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PERSPECTIVE

The backreaction problem for black holes in semiclassical gravity

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Abstract
The question of black hole evaporation is reviewed in the framework of quantum field theory in curved spacetimes and semiclassical gravity. We highlight the importance of taking backreaction effects into account to have a consistent picture of the fate of gravitational collapse in this framework. We describe the difficulties of solving the backreaction semiclassical equations due to practical complications of renormalizing the stress-energy tensor of quantum fields in general 3+1 spacetimes. We end with some personal views and plans on the subject.

Keywords Black holes · quantum fields · Hawking radiation · semiclassical gravity · backreaction

Contents

1 Hawking radiation and black hole evaporation

Quantum field theory (QFT) predicts that any dynamical spacetime is able to spontaneously excite particle pairs out of the quantum vacuum [1–3]. In this framework, Hawking found that the formation of a spherically symmetric black hole (BH) of mass M by gravitational collapse, will emit a steady flux of particles with a thermal spectrum at sufficiently late times, with temperature $T = \frac{\hbar c^3}{8\pi G M k_B}$, and irrespectively of the details of the original star [4–8]. Immediately after, this outgoing flux was found to be compensated by a negative-energy steady flux across the horizon [9–12], which

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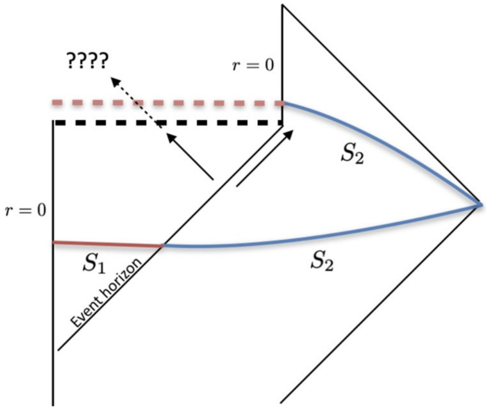

Fig. 1 Penrose diagram for black evaporation conjectured in [5, 17]

is expected to backreact on the BH and decrease its mass. For macroscopic BHs, quasi-stationary calculations show that this ingoing flux makes the horizon shrink adiabatically in time following the Stefan-Boltzmann law [13–16]. This is, the BH evaporates thermally.

The quanta emitted to infinity at late times are quantum-correlated with those particles that fall into the BH horizon [6, 7]. In other words, the Hawking radiation is entangled with degrees of freedom of the quantum field inside the BH. If the result of BH evaporation leads to a spacetime with Penrose diagram given by Fig. 1, where the BH interior is completely excised, as conjectured originally in [5], then these correlations are lost after the evaporation, and the quantum field would evolve from an initial pure vacuum state to a mixed thermal state at late times. Based on this, some people (initially Hawking [17]) have suggested that BH evaporation leads to a unitarity problem in QFT, and speculated on how information may be restored in quantum gravity (see [18] for a review).

The BH evaporation picture of Fig. 1 was only conjectured based on the quasi-stationary/test field approximations, where the renormalized stress-energy tensor of the quantum fields $\langle T_{ab}\rangle$ which sources Einstein's equations is calculated on a fixed, classical BH background. To figure out the final state of gravitational collapse and address the information issue, the problem of backreaction must be taken more seriously, both in the interior and exterior regions of the classical collapsing matter. Despite that the vacuum energy/stresses of a quantum field are negligible for astrophysical (solar-mass) scales, they increase with spacetime curvature, roughly as $\hbar R^{abcd} R_{abcd}$[1]. Therefore, in a spacelike hypersurface close to the classical curvature singularity during gravitational collapse, the effects of $\langle T_{ab}\rangle \neq 0$ are no longer negligible, and can drastically change the original causal structure of the spacetime! There is no reason to expect that Fig. 1 will hold. In particular, the classical trapped region may dissappear before

[1] In particular, the flux component in the exterior (Hawking radiation), while locally negligible, is a steady flux that may seriously affect the background geometry in the long term.

a spacelike/null singularity or Cauchy horizon ever forms[2]. What is, ultimately, the actual end point of gravitational collapse of stars?.

Although we lack a full theory of quantum gravity to properly answer the question of BH evaporation, one can still expect to obtain, from a qualitative viewpoint, a physically realistic and self-consistent solution for BH evaporation by solving the so-called semiclassical Einstein's equations. These are simply the ordinary Einstein's equations, but sourced by the quantum vacuum energy of quantum fields: $G_{ab} = 8\pi G\langle 0|T_{ab}|0\rangle$[3]. Interestingly, in QFT the classical energy conditions do not hold [23]. In fact, many examples of negative energy densities are known [24–27]. Consequently, singularity theorems in classical General Relativity do not apply anymore [28–32] (see [33] for a detailed historical treatment). Can semiclassical effects avoid the emergence of singularities in gravitational collapse, and lead to a regular BH spacetime where information is not 'lost'?.

2 The renormalized stress-energy tensor: a short review

Unfortunately, solving the semiclassical Einstein's equations is a formidable task. The problem resides in the difficulties for calculating the renormalized stress-energy tensor in general curved spacetimes, as a consequence of which $\langle 0|T_{ab}|0\rangle$ remains as an unknown functional of the spacetime metric and quantum state in the semiclassical field equations.

For definiteness, let $\phi(x)$ represent a massless, minimally coupled scalar field on a general curved spacetime [34–40], with metric g_{ab} and Levi-Civita connection ∇_a. As is well-known, Fock quantization gives rise to infinitely-many Hilbert space representations of the canonical commutation relations, which translates into infinitely-many possible choices of vacuum states in the quantum theory [41]. To each quantum state $|0\rangle$ we can assign a local notion of energy and stress for the quantum field, by calculating the *expectation value* of the stress-energy tensor, $\langle 0|T_{ab}|0\rangle$. Typically, the way to do this is to promote the classical expression, $T_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi$, to the quantum theory as an operator acting on the Fock space. Unfortunately, since the classical formula is quadratic in the fields, and fields in the quantum theory are well-defined only as operator-valued distributions [42] (not simply as operators), this formula involves taking the product of two distributions at the same spacetime point, which is mathematically ill-defined. As a consequence, the vacuum expectation value of this formal expression gives rise to several ultraviolet (UV) divergences, i.e. integrals and sums that are ill-defined in the high-frequency limit of the field modes.

To get a physically sensible result, a prescription to regularize and renormalize these divergences is required. Since UV divergences arise from the high-frequency

[2] The vacuum expectation value of the stress-energy tensor also tends to diverge near Cauchy horizons, see [19] and references therein.

[3] In this semiclassical regime fluctuations of the stress-energy tensor are assumed to be negligible compared to its mean value. This is not expected to be accurate near curvature singularities. Still, the semiclassical framework can give us useful insights, so it is worth exploring its implications even near curvature singularities in lack of a complete theory of quantum gravity (see e.g. [20–22]). In particular, it is important to study its self-consistency regarding the information loss issue.

component of the field modes, which only probe the geometry in the immediate vicinity of the spacetime point of interest, it seems natural to split the two points in the quadratic expression to regulate the UV divergences. Indeed, if the two field distributions in T_{ab} are evaluated at slightly separated points, the product is now mathematically well-defined. This is known as point-splitting regularization [43, 44]. Now, to get a finite quantity for the stress-energy tensor, the standard prescription is to subtract the singular behaviour of the two-point function as the two point merge,

$$\langle T_{ab}(x)\rangle := \lim_{x'\to x}\left\{\nabla_a\nabla_{b'}-\frac{1}{2}g_{ab'}g^{cd'}\nabla_c\nabla_{d'}\right\} \left[\langle\{\hat\phi(x),\hat\phi(x')\}\rangle-\langle\{\hat\phi(x),\hat\phi(x')\}\rangle_{\rm sing}\right], \tag{1}$$

where brackets denote symmetrization, and the limit is taken along a geodesic that connects the two points (which is unique for sufficiently close points [45]). In Minkowski space we can take $\langle\hat\phi(x)\hat\phi(x')\rangle_{\rm sing}=\langle 0_M|\hat\phi(x)\hat\phi(x')|0_M\rangle$, where $|0_M\rangle$ is the usual Minkowski vacuum. In any curved spacetime, the short-distance structure of the two-point function $\langle\hat\phi(x)\hat\phi(x')\rangle$ can be obtained as an asymptotic series by solving iteratively the field equation, and taking $\langle 0_M|\hat\phi(x)\hat\phi(x')|0_M\rangle$ as a starting point. This is the DeWitt-Schwinger expansion [34, 46].

This prescription manages to remove the UV divergences so long the two-point function has the "Hadamard" form:

$$\langle 0|\hat\phi(x)\hat\phi\left(x'\right)|0\rangle\sim\frac{U\left(x,x'\right)}{\sigma\left(x,x'\right)}+V\left(x,x'\right)\log\sigma\left(x,x'\right)+W\left(x,x'\right), \tag{2}$$

where $\sigma(x,x')$ represents the geodesic separation between the two points, which is zero for $x'=x$, and $U(x,x')$, $V(x,x')$, $W(x,x')$ are smooth. For this reason, only those quantum states satisfying (2) are regarded as physically admissible. Any spacetime that evolves from a static regime (like a collapsing star) admits such class of quantum states [47, 48].

Although the full two-point function depends on the choice of quantum state, the short-distance singular structure $\langle\hat\phi(x)\hat\phi(x')\rangle_{\rm sing}$ is independent of it, i.e. $U(x,x')$, $V(x,x')$ only depend on the local spacetime geometry, making this prescription useful in any curved spacetime (i.e. we don't need any fiducial state, like $|0_M\rangle$ in Minkowski). The regulated UV divergences in $\langle T_{ab}(x)\rangle$ can be further reabsorbed in the coupling constants of the semiclassical equations (cosmological constant, gravitational constant, etc), yielding a fully satisfactory renormalized theory [44] (see also [49]). Furthermore, it can be proven that (1) is the unique prescription (up to minor ambiguities) consistent with locality, causality, and stress-energy conservation [50].

While conceptually successful, the practical implementation of this prescription is technically challenging. This is because, in most cases, practical calculations require solving the field modes numerically, however the limit in (1) cannot be carried out using numerical techniques. To obtain explicit expressions for $\langle T_{ab}(x)\rangle$ we need more efficient prescriptions which can be adapted in a computer for numerical calculations.

Several methods are available today (mathematically equivalent to point-splitting) that can do this for some class of fixed spacetime metrics and vacuum states. For instance, for Fridman-Lemaitre-Robertson-Walker (FLRW) spacetimes in Cosmology, there is the so-called "adiabatic" method [51–63]. On the other hand, for different stationary BHs we have the "pragmatic mode-sum" method [12, 19, 64–71]; different "euclidean space" techniques [72–79]; and a generalization of the "adiabatic" method for the interior of the Schwarzschild BH [80].

These alternative prescriptions produce explicit expressions of the renormalized stress-energy tensor in particular spacetimes with high degree of symmetry. However, solving the semiclassical Einstein's equations requires knowledge of $\langle 0|T_{ab}|0\rangle$ as a function of the spacetime metric, but we still lack a general formula.

In view of these complications, and because the causal structure of a spherically-symmetric gravitational collapse is two-dimensional, a number of effective 1+1 models have alternatively been explored to gain further insights into the dynamics of BH evaporation (see [81] and references therein). In sharp contrast with the four-dimensional case, in 1+1 dimensions it is possible to derive an exact formula for the renormalized stress-energy tensor of conformal fields in any curved spacetime. Namely, if the spacetime metric is written as $ds^2_{(2)} = e^{2\rho} ds^2_{(2),\mathrm{flat}}$ for some conformal factor ρ (recall that every 2-dimensional spacetime is conformally flat) then, for a *conformally static vacuum* $|0_\rho\rangle$ we can write [9, 54, 81–85]

$$\langle 0_\rho|T^{(2)}_{ab}|0_\rho\rangle = \frac{\hbar R_{(2)}}{48\pi} g^{(2)}_{ab} + \frac{\hbar}{12\pi}\left[\nabla_a\nabla_b\rho + \nabla_a\rho\nabla_b\rho - \frac{1}{2} g^{(2)}_{ab}(\nabla_c\nabla^c\rho + \nabla_c\rho\nabla^c\rho)\right], \tag{3}$$

where $R_{(2)}$ is the 2-dimensional Ricci scalar. This observation can motivate us to reduce a QFT on a spherically-symmetric 3+1 spacetime down to an effective 1+1 theory, so as to take advantage of Eq. (3) or similar. This is known as *dimensional reduction*. Starting from the Einstein-Hilbert action and a minimally coupled scalar field f, working with a spherically-symmetric metric $ds^2 = ds^2_{(2)} + r^2 d\Omega^2$ with radial variable $r \equiv \kappa^{-1} e^{-\phi}$, and expanding the scalar field f in spherical harmonics, then the integration of the angular degrees of freedom in the action results in [81, 86]

$$S = \frac{1}{2\kappa^2}\int d^2x\sqrt{-g_{(2)}}e^{-2\phi}\left[\frac{R_{(2)}}{2} + \nabla_a\phi\nabla^a\phi + \kappa^2 e^{2\phi} - \nabla_a f_0 \nabla^a f_0\right] + \cdots, \tag{4}$$

where dots denote contributions $\ell \geq 1$ of the field's harmonic modes f_ℓ. The restriction to the $\ell = 0$ sector yields an effective 2-dimensional theory which is called the *s-wave approximation*. With this truncation, the 4-dimensional stress-energy tensor for f is given by $T_{ab} = \frac{T^{(\ell=0)}_{ab}}{4\pi r^2}$. If f_0 is now quantized, we may use (3) to obtain an s-wave approximation for $\langle T_{ab}\rangle$. (This is called the Polyakov approximation; a more accurate s-wave approximation which generalizes formula (3) by incorporating the coupling with ϕ in (4) is also available [81]). This strategy has been adopted to gain some insights in semiclassical gravity [87–91].

Unfortunately, the backreaction equations derived from this effective 2-dimensional model are still challenging to solve due to the explicit coupling of the scalar field f_0 with ϕ. To study BH formation and evaporation, slightly different toy models have been analyzed instead, which can be related to (4) under a "*near-horizon*" approximation. The most popular one is the Callan-Giddings-Harvey-Strominger "stringy" model (see [81] and references therein),

$$S_{CGHS} = \int d^2x\sqrt{-g_{(2)}}\left\{e^{-2\phi}\left[\frac{R_{(2)}}{2} + 2\nabla_a\phi\nabla^a\phi + 2\kappa^2\right] - \frac{1}{2}\nabla_a f_0\nabla^a f_0\right\}. \tag{5}$$

This theory can now be solved exactly. Most up-to-date analytical and numerical calculations indicate that, despite the formation of a singularity, BH evaporation in this framework does not involve information loss, in the sense that the S-matrix is unitary [92, 93].

Despite the interest of these results, the accumulation of so many hypothesis / approximations in this 2-dimensional approach raises some questions. To which extent does the 1+1 framework provide a reliable picture of gravitational collapse in 3+1 semiclassical gravity?

The reliability of the s-wave approximation (4) and the corresponding stress-energy formula $T_{ab} = \frac{T_{ab}^{(\ell=0)}}{4\pi r^2}$ lies in its ability to reproduce the $\ell = 0$ contribution of the Hawking luminosity at future null infinity, as well as the $\ell = 0$ part of the late-time negative-energy flux across the horizon [81]. However, it is known that dimensional reduction does not commute with quantization, not even in flat space [94, 95]. This is, although we can regard a classical scalar field f as an infinite collection of 2-d fields $\{f_\ell(t,r)\}_{\ell=0}^{\infty}$ by expanding it in spherical harmonics, $f = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} f_\ell(t,r)\frac{Y_{\ell m}(\theta,\phi)}{r}$, the quantization of f on a 3+1 spacetime is not equivalent to the quantization of the modes f_ℓ on the reduced 1+1 spacetime. In particular, using (1) separately for both f and f_ℓ, it can be shown that [94, 95]

$$\langle T_{ab}(t,r,\theta,\phi)\rangle \neq \sum_{\ell=0}^{\infty}\frac{(2\ell+1)}{4\pi r^2}\langle T_{ab,\ell}(t,r)\rangle = \frac{1}{4\pi r^2}\langle T_{ab,0}(t,r)\rangle + \cdots, \tag{6}$$

where dots denote $\ell \geq 1$ contributions. This result shows that dimensional reduction only provides a *naive* s-wave approximation of the actual renormalized stress-energy tensor of a quantum field. Therefore, although it does recover the s-wave part of the Hawking luminosity at infinity, it can dramatically fail to describe the s-wave part of $\langle T_{ab}\rangle$ in other spacetime regions where gravity is strong, like in a neighborhood of curvature singularities. Besides this issue, higher ℓ contributions become relevant near the curvature singularity, so even the actual s-wave approximation can fail to correctly capture semiclassical effects. Not to talk about the reliability of "near-horizon approximations" that are used to justify toy models like (5). Overall, my opinion is that any conclusion extracted or inspired from these 1+1 approximations should perhaps be taken with a grain of salt.

3 Some personal ideas and future prospects

The problematic discussed in this paper about the renormalized stress-energy tensor in 3+1 spacetimes is a long-standing one. Unfortunately, it does not look that a fully satisfactory answer is going to appear in the near future with current methodology (mostly concerned in obtaining numerical results for concrete gravitational backgrounds). If we aim/hope to make some progress in the backreaction problem, I believe that radically new and fresh ideas need to be discussed and proposed.

To my view, one interesting idea is to look for strategies that may allow us to generalize the well-known formula (3) to 3+1 spacetimes. All current derivations of this result rely crucially on the conformal flatness property of 1+1 spacetimes [9, 54, 82–85], and the lack of this property in 3+1 backgrounds has prevented, perhaps, a generalization of these calculations. We have recently deduced an explicit analytical expression for the renormalized stress-energy tensor in 1+1 dimensions, whose explicit calculation does not require assuming the hypothesis of conformal flatness, and which recovers the standard Eq. (3) in the conformal gauge [96]. This strategy can in principle be generalized to 3+1 spacetimes, although it is technically much more involved. We are currently exploring this computation and expect to reach a conclusion in the near future.

Another possible way to proceed is to reformulate the standard renormalization prescription for (1) in a manifest 3+1 covariant language. The choice of quantum state might be connected to a choice of spacetime foliation. If this is possible, it may be used to solve the semiclassical Einstein's equations using standard techniques in numerical relativity. I am not aware of any previous attempt to develop this idea.

To address the backreaction problem it can also be useful to deviate the focus from the explicit calculation of the renormalized stress-energy tensor. For instance, instead of focusing in getting $\langle T_{ab}\rangle$ as a functional of the metric, we can perhaps leave it as an unknown in the backreaction equations, and try to solve the full set of variables imposing some physically motivated constraints between them. This idea has been explored and developed for static and spherically-symmetric spacetimes in [97] using the trace anomaly [98–102] and an isotropy condition for the renormalized stress-energy components [10].

In connection with the above, we have also attempted to study a semiclassical version of the dynamical Oppenheimer-Snyder (OS) collapse model without solving explicitly the field equations [103]. Let me recall the causal structure of the classical OS gravitational collapse, given in the left panel of Fig. 2. This model consists of two solutions of the Einstein's equations that are matched through the surface of the star. On the one hand, the interior is modeled by a homogeneous and isotropic dust fluid, giving rise to a FLRW metric, whose scale factor describes the collapse of the star. On the other hand, the exterior vacuum is described by the Schwarzschild line element. The causal diagram is characterized by two apparent horizons, one in the interior (timelike) and another one in the exterior (the null event horizon), which enclose a trapped region with a curvature singularity in the future. The first marginally trapped surface forms when the radius of the star reaches the Schwarzschild radius. As time evolves further the compactness of the star increases monotonically in time, until the singularity is formed in finite time.

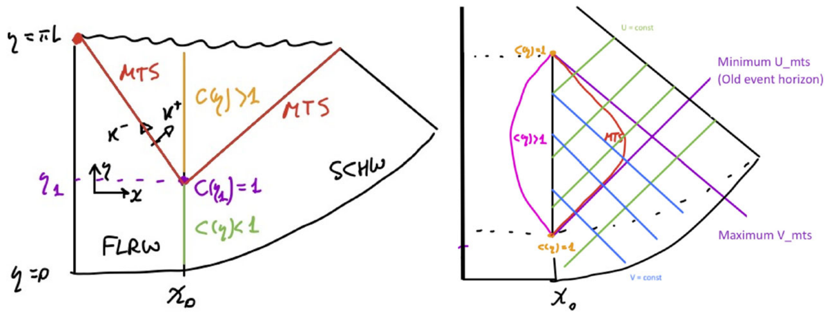

Fig. 2 Left: Penrose diagram for the causal structure of the Oppenheimer-Snyder gravitational collapse. Right: Penrose diagram inferred in [96] for a gravitational collapse in semiclassical gravity

In semiclassical gravity we have to solve again the field equations with the addition of the renormalized stress-energy tensor of a quantum field. In sharp contrast to the classical solution, the interior spacetime will fail to be homogeneous. Even though the classical metric in the interior is homogeneous and isotropic, to specify the vacuum state we need to impose some boundary conditions for the field modes on the surface of the star, which will in general introduce inhomogeneities through backreaction. This is, unlike in cosmological studies, the most natural vacuum state of the quantum field inside the star will only be isotropic, but not homogeneous. This make things significantly harder.

By working out the semiclassical Einstein's equations (using some results for $\langle T_{ab} \rangle$ in specific inhomogeneous FLRW metrics, and some approximations) we find that some solutions may admit a Penrose diagram which is topologically equivalent to the right panel of Fig. 2. In this diagram, the two apparent horizons of the classical OS model are now radially bounded from below and from above, and eventually reconnect again in the future, enclosing a compact and regular trapped region. Physically, the radius of the star (compactness) no longer decreases (increases) monotonically, but rather reaches a minimum (maximum) value, when the star undergoes a bounce. Interestingly, this bounce occurs at a time when the compactness of the star is again below 1. Thus, the trapped region must have fully vanished or "evaporated" before, when the radius of the star reaches back the Schwarzschild radius from below. Since the formation of trapped surfaces in this picture is bounded in a compact region of spacetime, in this solution information falling in the trapped region is not lost, it is only kept captive during a large amount of time, and it eventually goes out.

This global causal structure has been widely expected and conjectured for a long time in different frameworks of quantum gravity [104–108] (for a review see [109, 110] and references therein). So far, our findings indicate that a spacetime bounce after a trapped region "evaporates" can be predicted at the semiclassical level, during the late stages of gravitational collapse of stars. We hope to confirm these expectations with more accurate calculations.

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Declaration

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