Dual Formulation for Higher Spin Gauge Fields in AdS

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The aim of this work is to extend the analysis of [1] to the $(A)dS_d$ background to show in particular that, for non-zero background curvature λ^2, the equations for spin $s \geq 2$ frame-like fields are algebraic constraints $\lambda^2 e + \partial\omega = 0$ allowing e to be expressed in terms of derivatives of the Lorentz connection-type fields ω. Plugging the resulting expression back into the original HS action one obtains equivalent description of the HS dynamics in terms of ω. The main comment of these notes is that the flat limit description of [1] results from our description within the stationary phase expansion with respect to small λ^2.

As shown in [2], $4d$ gravity with the negative cosmological constant can be formulated in terms of gauge fields of the AdS_4 algebra $o(3,2)$. It was shown in [2] that, up to a topological term, Einstein-Hilbert action with the cosmological constant admits an equivalent form

$$S^M = -\frac{\lambda^{-2}}{4\kappa^2}\int \varepsilon_{abcd}\,\mathcal{R}^{ab}\wedge\mathcal{R}^{cd}\,, \tag{1}$$

with

$$\mathcal{R}^{ab} = d\Omega^{ab} + \Omega^a{}_c\wedge\Omega^{cb} - \lambda^2 E^a\wedge E^b\,, \tag{2}$$

where E^a is the frame 1-form and Ω is the Lorentz connection 1-form.

The straightforward d-dimensional generalization of the $4d$ MacDowell-Mansouri pure gravity action is [3]

$$S = -\frac{\lambda^{-2}}{4(d-3)!}\int \varepsilon_{c_1\ldots c_{d-4}abfg}E^{c_1}\wedge\ldots\wedge E^{c_{d-4}}\wedge\mathcal{R}^{ab}\wedge\mathcal{R}^{fg}. \tag{3}$$

The perturbative expansion near the AdS background which is the most symmetric solution of (3) reads

$$\Omega = \omega_0 + \omega, \qquad E = h + e,$$

where e and ω are dynamical (fluctuational) parts of the gauge fields whereas h and ω_0 define AdS background. Then the linear part of $\mathcal{R}^{ab}$ is

$$R^{ab} = D\omega^{ab} - \lambda^2(h^a\wedge e^b - h^b\wedge e^a), \tag{4}$$

with

$$D\omega^{ab} = d\omega^{ab} + \omega_0{}^a{}_c\wedge\omega^{cb} + \omega_0{}^b{}_c\wedge\omega^{ac}.$$

Hence, d-dimensional free action of a massless spin-2 field is

$$S^{(2)} = -\frac{\lambda^{-2}}{4(d-3)!}\int \varepsilon_{c_1\ldots c_{d-4}abfg}h^{c_1}\wedge\ldots\wedge h^{c_{d-4}}\wedge R^{ab}\wedge R^{fg}. \tag{5}$$

From (4) and (5) one obtains

$$S^{(2)}[\omega,e] = -\frac{\lambda^{-2}}{4(d-3)}\int d\mu\,[\delta^{cdhj}_{abfg}D_c\omega_{d|}{}^{ab}D_h\omega_{j|}{}^{fg}$$
$$-8\lambda^2(d-3)(\omega_{c|}{}^{ab}+\delta^a_c\omega_{d|}{}^{bd}-\delta^b_c\omega_{d|}{}^{ad})D_a e_b{}^c$$
$$+4\lambda^4(d-3)(d-2)(e_a{}^a e_b{}^b - e_a{}^b e_b{}^a)]. \tag{6}$$

Let us introduce the field $Y_{ab|c}$

$$Y_{ab|c} = \omega_{c|ab} + \eta_{ac}\omega_{d|b}{}^d - \eta_{bc}\omega_{d|a}{}^d.$$

Up to a total derivative, the action (6) can be rewritten as

$$S^{(2)}[Y,e] = \int d\mu\,[Y_{ab|c}Y^{ac|}{}_b - \frac{1}{d-2}Y_{ab|}{}^bY^{ac|}{}_c - 2D_aY^{ab|}{}_c e_b{}^c$$
$$-\lambda^2(d-2)(e_a{}^a e_b{}^b - e_a{}^b e_b{}^a)]. \tag{7}$$

Variation of the action (7) with respect to $e_a{}^c$ gives

$$\lambda^2 e_a{}^b = -\frac{1}{d-2}\left[D_cY^{bc|}{}_a + \frac{1}{d-1}\delta^b_a D_cY^{cd|}{}_d\right]. \tag{8}$$

Using (8) one can get rid of the frame field in (7) to obtain the second-order action

$$S^{(2)}[Y] = \int d\mu\,\left[Y_{ab|c}Y^{ac|}{}_b - \frac{1}{d-2}Y_{ab|}{}^bY^{ac|}{}_c - \frac{\lambda^{-2}}{d-2}D_aY^{ab|}{}_cD_eY^{ec|}{}_b\right.$$
$$\left.+\frac{\lambda^{-2}}{(d-2)(d-1)}D_aY^{ab|}{}_bD_cY^{cd|}{}_d\right]. \tag{9}$$

The generating functional $\mathcal{Z}$ for the action (9) is

$$\mathcal{Z} = N\int \mathcal{D}Y\, f[Y]\, e^{i\widetilde{S}[Y]}, \tag{10}$$

where N is a normalization factor,

$$f[Y] = e^{i\int d\mu\,(Y_{ab|c}Y^{ac|}{}_b - \frac{1}{d-2}Y_{ab|}{}^bY^{ac|}{}_c)}$$

and

$$\widetilde{S}[Y] = \frac{\lambda^{-2}}{d-2}\int d\mu\,(-D_aY^{ab|}{}_cD_eY^{ec|}{}_b + \frac{1}{d-1}D_aY^{ab|}{}_bD_cY^{cd|}{}_d).$$

To evaluate this integral in the flat limit $\lambda\to 0$ one can apply the stationary phase method. Stationary points of $\widetilde{S}[Y]$ satisfy the equation

$$D_bD_aY^{ad|}{}_c - \delta^d_c D_bD_aY^{ae|}{}_e - (b\leftrightarrow c) = 0. \tag{11}$$

It is readily seen that, up to the terms proportional to λ^2 which vanish in the flat limit, (11) admits a solution of the form

$$Y_{ab|}{}^d = D_eY^e{}_{ab|}{}^d, \tag{12}$$

where $Y_{eab|d} = Y_{[eab]|d}$ and [...] mean antisymmetrization.

The key point is that the solutions (12) are dominating because $D_a Y_c^{ab|} = \lambda^2 Y_{ec}{}^{b|e}$ and therefore $\bar{S}[Y] \sim \lambda^2$. Contribution from other stationary solutions is suppressed when $\lambda \to 0$ since $\bar{S}[Y] \sim \lambda^{-2}$ (because of the boundary terms).

Thus, the flat space action is equivalent to

$$S_{fl}[Y] = \int d^d x(\partial_d Y^d{}_{ab|c} \partial_e Y^{eac|}{}_b - \frac{1}{d-2} \partial_d Y^d{}_{ab|}{}^b \partial_e Y^{eac|}{}_c) \,, \tag{13}$$

which is just the action obtained in [1].

We see that in our approach the dual flat space action results from some sort of quasi-classical approximation with λ being a counterpart of Planck constant in quantum mechanics.

The case of higher spin fields is analysed in the same way using technique elaborated in [4].

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