LOW-ENERGY GAUGE COUPLINGS AND THE MASS GAP OF N = 1 SUPERSYMMETRY

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ABSTRACT

The low-energy values of the gauge coupling constants are very insensitive to those at a large energy scale if the corresponding theory is asymptotically divergent. This in general requires more than the standard three generations of quarks and leptons. We explore this scenario within the extension of the standard model provided by N = 1 supersymmetry with n quark and lepton generations. We introduce as a parameter the mass threshold M_{ss} of supersymmetric partners of ordinary particles. We find that, if all gauge theories enter the non-perturbative regime at an energy of the order of 10^{17} GeV, the low-energy values of gauge coupling constants are well reproduced with five quark and lepton generations and $M_{ss} < 5$ TeV. Furthermore, all fermion masses should be below 250-300 GeV.

A large fraction of theorists is becoming accustomed to the idea that superstrings at an energy scale of the order of the Planck mass will provide the ultimate theory of the world. However, there is a scenario according to which the values of the gauge coupling constants of the SU(3)×SU(2)×U(1) group at low energy (i.e., at about the Fermi scale) are largely insensitive to those at a very large scale and, therefore to some extent independent of our understanding of the physics up there[1]. This can happen if all known gauge theories behave at energies larger than the Fermi scale as asymptotically divergent theories and not as asymptotically free. This is simple to understand: let us consider the relation provided by the renormalization group between the gauge couplings at a low-energy scale (μ) and those at a larger one (Λ)

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\Lambda)} + c \ln(\Lambda/\mu) \tag{1}$$

The above expression is only correct at one-loop level but still suitable for our illustration purposes. The sign of the constant c in Eq. (1) determines whether the theory is asymptotically free (minus) or not (plus). In the first case, the value of the gauge coupling at low energy ($\alpha(\mu) \gg \alpha(\Lambda)$) is obtained as the difference of two large numbers: $1/\alpha(\Lambda)$ and $\ln(\Lambda/\mu)$. Indeed $\alpha(\Lambda) \ll \alpha(\mu)$ from which follows:

$$\frac{1}{\alpha(\mu)} \ll \frac{1}{\alpha(\Lambda)} \text{ or } \ln(\Lambda/\mu) \tag{2}$$

Any uncertainty in the value of $\alpha(\Lambda)$ will therefore be amplified when it is propagated to $\alpha(\mu)$. The opposite is true for divergent theories where $\alpha(\mu) \ll \alpha(\Lambda)$ and therefore $1/\alpha(\mu) \gg 1/\alpha(\Lambda)$, $1/\alpha(\mu) \sim O(c\ln \Lambda/\mu)$. In this case the value of $\alpha(\mu)$ is essentially determined by the second term in the right-hand side of Eq. (1) if one can neglect, with respect to it, $1/\alpha(\Lambda)$, i.e., if the theory is entering a non-perturbative regime ($\alpha(\Lambda) \sim O(1)$). The constant c appearing in Eq. (1) depends upon the gauge group and the particle content of the theory.

At "our" energies (of the order of the W mass) the SU(3) and SU(2) sectors are asymptotically <u>free</u>: one needs new particles in order to obtain the behaviour of an asymptotically divergent theory at large energies. The demand in terms of new quark and lepton generations within the standard model is very high (a total of 8-9 generations). A more economical solution is provided by the N = 1 supersymmetry version of the standard model when only a total of five quark and lepton generations is required[2]. Indeed, in this case, one gets "for free" the superpartners of ordinary particles. This also provides a solution of the well-known hierarchy problem. Without supersymmetry, the corrections to the curvature of the Higgs potential are of the form:

$$m^2 = m_0^2 + \alpha C \Lambda^2 \tag{3}$$

with C a model-dependent constant, and extraordinary fine tuning is required to keep m as low as the Fermi scale:

$$m \simeq \langle H \rangle_0 \simeq G_F^{-1/2} \ll \Lambda \tag{4}$$

Supersymmetry turns Eq. (3) into:

$$m^2 = m_0^2 + \alpha C \Delta M^2 \tag{5}$$

where ΔM is the mass gap between normal particles and their supersymmetric partners. With:

$$\Delta M \lesssim (\alpha G_F)^{-1/2} \simeq 3 \text{ TeV} \tag{6}$$

the inequality in Eq. (4) is not spoiled by higher order corrections.

The purpose of this lecture, which is based on a work done in collaboration with L. Maiani[3], is to analyze the low-energy values of gauge couplings when all gauge theories are divergent at a large energy scale within the N = 1 supersymmetric extension of the standard model. In particular, we consider an SU(3)×SU(2)×U(1), supersymmetric gauge theory with n supersymmetric generations and two Higgs chiral supermultiplets. We assume that supersymmetry is broken by soft[5] mass terms, which split squarks and sleptons from their fermionic counterparts and give masses to the gauginos. All these mass terms are independent of each other, but, for simplicity, we will assume that they are of the same order of magnitude. Thus, sparticles and gauginos start contributing to the β-functions above a single threshold, M_{ss}. Quarks and leptons take masses from the Yukawa couplings to the Higgs doublets. The latter couplings cannot diverge too early, which gives a general bound[4] of the order of 200-250 GeV to quark and lepton masses. To be definite, we assume that quarks and leptons of the yet unseen, n-3, generations have a mass above the W-mass and smaller than the "Fermi scale" Λ_F:

$$\Lambda_F \cong 250 \text{ GeV} \tag{7}$$

Similar bounds apply to the Higgs scalar masses. Excluding ad hoc cancellations, this implies the whole Higgs supermultiplet to be below Λ_F.

Present experiments determine the values of the gauge couplings at a mass scale of the order of M_W. For colour interactions, we take:

$$\alpha_3(M_W) = .12^{+.01}_{-.02} \tag{8}$$

corresponding to[5)]

$$\Lambda_{QCD} = 150^{+150}_{-100}\ \mathrm{MeV} \tag{9}$$

and

$$\alpha_{e.m.}(M_W) = .00772 \tag{10}$$

$$\sin^2\theta_W(M_W) = .231 \pm .007 \tag{11}$$

The latter value results from the average of the CDHS[6)] and CHARM[7)] results:

$$\sin^2\theta_W(M_W)\big|_{CDHS} = 0.227 \pm 0.005 \pm 0.005 \tag{12}$$

$$\sin^2\theta_W(M_W)\big|_{CHARM} = .236 \pm .006$$

Evolution of the above values from M_W to 250 GeV (Λ_F) has been computed with three or five quark and lepton generations, and the mean value of the two results has been taken as a reference value, to be compared with the $\alpha_i(\Lambda_F)$, i = 1,2,3, given by our calculation.

The reference values should be attained - within the errors - by the coupling constants running "backward" from the large energy scale down to Λ_F. Before they get to Λ_F they pass the mass threshold M_{ss} below which the supersymmetric partners do not contribute anymore to the beta function. Therefore, their values at Λ_F depend upon M_{ss}. The evolution has been done numerically using formulas which are correct up to two loops. The values of the coupling constant at Λ can be varied between 1 and 10 without affecting significantly the results. The relevant parameters are n (the number of generations), Λ the large energy scale and M_{ss}. n is essentially fixed to the first two integers for which colour interactions above M_{ss} are not asymptotically free to one loop, n = 5,6. Larger values would make colour interactions to diverge too early, for $M_{ss} \ll \Lambda$. For n = 5 or 6, we have varied independently Λ and M_{ss}, in the range:

$$10^{14}\,\mathrm{GeV} < \Lambda < 10^{19}\,\mathrm{GeV}$$
$$\Lambda_F < M_{ss} < 10^{7}\,\mathrm{GeV} \tag{13}$$

Our results for n = 5 generations are illustrated in Figs. 1 to 3. A good, simultaneous fit to the gauge couplings is obtained for:

$$\Lambda = 1 \cdot 10^{17}\,\mathrm{GeV}$$
$$M_{ss} = 2\,\mathrm{TeV} \tag{14}$$

The no-threshold situation[4)], $M_{ss} \approx \Lambda_F$, still leads to an acceptable solution. Increasing Λ, the curve giving $\sin^2\theta_W$ as a function of M_{ss} remains quite stable, while α_3 and $\alpha_{e.m.}$ shift both downwards. As their steepness is quite different, the triple coincidence is lost at some value of Λ. The opposite effect on α_3 and $\alpha_{e.m.}$ is produced when lowering Λ.

In conclusion, for n = 5, we can reproduce satisfactorily the values of the gauge couplings, for:

$$10^{16}\,\mathrm{GeV} \leq \Lambda \leq 5 \cdot 10^{17}\,\mathrm{GeV} \tag{15}$$

and for

$$M_{ss} \leq 5\,\mathrm{TeV} \tag{16}$$

No lower bound to M_{ss} results.

We find it quite satisfactory that a good fit to the low energy couplings requires a value of the mass-gap of N = 1 supersymmetry of the order given in Eq. (16), and well consistent with the value of M_{ss} needed to stabilize the low energy scale, Eq. (6).

The case n = 6 also admits a solution but only for considerably larger values of M_{ss}:

$$M_{ss} \simeq 10^3\,\mathrm{TeV} \tag{17}$$

The reason is that colour interactions diverge more rapidly above M_{ss}. This must be compensated by delaying the mass-scale where all particles contribute, i.e., by increasing M_{ss}. In contrast with the previous case, the value Eq. (17) seems too large to be reconciled with the bound in Eq. (6).

In conclusion, the idea that gauge interactions are not asymptotically free at large energy, but rather diverge at a common energy scale Λ and that $N = 1$ supersymmetry holds below, leads to a good fit to the low-energy gauge couplings for five supersymmetric generations. At the same time, a very significant upper bound can be put on the mass gap of $N = 1$ supersymmetry, M_{ss}, less than a few TeV. This is remarkably consistent with what is needed for the hierarchy problem, and very promising for the next generation of accelerators. The range of values which we find for Λ is close enough to the Planck mass, not to exclude that the strong regime of the gauge interactions is related to a further unification which includes gravity.

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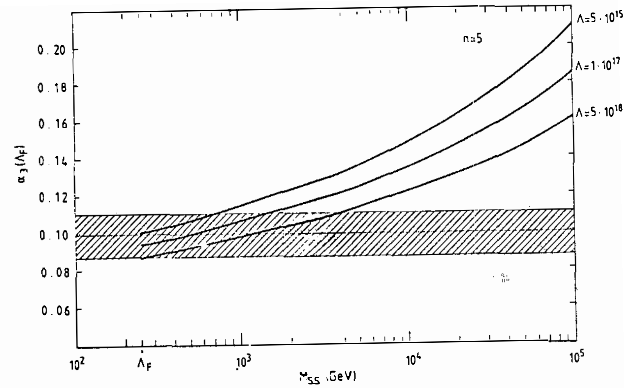
FIGURE CAPTIONS

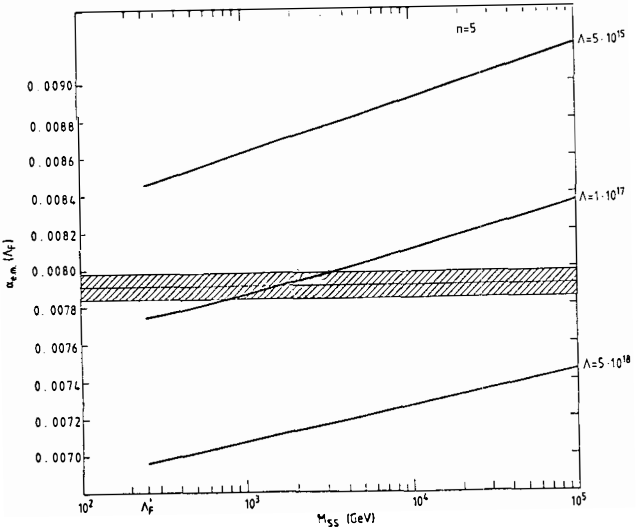
Fig. 1 The strong coupling constant, α_3, at Λ_F = 250 GeV as a function of the supersymmetry threshold M_{ss} for five generations of quarks and leptons. Values of the ultra-violet cut-off, Λ, are indicated. The shaded region represents the range allowed by present experimental data; see the Table.

Fig. 2 Same as Fig. 1, for the electromagnetic coupling, $\alpha_{e.m.}$.

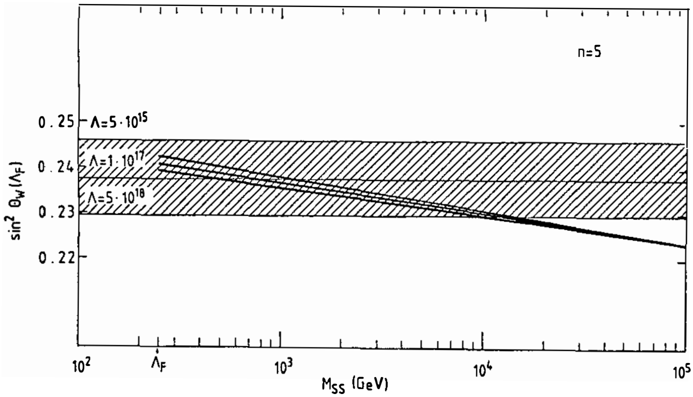
Fig. 3 Same as Fig. 1, for the weak mixing parameter, $\sin^2\theta_W$.



- Figure 1 -



- Figure 2 -


n=5
0.25
0.24
0.23
0.22
Λ=5·10^15
Λ=1·10^17
Λ=5·10^18
sin² θ_W(Λ_F)
10²
Λ_F
10³
10⁴
10⁵
M_SS (GeV)

- Figure 3 -