Observational constraints on assisted k-inflation

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Abstract

The k-inflation models can give rise to large non-Gaussianities of primordial density perturbations because the field propagation speed is different from the speed of light. For assisted k-inflation models in which multiple scalar fields join an effective single-field attractor, we evaluate three observables: (i) the spectral index n_s of curvature perturbations, (ii) the tensor-to-scalar ratio r, and (iii) the non-Gaussianity parameter f_{nl}. This will be useful to constrain such models from future high-precision observations of the temperature anisotropies in Cosmic Microwave Background (CMB).

1 Introduction

Inflation has been the backbone of the high-energy cosmology over the past 30 years. Originally it was proposed to solve the horizon, flatness and monopole problems plagued in Big Bang cosmology [1]. Furthermore, inflation generally predicts almost scale-invariant adiabatic density perturbations [2]. This prediction is consistent with the observations of the CMB temperature anisotropies measured by WMAP [3]. One can distinguish between a host of inflationary models by comparing the theoretical prediction of the spectral index n_s of curvature perturbations and the tensor-to-scalar ratio r with observations.

In the next few years, the measurement of CMB temperature anisotropies by the Planck satellite will provide more high-precision observational data. In particular the non-Gaussianity parameter f_{nl} of primordial density perturbations may be constrained by about one order of magnitude better than the bounds constrained by WMAP. This will give additional important information to discriminate between many inflation models.

The standard inflation driven by a canonical scalar field ϕ with a potential $V(\phi)$ usually predicts small non-Gaussianities with $|f_{nl}| \ll 1$ for primordial perturbations [4, 5]. However the k-inflation models [6] described by the Lagrangian density $p(\phi, X)$, where $X = -(\nabla\phi)^2/2$ is the field kinetic energy, can give rise to large non-Gaussianities [7, 8]. This is related with the fact that for the Lagrangian including a non-linear kinetic term of X the propagation speed c_s is different from 1 (in the unit where the speed of light is $c = 1$).

In the models motivated by particle physics such as superstring and supergravity theories, there are many scalar fields that can be responsible for inflation (see e.g. [9]). In some cases, even if each field is unable to lead to cosmic acceleration, the presence of many fields allows a possibility for the realization of inflation through the so-called assisted inflation mechanism [10]. In Ref. [10] it was shown that many canonical fields with exponential potentials $V_i(\phi_i) = c_i e^{-\lambda_i \phi_i}$ evolve to give dynamics matching a single field with the effective slope $\lambda_{\rm eff} = \left(\sum_{i=1} 1/\lambda_i^2\right)^{-1/2}$. Since $\lambda_{\rm eff}$ is smaller than λ_i, the presence of multiple fields can lead to sufficient amount of inflation.

Ref. [11] showed that the Lagrangian density $p = \sum_{i=1}^{n} X_i g(Y_i)$, where $g(Y_i)$ is an arbitrary function in terms of $Y_i = X_i e^{\lambda_i \phi_i}$, leads to assisted inflation as in the case of the canonical fields with exponential potentials (see also Refs. [12, 13]). Moreover, since this assisted Lagrangian covers the k-inflation models such as the dilatonic ghost condensate [12] and the DBI inflation [14], the primordial non-Gaussianities can be large if $c_s^2 \ll 1$. We shall study the theoretical prediction of the three observables n_s, r, f_{nl} to confront assisted k-inflation models with observations.

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2 K-inflation model

The single-field k-inflation models with non-standard kinetic terms are described by the action [6]

$$S = \int d^4x \sqrt{-g} \left[M_{\rm pl}^2 R/2 + p(\phi, X) \right] , \tag{1}$$

where g is a determinant of the metric $g_{\mu\nu}$, $M_{\rm pl}$ is a reduced Planck mass, R is a scalar curvature, p is a general function in terms of the field ϕ and a kinetic term $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$. The pressure p and the energy density ρ of the scalar field are given, respectively, by

$$p = p(X, \phi) , \qquad \rho = 2Xp_{,X} - p , \tag{2}$$

where $p_{,X} \equiv \partial p/\partial X$.

In the flat Friedmann-Lemaître-Robertson-Walker (FLRW) background with a scale factor $a(t)$ the equations of motion are

$$3M_{\rm pl}^2 H^2 = \rho , \qquad \dot{\rho} + 3H(\rho + p) = 0 , \tag{3}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter. The field propagation is defined by

$$c_s^2 \equiv \frac{p_{,X}}{\rho_{,X}} = \frac{p_{,X}}{p_{,X} + 2Xp_{,XX}} , \tag{4}$$

If p has a non-linear term of X, then c_s is different from 1. It is convenient to introduce "slow variation parameters", as

$$\epsilon \equiv -\frac{\dot{H}}{H^2} , \qquad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} , \qquad s \equiv \frac{\dot{c}_s}{Hc_s} . \tag{5}$$

In Ref. [15] the primordial scalar power spectrum $\mathcal{P}_S$ was derived, as

$$\mathcal{P}_S = \frac{H^2}{8\pi^2 M_{pl}^2 \, c_s \, \epsilon} , \tag{6}$$

where the expression is evaluated at the time of horizon exit at $c_s k = aH$ (k is a comoving wavenumber). The spectral index n_s is

$$n_s - 1 = \left. \frac{d \ln \mathcal{P}_S}{d \ln k} \right|_{c_s k = aH} = -2\epsilon - \eta - s . \tag{7}$$

As long as $\epsilon, |\eta|, |s|$ are much smaller than 1, the scalar power spectrum is close to scale-invariant.

The tensor power spectrum $\mathcal{P}_T$ and its spectral index n_T are given by

$$\mathcal{P}_T = \frac{2H^2}{\pi^2 M_{pl}^2} , \qquad n_T = \left. \frac{d \ln \mathcal{P}_T}{d \ln k} \right|_{c_s k = aH} = -2\epsilon . \tag{8}$$

From Eqs. (7) and (8) the tensor-to-scalar ratio is

$$r \equiv \mathcal{P}_T/\mathcal{P}_S = 16\epsilon c_s = -8c_s n_T . \tag{9}$$

The primordial scalar non-Gaussianities can be evaluated by considering three-point correlation $\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3) \rangle$ for the curvature perturbation $\mathcal{R}$. In the equilateral case ($|\mathbf{k}_1| = |\mathbf{k}_2| = |\mathbf{k}_3|$) the non-Gaussianity parameter f_{nl} is given by [7, 8]

$$f_{nl}|_{\rm equilateral} = 0.28 \left(1 - 1/c_s^2\right) - 0.02(\epsilon/\epsilon_X) \, s + 1.53 \, \epsilon + 0.42 \, \eta , \tag{10}$$

where $\epsilon_X \equiv -(\dot{X}/H^2)(\partial H/\partial X)$. Note that our sign convention of f_{nl} coincides with that in the WMAP paper [3].

3 Observational constraints on inflation models

Let us proceed to recent observational constraints on the three inflationary parameters n_s, r, and f_{nl}. From the WMAP 7 year data combined with the distance measurements from the baryon acoustic oscillations in the distribution of galaxies and the Hubble constant (H_0) measurement, the scalar spectral index and the tensor-to-scalar ratio, in the case of no runnings of scalar and tensor perturbations, are constrained to be [3]

$$n_s = 0.963 \pm 0.012 \ (68\% \ \text{CL}) \,, \qquad r < 0.24 \ (95\% \ \text{CL}) \,. \tag{11}$$

The bound on the non-Gaussianity parameter $f_{nl}|_{\rm equil}$ is

$$f_{nl}|_{\rm equil} = 26 \pm 140 \ (68\% \ \text{CL}) \,. \tag{12}$$

We can distinguish between many inflation models by comparing the theoretical prediction of these parameters with observations.

4 Assisted k-inflation model

We consider the following Lagrangian density with n scalar fields

$$p = \sum_{i=1}^{n} X_i \, g(Y_i) \,, \tag{13}$$

where $X_i = -g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i/2$, $g(Y_i)$ is an arbitrary function in terms of $Y_i \equiv X_i e^{\lambda_i \phi_i}$ for each field, and λ_i's are constants. The multiple fields evolve to give dynamics matching an effective single-field model $p = Xg(Y)$ with [11]

$$\frac{1}{\lambda^2} = \sum_{i=1}^{n} \frac{1}{\lambda_i^2} \,. \tag{14}$$

Originally the Lagrangian density $p = Xg(Y)$ with $Y = Xe^{\lambda\phi}$ was derived for the existence of scaling solutions [12, 13], but it was later found that the multi-field system described by (13) can lead to assisted inflation with λ smaller than λ_i [11].

Considering autonomous equations of the effective single-field system with $p(X,\phi) = Xg(Y)$, one can show that there is an inflation attractor characterized by the condition $\dot{\phi}/H = \lambda/p_{,X}$ [11], i.e.

$$\lambda^2 = \frac{6\,(g + Yg')^2}{g + 2\,Yg'} \,, \tag{15}$$

where a prime represents a derivative with respect to Y. The stability of this solution requires that $\lambda^2 < 2\,(g + Yg')$. For given $g(Y)$, it follows that Y is constant. The parameters ϵ and c_s^2 are given by

$$\epsilon = \frac{3\,(g + Yg')}{g + 2\,Yg'} \,, \qquad c_s^2 = \frac{g + Yg'}{g + 5\,Yg' + 2\,Y^2 g''} \,, \tag{16}$$

which are functions of Y only. Hence we have $\epsilon = const.$ and $c_s^2 = const.$ on the attractor, which gives $\eta = 0$ and $s = 0$. Therefore the three inflationary observables reduce to

$$n_s = 1 - 2\,\epsilon \,, \quad r = 16\,\epsilon\, c_s \,, \quad f_{nl}|_{\rm equil} = 0.28\left(1 - 1/c_s^2\right) + 1.53\,\epsilon \,. \tag{17}$$

If $c_s^2 \ll 1$, then the non-Gaussianity parameter f_{nl} can be large ($|f_{nl}| \gg 1$).

The standard slow-roll inflation model with an exponential potential $V(\phi) = ce^{-\lambda\phi}$ corresponds to the choice $g(Y) = 1 - c/Y$. In this case we obtain the following relations

$$c_s^2 = 1 \,, \qquad \lambda^2 = \frac{6\,Y}{Y + c} \,, \qquad \epsilon = \frac{3\,Y}{Y + c} \,. \tag{18}$$

The three inflationary observables can be written in terms of a single parameter λ:

$$n_s = 1 - \lambda^2 \,, \qquad r = 8\lambda^2 \,, \qquad f_{nl} = 0.765\lambda^2 \,. \tag{19}$$

From the observational constraint (11) we require that $\lambda^2 \ll 1$ and hence $f_{nl} \ll 1$. On the other hand, the assisted k-inflation models based on the dilatonic ghost condensate [12] and the DBI inflation [14] can give rise to large f_{nl} accessible to the upper bound given in (12). We will leave the detailed analysis of such cases in a separate publication [16].

5 Conclusion

We have evaluated the three observables n_s, r, and f_{nl} for the assisted k-inflation model described by the effective single-field Lagrangian density $p(X, \phi) = Xg(Y)$ with $Y = Xe^{\lambda\phi}$. Since Y is constant on the assisted inflation attractor, the three observables can be expressed in terms of a single parameter: Y or λ. The field propagation speed c_s is also a function of Y only and is different from 1 in general. The non-Gaussianity parameter f_{nl} can be large in k-inflation models, whereas $f_{nl} \ll 1$ in standard slow-roll inflation since $c_s = 1$. It will be of interest to constrain the allowed parameter space of assisted k-inflation models from the joint analysis of WMAP and other observations.

References

[1] A. A. Starobinsky, Phys. Lett. B **91** (1980) 99; D. Kazanas, Astrophys. J. **241** L59 (1980); K. Sato, Mon. Not. R. Astron. Soc. **195**, 467 (1981); A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[2] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49** (1982) 1110; S. W. Hawking, Phys. Lett. B **115**, 295 (1982); A. A. Starobinsky, Phys. Lett. B **117** (1982) 175.

[3] E. Komatsu *et al.*, arXiv:1001.4538 [astro-ph.CO].

[4] N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **65**, 103505 (2002).

[5] J. M. Maldacena, JHEP **0305**, 013 (2003).

[6] C. Armendariz-Picon, T. Damour and V. F. Mukhanov, Phys. Lett. B **458**, 209 (1999).

[7] D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005).

[8] X. Chen, M. x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).

[9] D. H. Lyth and A. Riotto, Phys. Rept. **314**, 1 (1999).

[10] A. R. Liddle, A. Mazumdar and F. E. Schunck, Phys. Rev. D **58**, 061301 (1998).

[11] S. Tsujikawa, Phys. Rev. D **73**, 103504 (2006).

[12] F. Piazza and S. Tsujikawa, JCAP **0407**, 004 (2004).

[13] S. Tsujikawa and M. Sami, Phys. Lett. B **603**, 113 (2004).

[14] E. Silverstein and D. Tong, Phys. Rev. D **70**, 103505 (2004); M. Alishahiha, E. Silverstein and D. Tong, Phys. Rev. D **70**, 123505 (2004).

[15] J. Garriga and V. F. Mukhanov, Phys. Lett. B **458**, 219 (1999).

[16] J. Ohashi and S. Tsujikawa, *in preparation.*