

POSSIBILITY OF DARK MATTER DETECTION AT FUTURE e^+e^- COLLIDERS*

B. GRZADKOWSKI, M. IGLICKI, K. MEKALA, A.F. ZARNECKI

Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland

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In this paper, we discuss possibility of detecting signals of dark matter particles at future e^+e^- colliders. Two simple models of dark matter are considered, a vector one and a fermion one. Scanning the parameter space of the models, we are seeking regions allowed by current experimental constraints, that maximize cross section for DM production at future e^+e^- colliders. Could the signal of DM be statistically significant? Would it be possible to determine mass and spin of dark particles? It turns out that the answers depend on the parameters of the model of dark matter — both positive and negative conclusion can be consistent with current experimental data.

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1. Introduction

Currently, several e^+e^- colliders, such as CLIC [1], ILC [2], CEPC [3] and FCC-ee [4] are planned to be built. One of their research tasks will be to provide an environment to search for dark matter (DM) particles. Our goal is to estimate chances of DM-signal detection at these colliders. Assuming two simple models of DM, *i.e.* a vector DM (VDM) model and a fermion DM (FDM) model, described in Sections 2.1 and 2.2, respectively, we look for maximal possible value of production cross section (DM production process is described in Section 3) taking into account the current experimental constraints discussed in Section 4. Results are presented in Section 5.

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2. The dark matter models

2.1. Vector dark matter (VDM) model

In the VDM model, the Standard Model (SM) gauge group is extended by $U(1)_X$

$$\mathcal{G} = \mathcal{G}_{\text{SM}} \times U(1)_X. \quad (2.1)$$

All the SM particles are assumed to be neutral under $U(1)_X$. The gauge vector of $U(1)_X$, denoted by X_μ , can serve as a dark matter particle. In order to provide mass of the X_μ field, a complex scalar S , neutral under $\mathcal{G}_{\text{SM}}$ and whose $U(1)_X$ charge is equal to 1, is introduced.

After the spontaneous symmetry breaking, both S field and the standard Higgs field get non-zero vacuum expectations values (v_s and v , respectively):

$$S \rightarrow \frac{v_s + \phi + i\sigma}{\sqrt{2}}, \quad H \rightarrow \begin{pmatrix} \pi^+ \\ \frac{v+h+i\pi^0}{\sqrt{2}} \end{pmatrix}. \quad (2.2)$$

The would-be Goldstone bosons of H and S are eaten by the gauge fields, $W^\pm$, Z and X_μ . Non-zero v_s provides a mass term for X_μ . Neutral-component real-part fluctuations of the fields, ϕ and h , mix to give eigenstates of the mass-squared matrix, h_1 and h_2

$$\begin{pmatrix} h \\ \phi \end{pmatrix} = \mathcal{R} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad \mathcal{R} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}, \quad (2.3)$$

where the mixing angle α is between $-\pi/4$ and $\pi/4$. We identify h_1 with the known Higgs particle, hence $m_1 = 125$ GeV and $v = 246$ GeV. Values of m_2 and v_s are independent free parameters of the model.

The interaction vertices present in the model, that involve dark particles, are shown in Fig. 1.

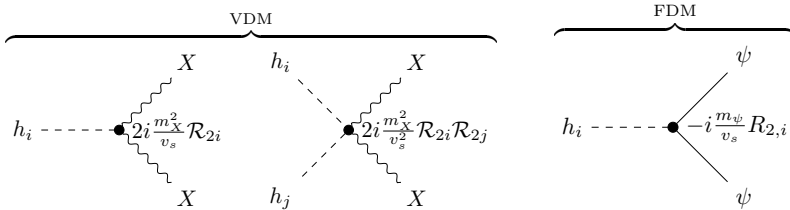


Fig. 1. Interaction vertices involving dark particles in the VDM (left and middle) and the FDM (right) model.

2.2. Fermion dark matter (FDM) model

In the FDM model, the gauge group of the SM is extended by $\mathbb{Z}_4$,

$$\mathcal{G} = \mathcal{G}_{\text{SM}} \times \mathbb{Z}_4. \quad (2.4)$$

As previously, all SM particles are neutral under $\mathbb{Z}_4$. A left-handed Dirac fermion χ of $\mathbb{Z}_4$ -charge 1 is introduced. Combination of the fermion field χ and its charge conjugate

$$\psi \equiv \psi^c \equiv \chi + \chi^c \quad (2.5)$$

is a Majorana mass eigenstate that serves as a dark particle in this model. In order to provide mass for ψ , a real scalar S of $\mathbb{Z}_4$ -charge 2 is introduced. After the spontaneous symmetry breaking

$$S \rightarrow v_s + \phi, \quad H \rightarrow \left(\begin{array}{c} \pi^+ \\ \frac{v+h+i\pi^0}{\sqrt{2}} \end{array} \right), \quad (2.6)$$

non-zero v_s provides a mass term for ψ through the allowed Yukawa interaction. The fluctuations ϕ and h mix to create eigenstates of the mass-squared matrix, h_1 and h_2 as described by (2.3). Again, we identify h_1 with the known Higgs particle ($m_1 = 125$ GeV, $v = 246$ GeV), while m_2 and v_s are free parameters of the model. A Yukawa interaction vertex is shown in Fig. 1.

2.3. Input parameters

Parameter spaces of the models are identical, hereafter we will adopt the following possibility, $(m_1, v, m_2, v_s, \sin \alpha, m_{\text{DM}})$, with $m_1 = 125$ GeV, $v = 246$ GeV and $m_{\text{DM}} = m_X$ or $m_{\text{DM}} = m_\psi$, depending on the model. This set completely defines both models. Parameters $m_2, v_s, \sin \alpha, m_{\text{DM}}$ are subject to experimental constraints that will be discussed in Section 4.

3. DM production at e^+e^- colliders

We consider following process of DM production: $e^+e^- \rightarrow \text{DM DM } Z$, where the DM is either X or ψ , see Fig. 2. The differential cross section for this process reads

$$\begin{aligned} \frac{d\sigma}{dm_{\text{rec}}^2} &= \frac{\sigma_{\text{SM}}(m_{\text{rec}})}{\pi} \sin^2 \alpha \cos^2 \alpha \frac{b(m_{\text{rec}}, m_{\text{DM}})}{v_s^2} \\ &\times \frac{m_{\text{rec}} (m_1^2 - m_2^2)^2}{\left[(m_{\text{rec}}^2 - m_1^2)^2 + (m_1 \Gamma_1)^2 \right] \left[(m_{\text{rec}}^2 - m_2^2)^2 + (m_2 \Gamma_2)^2 \right]}. \end{aligned} \quad (3.1)$$

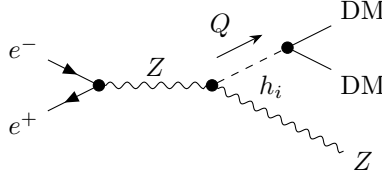


Fig. 2. Considered process of DM production in colliders.

Here, $m_{\text{rec}} \equiv \sqrt{Q^2}$ is the recoil mass of produced pair of dark particles and

$$\sigma_{\text{SM}}(m_{\text{rec}}) \equiv \frac{g_V^2 + g_A^2}{24\pi} \left(\frac{g^2}{\cos^2 \theta_W} \frac{1}{s - m_Z^2} \right)^2 \times \frac{\lambda^{1/2}(s, m_{\text{rec}}^2, m_Z^2) [12 s m_Z^2 + \lambda(s, m_{\text{rec}}^2, m_Z^2)]}{8s^2}, \quad (3.2)$$

$$b(m_{\text{rec}}, m_{\text{DM}}) \equiv \frac{m_i^3}{32\pi} \sqrt{1 - \frac{4m_{\text{DM}}^2}{m_{\text{rec}}^2}} \times \begin{cases} 2 \left[\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2} - 4 \left(\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2} \right)^2 \right] & (\text{FDM}) \\ 1 - 4 \frac{m_{\text{DM}}^2}{m_{\text{rec}}^2} + 12 \left(\frac{m_{\text{DM}}^2}{m_{\text{rec}}^2} \right)^2 & (\text{VDM}) \end{cases}. \quad (3.3)$$

Notice that because of the b factor, for the same set of input parameters, the differential cross section has different shape for the FDM and the VDM models. Hence, in principle, it is possible to distinguish fermion dark matter from vector dark matter if the shape of the distribution can be measured with sufficient precision. Moreover, the minimal total energy needed to produce DM is $\sqrt{s} = m_Z + 2m_{\text{DM}}$. Scanning the production cross section in the total energy of the collision $\sqrt{s}$ should allow to detect the position of this threshold and, therefore, determine the mass of the dark particle. Another possibility to measure the DM mass is to investigate the end point of the Z energy distribution.

If at least one of h_1 , h_2 can be produced on-shell and decay into DM, the cross section is dominated by the contribution of these on-shell decays

$$\sigma \approx \sigma_1 \mathbb{1}_{2m_{\text{DM}} < m_1 < \sqrt{s} - m_Z} + \sigma_2 \mathbb{1}_{2m_{\text{DM}} < m_2 < \sqrt{s} - m_Z}, \quad (3.4)$$

where

$$\begin{aligned} \sigma_1 &\equiv \sigma_{\text{SM}}(m_1) \cos^2 \alpha \text{BR}(h_1 \rightarrow \text{DM}), \\ \sigma_2 &\equiv \sigma_{\text{SM}}(m_2) \sin^2 \alpha \text{BR}(h_2 \rightarrow \text{DM}). \end{aligned} \quad (3.5)$$

4. Current experimental limits and constraints

4.1. Mixing angle and the branching ratio for invisible decays

Measurements at the LHC [5] provide an upper limit for the Higgs-sector mixing angle

$$\sin \alpha \lesssim 0.3. \quad (4.1)$$

The contributions of the on-shell- h_1 and on-shell- h_2 poles from Eq. (3.4), σ_1 and σ_2 , are limited by the null results of the search for invisible Higgs boson decays at CMS [6]

$$\sigma_1 < 0.19 \sigma_{\text{SM}}(m_1), \quad (4.2)$$

$$\log_{10} \left[\frac{\sigma_2}{\sigma_{\text{SM}}(m_2)} \Big|_{\sqrt{s}=13 \text{ TeV}} \right] < 0.0011 \frac{m_2}{1 \text{ GeV}} - 0.63. \quad (4.3)$$

Equation (4.2) comes from the approximate parametrization of the limiting curve shown in Fig. 7 from [6]. For $\sin \alpha < 0.3$ (see Eq. (4.1)), we have

$$\begin{aligned} \frac{\sigma_2}{\sigma_{\text{SM}}(m_2)} \Big|_{\sqrt{s}=13 \text{ TeV}} &= \text{BR}(h_2 \rightarrow \text{DM}) \sin^2 \alpha \\ &< 0.09 \\ &< 10^{0.0011 \frac{m_2}{1 \text{ GeV}} - 0.63}. \end{aligned} \quad (4.4)$$

Therefore, condition (4.2) is always satisfied, so we do not need to consider it separately.

4.2. Direct detection

Direct-detection experiments search for effects of scattering of dark particles on nuclei present in detectors (see Fig. 3). In our models, the cross section for such a scattering is given by

$$\sigma_{\text{DD}} = \frac{\mu^2 m_{\text{DM}}^2}{\pi v_s^2} \frac{(m_1^2 - m_2^2)^2}{m_1^4 m_2^4} \sin^2 \alpha \cos^2 \alpha \frac{m_N^2}{v^2} f_N^2, \quad (4.5)$$

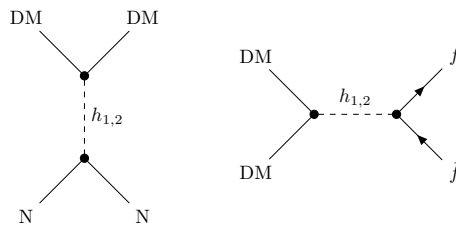


Fig. 3. DM direct-detection (left) and indirect-detection (right) processes present in our models.

where

$$\mu = \frac{m_N m_{\text{DM}}}{m_N + m_{\text{DM}}} \approx m_N \approx 0.94 \text{ GeV}, \quad f_N \approx 0.3. \quad (4.6)$$

Current limit for this cross section is based on null results of XENON1T [7]. Within considered range of masses of dark particles, it can be parametrized as

$$\frac{\sigma_{\text{DD}}}{1 \text{ cm}^2} \lesssim \frac{m_{\text{DM}}}{1 \text{ GeV}} \times 10^{-48.05}. \quad (4.7)$$

Comparison with Eq. (4.5) gives the following condition in terms of the input parameters

$$\frac{\sin^2 \alpha \cos^2 \alpha}{v_s^2} (m_1^2 - m_2^2)^2 < \frac{m_{\text{DM}}^4}{m_{\text{DM}}} \times 1.5 \times 10^{-6} \text{ GeV}^{-1}. \quad (4.8)$$

4.3. Indirect detection and the relic density constraints

Indirect-detection search for dark matter is based on the assumption that dark matter can annihilate and produce SM particles that could be observed directly (see Fig. 3). The thermally averaged cross section for the DM annihilation into a pair of SM fermions, calculated for temperature T , reads

$$\begin{aligned} \langle \sigma v \rangle_{\text{ID}}^{f\bar{f}} &= \frac{(m_{\text{DM}}^2 - m_f^2)^{3/2} (m_1^2 - m_2^2)^2 \sin^2 \alpha \cos^2 \alpha}{\left[(m_1^2 - 4m_{\text{DM}}^2)^2 + m_1^2 \Gamma_1^2 \right] \left[(m_2^2 - 4m_{\text{DM}}^2)^2 + m_2^2 \Gamma_2^2 \right]} \\ &\times \frac{1}{\pi} \frac{m_{\text{DM}} m_f^2}{v_s^2 v^2} \begin{cases} 1 & \text{VDM} \\ \frac{9}{4} \left(\frac{m_{\text{DM}}}{T} \right)^{-1} & \text{FDM} \end{cases} \\ &+ \left[\text{higher orders in } (m_{\text{DM}}/T)^{-1} \right]. \end{aligned} \quad (4.9)$$

Due to the m_f^2 factor, coming from the Yukawa coupling between SM fermions and the Higgs field, the dominating contribution is that with $b\bar{b}$ in the final state ($t\bar{t}$ is outside of the considered range of masses, as well as W^+W^- and ZZ). Notice that the thermally averaged cross section for annihilation of vector dark matter is, in the leading order, constant with respect to T , while in the case of fermion DM, it is proportional to temperature.

In the standard freeze-out mechanism, the relic density of dark matter scales as (see *e.g.* [8])

$$h^2 \Omega_0^{\text{DM}} \sim (n+1) (m_{\text{DM}}/T_f)^{n+1} / \sigma_0, \quad (4.10)$$

where it is assumed that the cross section for annihilation into the SM is given by

$$\langle \sigma v \rangle_{\text{ID}} = \sigma_0 (m_{\text{DM}}/T)^{-n} + \dots \quad (4.11)$$

(in the case of our models, $n = 0$ for VDM and $n = 1$ for FDM). To ensure relic density consistent with the value of 0.12, measured by the Planck Collaboration [9], the following equality must hold:

$$\langle\sigma v\rangle_{\text{ID}}\Big|_{\text{now}} = (T_0/T_{\text{f}})^n (n+1) \times 1.9 \times 10^{-9} \text{ GeV}^{-2}, \quad (4.12)$$

where T_0 is the current temperature of the DM halo, while T_{f} is the temperature at the moment of the freeze out, when the amount of DM stabilizes. Usually, $T_{\text{f}} \sim m_{\text{DM}}/25$, hence, using Eqs. (4.9) and (4.12), we obtain the following condition:

$$\begin{aligned} & \frac{\sin^2 \alpha \cos^2 \alpha}{v_s^2} (m_1^2 - m_2^2)^2 = 2.1 \times 10^{-5} \text{ GeV}^{-2} \\ & \times \frac{\left[(m_1^2 - 4m_{\text{DM}}^2)^2 + m_1^2 \Gamma_1^2 \right] \left[(m_2^2 - 4m_{\text{DM}}^2)^2 + m_2^2 \Gamma_2^2 \right]}{m_{\text{DM}} (m_{\text{DM}}^2 - m_b^2)^{3/2}} k, \end{aligned} \quad (4.13)$$

where $k = 1$ for the VDM and $k = \frac{8m_\psi}{9T_{\text{f}}} \approx 22$ for the FDM. For the VDM, the current value of the cross section corresponding to the correct relic density is consistent with the Fermi-LAT limit [10] only if

$$m_X \gtrsim 30 \text{ GeV}. \quad (4.14)$$

For the FDM, due to the T_0/T_{f} ratio in Eq. (4.12), the required current value of the cross section is orders of magnitude lower than the Fermi-LAT limit, so the range of m_ψ is not affected by this constraint.

5. Maximization of the cross section. Detectability of dark matter at future e^+e^- colliders

5.1. Maximization of the cross section

Constraints on the model parameters, adopted for our calculations, are the following:

- perturbativity: $\frac{m_{\text{DM}}}{v_s} < 4\pi$,
- the mixing angle: $\sin \alpha < 0.3$,
- the branching ratio of the SM Higgs particle into DM:

$$\text{BR}(h_1 \rightarrow \text{DM}) < 0.19, \quad (5.1)$$

— the relic density constraint:

$$\begin{aligned}
 & \frac{\sin^2 \alpha \cos^2 \alpha}{v_s^2} (m_1^2 - m_2^2)^2 \\
 &= \frac{\left[(m_1^2 - 4m_{\text{DM}}^2)^2 + m_1^2 \Gamma_1^2 \right] \left[(m_2^2 - 4m_{\text{DM}}^2)^2 + m_2^2 \Gamma_2^2 \right]}{m_{\text{DM}} (m_{\text{DM}}^2 - m_b^2)^{3/2}} \\
 & \times 2.1 \times 10^{-5} \text{ GeV}^{-2} \times \begin{cases} 1 & \text{(VDM)} \\ 22 & \text{(FDM)} \end{cases}, \quad (5.2)
 \end{aligned}$$

— the direct detection constraint:

$$\frac{\sin^2 \alpha \cos^2 \alpha}{v_s^2} (m_1^2 - m_2^2)^2 < \frac{m_2^4}{m_{\text{DM}}} \times 1.5 \times 10^{-6} \text{ GeV}^{-1}. \quad (5.3)$$

We use these constraints to reduce the number of independent model parameters in our study. For each set of (m_2, m_{DM}) parameters, v_s is calculated from Eq. (5.2) as a function of m_2 , m_{DM} and $\sin \alpha$. Then, the production cross section is maximized with respect to $\sin \alpha$. The calculations are performed assuming the total energy of the collision at the level of 240 GeV, as planned for the CEPC [3]. According to Eq. (3.1), lower total energy $\sqrt{s}$ results in higher cross section.

Using this procedure, we obtain results showed in Fig. 4. The cross section is maximal when h_1 and h_2 can be both on-shell, *i.e.* for $2m_{\text{DM}} < m_1, m_2$.

5.2. Detectability

The total luminosity of CEPC during 7-years running period is estimated as 5.6 ab^{-1} . This value multiplied by the cross section gives the number of DM-production events: 3.2×10^5 in the case of VDM and 3.3×10^5 in the case of FDM. Large number of events gives good statistics, in particular — small relative error.

The predicted amount of the background events ($e^+e^- \rightarrow Z\nu\bar{\nu}$), calculated using CalcHEP [11], is 2.8×10^6 . Hence, for the optimal set of input parameters the signal can be at the level of 10% of the background and the corresponding statistical significance of the observation is huge, about 180σ . Application of the dedicated event selection procedures can increase the significance even more and should allow to test a larger part of the model parameter space.

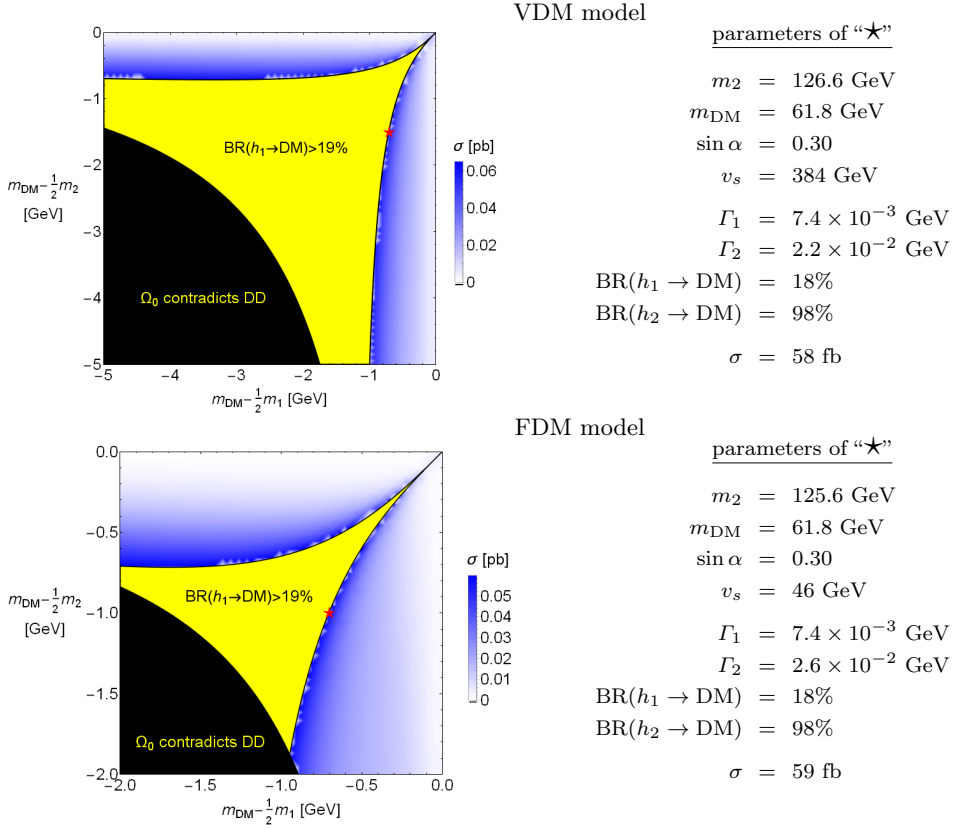


Fig. 4. Maximised cross section for DM production in e^+e^- colliders. Black: forbidden due to Eq. (5.3), light gray/yellow: forbidden due to Eq. (5.1), dark gray/bluish allowed. Star denotes an exemplary point nearby the maximum.

According to [3], decays of h_1 into dark matter should be detectable in CEPC if $\text{BR}(h_1 \rightarrow \text{DM}) \gtrsim 0.3\%$. Optimal parameters give this branching ratio comparable to the current CMS limit, *i.e.* 19%, which is much more than the sensitivity threshold.

Also h_2 decays into dark matter should be detectable if the input parameters are close to the optimal ones. For our models, quantity $\text{BR}(h_2 \rightarrow \text{DM}) \sin^2 \alpha$ is at the level of 8%, while detectors at future e^+e^- colliders are expected to be sensitive to values of the order of 1% [12].

6. Summary

Our analysis shows that if parameters of the dark matter models are close to the optimal set, provided in Section 5, DM could be easily detectable at future colliders. In principle, it could be possible to disentangle the VDM and the FDM models and measure the DM mass using analysis of the shape of differential cross section. However, non-optimal parameters can make all these tasks really tough, or even impossible.

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