Shock-waves in the gravitational wave compatible Horndeski theories with linear kinetic term

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Propagating shock-waves can be discussed in terms of junction conditions between space-time regions separated by a hypersurface. Recent observations of gravitational waves and their electromagnetic counterparts established that the former also propagate with the speed of light. Hence energetic gravitational waves could be perceived as shock-waves on null hypersurfaces. The most generic scalar-tensor theories with at most second order dynamics, the Horndeski-theories were severely constrained. We derive junction conditions across a null hypersurface for the subclass of allowed Horndeski-theories with linear kinetic term dependence, exploring a formalism based on a transverse null vector. We obtain a 2+1 decomposed generalised Lanczos equation, with the jump of the transverse curvature induced by both the distributional energy-momentum tensor of the wavefront of the shock-wave, and by the jump in the transverse derivative of the scalar. The surface density, current and pressure of the distributional light-like shock-wave and the transverse derivative of the scalar are also constrained by a scalar junction equation.

Keywords: Scalar-tensor gravity; junction conditions; null-hypersurfaces.

1. Introduction

In general relativity, matching two solutions of the Einstein field equations accross a hypersurface separating them is realized through the Israel junction conditions, provided the hypersurface is timelike or spacelike.[1] Across a boundary hypersurface, they prescribe the continuity of the induced metric and of the extrinsic curvature. The discontinuity of the extrinsic curvature is also admissible for hypersurfaces referred to as thin shells. The Lanczos equation relates the jump of the extrinsic curvature to the distributional stress-energy tensor $\mathscr{T}^{\mu\nu}$ of the thin shell.

Israel's procedure breaks down if the hypersurface is null. This is a physically relevant case, as impulsive shock-waves traveling at the speed of light are described by null hypersurfaces, as are event horizons. The generalization of the Israel junction conditions to arbitrary hypersurfaces has been developed by Barrabés and Israel.[2] If the hypersurface is specified to be null, then in Poisson's reformulation, the junction conditions take a particularily simple form.[3]

The most general scalar-tensor theory with second order equations of motion for both the scalar and the metric tensor was given by Horndeski.[4,5] Recent gravitational wave observations[6–12] have severely constrained this class. The subclass guaranteeing the propagation of the tensorial modes with the speed of light[13–16] has a Lagrangian for the scalar field often referred to as kinetic gravity braiding theory.[17,18] The coupling of the curvature with the scalar can be through an unspecified function of the scalar field, in the Jordan frame. Adding these contributions together we get a generalised kinetic gravity braiding class of scalar-tensor theories.

Junction conditions across timelike and spacelike hypersurfaces in the full Horndeski class have been found,[19,20] but the null case has not been discussed. This case is important, as all electromagnetic and gravitational waves propagate on the light-cone.

In this work we address the junction conditions accross null hypersurfaces in a subset of the generalised kinetic gravity braiding class of theories, namely those in which the Lagrangian is linear in the kinetic term $X = -\left(\nabla\phi\right)^2/2$:

$$L = B(\phi)X + V(\phi) - 2\xi(\phi)\Box\phi X + \frac{1}{2}F(\phi)R \tag{1}$$

Here B, V, ξ, F are arbitrary smooth functions of the dynamical scalar field ϕ. In the following we denote by $\langle Q\rangle$ and $[Q]$ the average and jump of the quantity Q across the null hypersurface. Greek and capital latin indices denote spacetime and two-dimensional spatial indices, respectively.

2. Junction conditions

Following Poisson,[3] we employ a pseudo-orthonormal basis with two null vectors N^μ and L^μ, normalized as $L^\mu N_\mu = -1$. Here N^μ is the normal (surface gradient, which is also tangent), while L^μ plays the role of the transverse vector, with respect to which we perform a (2+1)+1 decomposition. Across the hypersurface, the continuity of both the metric tensor $g_{\mu\nu}$ and scalar ϕ are required. Their first derivatives can have jumps $c_{\mu\nu}$ and ζ given by

$$[\partial_\kappa g_{\mu\nu}] = -N_\kappa c_{\mu\nu}, \tag{2}$$

$$[\partial_\mu \phi] = -N_\mu \zeta, \tag{3}$$

hence the second order derivatives of the metric and scalar fields acquire distributional contributions along the thin shell. The distributional equations of motion take the form

$$\mathscr{E}^{\mu\nu} = \frac{1}{2}\mathscr{T}^{\mu\nu} \tag{4}$$

$$\mathscr{E}^{\phi} = 0, \tag{5}$$

where $\mathscr{E}^{\mu\nu}$ and $\mathscr{E}^{\phi}$ are the singular parts of the tensorial and scalar Euler-Lagrange expressions, respectively. The distributional stress-energy-momentum tensor of the shockwave may be decomposed with respect to the basis $\{L, N, e_2, e_3\}$ (where e_2, e_3 are spatial basis vectors orthogonal to both L and N) as

$$\mathscr{T}^{\mu\nu} = \rho N^\mu N^\nu + j^A(N^\mu e^\nu_A + e^\mu_A N^\nu) + p^{AB} e^\mu_A e^\nu_B, \tag{6}$$

with the expressions ρ, j^A, p^{AB} interpreted as surface energy density, surface current and surface stress, respectively. These quantities could entirely be given in terms of internal hypersurface coordinates, hence they are calculatable even if the bulk coordinates are discontinuous across the hypersurface.

The explicit form of these components for the Lagrangian (1) have been derived[21] as

$$\rho = F(\phi)[\mathcal{K}_{AB}]q^{AB} + F'(\phi)[\phi_L] - 2\xi(\phi)[\phi_L^2]\phi_N, \tag{7}$$

$$j^A = -F(\phi)[\mathcal{K}_N{}^A] + 2\xi(\phi)\phi_N\phi^A[\phi_L], \tag{8}$$

$$p^{AB} = pq^{AB}, \tag{9}$$

$$p = F(\phi)[\mathcal{K}_{NN}] - 2\xi(\phi)[\phi_L]\phi_N^2, \tag{10}$$

where ϕ_L, ϕ_N are derivatives along L^μ and N^μ respectively, ϕ_A is the derivative along e^μ_A, $q_{AB} = g(e_A, e_B)$ is the two-dimensional induced metric, q^{AB} its inverse, capital latin indices are raised and lowered with the q metrics, and $\mathcal{K}$ is the "transverse curvature" defined as

$$\mathcal{K}_{ab} = e^\mu_a e^\nu_b \nabla_\mu L_\nu, \tag{11}$$

where $a, b = 1, 2, 3$ and $e^\mu_1 \equiv e^\mu_N = N^\mu$. The jump of the transverse curvature is related to the jump of the metric derivative as $[\mathcal{K}_{ab}] = \frac{1}{2}e^\mu_a e^\nu_b c_{\mu\nu}$ and the jump of the scalar field's transverse derivative is $[\phi_L] = \zeta$. Equations (7-10) represent the 2+1 decomposition of the tensorial equation (4), a generalization of the Lanczos equation.

The scalar equation (5) for the Lagrangian (1) becomes

$$\begin{aligned} 0 = {} & \xi(\phi)\phi_N^2 q^{AB}[\mathcal{K}_{AB}] - 2\xi(\phi)\phi_N\phi^A[\mathcal{K}_{NA}] + F'(\phi)[\mathcal{K}_{NN}] \\ & - 2\xi(\phi)[\phi_L]\phi_{NN} + 2\xi(\phi)\phi_N[\phi_L\mathcal{K}_{NN}] \end{aligned} \tag{12}$$

This equation contains the jumps of various components of the transverse curvature, which in principle are expressible[21] from the generalized Lanczos equations (7-10) in terms of the surface energy density, current, pressure and the jump of the transverse derivative of the scalar field. Hence, the scalar equation (12) could be perceived as a constraint on the distributional sources, mediated by the cubic derivative coupling ξ and the jump of the transverse derivative of the scalar and of its square.

The expression $[\phi_L\mathcal{K}_{NN}]$ can be transformed away[21] by a suitable gauge in the tetrad choice. The normal vector field is autoparallel[3]

$$N^\nu\nabla_\nu N^\mu = \kappa N^\mu \tag{13}$$

with the *non-affinity parameter* $\kappa = \mathcal{K}_{NN}$. If the null fields are rescaled as $\bar{N}^\mu = e^\alpha N^\mu$ and $\bar{L}^\mu = e^{-\alpha}L^\mu$ with some function α defined on the hypersurface, the non-affinity parameter transforms as

$$\bar{\kappa} = e^\alpha\left(N^\nu\nabla_\nu\alpha + \kappa\right). \tag{14}$$

It is possible to set

$$[\phi_{\bar{L}}\bar{\kappa}] = 0 \tag{15}$$

by solving the differential equation

$$\frac{\partial \alpha}{\partial \lambda} = -\langle \kappa \rangle - \frac{\langle \phi_L \rangle}{[\phi_L]}[\kappa], \tag{16}$$

where λ is a coordinate adapted to N^μ. Therefore, with this choice of gauge, the last term of Eq. (12) vanishes.

3. Concluding remarks

We have presented generic tensorial and scalar junction conditions across null hypersurfaces in generalized kinetic gravity braiding theories with linear kinetic term dependence.

The tensorial equation emerges as a generalized Lanczos equation, with the 2+1 decomposition (7-10) based on a pseudo-orthonormal basis in which the null vector is the normal. The jump of the scalar derivative along a second null vector, which is transverse to the hypersurface, also enters the generalized Lanczos equation.

The scalar equation (12) further constrains the energy, current, pressure of the shock-wave and the jump of the transverse derivative of the scalar field. In the absence of the cubic derivative coupling ξ, this simply reduces to the condition of vanishing isotropic pressure of the shock-wave, a property already revealed in the case of Brans-Dicke theory in the Jordan frame.[22]

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