GRAVITON MASS, QUINTESSENCE AND OSCILLATORY CHARACTER OF THE UNIVERSE EVOLUTION

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It is shown that using the relativistic field theory of gravity (RTG) and measured value of Ω_{tot} one can obtain the upper limit on the graviton mass with 95%C.L.: $m \leq 1.6 \cdot 10^{-66}$ [g]; within the (1σ) range its probable value is $m_g = 1.3 \cdot 10^{-66}$ [g]. It is pointed out that according to RTG the presence of the quintessence is necessary to explain the Universe accelerated expansion. Experimental data on the Universe age and dark matter density allow one to determine the range of possible values of the ν parameter in the equation of quintessence state and indicate characteristic time, which corresponds to the beginning and cessation of the accelerated expansion epoch, as well as the time period of the maximal expansion, which corresponds to the half-period of the oscillatory evolution of the Universe.

Показано, что, исходя из полевой релятивистской теории гравитации (РТГ) и измеренной величины Ω_{tot}, может быть получен на уровне 95% CL верхний предел на массу гравитона: $m \leq 1.6 \cdot 10^{-66}$ г, а в пределах (1σ) — ее вероятное значение $m_g = 1.3 \cdot 10^{-66}$ г. Указано, что, согласно РТГ, для объяснения ускоренного расширения Вселенной необходимо существование квинтэссенции. Использование экспериментальных данных о возрасте Вселенной и плотности холодной материи позволяет определить область возможных значений параметра ν в уравнении состояния квинтэссенции и указать времена, соответствующие началу и концу эпохи ускоренного расширения, а также время максимального расширения, соответствующее полупериоду осцилляционной эволюции Вселенной.

1. Introduction

The discovery of the Universe accelerated expansion [1] ÷ [3] has forced to revise many established ideas on the Universe content and character of its evolution. One of the popular explanations of the accelerated expansion is the assumption on the presence of the cosmological constant, Λ, that is equivalent to the existence of nonvanishing vacuum energy, ε_0, and related negative pressure $P_0 = -\varepsilon_0$ ([4] ÷ [8]). Such an assumption leads to unlimited inflationary expansion of the Universe (which rate, however, is at least by 60 orders of magnitude less than the initial inflationary expansion from the Plank scales assumed to solve the horizon problem and explain the flat geometry of the three-dimensional space). Another alternative explanation of the observed accelerated expansion involves the hypothesis of the existence of the special substance in the Universe — the quintessence [9] ÷ [11] with equation of state as follows:

$$P_q = -(1-\nu)\varepsilon_q \quad \left(0 < \nu < \frac{2}{3}\right), \tag{1}$$

where ε_q and P_q are the density of energy and quintessence pressure, correspondingly.

Relativistic field theory of gravity (RTG) [12, 13], which consider the gravitational field as a physical field in the Minkowsky space, is inconsistent with unlimited expansion of the Universe. Thus, as it was shown by V.P. Kalashnikov [14], the existence of quintessence (1) is essential to

explain the observed accelerated expansion of the Universe in framework of RTG. In this case, according to RTG, the cyclic evolution of the Universe will take place. The importance of this question and appearance of new experimental data impel us to come back to this problem.

In this paper it is shown that under the assumption of the quintessence existence the relativistic theory of gravity (RTG), in which there is no cosmological singularity and the flat character of the three-dimension space geometry is predicted unambiguously, leads to the fact that at $\nu > 0$ the current acceleration of the Universe expansion further should be changed into decreasing and then the expansion will stop; after that the "compression" up to some minimal value of the scale factor will start, then again the expansion cycle will take place. In §2 the basics of RTG are summarized. In §3 the consequences for evolution of the uniform and isotropic Universe following from these basic laws are scoped. In §4 we use recent experimental estimate on the cosmological parameter Ω_{tot} to put the 95% C.L. upper limit on the graviton mass, and its probable value is determined within the error range of (1σ). In §5 using this value we confine the region of the allowed values of the ν parameter, which agrees with the date on current age of the Universe and other cosmological parameters. In §6 we estimate the characteristic time, corresponding to the beginning and the cessation of currently observed expansion acceleration, as well as the possible period of the Universe oscillation.

2. Basics of RTG

RTG starts from the assumption that the gravitational field, as all other fields, develops in the Minkowsky space and that the tensor of the energy-momentum of all matter fields, including the gravitational field, is the source of this field. Such approach is concordant with modern gauge theories of the electroweak interactions and QCD, where conserving charges and their currents are the source of the vector fields. As the energy-momentum tensor is chosen to be the source of the gravitational field, the gravitational field itself should be described by symmetrical tensor of the second rank, $\varphi^{\mu\nu}$. Further this gives rise to a "geometrization" of the theory. The initial set of the RTG equations has the form [12, 13] ($\hbar = c = 1$):

$$(\gamma^{\alpha\beta} D_\alpha D_\beta + m_g^2) \cdot \tilde{\varphi}^{\mu\nu} = 16\pi G t^{\mu\nu}, \tag{2}$$

$$D_\nu \tilde{\varphi}^{\mu\nu} = 0, \tag{3}$$

where D_α is the covariant derivative in the Minkowsky space with metric tensor $\gamma_{\alpha\beta}$, $\tilde{\varphi}^{\mu\nu}$ and $t^{\mu\nu}$ are the densities of the gravitational field and total energy-momentum tensor, correspondingly:

$$\tilde{\varphi}^{\mu\nu} = \sqrt{-\gamma} \cdot \varphi^{\mu\nu}; \quad \gamma = \det(\gamma_{\mu\nu}) = \det(\tilde{\gamma}^{\mu\nu}), \quad t^{\mu\nu} = -2 \cdot \frac{\delta L}{\delta \gamma_{\mu\nu}},$$

where L is the density of the matter and gravitational field Lagrangian. Eq. (3) guarantees the conservation of the total energy-momentum tensor, singles out polarization states corresponding to the gravitons with the spin 2 and 0, and excludes the states with the spin 1 and $0'$ (analogously to the Lorentz condition, which excludes the photon with the spin 0). For equation set (2)–(3) to follow from the minimal action principle, i.e. for it to result from the Euler equations [1]

$$\frac{\delta L}{\delta \tilde{\varphi}^{\mu\nu}} = 0, \quad \frac{\delta L_M}{\delta \tilde{\varphi}_k} = 0, \tag{4}$$

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[1] Here $L_M(\tilde{\gamma}^{\mu\nu}, \tilde{\varphi}^{\mu\nu}, \tilde{\varphi}_k)$ is the matter Lagrangian density, which corresponds to the motion of the matter field φ_k in the gravitational field, L is the density of the full Lagrangian, which includes the Lagrangian of the gravitational field L_g itself.

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it is necessary and sufficient that the density of the $\tilde{\varphi}^{\mu\nu}$ tensor and density of the Minkowsky space metric tensor $\tilde{\gamma}^{\mu\nu}$ should enter into the matter Lagrangian in combination [12, 13]:

$$\tilde{\varphi}^{\mu\nu} + \tilde{\gamma}^{\mu\nu} = \tilde{g}^{\mu\nu}; \quad \tilde{g}^{\mu\nu} = \sqrt{-g} \cdot g^{\mu\nu}; \quad g = \det(\tilde{g}^{\mu\nu}) = \det g_{\mu\nu} \ .$$

Thus

$$L = L_g + L_M(\tilde{g}^{\mu\nu}, \tilde{\varphi}_k),$$

and the matter motion in the gravitational field looks like, as it would appear in the effective Riemann space with the metrics $g_{\mu\nu}$. It should be noted that all the crucial changes as compared with the Einstein general theory of relativity appear in the RTG due to the fact that the gravitational field is considered as the physical field in the Minkowsky space. Namely this approach essentially leads to the presence of the graviton. Gravity equations with nonvanishing graviton mass have been exploited early as well (see, for instance, [15]). However, they were written for inertial frames of reference only, since the special theory of relativity has been considered to be valid only for such frames. Therefore these equations naturally turned out not to be generally covariant, and due to this fact were not treated seriously. In its turn RTG takes into account the fact that in the Minkowsky space one can use any frames of reference, including accelerated ones, in which metric coefficients $\gamma_{\mu\nu}$ form the tensor with respect to arbitrary coordinate transformation. That is why Eqs. (2) and (3) are generally covariant.

The necessity to introduce the nonvanishing graviton mass in the gravity field theory is caused by the fact in the case of its absence the gravitational field $\varphi^{\mu\nu}$ (with the total energy-momentum tensor to be its source) obeys the group of gauge transformation [12, 13] (see also [16]), which presence leads to the fact that some physical observables (metric tensor of the effective Riemann space and its curvature among them) are dependent on the choice of the gauge. Introduction of the graviton mass breaks the gauge group and by this guarantees the independence of the physical observables on the any arbitrariness, meanwhile preserving the general covariant property of the gravity equations. The structure of the term in the gravitational field Lagrangian, which breaks gauge arbitrariness of the gravitational field by introducing a nonvanishing graviton mass, was unambiguously derived in papers [12, 13]. As the result, the equations of the gravitational field and matter takes the form

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{2}\left(\frac{m_g c}{\hbar}\right)^2 \left[g^{\mu\nu} + \left(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right] = 8\pi G T^{\mu\nu} \ , \tag{5}$$

$$D_\mu \tilde{g}^{\mu\nu} = 0, \tag{6}$$

where $R^{\mu\nu}$ and R are the corresponding curvatures in the effective Riemann space, and $T^{\mu\nu}$ is the energy-momentum tensor of the matter in the effective Riemann space

$$\sqrt{-g} \cdot T^{\mu\nu} = -2 \cdot \frac{\delta L_M}{\delta g_{\mu\nu}};$$

Eqs. (5)–(6) are generally covariant with respect to arbitrary coordinate transformations and form-invariant with respect tj the Lorentz transformations. Due to $m_g \neq 0$, the connection of the effective Riemann space with the metrics of the original Minkowsky space $\gamma_{\alpha\beta}$ in Eq. (5) is retained.

Eqs. (5)–(6) form the complete set of equations. It is necessary to stress out that here relation (6) is namely the **equation**, which is the consequence of the law of the total energy-momentum tensor conservation (or, that is equivalent, from Eq. (4) for the matter field), rather than any other additional condition. With current estimates on the probable graviton mass (see §4) Eqs. (5) and (6) are fully agree with all relativistic gravitational effects observed in the Solar system.

3. Evolution of the uniform and isotropic Universe according to RTG

For the uniform and isotropic Universe the interval between events in the effective Riemann space can be represented in the *Freedman-Robertson-Walker* metrics:

$$ds^2 = U(t) \cdot (dx^0)^2 - V(t) \cdot \left[\frac{dr^2}{1 - kr^2} + r^2(d\Theta^2 + \sin^2\Theta d\varphi^2) \right], \qquad (7)$$

where $k = 1, -1, 0$ for the closed (elliptic), open (hyperbolic), and flat (parabolic) Universe, correspondingly.

Eqs. (6) for metrics (7) takes the form:

$$\frac{\partial}{\partial t}\left(\frac{V^3}{U}\right) = 0, \qquad (8)$$

$$\frac{\partial}{\partial r}\left(r^2\sqrt{1 - kr^2}\right) - 2r(1 - kr^2)^{-1/2} = 0 . \qquad (9)$$

It follows from Eq. (8) that $V^3/U = \text{const}$, or

$$V = \beta U^{1/3}; \quad \beta = \text{const.} \qquad (10)$$

Eq. (9) is valid only for $k = 0$.

Thus, from (6) it follows immediately that **space geometry of the Universe has to be flat** (at that the initial inflationary expansion is not required). For the first time this result has been noticed in [17]. The fact that RTG results in the only (flat) solution ($k = 0$) for the uniform and isotropic Universe instead of three possible Freedman solutions is quite natural, as the set of equations (5)–(6) together with the equation of the state for $T^{\mu\nu}$ form a complete set of equations, which has the only solution.

Introducing the eigen time

$$d\tau = U^{1/2} \cdot dt$$

and notation of

$$a^2(\tau) = U^{1/3},$$

one can rewrite interval (7) in the following form

$$ds^2 = c^2 d\tau^2 - \beta a^2(\tau) \cdot \left[dr^2 + r^2(d\Theta^2 + \sin^2\Theta d\Phi^2) \right] . \qquad (11)$$

When expressions of (11) are used the equations of gravity (5) for uniform and isotropic Universe take the form [12, 13]

$$\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 = \frac{8\pi G}{3}\rho - \frac{1}{6}\left(\frac{m_g c^2}{\hbar}\right)^2\left(1 - \frac{3}{2\beta a^2} + \frac{1}{2a^6}\right), \qquad (12)$$

$$\frac{1}{a}\frac{d^2 a}{d\tau^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) - \frac{1}{6}\left(\frac{m_g c^2}{\hbar}\right)^2\left(1 - \frac{1}{a^6}\right), \qquad (13)$$

where ρ and P баre the total density of all types of matter and the pressure caused by the matter.

The β constant, which is determined by Eq. (10) and enters in Eq. (12), has a simple physical meaning. Considering the gravitational field $\varphi^{\mu\nu}$ as a physical field in the Minkowsky space it is necessary to require the fulfillment of the causality principle, which lies in the fact that the trajectory

of the particle subjected to the gravitational field action should not leave the limits of the light cone in the Minkowsky space. For interval (11) this condition leads to inequality

$$a^2(\tau) \cdot \left[a^4(\tau) - \beta\right] \leq 0.$$

Thus, the β constant determines the maximal value of the scale multiplier [12, 13].

$$a^4{}_{\max} = \beta.$$

It means that **according to RTG, the unlimited increase of the scale factor $a(\tau)$ is not possible**, i.e. the unlimited expansion of the Universe [2] is not possible.

According to Eq. (12) the nonvanishing graviton mass guarantee the fulfillment of this requirement in the case when the matter density ρ is decreasing function of the scale factor a. Minimal value of ρ corresponding to the cessation of the expansion ($da/d\tau = 0, \quad a \gg 1$), and accjrding to (12), is equal to

$$\rho_{\min} = \frac{1}{16\pi G} \left(\frac{m_g c^2}{\hbar}\right)^2. \tag{14}$$

If one rewrites the matter state equation in the form of (1), then, as it is known form the first law of the thermodynamics, it follows that the ρ dependence on the scale factor a will be as follows

$$\rho = \frac{\text{const}}{a^{3\nu}}, \tag{15}$$

where $\nu = 4/3$ for the relativistic matter (radiation and "light" neutrino) and $\nu = 1$ for baryon matter and dark cold matter. According to (13) the dark matter and radiation should lead to the expansion deceleration. To explain the observed acceleration it is necessary to assume the presence of the "dark" energy $\mathcal{E}_x$ in the Universe with

$$\left(\rho_x + \frac{3P_x}{c^2}\right) < 0 \ .$$

In this case from equation of state (1) one gets

$$\rho_x + \frac{3P_x}{c^2} = -2\rho_x \cdot \left(1 - \frac{3}{2}\nu\right),$$

and to have the acceleration it is necessary that

$$0 \leq \nu < \frac{2}{3}.$$

Value of $\nu = 0$ corresponds to the presence of the vacuum energy with the density $\mathcal{E}_{\text{vac}} = \rho_{\text{vac}} \cdot c^2$ and $P_{\text{vac}} = -\mathcal{E}_{\text{vac}}$. In this case ρ_{vac} does not depend on the scale factor, and for $\rho_{\text{vac}} > \rho_{\min}$ (where $\rho_{\min}$ is determined by Eq. (14)) the Universe expansion, according to (12), (13), is unlimited. Thus, the relativistic theory of gravity in the Minkowsky space is inconsistent with the presence of the vacuum energy $\varepsilon_{\text{vac}} \neq 0$. This is quite natural, since in the flat space the density of the vacuum energy can not be different from zero. From the point of view of RTG the acceleration of the Universe

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[2] We adhere to the traditional notation of the Universe "expansion", though, in reality, the Universe is infinite: the r coordinate in interval (17) varies in the range $0 < r < \infty$. The increase of the distance between galaxies, detected by means of the red shift and interpreted as the Doppler effect, is the consequence of the fact that the light signal emission from remote galaxies takes place in the gravitational field, which is more strong, than that one in the moment when the observer receives the signal.

expansion can be explained only by existence of quintessence (1) with parameter ν essentially greater than zero,

$$\nu > 0.$$

In this case the density of the dark matter energy should decrease with the increase of the scale factor according to the law in (15), and at values of the scale factor large enough the existence of the nonvanishing graviton mass should lead, according to Eqs. (12) and (13), to cessation of the Universe expansion, which than will change into its compression. This compression, in its turn, should stop when some minimal value of the scale factor $a_{\min} \neq 0$ is reached. Indeed, due to the fact that the left-hand side of equation (12) is positive defined, the negative term in the right-hand side, which increases at $a \to 0$ proportionally to $m_g^2 a^{-6}$, should be compensated by the increase of the density ρ (taking place in the radiation-dominant stage and proportional to $\rho \sim 1/a^4$). This could happen only for $a_{\min} \neq 0$. After value $a = a_{\min}$ is reached, the new stage of the Universe expansion should start. Thus, the structure of the term proportional to m_g^2 in Eqs. (12) as provides the cancellation of the cosmological singularity so excludes the possibility of the infinite Universe expansion. In other words, according to RTG, due to the nonvanishing graviton mass the Universe evolution should proceed in the oscillating regime. The experimental data accumulated so far allow one to estimate the possible value of the graviton mass and basing on this value estimate the possible oscillation period.

4. Ω_{tot} and graviton mass estimate

In 1970, just after the discovery of the relic radiation, R.A. Syunyaev and Ya.B. Zeldovich had undertook the detailed quantitative analysis of the processes taking place during the period of the hydrogen recombination and separation of the relic radiation from the matter [18]. In particular, they showed that adiabatic perturbations (sonic waves) in plasma in the epoch of recombination should lead to the angle anisotropy of the observed relic radiation, and studying this anisotropy one can determine experimentally the values of some important cosmological parameters (see alsoв earlier paper by J. Silk [19] and subsequent development of this idea in [20, 21, 22].

The question of the required accuracy of the angle correlation measurements for relic radiation spectrum was discussed in details in [23, 24]. Among all other cosmological parameters, which values are directly determined from the angle characteristics of the relic radiation spectrum, there is the Ω_{tot}^0 value, which is the ratio of the total density of all matter types (ρ) to current value of the critical density ρ_c^0, i.e.

$$\Omega_{\text{tot}}^0 = \rho/\rho_c^0 \quad \left(\rho_c^0 = 3H_0^2/8\pi G\,,\right)$$

where H_0 is the current value of the Hubble constant [25]:

$$H_0 = h \cdot (9.778\,13 \cdot 10^9 \text{ years})^{-1}, \quad h = 0.71 \pm 0.07.$$

In paper by A.H. Jaffe et al. [26] one can find the combined analysis of the BOOMERANG-98 [27] and Maxima-1 [28] experiments, which involves the earlier data of the COBE DMR [29] experiments, as well as the data resulting from the observations of the $SN1a$ supernovas [1, 2] and large scale structures of the Universe [31]. The results of the analysis [26] show that average values of Ω_{tot} for combination of different experiments systematically exceed the value $\Omega_{\text{tot}} = 1$ (see **Table 1**, [26]).

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Table 1 (from paper by A.H. Jaffe et al., [26].

	Ω_{tot}^0
B98 +DMR	$1.15^{+0.10}_{-0.09}$
MAXIMA-1+DMR	$1.01^{+0.09}_{-0.09}$
B98+MAXIMA1+DMR	$1.11^{+0.07}_{-0.07}$
CMB + LSS	$1.11^{+0.05}_{-0.05}$
CMB + SN1a	$1.09^{+0.06}_{-0.05}$
CMB + SN1a+LSS	$1.06^{+0.04}_{-0.04}$

At the confidence level of 68% the Ω_{tot} value, according to [26], is equal to

$$\Omega_{\text{tot}}^0 = 1.11 \pm 0.07, \tag{16}$$

while, according to the inflationary theory of the early Universe [4] $\div$ [8], the Ω_{tot}^0 value has to be equal to unit with a high degree of accuracy. Therefore, though the results of [26] at 95% CL

$$\Omega_{\text{tot}} = 1.11^{+0.13}_{-0.12} \tag{17}$$

do not contradict to the value of $\Omega_{\text{tot}}^0 = 1$, the fact itself of systematical excess of average values $\Omega_{\text{tot}}^0 > 1$ seems to be rather designing. Indeed, dividing both sides of Eq. (12), recalculated to the current moment, by the modern value of the Hubble constant, H_0^2, one can obtain the relation for $(a \gg 1)$ as follows

$$\Omega_{\text{tot}}^0 = 1 + f^2/6, \tag{18}$$

where $f = m_g c^2 / \hbar H_0$. It is convenient to rewrite the f parameter in the form of the ratio of the graviton mass to the m_H^0 value, which could be referred to as **the Hubble mass**:

$$m_H^0 = \frac{\hbar H_0}{c^2} = 3.8 \cdot 10^{-66} \cdot h \text{ [g]},$$

$$\boldsymbol{f = m_g / m_H^0}. \tag{19}$$

Eqs. (18)-(19) immediately give the feeling of the possible order of magnitude of the graviton mass. From values (17) and expression(18) it follows that the upper limit on the graviton mass is as follows

$$m_g \leq 1.2 \cdot m_H^0 \quad (95\% \text{CL}).$$

At the same time for the confidence level of 68% this does not exclude that, according to (16) $\div$ (18), the graviton mass is

$$m_g = (0.8^{+0.2}_{-0.3}) \cdot m_H^0 .$$

Recent preliminary data of the WMAP experiment [31] allow one to make more accurate estimate on the graviton mass. According to these data

$$\Omega_{\text{tot}}^0 = 1.02 \pm 0.02. \tag{20}$$

From this fact it follows that at the (2σ) level $f^2/6 < 0.06$, i.e.

$$m_g \leq 0.6 \cdot m_H^0 = 1.6 \cdot 10^{-66} \text{ [g]} \quad (\text{for} \quad h = 0.71).$$

But within the accuracy of (1σ) the upper limit $\Omega_{\text{tot}}^0 = 1.04$ from (20) coincides with the lower value in (16). This does not exclude the possibility

$$\frac{f^2}{6} = 0.04 \quad \text{and} \quad m_g \approx 0.5 \cdot m_H^0 = 1.3 \cdot 10^{-66} \text{ [g]}. \tag{21}$$

This value of the graviton mass we will use for further estimates.

5. The Universe age and the bounds on the quintessence parameter ν

As the evolution of the scale multiplier a from its minimal value $a_{\min}$ to the Freedman evolution regime takes a negligible time, and the duration of the radiation-dominant stage of expansion is, at least, 4 orders of magnitude less then the current age of the Universe, the definition of the latter can be started immediately from the matter-dominant stage, assuming that the density of the cold matter (including baryons) is equal $\rho_m = \rho_m^0 \frac{1}{x^3}$, where ρ_m^0 is the current density, and the x parameter is the ratio of the scale factor $a(\tau)$ to its current value a_0:

$$x = a(\tau)/a_0.$$

Analogously, the quintessence density can be presented in the following form

$$\rho_q = \frac{\rho_q^0}{x^{3\nu}} \ ,$$

where ρ_q^0 is its current value. So, Eq. (12) takes the form

$$\left(\frac{1}{x} \cdot \frac{dx}{d\tau} \right)^2 = H_0^2 \cdot \left(\frac{\Omega_m^0}{x^3} + \frac{\Omega_q^0}{x^{3\nu}} - \frac{f^2}{6} \right), \tag{22}$$

where

$$\Omega_m^0 = \frac{\rho_m^0}{\rho_c^0} \quad \text{and} \quad \Omega_q^0 = \frac{\rho_q^0}{\rho_c^0}.$$

From Eq. (22) it follows that

$$d\tau = \frac{1}{H_0} \frac{x^{1/2} dx}{\sqrt{F(x)}} \ ,$$

where

$$F(x) = \Omega_m^0 + \Omega_q^0 \cdot x^{3(1-\nu)} - \frac{f^2}{6} x^3. \tag{23}$$

Thus, the current age of the Universe, t_0, is determined by the integral

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{x^{1/2} dx}{\sqrt{F(x)}} \ , \tag{24}$$

and moments of time, which correspond to the beginning (t_1) and the cessation (t_2) of the currently observed acceleration, are as follows:

$$t_{1(2)} = \frac{1}{H_0} \int_0^{x_1(x_2)} \frac{x^{1/2} dx}{\sqrt{F(x)}} \ ,$$

where x_1 and x_2 are the roots of the equation

$$\Omega_m^0 - 2\Omega_q^0 \left(1 - \frac{3}{2}\nu \right) x^{3(1-\nu)} + \frac{f^2}{3} x^3 = 0 \ ,$$

which corresponds to the zero acceleration value, see (13). The maximal time of the expansion (half-period of the oscillation, $T_0/2$) is determined by the analogous integral

$$T_0/2 = \frac{1}{H_0} \int_0^{x_{\max}} \frac{x^{1/2} dx}{\sqrt{F(x)}} \ ,$$

where $x_{\max}$ is the root of the equation

$$F(x_{\max}) = 0 . \tag{25}$$

The Universe age $t_0 = (13.7 \pm 0.2) \cdot 10^9$ years, estimated in [31], allows one to put bounds on the allowed region for the quintessence parameter ν for nonvanishing value of the graviton mass. As in the WMAP experiment [31] the value, measured directly, is $\omega_m = \Omega_m^0 \cdot h^2 = 0.135_{-0.009}^{+0.008}$, and $\Omega_q^0 = \Omega_\Lambda^0 = \Omega_{\rm tot} - \Omega_m^0$ (where for the chosen graviton mass (21) $\Omega_{\rm tot} = 1.04$, so the Universe age in the region of $\bar\omega_m - \Delta\omega_m < \omega_m < \bar\omega_m + \Delta\omega_m$ acquires, according to Eqs. (23)-(24), an additional dependence – the dependence on h and ν. Here one can determine the allowed region for the ν parameter, which corresponds to the interval [31] of the current Universe age [3] . This allowed parameter region is shown in **Fig. 1** [4] .

It is interesting to note that chosen interval for the current Universe age requires, for $0.64 < h < 0.67$, the existence of the quintessence with $6\,\nu_{\min} > 0$. In principle, the ν value can be determined, if the higher accuracy for the Ω_m^0, Ω_Λ^0 values (assuming $\Omega_\Lambda^0 = \Omega_q^0$) and value of the acceleration q_0 will be achieved. According to (13)

$$q_0 = \frac{\ddot a_0}{a_0 \cdot H_0^2} = \left(1 - \frac{3}{2} \cdot \nu\right) \cdot \Omega_q^0 - \frac{\Omega_m^0}{2} - \frac{f^2}{6}. \tag{26}$$
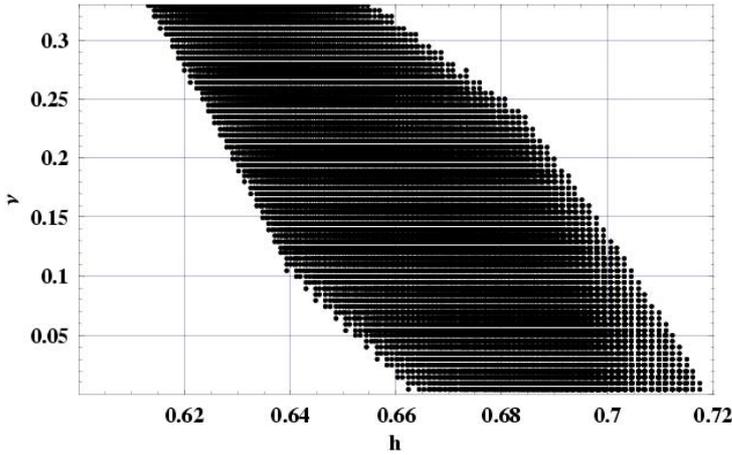

Figure 1. The range of the ν parameter variation for $\Omega_{\rm tot} = 1.04$; $0.126 \le \omega_m \le 0.143$, $13.5 < t_0 < 13.9$ GY.

Excluding the $f^2/6$ value from this relation, according to (18), one gets

$$\frac{3}{2} \cdot \nu \cdot \Omega_q^0 = 1 - q_0 - \frac{3}{2} \cdot \Omega_m^0. \tag{27}$$

The recent data do not contradict to the following condition:

$$q_0 < 1 - \frac{3}{2} \cdot \Omega_m^0,$$

which is necessary to fulfill the condition $\nu > 0$ in Eq. (27). If one adopts for the acceleration q_0 the value of $q_0 = 0.32 \pm 0.16$, then for average values $\Omega_m^0 = 0.27$ and $\Omega_q^0 = \Omega_\Lambda^0 = 0.73$ [31] from Eq. (27) one gets $\bar\nu = 0.25$, and within the range of (1σ): $0.05 < \nu < 0.43$.

6. Time moments corresponding to the beginning and the cessation of the accelerated expansion epoch. Oscillation period

Using the estimate of the possible graviton mass (21) and measured values of $\omega_m = \Omega_m^0 \cdot h^2 = 0.135_{-0.09}^{+0.08}$ [31], one can evaluate the current Universe age and time moments of the beginning

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[3] For distinctness we have chosen given interval for the Universe age as

$$13.5 \cdot 10^9 \le t_0 \le 13.9 \cdot 10^9 \text{ years}$$

despite the fact that its definition can not be considered as model independent (see N.P. Tkachenko, Preprint IHEP, 2003).

[4] According to the PDG data [25] the range for the h parameter should be confined as: $0.64 \le h \le 0.78$.

of the accelerated expansion (t_1) and its cessation (t_2) for different values of ν as a function of the h parameter (see **Fig. 2–3**).

In these figures one can see that the time of the accelerated expansion beginning (t_1) is not too sensitive to the graviton mass and ν parameter values, and it varies in the range of $(7 \div 8) \cdot 10^9$ years. Here the smallest value of $t_1 \approx 7 \cdot 10^9$ years corresponds to the largest value of the h parameter, which is compatible with the chosen interval of the Universe age. The appearance of the acceleration starting from $t_1 \approx 7 \cdot 10^9$ years explain well-known observable paradox, which lies in the fact that the Hubble expansion law becomes valid already for relatively small distances, distances of the order of few tens Mps. (see, for instance, [32]). With the ν increase the region of the h variation, which corresponds to the chosen interval of the Universe age, shifts to region of smaller values of h. For example, for $\nu = 0.05$ it is $0.65 \le h \le 0.71$, but for $\nu = 0.20$: $0.64 \le h \le 0.69$.

Time corresponding to the cessation of the accelerated expansion and the beginning of the deceleration, which leads to the cessation of the expansion, depends severely on the ν parameter value (see **Table 2**).
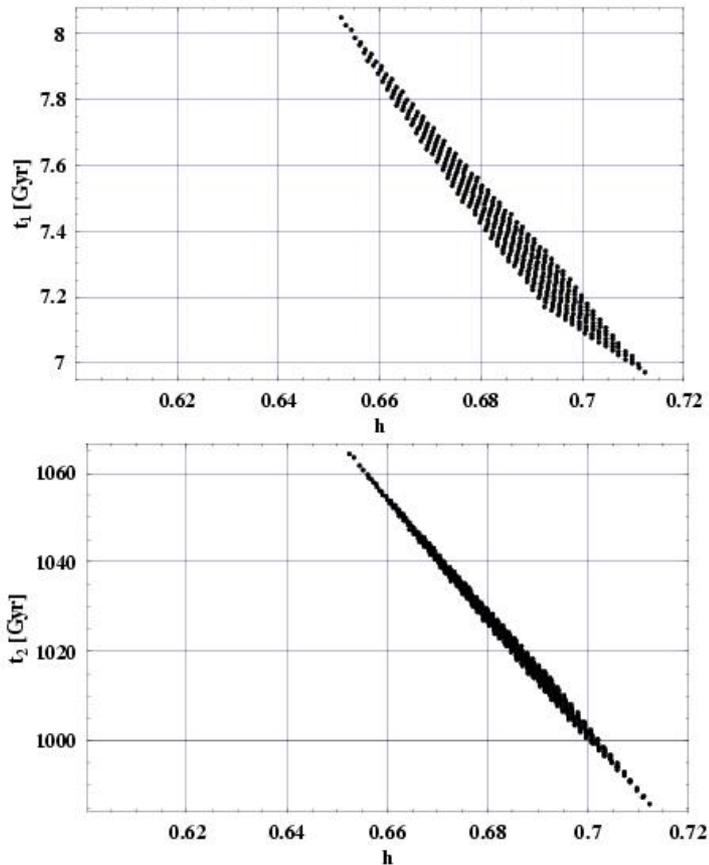

Figure 2. The dependences of the time of the beginning of the Universe accelerated expansion **(a)** and its cessation **(b)** on the h value for $\nu = 0.05$ at $\Omega_{\text{tot}} = 1.04$; $13.5 < t_0 < 13.9$ GY; $\nu = 0.05$; $0.126 < \omega_m < 0.143$.

Table 2. The time of the beginning of the Universe accelerated expansion, t_1, and the time of its cessation, t_2. The time of the maximal expansion (oscillation half-period), t_{max}, is given in billions of years.

	t_1	t_2	t_{max}
$\nu = 0.05$	7.0 - 8.2	980 - 1080	1220 - 1360
$\nu = 0.10$	7.0 - 8.2	440 - 485	620 - 685
$\nu = 0.15$	7.1 - 8.3	275 - 295	430 - 460
$\nu = 0.20$	7.1 - 8.3	190 - 205	325 - 347
$\nu = 0.25$	7.2 - 8.5	142 - 149	263 - 280
$\nu = 0.30$	7.5 - 8.7	109 - 113	227 - 235

As it was pointed out above, the field theory of gravity in the Minlowsky space (RTG) does not allow the unlimited expansion of the Universe. Therefore, from the point of view of RTG the only possibility to explain the acceleration observed is the existence of the quintessence or any other substance, which density decreases with the scale factor increase, but not faster than const/a^2). The cessation of the expansion follows from the existence of the nonvanishing graviton mass. Under this condition the minimal value of the matter density (14) is achieved.

The scale factor corresponding to the cessation of the expansion, $x_{\max}$, is determined by the root of Eq. (25), and at small ν with a high accuracy it is equal to

$$x_{\max} \simeq \left(\frac{\Omega_q^0}{f^2/6} \right)^{1/3\nu} .$$

In the given approximation it is related with the scale factor (x_2), which corresponds to the cessation of the accelerated expansion, as follows

$$x_2 = \left(1 - \frac{3}{2}\nu \right)^{1/3\nu} \cdot x_{\max} \approx \frac{x_{\max}}{\sqrt{e}}.$$

Figure 3. The same as in Fig. 2, but for $\nu = 0.20$.

The characteristic time, corresponding to the cessation of the expansion (oscillation half-period) for the chosen graviton mass (21) and for $\nu = 0.05$ ia about $1300 \cdot 10^9$ years, for $\nu = 0.10$ it is about $650 \cdot 10^9$ years, and for $\nu = 0.25$ – about $270 \cdot 10^9$ years (see **Fig. 4** and Table 2). It is interesting to note that the minimal density value (ρ_{min}) thus achieved does not depend on the time of the maximal expansion (t_{max}) and turns out not to be too small. Indeed, according to Eqs. (14), (18), and (19)

$$\frac{\rho_{min}}{\rho_c^0} \simeq \frac{f^2}{6} = \Omega_{tot}^0 - 1 \, ,$$

and for the chosen value of (21)

$$\rho_{min} \simeq 0.04 \rho_c^0 \, .$$

The idea of the oscillatory character of the Universe evolution was repeatedly adduced earlier, and it was stimulated mainly by the philosophic arguments (see, for instance, [33, 34]). One could expect such a regime, in principle, in the closed Freedman model with $\Omega_{\text{tot}} > 1$. However, there some problem: firts, the insurmountable difficulty related with the transition through the cosmological singularity, second, the arguments, connected with the increase of the entropy from cycle to
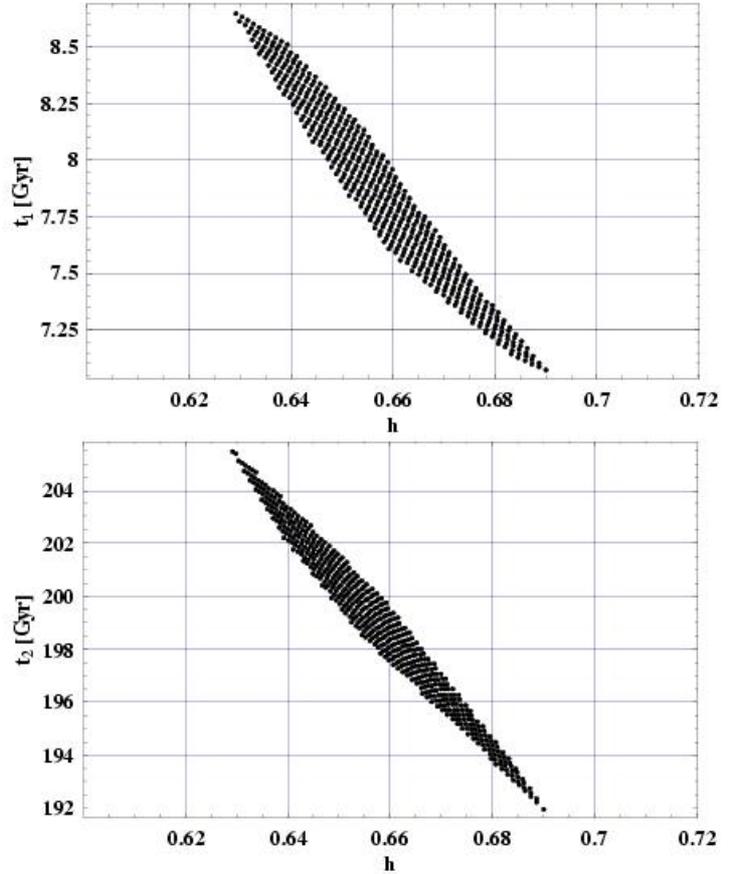
cycle [35]. In the RTG for the unlimited Universe the difficulties mentioned above are eliminated. For all that the oscillating behavior of the evolution for infinite number of preceding cycles provides the currently observed average homogeneity of the matter in the Universe at large scales.
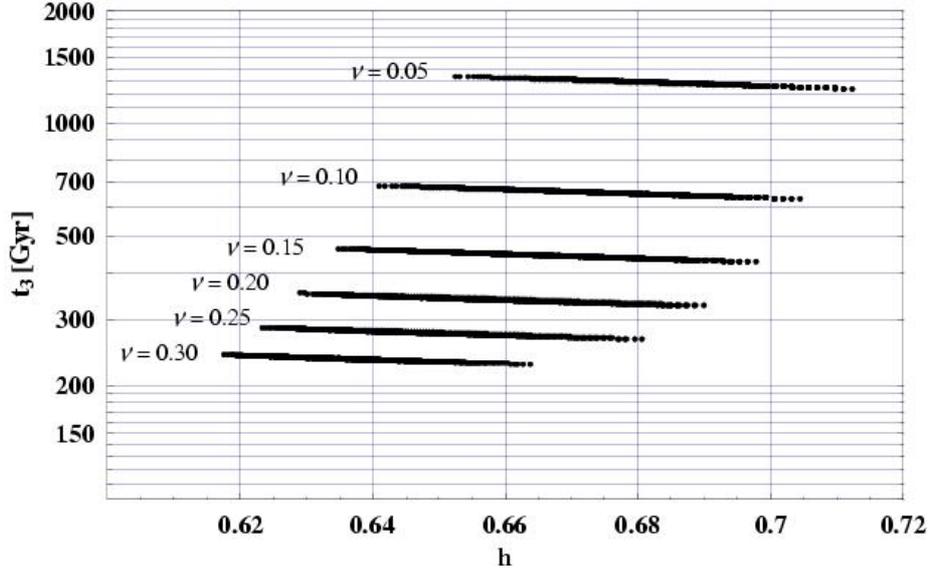

Figure 4. The dependence of the time of the maximal Universe expansion on the h value for different values of ν at $\Omega_{\text{tot}} = 1.04; 13.5 < t_0 < 13.9$ GY; $\nu = 0.05; 0.126 < \omega_m < 0.143$.

The attraction of the Universe oscillatory evolution was stressed out in recent paper [36]. Oscillatory regime is realized there by introducing the scalar field φ, which interacts with the matter, and using the idea of extra dimension. The authors suggest important arguments in favor of the fact that the phase of the accelerated expansion contributes to entropy conservation in the periodic evolution cycles. In RTG the oscillatory character of the Universe evolution is achieved as a result of fact that the gravitational field is considered as the physical field in the Minkowsky space and it is generated by the total energy-momentum tensor of all matter (see (5), (6)).

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References

[1]   Riess A.G. et al. Astron. J. **V.116**, 1009 (1999).

[2]   Perlmutter S. et al, Astrophys. J. **517**, 565 (1999).

[3]   P.M. Garnvich et al, Astrophys. J. **509** 74 (1998).

[4]   Guth A.H., Phys. Rev. **D23**, 347 (1981).

[5]   Kato S., MNRAS **195**, 467 (1981).

[6]   Linde A.D., Phys. Lett. **B108**, 389 (1982).

[7]   Albrecht A. and Steinhardt P.J., Phys. Rev. Lett. **48** P.1220 (1982).

[8]   Starobinskii A.A., Phys.Lett. **B117**, 175 (1982).

[9]   Caldwell R.R., Dave R. and Steinhardt P.J., Phys. Rev. Lett. **80** 1582 (1998).

[10]   Caldwell R.R. and Steinhardt P.J., Phys. Rev. **D57**, 6057 (1998).

[11]   Ostriker J.P. and Steinhardt P.J., Sci. Am. **284**, 46 (2001).

[12]   A.A. Logunov, M.A. Mestvirishvili *Relativistic theory of gravitation* (Nauka, Moscow, 1989).

[13]   A.A. Logunov *Graviataional field theory* (Nauka, Moscow, 2001); Logunov A.A. *The Theory of Gravity* (Nauka, Moscow 2001).

[14]   V.L. Kalashnikov, gr-qc/0109060; gr-qc/0202084.

[15]   Ogievetsky V.I. and Polubarinov I.V., Ann. of Phys. **35**, 167 (1965) and refs therein.

[16]   *Feynman Lectures on Gravitation* Addison-Wesley Publ. Comp., 1995.

[17]   A.A. Logunov, M.A. Mestvirishvili, TMF **61**, 327 (1984).

[18]   Sunyaev R.A. and Zeldovitch Ya.B., Astrophys. and Space Science, **7**, 1 (1970).

[19]   Silk J., Nature. **215** 1155 (1967); **21** 453 (1968); Astrophys. Journ. **151** 459 (1968).

[20]   Peebls P.J. and Yu J.T., Astrophys. Journ. ,**162**, 815 (1970).

[21]   Wilson M.L. and Silk J., Astrophys. Journ. **243**, 14 (1981).

[22]   Bond J.R. and Efstathiou G., Astrophys. Journ. **285**, 45 (1984); Mon Not. R. Astron. Soc. **226**, L655 (1987).

[23]   Yungman G. et al. Phys. Rev. Lett. **76** 1007 (1996.); Phys. Rev. **D54**, 1332 (1996).

[24]   Bond J.R., Efstathiou G.E. and Tegmark M. Mon Not. R. Astron. Soc. **291**, L33 (1997).

[25]   Particle Data Group. *Review of Particles Physics.* Phys. Rev. **D66**, N1-1 (2002).

[26]   Jaffe A.H., Ade P.A.R., Babbi A. et al., Phys. Rev. Lett. **86** 3475 (2001).

[27]   de Bernardis P. et al., Nature (London) **404**, 995 (2002).

[28]   Hanany S., Ade P., Babbi A. et al., Astrophys. J. **545**, L5 (2000).

[29]   Bennett C., Banday A., Gorski K.M. et al., Astrophys. J. **464**, L1 (1996).

[30]   Lange A.E. et al., astro-ph/0005004. [Phys. Rev. D...].
       Bond J.R. and Jaffe A.H. Philos, Trans. R. Soc. London **357**, 57 (1999).

[31]   Bennet C. et al. arXiv: astro-ph/0302207 v.2, 2003.

[32]   A.D. Chernin, UFN, **171**, 1153 (2001).

[33]   A.D. Sakharov, *Scientific papers* (Zentrcom., Moscow, 1985) pp. 269-299; JETF 83, 1233 (1982).

[34]   Aman J.M. and Markov M.A., TMF, **58** 163 (1984);
       Aman J.M. and Markov M.A., Ann. Phys. **155**, 333 (1984).

[35]   Tolman R.C. *Relativity, Thermodynamics and Cosmology.* (Oxford U. Press, Clarendon Press) 1934.

[36]   Steinhardt P.J. and Turok N. arXiv: hep-th/0111030, 2002. v.2.