

COSMIC STRINGS IN LABORATORY SUPERFLUIDS AND THE TOPOLOGICAL REMNANTS OF OTHER PHASE TRANSITIONS

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Superfluid phase transition in He^4 can be accomplished through pressure quench on a short (dynamical) timescale. "Vacuum condensate" of the new, broken symmetry phase is the wavefunction of the superfluid Bose condensate with a certain "trapped" distribution of vortex lines. Analogous phase transitions from the false to the true vacuum are expected to occur in the early universe. There they are thought to leave behind topological defects such as strings. I calculate the density of strings (*i.e.*, vortex lines) predicted by the application of the cosmological scenario to He^4 . The proposed experiment tests key elements of the standard cosmological scenario for creation of topological defects — known as the Kibble mechanism — in a cryogenic setting. Analogous experiments can be performed in superconductors as well as in either phase of He^3 , and have been already carried out by Yurke and his co-workers in liquid crystals.

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1. Introduction

In the standard "Big Bang" model of the early universe as well as in its new inflationary implementations, the symmetry of the vacuum is broken as the expansion causes a decrease of temperature. The new vacuum is presumably chosen locally, within a causally connected region. Such local choices usually result in topological defects — *e.g.*, domain walls, strings, and monopoles — which can be either useful or dangerous for a cosmological model. The purpose of this paper is to discuss a laboratory experiment in which an analog of astrophysically useful strings can be generated in a manner similar to the cosmological phase transition. The analogy between quenches in superfluid He^4 and string — generating phase transitions in

the early universe has been pointed out, some time ago, by this author [1]. However, in the original paper most of the attention was devoted to the case of annular geometry, which has the advantage of yielding a “clear”, conceptually simple and surprising prediction (the quench should induce rotation in the superfluid), but which is difficult to realize experimentally. In this paper I shall discuss in some detail another version of the same experiment: quench into a bulk superfluid. I shall argue that if the so-called “Kibble mechanism” for generation of cosmic strings in the early universe is essentially correct, then the rapid phase transition into superfluid He⁴ should result in a copious production of vortex lines, which are the superfluid analogues of strings.

An experiment successfully implementing the “bulk” version of the proposal of Ref. [1] has been by now carried out by Yurke and his co-workers [2], who studied symmetry breaking in a nematic liquid crystal which — as Yurke has realized — offers a somewhat more distant analogue of the cosmological phase transition than superfluids, but is significantly easier to work with. In particular, topological defects in liquid crystals are clearly visible, which alone is a major advantage. This pioneering experiment has been by now repeated by at least one more group [3], and I fully expect that research on “freezing out” of topological defects by rapid quenches will yield valuable insights into the dynamics of non-equilibrium phase transitions.

2. Vortex lines and cosmic strings

To establish an analogy between field-theoretic strings [4] and vortex lines in the superfluid [5] consider a second order phase transition described by the Landau–Ginzburg (L–G) theory with the potential contribution to the free energy density:

$$V = \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4. \quad (1)$$

In the context of cosmological phase transitions V is known as the effective potential [4]. Strings can exist when the field ψ is complex. The order parameter ψ is also complex, $\psi = |\psi| \exp(i\theta)$, in the superfluid He⁴. It will simplify our discussion, but not change its conclusions, to regard the order parameter in He⁴ as equal to the wave function of the Bose condensate. (The degree to which the two are equivalent is still a subject of some debate; see, *e.g.*, Lifshitz and Pitaevskii, Ref. [5]). Near the superfluid phase transition $\alpha(T) = \alpha'(T_\lambda - T)$, and α' , β are phenomenological constants.

It should be emphasized that the general scheme described above applies not just in the context of superfluid phase transitions, but is valid whenever Landau–Ginzburg theory is a reasonable approximation, that is,

in essence, to all second-order phase transitions. This should not be too surprising: If the universality of Eq. (1) suffices to justify the analogy between cosmological phase transitions and laboratory superfluids, it should also have no trouble accomodating other situations in which a non-conserved order parameter, second order phase transition, and a non-trivial symmetry breaking scheme are involved. I shall, however, continue to write specifically about superfluids and leave it to the reader to “analytically continue” these considerations to other interesting second-order phase transitions.

The superfluid wave function obeys the Schroedinger equation [5]:

$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2m}\nabla^2\psi + \mu\psi, \quad (2)$$

where m is the mass of the He^4 atom, and the chemical potential μ is the derivative of V with respect to the number density $|\psi|^2$. In L-G theory this leads to the Gross-Pitaevskii (G-P) equation:

$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2m}\nabla^2\psi - (\alpha - \beta|\psi|^2)\psi. \quad (3)$$

A relativistic equation with identical right-hand side describes the field ψ used to discuss formation of topological defects in the early universe [1-6].

Equation (3) can be simplified by introducing a *correlation length*; $\xi = \hbar/(2m\alpha)^{1/2}$, a *correlation time*; $\tau = \hbar/\alpha$, and an equilibrium value of ψ ; $\psi_E = (\alpha/\beta)^{1/2}$. In terms of the new variables $\rho = r/\xi$, $\eta = \psi/\psi_E$, the time-independent G-P equation reads:

$$\nabla^2\eta = (|\eta|^2 - 1)\eta. \quad (4)$$

Apart from the trivial case $|\eta|^2 = 1$ it has axially symmetric solutions of the form $\eta = \Psi(\rho)\exp(in\phi)$, where (ρ, ϕ, z) are cylindrical coordinates. Here n , the *winding number*, must be an integer; otherwise, η could not be singlevalued. The radial part of the physically relevant solution is regular near the origin ($\Psi \sim \rho^n, \rho \ll 1$) and approaches the equilibrium density at large distances ($\Psi^2 \cong 1 - n^2/\rho^2, \rho \gg 1$). The phase of the complete solution is $\theta = n\phi$ on any $\rho = \text{const} > 0$ circle, but remains undefined along the singular $\rho = 0$ axis [5].

The gradient of the phase gives the local superfluid velocity [5]:

$$\vec{v}_s = \frac{\hbar}{m}\vec{\nabla}\theta(\rho, \phi, z). \quad (5)$$

Therefore, the axially symmetric solution of Eq. (4) is a vortex with core of width ξ . The superfluid circulates around it with the velocity

$$v_\phi = n\frac{\hbar}{m}r^{-1}. \quad (6)$$

Inside the core the symmetric vacuum — normal fluid — makes up for the density deficit caused by the decrease of $|\psi|^2$. For energetic reasons vortex lines with $n = 1$ are preferred [5].

The analog of a vortex line in field theories relevant in the cosmological context is a string. It emerges as a solution of an equation which is essentially identical to Eq. (4) (see, *e.g.*, Nielsen and Olesen, Ref. [4]). Strings can exist when the first homotopy group $\Pi_1(G/H)$ — where G and H are the symmetry groups before and after the phase transition — is nontrivial [4, 6]. For superfluid HeII this is indeed the case, as $G/H = U(1)$ and $\Pi_1(G/H) = \mathbb{Z}$. A vortex line is a *global* string — *i.e.*, a string associated with the breaking of a global gauge symmetry. The alternative, *local* strings are like the flux lines in superconductor [4–6]. Local strings have a finite energy density per unit length, while global strings have energy density which diverges logarithmically with the upper cutoff (*e.g.*, with the string separation).

3. Kibble scenario in superfluid helium

The standard scenario for cosmological formation of both local and global strings asserts that while before the phase transition the field is, on the average, in the symmetric state, locally, within ξ -sized regions, the order parameter fluctuates and assumes values of the order of the broken symmetry ground state expectation value ψ_E [4, 6]. This initial, fluctuating state can be “frozen out” by a rapid phase transition [1]. In the new phase ψ will readjust dynamically so that the free energy is minimized, subject to the topological constraint imposed by the initial state: Inside each closed contour C along which θ changes by 2π there must be (at least one) string. It is worth stressing that the cosmological setting has only a very limited function in this scenario: It induces a rapid (nonequilibrium) phase transition — the expansion of the Universe causes a drop of temperature and can provide some effective viscosity — but other than that it plays no role whatsoever [1–4, 6].

The key element of the above model is the emergence of the characteristic scale of frozen out fluctuations. This should take place equally well in a rapid phase transition from the normal He⁴ to superfluid. Such a phase transition can be induced by a pressure quench. The phase diagram of He⁴ is shown in Fig. 1. The proposed quench trajectory is also indicated. One of the key advantages of this experiment over other possible condensed matter phase transitions is the possibility of inducing it on a short, dynamical sound-crossing timescale. Moreover, the velocity of the second sound c_2 , which determines the speed with which the phase readjusts, is much smaller than the velocity of the ordinary first sound c_1 , which limits the speed of the

quench [5]. In this sense, one can induce the phase transition on a timescale which is “supersonic” in terms of the speed with which perturbations of ψ can propagate.

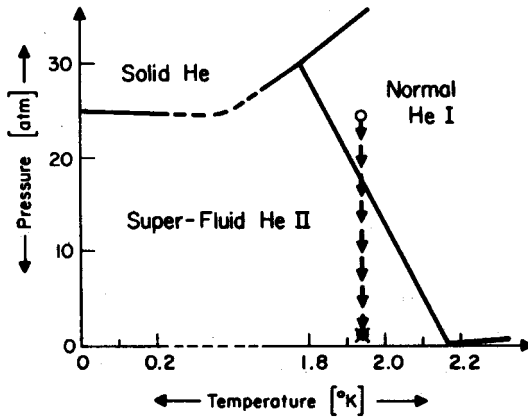


Fig. 1. The phase diagram of He^4 and the trajectory of the quench in the proposed experiment.

To estimate the expected density of vortex lines in HeII one must calculate the characteristic correlation length d which is “frozen out” in course of the nonequilibrium phase transition. Let us first note that an *instantaneous* phase transition would freeze out *preexisting* fluctuations with the initial correlation length. What happens if a phase transition occurs on a finite timescale? To calculate d I assume that in the course of the quench the pressure is lowered uniformly throughout the volume, and that the dimensionless relative temperature

$$\epsilon(T) = \frac{T - T_\lambda}{T_\lambda} \quad (7)$$

is proportional to time; $\epsilon(t) = t/\tau_Q$. If the system could equilibrate (in our case, for T sufficiently close to T_λ it cannot) the correlation length would reach [5, 7]

$$\xi = \xi_0 |\epsilon|^{-\nu}. \quad (8)$$

For L-G theory in He^4 $\xi_0 \cong 5.6[\text{\AA}]$ and $\nu = 0.5$ [5]. Measurements yield $\xi_0 \cong 4[\text{\AA}]$ and $\nu \cong 0.67$ [7], in better accord with the renormalization group (R-G) approach.

Actual correlation length of the order parameter is given by Eq. (8) only as long as the quench-induced evolution of the system is slow in comparison with the time-scale τ on which fluctuations of ψ can readjust. Consider

a quench slow enough so that this assumption is indeed correct near the initial high-pressure point. The condition for such a quasi-equilibrium is to allow fluctuations to interact. This implies that the size of the "sonic horizon" $h(\Delta t)$, the distance to which influence of a perturbation can spread within some time interval Δt , ought to be large compared with equilibrium correlation size ξ , if Δt is the time interval over which ξ changes appreciably. A natural estimate of Δt is then:

$$\Delta t = \left| \frac{\xi}{\dot{\xi}} \right| = |\dot{t}|. \quad (9)$$

The additional piece of information needed to calculate the radius of the sonic horizon is the characteristic velocity u , which determines the speed with which perturbations of ψ can spread. In the superfluid the speed of the second sound c_2 is the only choice for u . Above T_λ the rate with which ψ can change is presumably of a similar order. Therefore, even if the system was initially in quasi-equilibrium, $\xi \ll h(t)$, as the pressure is lowered and as ϵ approaches zero, equilibrium correlation length will become equal to the radius of the sonic horizon at some instant \hat{t} :

$$\xi(\hat{t}) = h(\hat{t}). \quad (9a)$$

This equality will be, at least formally, satisfied twice: first when $T > T_\lambda$ and $\hat{t}_- < 0$, and subsequently for $T < T_\lambda$, with $\hat{t}_+ > 0$. Inside the time interval $[\hat{t}_-, \hat{t}_+]$ evolution of ψ is slow compared to the available amount of time, as $\tau > \hat{t}_+ - \hat{t}_-$. Therefore, the correlation length corresponding to the time instant \hat{t} is a reasonable estimate of the frozen-out scale of fluctuations:

$$d = \xi(\hat{t}). \quad (9b)$$

If the values of ξ corresponding to \hat{t}_+ and \hat{t}_- were not equal (below we shall assume that they are), one would have to use the larger one. Eq. (9a) can be now regarded as an approximate formula for \hat{t} , which, in turn, can be used to obtain d .

Using either the L-G theory ($u_{LG} = u_0 \epsilon^{0.5}$, $u_0 = 70$ [m/s]) or the more accurate R-G approach consistent with the experimental data¹ yields;

$$\hat{t} = \sqrt{\frac{\xi_0}{v_0} \tau_Q} = \sqrt{\tau_0 \tau_Q}. \quad (10)$$

¹ The exponent in Eq. (10) will be actually somewhat different from 0.5 if one were to use experimental fits rather than either L-G or R-G values. This difference will be of little consequence in the estimates given here.

The characteristic cell size is now given by Eq. (9b):

$$d_{LG}[\text{\AA}] \sim 5.6 \left(\frac{\tau_Q}{\tau_0} \right)^{1/4}, \quad (11a)$$

$$d_{RG}[\text{\AA}] \sim 4 \left(\frac{\tau_Q}{\tau_0} \right)^{1/3}. \quad (11b)$$

The characteristic timescale ($\tau_0 = \xi_0/v_0 \cong 0.85 \times 10^{-11}$ [s]) is almost identical in either the L-G or R-G case. Values of the quench timescale τ_Q can vary between very short 10^{-6} [s] (\sim first sound crossing time for a fraction of a millimeter thick capillary) and very long (10^6 [s] or even more). These extreme values correspond to $d_{LG} \sim 100[\text{\AA}]$ ($d_{RG} \sim 200[\text{\AA}]$) and $d_{LG} \sim 10^{-3}$ [cm] ($d_{RG} \sim 2 \times 10^{-2}$ [cm]), respectively. The domain size can be now employed to estimate the vortex line density.

Before we proceed any further, it is important to emphasize that our calculation — in spite of the use of equilibrium concepts such as correlation length — assesses the impact of the *nonequilibrium* nature of the phase transition induced by the pressure quench. In a sense, we have split the quench-induced evolution of the system into three epochs: (1) Far enough above T_λ , where intrinsic timescales and sizes of the fluctuations are sufficiently small so that the evolution of the state of the normal fluid can “keep up” with the change of the global thermodynamic properties, such as pressure and/or temperature; (2) epoch when T is so near T_λ that the system is too sluggish (as a consequence of critical slowing down) to “keep up”, and (3) region sufficiently below T_λ to have a well-defined Bose condensate wave function. The equilibrium correlation length — we have assumed — is a valid estimate of the actual spatial correlations in epochs (1) and (3), but *not* in epoch (2). Size d of the frozen-out domains is determined by the correlation length on the border between the near-equilibrium (1) and (3), and the full-fledged nonequilibrium, (2). This reasoning does not depend on the details of the model one uses to describe the system in the vicinity of the phase transition: Qualitative predictions will be the same for L-G or R-G theory. It is also useful to emphasize the analogy between the sonic horizon in He⁴ and the causal horizon in the early universe: they play the same role in limiting the size of dynamically correlated regions.

We are now ready to estimate the density of vortex lines induced by the quench. The phase of the Bose condensate varies significantly between domains with a volume $\sim d^3$. This leads to a typical specific vorticity ℓ defined as the length of the strings per unit volume [1–4, 6],

$$\ell = \frac{\kappa d}{d^3} = \frac{\kappa}{d^2}. \quad (12)$$

Using the values of d obtained before and taking the proportionality coefficient $\kappa \sim 1.0$, one obtains $\ell \sim 10^{12} - 10^3 [\text{cm}^{-2}]$.

It is useful to note that the above estimate depends on the assumption that one begins and ends the quench in a nearequilibrium region with $\xi \ll d$. It is possible to imagine experiments starting or terminating the quench sufficiently close to T_λ that ξ is comparable with or larger than d corresponding to the rate at which the quench occurs. In particular, if one starts with $T > T_\lambda$ and $d > \xi_i$, one can expect that the initial correlation length will be frozen out. Thus, density of vortex lines should be estimated using ξ_i rather than d .

Similarly, if the quench terminates at $T < T_\lambda$, but with $\xi_f > d$, separate vortices will not be able to form. Again, the relevant scale for the evaluation of ℓ would be ξ_f rather than d .

4. Evolution of the vortex line network and other complications

It is reasonable to assume — and has been borne out in computer simulations [8] — that the string network formed in a process described above is much like the network of random walks on a lattice with spacing $\sim d$. In particular, the typical size R of the string — the straight line distance between its two points — increases with the actual length of the string L as $R \sim \sqrt{Ld}$. Moreover, in 3 dimensions $\sim 20 - 30\%$ of random walks return to their origin [9], and by the same token, a similar percentage of strings form closed loops. Consequently, in a tangled Brownian network of vortex lines cancellation of vortices with opposite senses of rotation, straightening of entangled vortex lines due to the line tension, and shrinking of the loops may all decrease vorticity even before its measurement (e.g., by means of second sound attenuation) is accomplished. Therefore, vorticity detected in such experiments may be orders of magnitude below the estimate obtained from Eqs (11) and (12). Here let us only note that the specific vorticity induced by turbulent fluid motions of HeII obeys the evolution equation [10] $\dot{\ell} \sim -\ell^2$. The specific vorticity of the Brownian network is likely to obey a similar evolution equation. Therefore, one should be able to find evidence for vortex production even some time after the quench. Indeed, this was precisely the strategy adapted in Ref. [2], where the authors have investigated relaxation of the tangled network of topological defects and proved that — as expected — it obeys the scaling solution.

An even more interesting complication may arise in the course of a rapid quench as a result of the large specific vorticity implied by Eq. (12). Vortex line cores contain only normal He⁴ and no superfluid. Therefore, the order parameter ψ must vanish on them. If the lines were stationary, one could regard the Brownian network as a “porous substance” with the

characteristic length scale d and a rather unusual topology, imposing an interesting “distributed” boundary condition $\psi = 0$ along the lines. We shall assume that this heuristic picture is qualitatively correct. The superfluid phase transition occurs in a porous geometry with characteristic length scale d at a depressed temperature T_d , with the relative temperature of the phase transition ϵ shifted from $\epsilon = 1$ by:

$$\Delta_\epsilon = \frac{T_\lambda - T_d}{T_\lambda} = \left(\frac{\xi_0}{d} \right)^{1.5}. \quad (13)$$

Employing our above, heuristic approach and using estimates of $d \sim 10^3 \text{ \AA}$ corresponding to relatively rapid quenches one arrives at $\Delta_\epsilon = 10^{-3}$. It should be emphasized that the actual depression of ϵ may be larger than this estimate, as the values of d obtained from Eq. (11) represent an “upper bound” on the frozen-out size of the correlation length. The net effect of this phenomenon would be then to depress somewhat the temperature corresponding to the immediate transition to superfluidity — or, alternatively — to delay the onset of the superfluid phase transition.

A large specific vorticity ℓ corresponds to a significant rotation on the microscopic level in the superfluid. Moreover, the phase transition from normal to the superfluid He^4 is of the first order in a rotating vessel [11]. Therefore, one may also look for some signatures of the first order transitions — such as hysteresis — in the quench. Quantitative estimates of this effect are rather difficult in the contemplated experiments as the first order phase transition was investigated so far only in case of uniform, large scale rotation. However, a time lag in the onset of superfluidity may be a hint that the transition is of the first order.

5. Discussion

One way of looking at the spontaneous generation of vorticity in a superfluid with complex order parameter ψ is to consider it in terms of a spatial distribution of the winding number deposited by thermal fluctuations in a system undergoing a rapid second order phase transition. This analogy — useful already in the present discussion of He^4 — can be further extended to superconductors [12], where a rapid phase transition should “freeze out” random number of magnetic flux quanta, as well as to either phase of the superfluid He^3 , where the order parameter is far more complicated and the resulting topological defects far more complex [12, 13].

An equally valid and even more intriguing point of view of the rapid phase transition recognizes that the order parameter ψ is a wave function. Therefore, one can imagine a situation where, following the quench,

many competing wavefunctions with significantly different distributions of the winding number could emerge from the given initial pre-quench state. It would be therefore difficult to rule out the possibility that at least for a short instant following the quench the wavefunction/order parameter will be in a superposition of several winding number configurations.

Superfluid phase transition in an annular geometry [1] offers the simplest case of such "macroscopic quantum superposition". There one can, for instance, consider ψ given by:

$$\psi = a_1\psi_{n_1} + a_2\psi_{n_2}, \quad (14)$$

where ψ_{n_1} and ψ_{n_2} have distinct winding numbers, n_1 and n_2 . The order parameter with the final, unique winding number is chosen by the combination of environmental effects [15, 16] and microscopic interactions of He^4 atoms. Behavior of ψ may, or may not, be approximately described by the Gross-Pitaevskii type of equation with an appropriate damping term. For lack of space, we shall not attempt to explore these issues here. It is, however, important to point out that a similar problem — emergence of a definite state of the vacuum from a pre-transition mixture — is also encountered in the discussion of cosmological phase transitions. One may, therefore, hope that the results obtained in laboratory superfluid condensates may shed a new and much needed light on cosmological phase transitions into vacua with broken symmetries.

It is worth stressing that regardless of the relevance of the experiments proposed above for the cosmological scenarios [1-4, 6], the issue of the vorticities and superfluid flows induced by a rapid phase transition is of interest in itself. Some evidence for vortex line creation in superfluid phase transition may be already at hand².

The basic tenets of the scenario outlined above have been already confirmed by experiments in nematic liquid crystals [2, 3]. Superconductors (with local gauge symmetry) and superfluid He^3 (with tensor order parameter) offer somewhat similar, but different in many essential aspects, testing grounds for our scenarios of cosmological phase transitions.

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² D.D. Awschalom, K.W. Schwarz, *Phys. Rev. Lett.* 49 (1984) note that after a passage through T_λ superfluid contained vortex line density much in excess of the final remnant vorticity.

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