase transition (LGPT) exhibits in HIC around hundred MeV/nucleon [1, 2, 3, 4]. Multifragmentation and LGPT have become the most important subjects in heavy ion collision at intermediate energies in the past years.

Viscosity is a measure of the resistance of a fluid which is being deformed by either shear or tensile stress. The less viscous the fluid is, the greater its ease of movement (fluidity). Viscosity describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction. In ultra-relativistic heavy ion collision, hydrodynamic model has been used to study the Quark Gluon Plasma (QGP) phase and critical phenomenon [5, 6, 7, 8, 9, 10]. It is found that QGP has small viscosity and behaves like a perfect fluid. Only few efforts has been devoted to the study of viscosity for nuclear matter formed in intermediate energy heavy ion collisions [11, 12, 13, 14, 15]. Empirical observation of temperature dependence of shear viscosity to entropy density ratio exhibits a minimum at the critical point of phase transition [16]. A lower bound of this ratio ($\eta/s \geq 1/4\pi$) is speculated to be valid universally according to certain gauge theory (Kovtun-Son-Starinets (KSS) bound) [17, 18].

By using the Boltzmann-Uehling-Uhlenbeck (BUU) model, we have studied the transport coefficients, like viscosity of nuclear matter formed during heavy ion collisions at intermediate energies. The rest of the paper is organized as follows: In Sec. 2, we briefly introduce BUU model and describe the conditions for system equilibrium. In Sec. 3, Green-Kubo method is given for calculating the viscosity coefficients. We give the results and discussions in Sec. 4. Finally, we give a brief summary in Sec. 5.

2. BUU model and system equilibration

The BUU model is a one-body microscopic transport model based upon the Boltzmann equation which reads[19, 20, 21]:

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f = \frac{4}{(2\pi)^3} \int d^3p_2 d^3p_3 d\Omega$$
$$\frac{d\sigma_{NN}}{d\Omega} V_{12} \times [f_3 f_4 (1-f)(1-f_2) - f f_2 (1-f_3)(1-f_4)]$$
$$\delta^3(p + p_2 - p_3 - p_4).$$

It is solved with the method of Bertsch and Das Gupta [22]. In the above equation, $\frac{d\sigma_{NN}}{d\Omega}$ and V_{12} are in-medium nucleon-nucleon cross section and relative velocity of the two colliding nucleons, respectively, and U is the mean field potential including the isospin-dependent term:

$$U(\rho, \tau_z) = a(\frac{\rho}{\rho_0}) + b(\frac{\rho}{\rho_0})^\sigma + C_{sym} \frac{(\rho_n - \rho_p)}{\rho_0} \tau_z, \quad (1)$$

where ρ_0 is the normal nuclear matter density; ρ, ρ_n, and ρ_p are the nucleon, neutron and proton densities, respectively; τ_z equals 1 or -1 for neutrons and protons, respectively; The coefficients a, b and σ are parameters for nuclear equation of state. Two sets of mean field parameters are used in this work, namely the soft EOS with the compressibility K of 200 MeV (a = -356 MeV, b = 303 MeV, $\sigma = 7/6$), and the hard EOS with K of 380 MeV (a = -124 MeV, b = 70.5 MeV, σ = 2). C_{sym} is the symmetry energy strength due to the density difference of neutrons and protons in nuclear medium, here C_{sym} = 32 MeV is used.

In this work, we investigate the process of Au+Au central collision (impact parameter $b = 0$) in a spherical volume (radius=5 fm). First we check the temperature and dynamics evolution of the system. The dynamics condition could be defined as

$$R_p = \frac{2}{\pi} \frac{R_\parallel}{R_\perp} \quad (2)$$

where $R_\parallel =< \sqrt{p_x^2 + p_y^2} >$ and $R_\perp =< \sqrt{p_z^2} >$ are calculated by the momentum of nucleons. The time evolution of R_p is shown in Figure 1. When R_p is approaching to 1, it indicates that

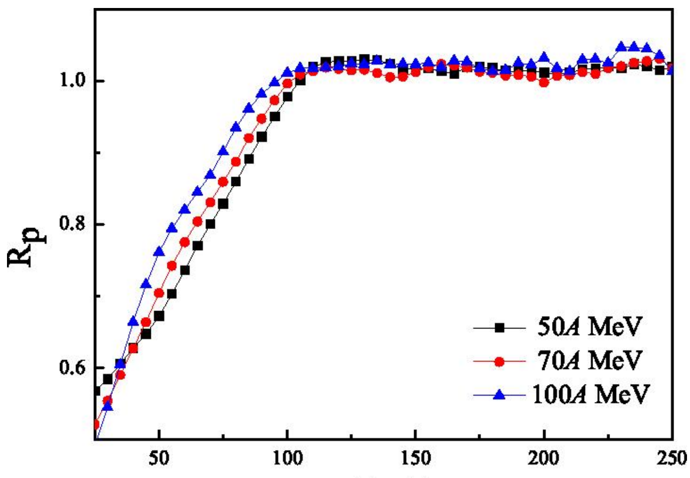

Figure 1. R_p as a function of time for different incident energy $E = 50, 70, 100$ AMeV.

the collision system is under equilibrium. The time evolution of temperature is also used to judge the state of equilibration. Temperature of the system can be derived from the momentum fluctuations of particles in the center of mass frame of the source [23]. The variance σ^2 is obtained from the Q_z distribution through

$$\sigma^2 = < {Q_z}^2 > - < Q_z >^2 \tag{3}$$

Q_z is the quadrupole moment defined by $Q_z = 2{p_z}^2 - {p_x}^2 - {p_y}^2$, and p_x, p_y, p_z is extracted from the phase space of BUU simulation. For the whole system, the mean value of Q_z equals zero, thus the second term vanishes. Q_z^2 is described by

$$< {Q_z}^2 > = \int d^3p (2{p_z}^2 - {p_x}^2 - {p_y}^2)^2 f(p) \tag{4}$$

Assuming a Maxwellian distribution for the nucleon momentum distribution,

$$f(p) = \frac{1}{2\pi m T^{3/2}} e^{-\frac{{p_x}^2 + {p_y}^2 + {p_z}^2}{2mT}} \tag{5}$$

We can obtain the following relation between $< Q_z^2 >$ and T after Gaussian integral of eq.(4),

$$< {Q_z}^2 > = 4m^2 A^2 T^2 \tag{6}$$

where m is the mass of a nucleon and A is the mass number of the fragment. For a nucleon system, we have $A = 1$ and can calculate the temperature using this equation. Figure 2 shows the temperature's evolution after 25 fm/c, it is seen that temperature reaches a maximum around 50 fm/c when the system is in the most compressible stage and then it starts to cool down when the system expands. Later on the system tends to a thermodynamic equilibrium. Different incident energy has different equilibrium temperature according to this figure. For an equilibrated system, the kinetic energy distributions approach the Boltzmann distribution as time increases [24]. When the collision system approaches a equilibrate state, we can investigate the viscosity coefficient in system.

3. Shear viscosity coefficient and entropy density

Viscosity is one of the transport coefficients which characterize the dynamical fluctuation of dissipative fluxes in a medium. Transport coefficients can be measured, as in the case of condensed matter applications. Also, they should be in principle calculable from the first principle. Monte-Carlo simulation is a powerful tool when studying transport coefficients using Green-Kubo relations [25, 26]. In high energy heavy-ion collisions, calculation of transport coefficients of shear viscosity for a binary mixture [27], and calculation of coefficient of a hadrons gas has been studied [24, 28]. The situation of nuclear gas in intermediate energy heavy ion collisions is similar to hadrons gas. To study the extended irreversible dynamic processes, we use the Kubo fluctuation theory to extract transport coefficients [29]. The formula relates linear transport coefficients to near-equilibrium correlations of dissipative fluxes and treats dissipative fluxes as perturbations to local thermal equilibrium. The Green-Kubo formula for shear viscosity is defined by

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt < \pi_{ij}(0,0) \pi_{ij}(\vec{r}, t) > \tag{7}$$

where T is the equilibrium temperature of the system, t is time after equilibration (the above formula defines $t = 0$ as the time the system equilibrating), and $< \pi_{ij}(0,0)\pi_{ij}(\vec{r}, t) >$ is the

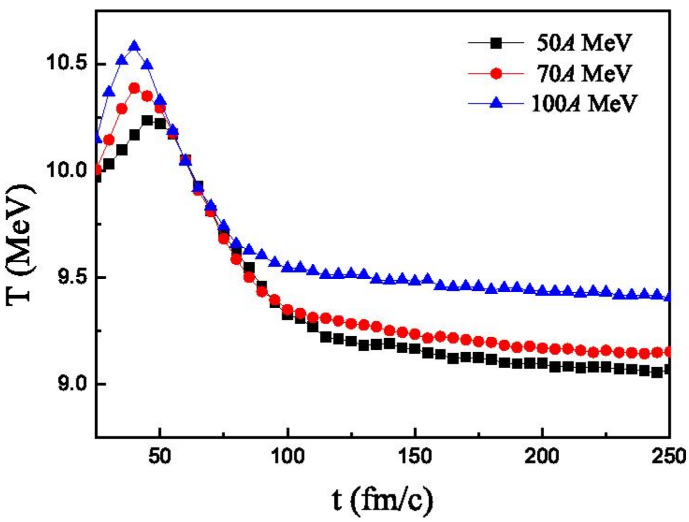

Figure 2. Temperature as a function of time for different incident energy $E = 50, 70, 100$ AMeV.

shear component of the energy momentum tensor. The expression for the energy momentum tensor is defined by $\pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij}T^i_i$, the momentum tensor is [24]

$$T_{ij}(r,t) = \int d^3p \frac{p^i p^j}{p^0} f(x,p,t) \tag{8}$$

where $f(x,p,t)$ is the phase space density of particles in the system. In order to compute an integral, we assume that nucleons are uniformly distributed in the space. In the meanwhile, the isolated spherical volume (radius is 5 fm) is fixed, so the viscosity becomes

$$\eta = \frac{V}{T} < \pi_{ij}(0)^2 > \tau_\pi \tag{9}$$

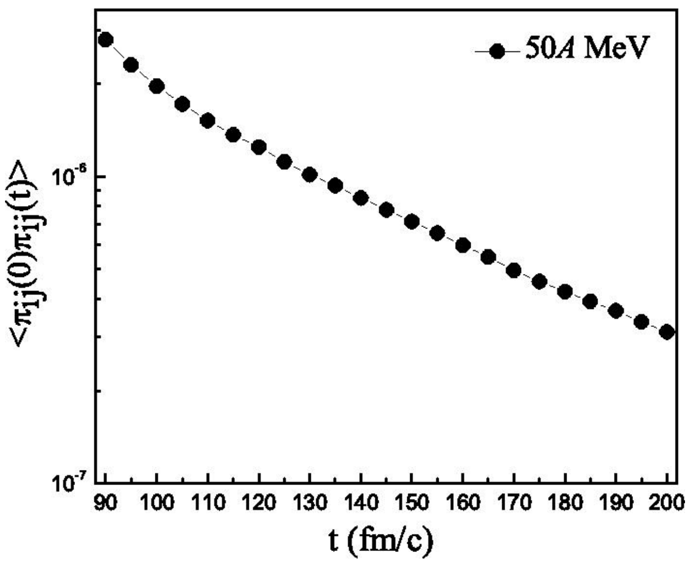

Figure 3. $< \pi_{ij}(0)\pi_{ij}(t) >$ as a function of time for incident energy $E = 50$ AMeV.

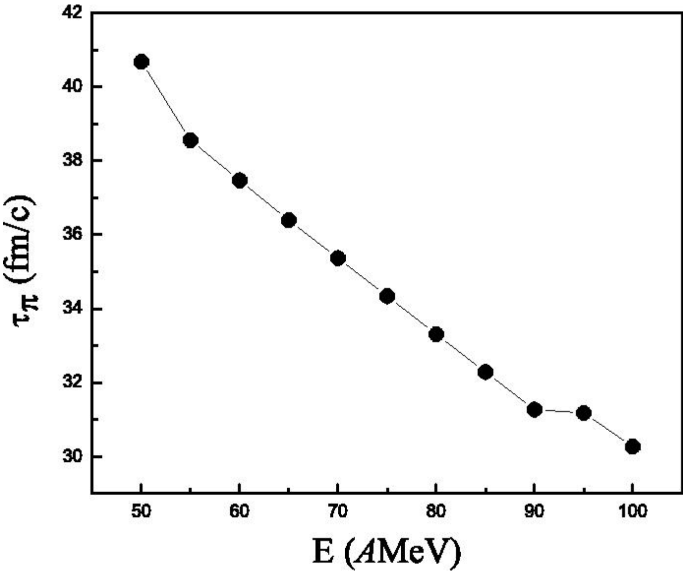

Figure 4. The relaxation time for the shear flux as a function of incident energy.

where τ_π is determined by $< \pi_{ij}(0)\pi_{ij}(t) > \propto \exp(-\frac{1}{\tau_\pi})$. The time dependence of $< \pi_{ij}(0)\pi_{ij}(t) >$ is shown in Figure 3. From this relation we will extract the relaxation times by an exponential function fit. Figure 4 shows the obtained relaxation time. It seems that the shear relaxation time also decreases with incident energy.

To calculate entropy, thermodynamic variables like energy density and pressure should be calculated. Energy density inside a volume with a radius r_0 can be defined as

$$\varepsilon = \frac{1}{V} \sum_{r_i < r_0} E_i, \tag{10}$$

where E_i is $\sqrt{p_i^2 + m_i^2}$, r_i is the position of the i-th nucleon in the center of mass and r_0 is the selected radius (here we set $r_0 = 5$ fm) and pressure can be defined as

$$P = \frac{1}{3V} \sum_{r_i < r_0} \frac{p_i^2}{E_i}. \tag{11}$$

After we get the energy density, pressure and temperature, entropy density can be calculated by the Gibbs formula

$$s = \frac{\varepsilon + P - \mu_n \rho}{T}, \tag{12}$$

where μ_n is the nucleon chemical potential and ρ is nucleon density of system within the given sphere. In principal, once we have the temperature T and $f(p)$, we can fit to a Fermi-Dirac function to extract the chemical potential. However, for simplicity, μ_n is taken to be 20 MeV in the present calculation [30]. We have also checked the results with zero chemical potential, this will increase the entropy about 8% and then lead to the decreasing of η/s about 8%. But this does not change our conclusions of this work.

4. Calculation results and discussion

Using the above method, we show the value of η/s as a function of incident energy after the studied system has been in equilibrium in Figure 5. The two sets of nuclear equation of state are used, ie. hard and soft potentials. The η/s value shows a rapid decrease as the increasing of incident energy up to $E < 70A$ MeV and then drops slowly to a value close 0.5 when $E > 70A$

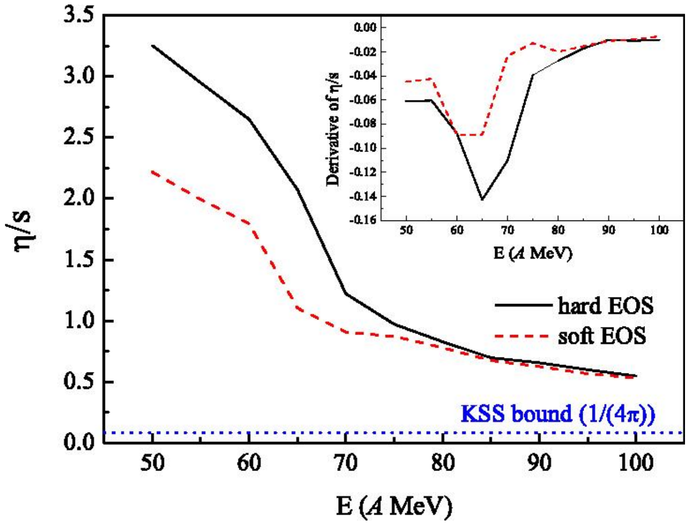

Figure 5. η/s as a function of incident energy for hard and soft potential. The inset show the derivative of η/s with respect to the incident energy.

MeV. The continuous drop of the ratio of shear viscosity to entropy density does not show a minimum at a certain beam energy, which indicates no obvious phase change or critical behavior in the present model. Since the phase transition behavior cannot be described in the BUU model due to its one-body theory characteristic. Actually, when we calculate the differential values of η/s versus the beam energy (see the inset of Figure 5), it seems a turning point around $E \sim 65A$ MeV. This turning point could indicate the change of dynamical behavior of system, in other words, other mechanisms may be needed to be taken into account in the model especially at higher beam energies, eg. multifragmentation [31]. In the present BUU calculation, all calculated values of η/s are well above the conjectured KSS lower bound of $1/4\pi$ [17, 18]. By comparison, we see that they are not drastically different either from the RHIC results [6, 32] or from the results of the usual finite nuclei at low temperature from the widths of giant vibration states in nuclei [15]. Another interesting point in Figure 5 is that η/s shows EOS sensitivity in lower beam energy: hard EOS displays larger η/s than the soft one, i.e. larger compressibility of nuclear matter can lead to higher η/s value.

5. Summary

In summary, we have studied thermodynamic variables as well as viscosity for heavy ion collisions at intermediate energy by using the BUU model. The Green-Kubo relation has been applied for the nucleonic matter in a fixed volume when the system has been in equilibrium during central heavy-ion collisions of Au + Au. It is found that the ratio of shear viscosity to entropy density η/s decreases quickly before $70A$ MeV and then drops slowly towards a smaller value of η/s around 0.5 at higher beam energy. The value of η/s are close to the RHIC results or the results of the usual finite nuclei at low temperature. However, no obvious minimum η/s value occurs within the investigated energy range. This may indicate that the viscosity of nucleonic matter does not describe the liquid-gas phase transition behavior, due to lacking of dynamical fluctuation and correlation in the BUU model. Therefore, other models which can incorporate liquid gas phase transition should be checked for the shear viscosity and entropy density. For instance, it will be very interesting to use quantum molecular dynamics-type model to check if a minimum of η/s will occur around liquid-gas phase transition. Of course, experimental studies on shear viscosity is more important to demonstrate the relation of η/s and liquid-gas transition point.

6. ACKNOWLEDGEMENTS
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