

## The effective dipole-type field approach and the dual Higgs model

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**Abstract.** We consider the dual picture of the Yang–Mills theory at large distances. The dual Higgs model is reformulated in terms of two-point Wightman functions with the equations of motion involving higher derivatives.

### 1. Introduction

The nature of confining forces in gauge theories has attracted a lot of attention in recent years. We consider a model (in four-dimensional space-time (4d)) based on the dual description of a long-distance Yang–Mills (LDY–M) theory which can provide quark confinement in a system of static test charges. This article pursues the idea that the vacuum of quantum Yang–Mills (Y–M) theory is realized by a condensate of monopole–antimonopole pairs [1]–[4]. Since there are no monopoles as classical solutions with finite energy in a pure Y–M theory, it has been suggested by 't Hooft [5] that one might go to the Abelian projection where the gauge group  $SU(2)$  is broken by a suitable gauge condition to its Abelian subgroup  $U(1)$ . Now there is the well-known assertion that the interplay between a quark and an antiquark is analogous to the interaction between a monopole and an antimonopole in a superconductor.

It is known that the topology of the Y–M  $SU(N)$  manifold and that of its Abelian subgroup  $[U(1)]^{N-1}$  are different, and new topological objects can appear if the local gauge transformation of some gauge function is introduced in our model, e.g., the field strength tensor for the gauge field  $A_\mu(x)$  in quantum chromodynamics (QCD) with  $D_\mu(x) = \partial_\mu + ieA_\mu(x)$ :

$$F_{\mu\nu}(x) = \frac{1}{ie}([D_\mu(x), D_\nu(x)] - [\partial_\mu, \partial_\nu]),$$

which transforms with the gauge function  $\Omega(x)$  as

$$\begin{aligned} F_{\mu\nu}(x) &\rightarrow F_{\mu\nu}^{\Omega}(x) = \Omega(x)F_{\mu\nu}(x)\Omega^{-1}(x) \\ &= \partial_{\mu}A_{\nu}^{\Omega}(x) - \partial_{\nu}A_{\mu}^{\Omega}(x) + ie[A_{\mu}^{\Omega}(x), A_{\nu}^{\Omega}(x)] - \frac{1}{ie}\Omega(x)[\partial_{\mu}, \partial_{\nu}]\Omega^{-1}(x). \end{aligned} \quad (1)$$

The last term in (1) reflects the singular character of the above-mentioned gauge transformation. One can identify the Abelian projection by making the replacement

$$F_{\mu\nu}^{\Omega}(x) \rightarrow F_{\mu\nu}^{\alpha}(x) = \partial_{\mu}A_{\nu}^{\alpha}(x) - \partial_{\nu}A_{\mu}^{\alpha}(x) - \frac{1}{ie}\Omega(x)[\partial_{\mu}, \partial_{\nu}]\Omega^{-1}(x),$$

where  $A_{\mu}^{\Omega} \rightarrow A_{\mu}^{\alpha}$ , while the label  $\alpha$  reflects the Abelian world. This leads to the Dirac string and magnetic current

$$J_{\mu}^{mon}(x) = -\frac{1}{2ie}\epsilon_{\mu\nu\rho\sigma}\partial_{\nu}\Omega(x)[\partial_{\rho}, \partial_{\sigma}]\Omega^{-1}(x)$$

in the Abelian gauge sector.

Formally, a gauge group element which transforms a generic  $SU(3)$  connection onto the gauge-fixing surface in the space of connections is not regular everywhere in space-time. The projected (or transformed) connections contain topological singularities (or defects). Such a singularity may form the worldline(s) of magnetic monopoles. Hence, this singularity leads to the monopole current  $J_{\mu}^{mon}$ . This is a natural way of performing the transformation from the Y-M theory to a model dealing with Abelian fields.

Analytical models of the dual QCD with monopoles have been intensively investigated [6]–[8]. The monopole confinement mechanism is confirmed by many lattice calculations (see reviews [9] and references therein). We study the Lagrangian model where the fundamental variables are an octet of dual potentials coupled minimally to three octets of monopole (Higgs) fields. The dual gauge model is studied at the lowest order of the perturbative series using canonical quantization. The basic manifestation of the model is that it generates the equations of motion, where one of them for the scalar (Higgs) field is the equation involving higher derivatives. Our aim is to apply the previous results [10]–[16] on Wightman functions in  $4d$  models, with equations of motion (for the scalar fields) involving the dipole-type structure at least, to the model with monopoles. The monopole fields obeying such equations are classified by their two-point Wightman functions (TPWF). In the scheme presented in this paper, the flux distribution in the tubes formed between two heavy colour charges is understood via the following statement: the Abelian monopoles are excluded from the string region while the Abelian electric flux is squeezed into the string region. In our model, we use the dual gauge field  $\hat{C}_{\mu}^a(x)$  and the monopole field  $\hat{B}_i^a(x)$  ( $i = 1, \dots, N_c(N_c - 1)/2$  and  $a = 1, \dots, 8$  is a colour index), which are relevant modes for infrared behaviour. The local coupling of the  $\hat{B}_i^a$ -field to the  $\hat{C}_{\mu}^a$ -field provides the mass of the dual field and, hence, a dual Meissner effect. The commutation relations, TPWF and Green functions, as well-defined distributions in the space  $S(\mathbb{R}^d)$  of complex Schwartz test functions on  $\mathbb{R}^d$ , will be defined in the following text.

## 2. The Lagrangian density

Let us consider the Lagrangian density (LD)  $L$  of the  $U(1) \times U(1)$  dual Higgs model corresponding to the LDY-M theory [8]:

$$L = 2 \text{Tr}[-\frac{1}{4}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + \frac{1}{2}(D_{\mu}\hat{B}_i)^2] - W(\hat{B}_i), \quad (2)$$

where

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{C}_\nu - \partial_\nu \hat{C}_\mu - ig[\hat{C}_\mu, \hat{C}_\nu],$$

$$D_\mu \hat{B}_i = \partial_\mu \hat{B}_i - ig[\hat{C}_\mu, \hat{B}_i],$$

$\hat{C}_\mu$  and  $\hat{B}_i$  are the  $SU(3)$  matrices,  $g$  is the gauge coupling constant;  $\hat{C}_\mu = \lambda^8 C_\mu$  ( $\lambda^a$  is the generator of  $SU(3)$ ).  $\hat{F}_{\mu\nu}$  becomes the Abelian tensor with  $C_\mu$  being the dual to an ordinary vector potential. The Higgs fields develop their vacuum expectation values (v.e.v.)  $\hat{B}_{0i}$  and the Higgs potential  $W(\hat{B}_i)$  has a minimum at  $\hat{B}_{0i}$ . The v.e.v.  $\hat{B}_{0i}$  produce a colour monopole-generating current confining the electric colour flux. The interaction between a quark  $Q$  and antiquark  $\bar{Q}$  is provided by  $\hat{C}_\mu$  via a Dirac string tensor [17]  $\hat{\tilde{G}}_{\mu\nu}$  having the same colour structure as  $\hat{C}_\mu$ , namely  $\hat{\tilde{G}}_{\mu\nu} = \lambda^8 \tilde{G}_{\mu\nu}$ . A bound state of  $Q$  and  $\bar{Q}$  occurs with the replacement  $\hat{F}_{\mu\nu} \rightarrow \lambda^8 G_{\mu\nu}$ , where  $G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu + \tilde{G}_{\mu\nu}$ . Following [8], we choose the colour structure for  $\hat{B}_i$ -fields

$$\hat{B}_1 = \lambda_7 B_1 - \lambda_6 \bar{B}_1, \quad \hat{B}_2 = -\lambda_5 B_2 + \lambda_4 \bar{B}_2, \quad \hat{B}_3 = \lambda_2 B_3 - \lambda_1 \bar{B}_3.$$

The effective potential looks as follows (see also [8]):

$$W(B, \bar{B}, B_3) = \frac{2}{3} \lambda \{ 11[2(B^2 + \bar{B}^2 - B_0^2)^2 + (B_3^2 - B_0^2)^2] + 7[2(B^2 + \bar{B}^2) + B_3^2 - 3B_0^2]^2 \},$$

where  $\lambda$  is dimensionless. The LD (2) becomes

$$L(\tilde{G}_{\mu\nu}) = -\frac{1}{3} G_{\mu\nu}^2 + 4|(\partial_\mu - igC_\mu)\phi|^2 + 2(\partial_\mu \phi_3)^2 - W(\phi, \phi_3), \quad (3)$$

where  $\phi \equiv \phi_1 = \phi_2 = B - i\bar{B}$ ,  $\phi_3 = B_3$ ; the generating current of (3) is nothing but the monopole current confining the electric colour flux  $J_\mu^{mon} = (2/3)\partial^\nu G_{\mu\nu}(x)$ . The formal consequence of the  $J_\mu^{mon}$ -conservation,  $\partial^\mu J_\mu^{mon} = 0$ , means that monopole currents form closed loops.

Since  $\phi_1$  and  $\phi_2$  couple to  $C_\mu$  in the same way [8], we choose  $B(x) = b(x) + B_0$ ,  $\bar{B}(x) = \bar{b}(x)$ ,  $B_3(x) = b_3(x) + B_0$  with  $\langle B(x) \rangle_0 \neq 0$ ,  $\langle \bar{B}(x) \rangle_0 \neq 0$ ,  $\langle B_3(x) \rangle_0 \neq 0$ . In terms of the new fields  $b, \bar{b}, b_3$ , the LD (3) is divided into two parts:  $L = L_1 + L_2$  where  $L_1$  is of lowest order in  $g$  and  $\lambda$ , and with the minimal weak interaction looks as follows:

$$L_1 = -\frac{1}{3} G_{\mu\nu}^2 + 4[(\partial_\mu b)^2 + (\partial_\mu \bar{b})^2 + \frac{1}{2}(\partial_\mu b_3)^2] + m^2 C_\mu^2 - \frac{4}{3} \mu^2 (50b^2 + 18b_3^2) + 8m\partial_\mu \bar{b} C_\mu.$$

Here,  $m \equiv gB_0$  and  $\mu \equiv \sqrt{2\lambda}B_0$ . The equations of motion for the fields  $b, \bar{b}, b_3$  and  $C_\mu$  are

$$\begin{aligned} (\Delta^2 + \mu_1^2)b(x) &= 0; \\ \Delta^2 \bar{b}(x) + m(\partial \cdot C) &= 0; \\ (\Delta^2 + \mu_2^2)b_3(x) &= 0; \\ (\Delta^2 + m_1^2)C_\mu(x) - \partial_\mu(\partial \cdot C) + 12m\partial_\mu \bar{b}(x) - \partial^\nu \tilde{G}_{\mu\nu}(x) &= 0, \end{aligned} \quad (4)$$

where  $\mu_1^2 = (50/3)\mu^2$ ,  $\mu_2^2 = 12\mu^2$ ,  $m_1^2 = 3m^2$ . The formal solution of equation (4) looks as follows:

$$C_\mu(x) = \alpha \partial^\nu \tilde{G}_{\mu\nu}(x) - \beta \partial_\mu \bar{b}(x),$$

with  $\alpha \equiv (3m^2)^{-1}$ ,  $\beta \equiv 4/m$ . We obtain that the dual gauge field is defined via the divergence of the Dirac string tensor  $\tilde{G}_{\mu\nu}(x)$  shifted by the divergence of the scalar field  $\bar{b}(x)$ . For large enough

$\vec{x}$ , the monopole field is going to its v.e.v. while  $C_\mu(\vec{x} \rightarrow \infty) \rightarrow 0$  and  $J_\mu^{mon}(\vec{x} \rightarrow \infty) \rightarrow 8m^2C_\mu$ . This implies that in the  $d = 2h$  dimensions the  $\bar{b}(x)$  field obeys the equation

$$\Delta^{2h}\bar{b}(x) \simeq 0, \quad h = 2, 3, \dots, \quad (5)$$

but  $\Delta^{2h}\bar{b}(x) \neq 0$ . Here, the solutions of equation (5) obey locality, Poincaré covariance and spectral conditions, and look like the dipole ‘ghosts’ at  $h = 2$ .

We define TPWF  $W_h(x)$  in  $d = 2h$  dimensions,  $W_h(x) = \langle \bar{b}(x)\bar{b}(0) \rangle_0$ , as the distribution in the Schwartz space  $S'(\mathbb{R}^{2h})$  of moderate distributions on  $\mathbb{R}^{2h}$  which obeys the equation

$$\Delta^{2h}W_h(x) = 0. \quad (6)$$

The general solution of (6) should be Lorentz invariant and is given in the following form [10] at  $h = 2$ :

$$W_2(x) = a_1 \ln \frac{l^2}{-x_\mu^2 + i\epsilon x^0} + \frac{a_2}{x_\mu^2 - i\epsilon x^0} + a_3, \quad (7)$$

where  $a_i$  ( $i = 1, 2, 3$ ) are the coefficients,  $l$  is an arbitrary length scale. The coefficients  $a_1$  and  $a_2$  in (7) can be fixed using the canonical commutation relations (CCR)  $[C_\mu(x), \pi_{C_\nu}(0)]|_{x^0=0} = ig_{\mu\nu}\delta^3(\vec{x})$  and  $[\bar{b}(x), \pi_{\bar{b}}(0)]|_{x^0=0} = i\delta^3(\vec{x})$ , respectively, with  $\pi_{C_\mu}(x) = -\frac{4}{3}G_{0\mu}(x)$  and  $\pi_{\bar{b}}(x) = 8[\partial^0\bar{b}(x) + mC^0(x)]$ . The standard commutator for the scalar field  $\bar{b}$  is

$$[\bar{b}(x), \bar{b}(0)] = (2\pi)^2 i[4a_1 E_2(x) + a_2 D_2(x)],$$

where  $E_2(x) = (8\pi)^{-1} \operatorname{sgn}(x^0)\theta(x^2)$  and  $D_2(x) = (2\pi)^{-1} \operatorname{sgn}(x^0)\delta(x^2)$  are taken into account. The direct calculation leads to  $a_1 = (m^2/48\pi^2)$  and  $a_2 = -(1/24\pi^2)$ .

The propagator of the  $\bar{b}$ -field in  $S'(\mathbb{R}_4)$  is

$$\hat{\tau}_2(p) = \operatorname{weak} \lim_{\tilde{\kappa}^2 \ll 1} \frac{i}{3(2\pi)^4} \left\{ m^2 \left[ \frac{1}{(p^2 - \kappa^2 + i\epsilon)^2} + i\pi^2 \ln \frac{\kappa^2}{\tilde{\mu}^2} \delta_4(p) \right] - \frac{1}{2} \frac{1}{p^2 - \kappa^2 + i\epsilon} \right\}. \quad (8)$$

Here,  $\kappa$  is a parameter of the representation and not an analogue of the infrared mass;  $\tilde{\kappa}^2 \equiv \kappa^2/p^2$ .

To define the commutation relation  $[C_\mu(x), C_\nu(y)]$ , let us consider the canonically conjugate pair  $\{C_\mu, \pi_{C_\nu}\}$ :

$$[\frac{4}{3}C_\mu(x), \partial_\nu C_0(0) - \partial_0 C_\nu(0) - g_{0\nu}(\partial \cdot C(0)) + \Delta_{0\nu}(0)]|_{x^0=0} = ig_{\mu\nu}\delta^3(\vec{x}), \quad (9)$$

where  $\Delta_{\mu\nu}(x) = g_{\mu\nu}(\partial \cdot C(x)) - \tilde{G}_{\mu\nu}(x)$  tends to zero as  $x \rightarrow 0$  and the Dirac string tensor  $\tilde{G}_{\mu\nu}(x)$  obeys the equation  $[\Delta^2 + (3m)^2]\tilde{G}_{\mu\nu}(x) = 0$ . Obviously, the following form of the free- $C_\mu$ -field commutator:

$$[C_\mu(x), C_\nu(0)] = ig_{\mu\nu}[\xi m_1^2 E_2(x) + cD_2(x)], \quad (10)$$

ensures the CCR (9) at large  $x_\mu^2$  with both  $\xi$  and  $c$  (in (10)) being real arbitrary numbers but  $\xi = \frac{3}{4} - 4c$ .

The free dual gauge field propagator in  $S'(\mathbb{R}_4)$  in any local covariant gauge is given by

$$\hat{\tau}_{\mu\nu}(p) = \int d^4x \exp(ipx) \tau_{\mu\nu}(x) = i \left[ g_{\mu\nu} - \left( 1 - \frac{1}{\zeta} \right) \frac{p_\mu p_\nu}{p^2 + i\epsilon} \right] [\xi m_1^2 \hat{t}_1(p) + c\hat{t}_2(p)], \quad (11)$$

where

$$\tau_{\mu\nu}(x) = \frac{ig_{\mu\nu}}{(4\pi)^2} \left[ \xi m_1^2 \ln(-\tilde{\mu}^2 x_\mu^2 + i\epsilon) + \frac{c}{x_\mu^2 + i\epsilon} \right];$$

$$\hat{t}_1(p) = \text{weak} \lim_{\kappa^2 \ll 1} \left[ \frac{1}{(p^2 - \kappa^2 + i\epsilon)^2} + i\pi^2 \ln\left(\frac{\kappa^2}{\tilde{\mu}^2}\right) \delta_4(p) \right];$$

$$\hat{t}_2(p) = \text{weak} \lim_{\kappa^2 \ll 1} \frac{1}{2} \frac{1}{(p^2 - \kappa^2 + i\epsilon)}.$$

The gauge parameter  $\zeta$  in (11) is a real number. The following requirement:  $(\Delta^2)^2 \tilde{\tau}_{\mu\nu}(x) = i\delta_4(x)$  on the Green function  $\tau_{\mu\nu}(x)$  leads to the constant  $c$  having to be equal to zero and  $\tilde{\tau}_{\mu\nu}(x) = \tau_{\mu\nu}(x)/(\xi m_1^2)$ .

### 3. An approximate solution

To obtain an approximate topological solution for this dual model, we fix the equations of motion:

$$\partial^\nu G_{\mu\nu} = 6ig[\phi^*(\partial_\mu - igC_\mu)\phi - \phi(\partial_\mu + igC_\mu)\phi^*], \quad (12)$$

$$(\partial_\mu - igC_\mu)^2\phi = \frac{2}{3}\lambda(32B_0^2 - 25|\phi|^2 - 7\phi_3^2)\phi, \quad (13)$$

where  $\phi(x)$  is decomposed as  $\phi(x) = (1/\sqrt{2}) \exp(if(x))[\chi(x) + B_0]$  using the new scalar variables  $\chi(x)$  and  $f(x)$ . The equation of motion (12) transforms into the following one:

$$\partial^\nu G_{\mu\nu} = 6g(\chi + B_0)^2(gC_\mu - \partial_\mu f)$$

which means that the  $\bar{b}(x)$  field is nothing but a mathematical realization of the ‘massive’ phase  $B_0f(x)$  at large enough  $\vec{x}$ , i.e.,  $\bar{b}(x) \simeq (B_0/2)S(x)f(x)$  and  $S(x) \equiv (1 + \chi(x)/B_0)^2$ . Integrating out  $G_{\mu\nu}$  over the 2D surface element  $\sigma^{\mu\nu}$  in the flux  $\Pi = \int G_{\mu\nu}(x) d\sigma^{\mu\nu}$ , we conclude that the phase  $f(x)$  is varied by  $2\pi n$  for any integer number  $n$  associated with the topological charge [18] inside the flux tube. Using the cylindrical symmetry we arrive at the field equation ( $\tilde{C} \rightarrow (\tilde{C}(r)/r)\vec{e}_\theta$ ):

$$\frac{d^2\tilde{C}(r)}{dr^2} - \frac{1}{r} \frac{d\tilde{C}(r)}{dr} - 3m^2[3 + 2S(r)]\tilde{C}(r) + 6nmB_0S(r) = 0$$

with asymptotic transverse behaviour of its solution:

$$\tilde{C}(r) \simeq \frac{4n}{7g} - \sqrt{\frac{\pi mr}{2k}} e^{-kmr} \left(1 + \frac{3}{8kmr}\right), \quad k \equiv \sqrt{21}.$$

The field equation (13) is given by

$$\frac{d^2\hat{\chi}(r)}{dr^2} + \frac{1}{r} \frac{d\hat{\chi}(r)}{dr} - \left\{ \left[ \frac{n - g\tilde{C}(r)}{r} \right]^2 + \frac{25}{3}\lambda[2B_0^2 - \hat{\chi}^2(r)] \right\} \hat{\chi}(r) \simeq 0,$$

where  $\hat{\chi}(r) = \chi(r) + B_0$ . The profile of the colour electric field in the flux tube at large  $r$  looks as follows:

$$E_z(r) = \sqrt{\frac{\pi m}{2kr}} \left( km - \frac{1}{2r} \right) e^{-kmr}.$$

#### 4. The confinement potential in analytic form

Now, our aim is to obtain the confinement potential in an analytic form for the system of interacting static test charges of a quark and an antiquark. According to the distribution (8), the static potential in  $\mathbb{R}^3$  is a rising function with  $r = |\vec{x}|$  [14, 15]:

$$P_{stat}(r) \sim \frac{1}{2^{2h}\pi^{3/2}} \frac{1}{(h-1)!} \Gamma(3/2-h) r^{2h-3},$$

and the Fourier transformation of the analytic function  $r^\sigma$  at  $\sigma \neq -d, -d-2, \dots$  is [15]

$$F\{r^\sigma\} = \left(\frac{4\pi}{p^2}\right)^{(\sigma+d)/2} \pi^{-\sigma/2} \frac{\Gamma[(\sigma+d)/2]}{\Gamma(-\sigma/2)}.$$

We define the static potential as  $P_{stat}(r) = \lim_{T \rightarrow \infty} [T^{-1} \cdot A(r)]$  and the action  $A(r)$  is given by the colour source current part of the LD  $L(p) = -\vec{j}_\alpha^\mu(-p)\hat{\tau}_{\mu\nu}(p)\vec{j}_\alpha^\nu(p)$  with the quark current  $\vec{j}_\alpha^\mu(x) = \vec{Q}_\alpha g^{\mu 0}[\delta_3(\vec{x} - \vec{x}_1) - \delta_3(\vec{x} - \vec{x}_2)]$ . Here,  $\vec{Q}_\alpha = e\vec{\rho}_\alpha$  is the Abelian colour electric charge of a quark while  $\vec{\rho}_\alpha$  is the weight vector of the  $SU(3)$  algebra:  $\rho_1 = (1/2, \sqrt{3}/6)$ ,  $\rho_2 = (-1/2, \sqrt{3}/6)$ ,  $\rho_3 = (0, -1/\sqrt{3})$  [18];  $\vec{x}_1$  and  $\vec{x}_2$  are the position vectors of a quark and an antiquark, respectively.

As a consequence of the dual field propagator (11), and using the following representation in the sense of generalized functions [11]:

$$\begin{aligned} \text{weak } \lim_{\tilde{\kappa}^2 \ll 1} & \left[ \frac{1}{(p^2 - \kappa^2 + i\epsilon)^2} + i\pi^2 \ln \frac{\kappa^2}{\tilde{\mu}^2} \delta_4(p) \right] \\ &= \frac{1}{4} \frac{\partial^2}{\partial p^2} \frac{1}{-p^2 - i\epsilon} \ln \frac{-p^2 - i\epsilon}{\tilde{\mu}^2} = \frac{1}{2} \frac{1}{(p^2 + i\epsilon)^2} \left( 5 - 3 \ln \frac{-p^2 - i\epsilon}{\tilde{\mu}^2} \right), \end{aligned}$$

we get

$$P_{stat}(r) = -\frac{\vec{Q}^2}{16\pi} \left[ \xi m^2 r (-12.4 + 6 \ln \tilde{\mu} r) + O\left(\frac{c}{r}\right) \right]. \quad (14)$$

Hence, the string tension  $a$  in  $P_{stat}(r) \sim ar$  emerges as

$$a \simeq \frac{3}{4} \frac{\pi}{g^2} m^2 = \frac{3}{4} \alpha_s m^2,$$

where the logarithmic correction in (14) has been ignored at large distances  $r \sim \tilde{\mu}^{-1}$ . We obtain  $a \simeq 0.18 \text{ GeV}^2$  for the mass of the dual  $C_\mu$ -field  $m = 0.8 \text{ GeV}$ , and  $\alpha_s = \pi/g^2 = e^2/(4\pi) = 0.37$ , fixed from fitting the heavy quark-antiquark spectrum [19]. The string tension  $a_{fixed} = 1.0 \text{ GeV fm}^{-1}$  extracted from the Regge slope of the hadrons [20] then determines the fixed value of the mass  $m_{fixed} = 0.85 \text{ GeV}$ . Performing a formal comparison, let us recall that the string tensions in papers [21] and [22] are equal to  $a = (430 \text{ MeV})^2$  and  $(440 \text{ MeV})^2$ , respectively. We found that for a sufficiently long string the  $\sim r$  behaviour of the static potential is dominant; for a short string the singular interaction provided by the second term in (14) becomes important if the average size of the monopole is even smaller.

## 5. Conclusions

Finally, some conclusions can be stated:

- (a) We have actually derived the analytic expressions for the propagators of both the  $\bar{b}$ -field (8) and the dual gauge boson field (11) in  $S'(\mathbb{R}_4)$ . Our result should be regarded as the distributions (8) and (11) in a weak sense.
- (b) The dual gauge bosons become massive due to their interaction with scalar field(s). But not every scalar species becomes massive, since the symmetry-breaking pattern is  $SU(3) \rightarrow U(1) \times U(1)$  and one scalar field remains massless. We see that the fields  $b(x)$  and  $b_3(x)$  receive their masses, and the  $\bar{b}(x)$  field—in combination with  $\partial^\nu \tilde{G}_{\mu\nu}(x)$ —forms the vector field  $C_\mu(x)$  obeying the equation of motion for the massive vector field with the mass  $m = gB_0$ . The solution for the  $\bar{b}(x)$  field can be identified as a ‘ghost’-like particle in a similar manner.
- (c) The form of the potential (14) grows linearly with the distance  $r$  apart from a logarithmic correction.
- (d) The string tension  $a$  obtained is quite close to a phenomenological value.
- (e) Since no real physics can depend on the choice of the gauge group (where the Abelian group appears as a subgroup), there seems to be a new mechanism of confinement [23].

## References

- [1] Nambu Y 1974 *Phys. Rev. D* **10** 4262
- [2] Mandelstam S 1976 *Phys. Rep. C* **23** 245  
Mandelstam S 1979 *Phys. Rev. D* **19** 2391
- [3] Polyakov A M 1977 *Nucl. Phys. B* **120** 429
- [4] 't Hooft G 1978 *Nucl. Phys. B* **138** 1
- [5] 't Hooft G 1981 *Nucl. Phys. B* **190** 455
- [6] Suzuki T 1988 *Prog. Theor. Phys.* **80** 929  
Suzuki T 1989 *Prog. Theor. Phys.* **81** 752  
Maedan S and Suzuki T 1989 *Prog. Theor. Phys.* **81** 229
- [7] Suganuma H, Sasaki S and Toki H 1995 *Nucl. Phys. B* **435** 207
- [8] Baker M *et al* 1996 *Phys. Rev. D* **54** 2829
- [9] Suzuki T 1993 *Nucl. Phys. B* **30** 176  
Polikarpov M I 1997 *Nucl. Phys. B* **53** 134  
Di Giacomo A 1998 *Prog. Theor. Phys. Suppl.* **131** 161  
Chernodub M N *et al* 1998 *Prog. Theor. Phys. Suppl.* **131** 309  
Haymaker R W 1999 *Phys. Rep.* **315** 153
- [10] Ferrari R 1974 *Nuovo Cimento A* **19** 204
- [11] Zwanziger D 1978 *Phys. Rev. D* **17** 457
- [12] D'Emilio E and Mintchev M 1979 A gauge model with perturbative confinement in four dimensions *IFUP Report* Pisa
- [13] D'Emilio E and Mintchev M 1980 *Phys. Lett. B* **89** 207
- [14] Kozlov G A, Kuleshov S P and Zaikov R P 1980 On the linear quasipotential in the framework of quantum field theory *JINR Report* E2-80-352, Dubna
- [15] Kozlov G A 1992 *Nuovo Cimento A* **105** 139
- [16] Kozlov G A 1994 *Nuovo Cimento A* **107** 819  
Kozlov G A 1994 *Nuovo Cimento A* **107** 1739

- [17] Dirac P A M 1948 *Phys. Rev.* **74** 817
- [18] Koma Y, Saganuma H and Toki H 1999 *Phys. Rev. D* **60** 074024
- [19] Baker M, Ball J S and Zachariasen F 1995 *Phys. Rev. D* **51** 1968
- [20] Sailer K *et al* 1991 *J. Phys. G: Nucl. Phys.* **17** 1005
- [21] Baker M, Brambillia N, Dosch H G and Vairo A 1998 *Phys. Rev. D* **58** 034010
- [22] Chernodub M N 2000 *Phys. Lett. B* **474** 73
- [23] Faddeev L D and Niemi A J 1999 *Phys. Rev. Lett.* **82** 1624  
Faddeev L D and Niemi A J 1999 *Phys. Lett. B* **449** 214  
Shabanov S V 1999 *Phys. Lett. B* **463** 263  
Cho Y M 2000 *Phys. Rev. D* **62** 074009