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Parity Doubling in Dense Baryonic Matter as an Emergent Phenomenon and Pseudo-Conformal Phase

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Abstract: The star matter composed of nucleons deep inside compact stars, such as neutron stars, is believed to be very dense, such that various types of new concepts and physical phenomena are naturally expected due to the nontrivial strong correlations between hadrons. The possibility of revealing the hidden scale symmetry in dense baryonic matter has been discussed recently, to uncover the pseudo-conformal phase in dense star matter. In the pseudo-conformal phase, the trace of the energy–momentum tensor becomes density-independent, and the speed of sound approaches the conformal velocity in scale symmetric matter. Interestingly, it is also observed that the effective nucleon mass becomes a density-independent finite quantity, which can be identified as the chiral invariant mass of the parity doublet model, indicating that the parity doubling is an emergent phenomenon. In this paper, we will discuss how parity-doubling symmetry emerges inside the core of a compact star as a consequence of the interplays between ω vector mesons and nucleons (or dilaton, χ, equivalently) and between the chiral symmetry and the scale symmetry.

Keywords: scale invariance; chiral symmetry; parity doubling; pseudo-conformal phase; vector meson-dilaton interplay; compact star matter; speed of sound

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1. Introduction

Scale invariance would be exact if all elementary particle masses (more generally, all dimensionful couplings) vanished [1]. In the real world, scale transformation is not a symmetric transformation. But in situations where the effects of the masses are not important, such as extremely high-energy phenomena, the role of scale symmetry, when properly formulated, can be studied systematically, even together with small symmetry-breaking effects.

Nuclear matter at a much higher density than normal nuclear density, n_0, has been of great interest, because recent observations [2,3] imply the possibility of a high-density regime $n \gtrsim 6n_0$ at the core of neutron stars. These density regimes have not been fully explored theoretically or experimentally. Kinematically, the Fermi momentum of nuclear matter at a density of $6n_0$ is $k_F \sim 0.65 m_N$. Although it is not high enough for the nucleon mass to be ignored, nontrivial strong correlations of nuclear matter invoke the quest for scale symmetry in such a high-density regime. If the effect of strong correlations should be such that perturbative QCD works in terms of quark degrees of freedom, then the scale symmetry would be apparent modulo current quark masses, but there is some doubt about whether the corresponding density is sufficiently reached at the core of the neutron star for the explicit appearance of quarks and gluons.

The relevant degrees of freedom of dense star matter deep inside compact stars are supposed to be hadrons (nucleons and mesons) appearing in the appropriate effective theories. Let us suppose that, at a higher density, the strong correlations of nucleons lead the system to reveal the hidden underlying scale symmetry. There would be excitations in a scalar channel and dilaton fields, such that scale invariance would be realized formally in an effective Lagrangian with dilatons. One of the interesting results from effective field theory with the scale symmetry implemented by the dilaton field [4,5] is that the trace of the energy–momentum tensor (TEMT) becomes density-independent in a higher–density regime relevant to star matter. In the mean field approach, the vacuum expectation value of the dilaton is density-independent, which leads to the density–independent TEMT and the density-independent effective nucleon mass. In this formulation, the nucleon mass is basically due to the spontaneous symmetry breakings of the chiral and scale symmetry. However, it turns out that the nucleon mass at a higher density is supposed to be mainly from the vacuum expectation value of the dilaton developed for the spontaneously broken phase of the scale symmetry. These observations lead to two interesting possibilities: pseudo-conformality and parity doubling in dense baryonic matter.

Although the non-zero value of TEMT seems to indicate the apparent violation of the scale-invariance of the system of nuclear matter, the nature of the underlying scale symmetry, which is partly supported by recent discussions on the infrared conformal window of QCD [6–11], can emerge in a quite different way, i.e., pseudo-conformality [12,13] in dense star matter in the spontaneously broken phase. One of the useful dynamical properties of the system is the speed of sound at which local fluctuations of star matter will be traveling, depending on the equation of state. The speed of sound can provide information from which we can infer the state of matter. The ratio of the pressure, representing the restoring force of star matter, to the energy density, the inertia of the star matter, determines the speed of sound, v_s,

$$v_s^2 = c^2 \frac{dP}{d\epsilon} \tag{1}$$

where c is the speed of light. P and ϵ are the pressure and energy density, including the particle rest mass, respectively.

It is observed [5] that TEMT is found to be approaching a constant which, interestingly, leads the speed of sound to be that of scale symmetric matter, with a conformal velocity of $v_c^2 = 1/3c^2$. This is basically due to the density-independent vacuum expectation value of the dilaton. It is interpreted as an indication that the hidden scale symmetry is emerging in high-density star matter, disguised in the form of the conformal speed of sound. It is referred to as the pseudo-conformal phase [12].

Another interesting observation in the pseudo-conformal phase is that the nucleon has a nonvanishing mass, which seems to be unconnected to the spontaneous breaking of the chiral symmetry. This is due to the dilaton condensation. The presence of the finite nucleon mass in the nucleon sector seems to be inconsistent with the chiral invariance of the system, which is believed to be the symmetry (spontaneously broken or restored) modulo current quark masses of the QCD. These observations indicate that the proper way to describe the chiral symmetric nucleonic sector is to implement the parity doubling structure [14–17]. For a parity doublet, the negative parity nucleon (N_-) is needed in the matter, together with the positive parity nucleon (N_+; proton and neutron). Now, it is possible because there can be various particle excitations when nucleons get closer with strong correlations. The effect of excitations would be such that hidden symmetries of the system can be manifested. It turns out that the density-independent nucleon mass corresponds to the invariant mass m_0 of the parity doublet model. The star matter at a high density can provide us with the possibility of uncovering the parity-doubled structure of the dense nucleonic matter, which has not been transparent enough to be observed or to be necessarily implemented in describing hadronic matter in the low-density regime. It is a sort of interplay between the chiral symmetry and scale symmetry that has not yet been rigorously formulated. In this work, by systemically reassessing the previous results of the pseudo-conformal phase and the mean field calculations, we find that the interplay can be formulated properly in the scale-invariant parity doublet model with two dilatons (soft dilaton χ_s and hard dilaton χ).

It is to be noted that the discussions are based on the effective Lagrangian, which is constructed with chiral and scale symmetry, adopted for the compact star matter in which both symmetries are spontaneously broken. Most of the detailed calculations and conventions are from the works by Paeng et al. [4,5,18] and later developments [19–22].

The trace of the energy–momentum tensor becomes density-independent at higher density. Its implications on the speed of sound and on the emergence of pseudo-conformality will be discussed in Section 2. In Section 3, the parity doubling is discussed as an emergent phenomenon in the pseudo-conformal phase of the strongly correlated compact star matter in mean field approach. The role of the chiral symmetry and the construction of the parity doublet model with two dilatons supported by RG analysis are discussed in detail. A summary and remarks on the possible implication on the correspondence of hadrons to quark degrees of freedom are given in Section 4.

2. Dense Star Matter and Pseudo-Conformality at Higher Density

In [5], the effective field theory, which would be relevant in the highly dense regime, has been developed. In addition to the nucleon and the pseudo-scalar meson, pion π, appearing in the standard chiral perturbation theory, a dilaton field χ is introduced as a conformal compensator field to implement the scale invariance in the Lagrangian for massive matter fields. Vector mesons, ρ and ω, are introduced in the frame work of the hidden local symmetry [23–25]. The formulation of effective field theory, referred to as dBHLS Lagrangian (with d standing for dilaton χ, B for baryon, and HLS for hidden local symmetry), and the numerical results of the physical quantities (for example, symmetry energy, effective nucleon mass, equation of state, speed of sound, and neutron star properties (mass and tidal deformability)) are discussed in [5].

From the analysis of the effective theory using the full "V_{lowk} -RG" treatment in [5], it is observed that TEMT becomes an density-independent finite quantity. The results are reproduced here in Figure 1 (Figure 4 in [5]). The composition of proton number density (n_p) and neutron number density (n_n) is denoted by $\alpha(=(n_n - n_p)/n)$, where $n(= n_n + n_p)$ is the total nucleon number density. Two cases for symmetric nuclear matter $\alpha = 0$ and pure neutron matter $\alpha = 1$ are shown among the available compositions inside a neutron star. In fact, the realistic neutron star has a composition between those special cases, $0 < \alpha < 1$. The density-independence of TEMT at a higher density is expected to be valid for the realistic neutron star matter.

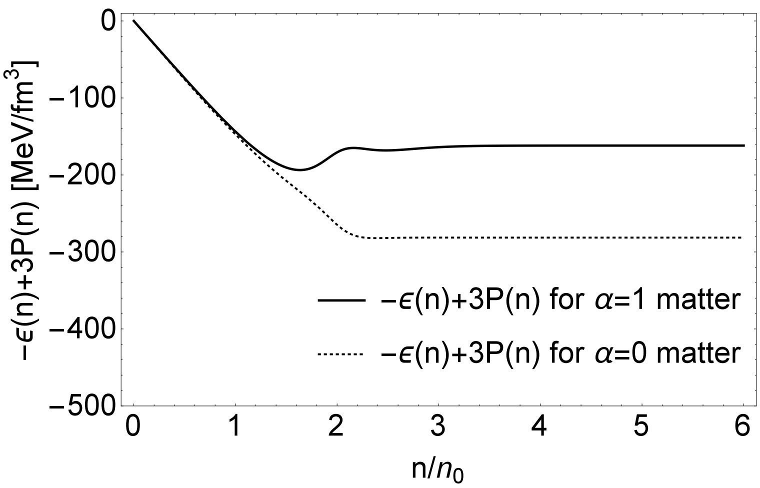

Figure 1. $-\epsilon(n) + 3P(n) = -$TEMT vs. density for $\alpha = 0$ (symmetric nuclear matter) and $\alpha = 1$ (pure neutron matter).

It is interesting to see that, for the higher density, $n > n_A$ (here, $n_A \sim 2n_0$; the exact numerics of n_A is dependent specifically on the model considered), the numerical results of energy density, ϵ, and pressure, P, can be captured very well by the simple formulae

$$\epsilon = Bn^{4/3} + D, \quad (2)$$

$$P = \frac{1}{3}Bn^{4/3} - D, \quad (3)$$

where B and D are density-independent parameters determined numerically [5]. Then, the trace of the energy–momentum tensor is simply given by

$$\theta^\mu_\mu = \epsilon - 3P = 4D, \tag{4}$$

which clearly shows the density-independence of TEMT. As an example, for symmetric matter, $D = 70.4\,\mathrm{MeV/fm^3}$.

Among the useful dynamical properties of the system, we consider the speed of sound inside compact star matter. The general expression for the speed of sound, v_s, is

$$v_s^2 = c^2 \frac{dP}{d\epsilon} = c^2 \frac{dP}{dn} / \frac{d\epsilon}{dn}, \tag{5}$$

where c is the speed of light. $\frac{dP}{dn}$, representing the compressibility of the fermion system, determines the restoring force, while the energy density, ϵ, including the particle's rest mass, determines the inertia. This is the speed at which small local fluctuations of a star matter are traveling. It depends on the equation of state and, therefore, on the underlying hidden scale symmetry we are exploring. In a high density regime in Figure 1, the variation in the trace of the energy–momentum tensor in Equation (4) with respect to density is zero,

$$\frac{\partial}{\partial n}\theta^\mu_\mu \quad = \quad 0. \tag{6}$$

Then, we obtain

$$\frac{\partial \epsilon}{\partial n}\left(1 - 3\frac{v^2}{c^2}\right) = 0, \tag{7}$$

and the speed of sound becomes the conformal velocity v_c,

$$\frac{v_c^2}{c^2} = \frac{1}{3}, \tag{8}$$

provided that there is no extremum in the energy density, $\frac{\partial \epsilon}{\partial n} \neq 0$.

It is an old idea [1] that, at high energies, masses of particles become unimportant, and the scale invariance of the system can be inferred, while explicit scale symmetry breaking terms can be treated perturbatively in this kinetic regime, namely a conformal window. Alternatively, it is also interesting to see whether the high density of nuclear matter can be a possible window for investigating the effect of the scale symmetry. The divergence of dilatation current, s^μ, induced by scale transformations, is given by the trace of the energy–momentum tensor, θ^μ_μ,

$$\partial_\mu s^\mu = \theta^\mu_\mu. \tag{9}$$

For the system with conformal invariance, it becomes zero,

$$\theta^\mu_\mu = 0. \tag{10}$$

Then, the speed of sound in a conformally invariant system is $1/\sqrt{3}$, and the conformal velocity is v_c. As an example for the conformal window, let us consider a noninteracting degenerate fermion matter at zero temperature. The energy density and the pressure are given by

$$\epsilon_{free} \quad = \quad \frac{1}{4\pi^2}\left[2E_F^3 k_F - m_N^{*2} E_F k_F - m_N^{*4} \ln\left(\frac{E_F + k_F}{m_N^*}\right)\right], \tag{11}$$

$$P_{free} \quad = \quad \frac{1}{4\pi^2}\left[\frac{2}{3} E_F k_F^3 - m_N^{*2} E_F k_F + m_N^{*4} \ln\left(\frac{E_F + k_F}{m_N^*}\right)\right]. \tag{12}$$

At a high density, where the Fermi momentum, k_F, is much larger than the mass,

$$k_F \gg m, \tag{13}$$

the energy density and the pressure becomes in the leading order,

$$\epsilon \rightarrow \frac{1}{2\pi^2}k_F^4, \tag{14}$$

$$P \rightarrow \frac{1}{6\pi^2}k_F^4 = \frac{1}{3}\epsilon. \tag{15}$$

Therefore, one can see that the high density regime is a possible conformal window, where $\theta_\mu^\mu = \epsilon - 3P \rightarrow 0$, and the speed of sound approaches conformal velocity v_c. However, one can see that, even in the high energy limit, the trace of the energy–momentum tensor does not vanish exactly. If we keep the terms beyond leading order, in the limit $k_F \gg m$,

$$\theta_\mu^\mu \rightarrow \frac{m^2}{\pi^2}k_F^2, \tag{16}$$

which is nonvanishing and of order $\mathcal{O}(m^2 k_F^2)$. It is interesting to note that the speed of sound can have a conformal velocity limit, even though the trace of energy–momentum tensor does not vanish exactly. This tells us that the effect of the finite mass would not affect the speed of sound reaching the conformal velocity in the conformal window ($k_F \gg m$, in this case). It suggests that a useful quantity to properly discuss the conformal limit at a high density, as far as conformal velocity is concerned, is not the energy–momentum tensor itself, but might be the ratio of the trace of the energy–momentum tensor to the energy density, Δ, as proposed in [26],

$$\Delta \equiv \frac{\theta_\mu^\mu}{3\epsilon}, \tag{17}$$

which has a proper limit in the conformal window, $k_F \gg m_N$,

$$\Delta \rightarrow \mathcal{O}(\frac{m^2}{k_F^2}) \rightarrow 0. \tag{18}$$

The speed of sound approaches v_c in the conformal limit, $\Delta \rightarrow 0$.

For the compact-star matter, the core densities are supposed to be 5–10 n_0, depending on the models. The Fermi momentum for $n_{core} = 6n_0$, as an example, is given by

$$k_F = \left[\frac{3\pi^2}{2}n_{core}\right]^{1/3} \sim 0.65 m_N, \tag{19}$$

which does not seem to be high enough to be a conformal window, $k_F \gg m_N$ or $\Delta \rightarrow 0$.

However, since it is a criterion relevant to the non-interacting massive fermions, the same criterion cannot be simply imposed on the strongly interacting fermionic matter at the core of a compact star. A conformal window may appear in a different context. Alternatively, we can take the speed of sound as a physical quantity that can quantify the conformal window as $v_s \rightarrow v_c$, rather than $\theta_\mu^\mu \rightarrow 0$ or $\Delta \rightarrow 0$. In fact, it is observed that the trace of the energy–momentum tensor appears to be constant, as shown in Equation (4). This is a result of many body effects in nuclear matter, which happens curiously at moderately high density, $n > n_A$. Interestingly, one can notice that the density-independent feature of TEMT for the dense hadronic matter reveals the same conformal velocity in Equation (8), and it can be taken as a signature for the conformal window, even though $\Delta \neq 0$. It is dubbed as the pseudo-conformality of the dense hadronic matter. It is the many body effect, specifically the interplay between the omega meson excitation and nucleons in the dense medium, that reveals the conformal velocity for $n > n_A$ [27–29]. This is considered one of the characteristics of the underlying scale symmetry in dense matter. This feature accounts for the emergent pseudo-conformal symmetry in the compact-star matter, and suggests that the core of the compact star provides a new window for investigating the effect of the scale symmetry (spontaneously broken) hidden in the dense hadronic matter. The numerical results of the physical quantities (for example, symmetry energy and neutron star properties (mass and tidal deformability)) are consistent with the present terrestrial laboratory experiments and astrophysical observations related to neutron stars, as discussed in [5]. In this sense, the pseudo-conformal phase in its core can be considered one of the viable models for the neutron star matter, which include traditional mean-field approaches with conventional hadronic degrees of freedom.

3. Parity Doubling in Mean Field Approach

In this section, the results obtained by using the renormalization group (RG) treatment with "V_{lowk}–RG" described in the previous section will be discussed in the mean field approach, in order to further determine the interplay between the vector meson, dilation, and parity doubling as an emergent symmetry. To make the discussion simple, we consider the symmetric nuclear matter $\alpha = 0$. We suppose that, in the mean field calculation, the most relevant fields at highly dense nuclear matter are the vector meson ω, the nucleon N, and the dilaton χ as a conformal compensator field. We take the simple Lagrangian, a truncated form of the Lagrangian developed in [5] and referred to as Landau–Fermi-liquid fixed-point approach in the reviews [20,21] for a symmetric matter, $<\rho>=0$ and $<\pi>=0$,

$$\mathcal{L}' = \mathcal{L}'_M + \mathcal{L}'_N, \tag{20}$$

where

$$\begin{aligned}\mathcal{L}'_M &= -\frac{1}{2}\mathrm{tr}\left[\omega_{\mu\nu}\omega^{\mu\nu}\right] + \frac{F_{\sigma\omega}^2}{2F_\chi^2}\chi^2\left(\frac{\partial_\mu\sigma_\omega}{F_{\sigma\omega}} - g_\omega^*\omega_\mu\right)^2 \\ &\quad + \frac{1}{2}\partial_\mu\chi\cdot\partial^\mu\chi - V(\chi),\end{aligned} \tag{21}$$

and

$$\mathcal{L}'_N = \bar{N}i\left(\partial_\mu - ig_\omega^*\frac{\omega_\mu}{2}\right)N - \bar{N}m_N\frac{\chi}{F_\chi}N + g_{V\omega}^*\bar{N}\gamma^\mu\left(\frac{\partial_\mu\sigma_\omega}{2F_{\sigma\omega}} - g_\omega^*\frac{\omega_\mu}{2}\right)N, \tag{22}$$

where σ_ω is a would-be Nambu–Goldstone boson of hidden local symmetry that will be Higgsed away, and $f_{\sigma\omega}$ is a corresponding decay constant [4]. Now, the mean field calculation is performed in the background of degenerated nucleons at $T = 0$. The nucleon number density n is given by

$$n = \frac{2}{3\pi^2}k_F^3, \tag{23}$$

where k_F is a Fermi momentum. The relevant mean field variables are $\chi^*(=<\chi>)$ and $\omega^*(=<\omega_0>)$. Their density dependencies are denoted by * and

$$\chi^*|_{n=0} = F_\chi. \tag{24}$$

The coupling constants, $g_{V\omega}^*$ and g_ω^*, are also density-dependent quantities. The nucleon mass dressed with the compensator field is now density-dependent linearly in χ^*,

$$m_N^* = m_N\frac{\chi^*}{F_\chi}, \tag{25}$$

where m_N is the nucleon mass in free space. From Equation (20) the thermodynamic potential Ω can be obtained as given by

$$\begin{aligned}\Omega/V &= \frac{1}{4\pi^2}\left[2E_F^3 k_F - m_N^{*2} E_F k_F - m_N^{*4}\ln\left(\frac{E_F+k_F}{m_N^*}\right)\right] \\ &\quad +(g_\omega^*(g_{V\omega}^*-1)\omega_0^* - \mu)n - \frac{1}{2}F_{\sigma\omega}^2 g_\omega^{*2}\frac{\chi^{*2}}{F_\chi^2}\omega_0^{*2} + V(\chi^*), \end{aligned} \tag{26}$$

where μ is the chemical potential and

$$E_F = \sqrt{m_N^{*2} + k_F^2}. \tag{27}$$

Energy density ϵ and pressure P are given by

$$\begin{aligned}\epsilon &= \frac{1}{4\pi^2}\left[2E_F^3 k_F - m_N^{*2} E_F k_F - m_N^{*4}\ln\left(\frac{E_F+k_F}{m_N^*}\right)\right] \\ &\quad +g_\omega^*(g_{V\omega}^*-1)\omega_0^* n - \frac{1}{2}F_{\sigma\omega}^2 g_\omega^{*2}\frac{\chi^{*2}}{F_\chi^2}\omega_0^{*2} + V(\chi^*), \end{aligned} \tag{28}$$

$$\begin{aligned}P &= \frac{1}{4\pi^2}\left[\frac{2}{3}E_F k_F^3 - m_N^{*2} E_F k_F + m_N^{*4}\ln\left(\frac{E_F+k_F}{m_N^*}\right)\right] \\ &\quad +\frac{1}{2}F_{\sigma\omega}^2 g_\omega^{*2}\frac{\chi^{*2}}{F_\chi^2}\omega_0^{*2} - V(\chi^*)\,. \end{aligned} \tag{29}$$

ϵ and P obtain nontrivial contributions from the hadronic interactions, in addition to non-interacting fermion contributions represented by the first parts in Equations (26), (28) and (29).

From the stationary conditions of the thermodynamic potential, the gap equations for χ^* and ω^* can be obtained,

$$\frac{m_N^2\chi^*}{\pi^2 F_\chi^2}\left[k_F E_F - m_N^{*2}\ln\left(\frac{k_F+E_F}{m_N^*}\right)\right] - \frac{F_{\sigma\omega}^2}{F_\chi^2}g_\omega^{*2}\omega_0^*\chi^* + \left.\frac{\partial V(\chi)}{\partial\chi}\right|_{\chi=\chi^*} = 0, \tag{30}$$

$$g_\omega^*(g_{V\omega}^*-1)n - F_{\sigma\omega}^2 g_\omega^{*2}\frac{\chi^{*}2}{F_\chi^2}\omega_0^* = 0\,. \tag{31}$$

Then, the trace of the energy–momentum tensor, θ_μ^μ, is obtained,

$$\theta_\mu^\mu = \epsilon - 3P \tag{32}$$

$$= \frac{m^2}{\pi^2}\left[E_F k_F - m^2\ln\left(\frac{k_F+E_F}{m}\right)\right] - F_{\sigma\omega}^2 g_\omega^2\frac{\chi^{*2}}{f_\chi^2}\omega_0^{*2} + 4V(\chi^*) \tag{33}$$

$$= 4V(\chi^*) - \chi\left.\frac{\partial V(\chi)}{\partial\chi}\right|_{\chi=\chi^*}, \tag{34}$$

where Equation (30) has been used in the last step. There is no contribution from matter fields in Equation (34), because the Lagrangian is scale invariant for the matter fields by construction. The Fermi surface does not spoil scale symmetry.

The nonvanishing TEMT would be a result solely from the symmetry-breaking nature of the dilaton potential, $V(\chi)$. This shows that the scale invariant formalism provides a simple description that the symmetry breaking due to the finite nucleon mass can be transferred onto the spontaneous symmetry breaking of the scale symmetry, represented by the dilaton potential. TEMT in Figure 1 is essentially that of Equation (34). Hereafter, the dilaton potential is taken as in the standard form [30],

$$V(\chi) = -\frac{m_{\chi^2} F_\chi^2}{8}\left((\frac{\chi^*}{F_\chi})^4 \left[\frac{1}{2} - \ln \frac{\chi^{*2}}{F_\chi^2} \right] - \frac{1}{2} \right), \tag{35}$$

Then, we obtain

$$\theta_\mu^\mu = \frac{m_\chi^{*2} F_\chi^2}{4}\left[1 - \frac{\chi^{*4}}{F_\chi^4} \right]. \tag{36}$$

The density dependence of the dilaton field, χ^*, determines the evolution of TEMT with density in Equation (34) and the effective nucleon mass in Equation (25). χ^* is determined by the gap equations. Using Equation (31), Equation (30) becomes

$$\frac{m_N^2 \chi^*}{\pi^2 F_\chi^2}\left[k_F E_F - m_N^{*2} \ln\left(\frac{k_F + E_F}{m_N^*} \right) \right] - \frac{\left(g_{V\omega}^* - 1\right)^2 n^2}{F_{\sigma\omega}^2 / F_\chi^2 \chi^{*3}} + \left. \frac{\partial V(\chi)}{\partial \chi} \right|_{\chi = \chi^*} = 0. \tag{37}$$

The first part represents the dilaton coupling to Fermi gas, and the third term is from the dilaton potential. The second term is due to ω-nucleon (dilaton) coupling. One can see that the density dependencies of the dilaton mass and the nucleon mass are a result of the interplay between the omega meson and the dilaton. The details are given in [5,18], where the density dependence of the ω-nucleon coupling, $g_{V\omega}^*$, is taken as a parametric form to find the solutions,

$$\frac{g_{V\omega}^* - 1}{g_{V\omega} - 1} = \frac{1}{1 + Bn/n_0}. \tag{38}$$

Interestingly, the interplay [18] is such that we can obtain a solution of χ^* with $B = 0.261$, which becomes density-independent at higher density $n > n_A (\sim 2n_0)$, as shown in Figure 2.

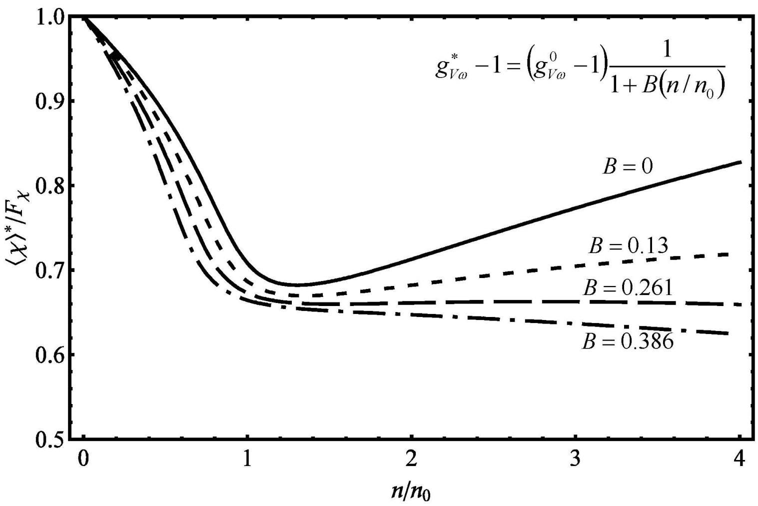

Figure 2. The ratio $m_N^*/m_N \approx \langle\chi\rangle^*/\langle\chi\rangle_0$ as a function of density for varying density dependence of $g_{V\omega}^*$. It stays more-or-less constant above that density. The Figure is borrowed from [18].

We can notice that TEMT in Equation (34), which is a function of χ, is density-independent, as shown in Figure 1, supporting the pseudo-conformality in the previous section. It is also to be noted that the density-independent part D in the energy density is well explained in terms of the density-independent dilaton condensate χ^* in Equation (36). In this effective theory of star matter, formulated with guidance of the scale invariance, which is supposed to be broken spontaneously, $\chi^* \neq 0$, TEMT is nonvanishing. This is equivalent to the absence of the exact scale symmetry. However, the density-independence of χ^* might be one of the ways how the hidden scale symmetry in spontaneously broken phase is manifested itself and emerging in dense matter.

In the pseudo-conformal phase discussed above, the nucleon obtains a nonvanishing mass in Equation (25). The nonvanishing mass apparently violates the chiral symmetry, which is believed to be the underlying symmetry of the hadronic matter. In the scheme of linear sigma model like Gell-Mann–Lévy type [31], the finite mass of nucleon is not contradictory with the chiral symmetry, since the largest part of nucleon mass is due to the spontaneous symmetry breaking of the chiral symmetry. It is expected that the chiral limit is reached with the vanishing chiral condensate at high density. Then, the chiral symmetry is getting restored with decreasing nucleon mass with density. On the other hand, the nonvanishing nucleon mass, which stays constant on the contrary to the expectation of decreasing mass with density toward the chiral restoration, seems to be inconsistent with the chiral symmetry. It is found to be largely due to the χ-condensate (χ^*) of the spontaneously broken scale symmetry, rather than the chiral condensate. The χ-condensate at higher density has no apparent connection to the chiral symmetry breaking (the interplay between chiral symmetry and scale symmetry has been discussed in the different context [32]).

However, the finite mass of the nucleon can be formulated in a chiral invariant way, which has been explored in the framework of the parity doubling model (PDM) [14]. In PDM, the chiral invariance in the nucleon sector can be implemented if there are excitations of parity odd nucleons at high density. There can be various excitations when nucleons get closer with strong correlations. The excitations would be such that the hidden symmetries of the system can be manifested. In dense hadronic matter, we consider excitations of the dilaton and the odd parity nucleons (parity partner of nucleon in PDM) for the scale symmetry and the chiral symmetry, respectively. In PDM formulation, the effective Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_N^{pdm} &= \bar{B}i\left(\partial_\mu - ig_\omega^*\frac{\omega_\mu}{2}\right)B + g_{V\omega}^*\bar{B}\gamma^\mu\left(\frac{\partial_\mu\sigma_\omega}{2F_{\sigma\omega}} - g_\omega^*\frac{\omega_\mu}{2}\right)B \\ &\quad +\mathcal{L}_{nucleon\,mass}, \end{aligned} \tag{39}$$

where $B = (B1, B2)$ denotes the nucleon in the parity doublet in the chiral eigenstate. ρ_i are the Pauli matrices in the parity pair space, and the mass term corresponding to $\bar{N}m_N^*N$ is given by the parity doublet mass term [18,33], $\mathcal{L}_{nucleon\,mass}$, in the PDM Lagrangian,

$$\mathcal{L}_{nucleon\,mass} = \ldots - g_1\sqrt{\kappa}\chi\bar{B}B + g_2\sqrt{\kappa}\chi\bar{B}\rho_3 B - i\bar{m}_0\bar{B}\rho_2\gamma_5 B\,, \tag{40}$$

where g_1 and g_2 are dimensionless parameters for the contribution from the spontaneously broken chiral symmetry signified by the pion decay constant F_π in $\kappa = (F_\pi/F_\chi)^2$. As in the previous section, to make Equation (40) scale invariant, we introduce a dilaton χ as a conformal compensator for two dimensionful couplings: the pion decay constant F_π and the chiral-invariant mass (m_0) term. $\bar{m}_0 = (\chi/F_\chi)m_0$ is supposed to be from the scale symmetry breaking. Here, the mean fields of π and ρ are suppressed.

Baryons B are not mass eigenstates because of the last term in Equation (40). After diagonalizing the mass matrix, two mass eigenstates are identified as the positive parity nucleon, N_+, and its chiral partner, N_-. Their masses are given by

$$m_{N_\pm}^* = \left[\mp g_2\kappa + \sqrt{(g_1\kappa)^2 + (\frac{m_0}{F_\chi})^2}\right]\chi^*. \tag{41}$$

It is to be noted that they are functionals of a dilaton, such that it transforms as scale dimension 1. The density-dependent nucleon mass (m_N^*), which is now $m_{N_+}^*$ in Equation (41), has the same form as in Equation (25). Here, the nucleon mass in free space is given by

$$m_N = m_{N_+} = \left[-g_2F_\pi + \sqrt{(g_1F_\pi)^2 + m_0^2}\right], \quad (42)$$

where

$$\chi^*|_{n=0} = F_\chi. \quad (43)$$

Hence, the density dependence of χ^* is essentially the same as in Figure 2. Therefore, the nonvanishing nucleon mass in the pseudo-conformal phase is considered to be the evidence for the emergence of the parity doubling structure subjected to the chiral symmetry.

However, the formulation with a single dilaton is found to be not suitable to implement the results of the renormalization group (RG) analysis. In RG analysis [18,34] of PDM model in which the scale symmetry is not implemented, it is observed that the mass parameters have fixed point values. Two mass parameters, M_s and M_D, are considered,

$$M_S = \frac{1}{4}(m_{N_+} + m_{N_-})^2 = (g_1F_\pi)^2 + m_0^2 \quad (44)$$

$$M_D = \frac{1}{4}(m_{N_+} - m_{N_-})^2 = (g_2F_\pi)^2. \quad (45)$$

The RG flow of M_D has a fixed point, $M_D \to 0$, which is equivalent to the RG flow for g_2F_π, $g_2F_\pi \to 0$. We can expect a similar flow, $g_1F_\pi \to 0$, which implies the RG flow of M_S toward a fixed point m_0^2, $M_S \to m_0^2$. We propose these RG flows can be copied by the density-dependent dilaton condensate in dilatonic PDM (see Equation (39)). In fact, the density-independent χ^* at high density corresponds to the fixed point of M_S. However, with a single χ, only the RG flow of M_S, which goes to zero, can not be explained. χ is a compensator field suitable only for the m_0 part in Equation (39). Hence, a different kind of dilaton, χ_s, as a compensator for F_π terms, is needed with vanishing condensate at a higher density. It is analogous to the σ field in the Gell-Mann–Lévy-type model. In the linearized scheme suitable for the study of dynamical aspects of chiral symmetry, the vacuum expectation value of the scalar field identified as χ_s becomes zero at chiral symmetry restoration,

$$<\sigma>\to 0 \text{ equivalently } <\chi_s>\to 0, \quad (46)$$

which corresponds to the dilaton limit [35] in which ρ-nucleon coupling is supposed to no longer be active.

Now, it is proposed that Equation (40) should be modified to incorporate two dilatons, χ_s and χ for chiral condensate and chiral-invariant mass, respectively:

$$\mathcal{L}_{nucleon\ mass} \to \ldots - g_1\sqrt{\kappa}\chi_s\bar{B}B + g_2\sqrt{\kappa}\chi_s\bar{B}\rho_3B - i\tilde{m}_0\bar{B}\rho_2\gamma_5B\,, \quad (47)$$

where $\sqrt{\kappa} = \frac{\chi_s}{F_{\chi s}}$ and $\tilde{m}_0 = (\chi/F_\chi)m_0$. The mass formula Equation (41) is changed accordingly,

$$m^*_{N_\pm} = \left[\mp g_2\kappa\chi_s^* + \sqrt{(g_1\kappa\chi_s^*)^2 + (\frac{m_0}{F_\chi}\chi^*)^2}\right], \quad (48)$$

and we obtain, in the pseudo-conformal phase ($\chi_s* \to 0$),

$$m^*_{N_\pm} \to \tilde{m}^*_0 = \frac{m_0}{F_\chi}\chi^*. \quad (49)$$

One can see a part of nucleon mass is generated dynamically via spontaneous symmetry breaking of the chiral symmetry dialed through χ_s (what is observed in [18] is actually the behavior of χ at higher density, not χ_s. We do not have the mean-field result on χ_s, which does not couple to omega meson directly in this work, but χ_s is assumed to be sufficiently small enough to be ignored at the high density, $n > n_A$). The rest of the nucleon mass is unconnected with the chiral symmetry breaking. It is supposed to be the contribution from the spontaneous symmetry breaking of the scale symmetry represented by χ. The excitation of parity doublets in the compact star matter makes these features in the mass formula compatible with the chiral symmetry. Then, the nucleon masses approach a constant quantity represented by the chiral-invariant mass m_0 is the signature of the emerging parity doubling structure in a dense hadronic matter. The parity doubling is one of the emergent phenomena in the pseudo-conformal phase of the strongly correlated dense baryonic matter.

4. Summary and Discussion

In this work, the possibility of revealing the hidden scale symmetry in a dense baryonic matter is discussed in the pseudo-conformal phase, where the trace of the energy–momentum tensor becomes density-independent, and the speed of sound approaches the conformal velocity of the scale symmetric matter. It is interpreted as an indication that the hidden scale symmetry is emerging at high density star matter disguised in the form of the conformal speed of sound.

One of the interesting results of the dilatonic mean field calculation is that the effective nucleon mass obtains a nonvanishing density-independent constant value at a higher density regime relevant to the star matter. This is due to the nontrivial interplay between ω-nucleon (dilaton χ) coupling encoded in the asymptotic behavior of ω coupling constant $g_{V\omega}$ [12]. The trace of the energy–momentum tensor, which is a function of χ, becomes density-independent as well. Although TEMT is nonvanishing, the speed of sound approaches the conformal velocity, $v_s \to 1/\sqrt{3}$. One of the characteristics of underlying scale symmetry in dense matter, the conformal velocity, appears precociously for $n > n_A(\sim 2n_0)$. This feature accounts for the emergent pseudo-conformal symmetry in compact-star matter, and suggests that the core of the compact star provides a new window for investigating the effect of scale symmetry (spontaneously broken) hidden in dense medium.

The nucleon mass is largely due to the spontaneous breakings of chiral symmetry and scale symmetry. The nucleon mass at a higher density is supposed to be mainly from the vacuum expectation value of the dilaton developed in the spontaneously broken phase of the scale symmetry. The finite nucleon mass which is unconnected to chiral condensation is apparently chiral-symmetry-breaking. The excitation of parity doublets in the compact star matter makes the system chiral invariant. This is an interesting observation that, in dense matter, there is an additional interplay between scale and chiral symmetry. It can be understood that the constraint of the chiral symmetry induces the parity doublet excitations in the pseudo-conformal matter. The parity doubling is an emergent phenomenon in the pseudo-conformal phase of the strongly correlated compact star matter. It is considered basically as a result of additional excitations and the nontrivial interplays between them in a highly dense hadronic matter. The precise mechanism how particle excitations are such that the system becomes pseudo-conformal with parity doubled structure in nucleon sector is not clear yet, and how the populations of N_+ and N_- in parity-doubled structure evolve with density in the pseudo-conformal phase have not been properly discussed, while it has been discussed in various contexts either of PDM [36–38] or in the pseudo-conformality [4,5]. These are the interesting future directions toward the equation of state viable for the dense neutron star matter.

There has always been a quest for excitations of quarks or deconfinement in the extreme conditions, high temperature and/or high density. For the highly dense matter expected at core of compact stars, the analytic parameterization of the numerical result of the energy density in Section 2 is given by Equation (2). If we take it seriously as it is, one can notice that the first term in Equation (2) has the same density dependence as that of ideal relativistic quark matter, and D looks like a bag constant in the bag model [39,40]. The appearance of a bag-like constant in the dense hadronic matter [32] can be considered as an incomplete confinement, and leakage of the bag constant throughout the dense core. What is interesting in this approach is that, although no explicit QCD degrees of freedom are involved, the property of the equation of state in this approach is quite similar to that of "deconfined quark matter". This might be a useful hint for exploring the quark–hadron continuity in dense hadronic matter [13,41,42].

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