

## Triplet states in Lead isotopes

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### Introduction

In atomic nuclei with even numbers of neutrons and protons, the low-lying excitation spectrum is generally formed by nucleon pair breaking and nuclear vibrations or rotations. However, for certain numbers of protons and neutrons, a subtle rearrangement of only a few nucleons among the orbitals at the Fermi surface can result in microscopic shape change. In mean-field models, the  $0^+$  states observed at low energies are associated with coexisting energy minima which appear for different values of the axial quadrupole moment. The energies of different shape configurations are calculated using a nuclear potential with energies of the single particle orbitals depending on the deformation. In the context of the nuclear shell model, the emergence of low-lying excited  $0^+$  states is traced back to the proton particle-hole excitation across the  $Z = 82$  closed shell. The residual interaction between protons and neutrons is enhanced due to this cross-shell excitation, resulting in the lowering of the excited  $0^+$  states. In the vicinity of the  $N = 104$  mid-shell, the effect is strengthened and has a stronger impact on excitation energies [1]. The fact that this situation takes place in several lead isotopes makes this region more attractive to understand the phenomenon of coexistence experimentally and theoretically [2, 3]. We have done calculations for  $^{184-190}Pb$  using the Relativistic Hartree Bogoliubov (RHB) formalism [4] with finite-range (DD-ME2 and NL3\*) and zero-range (DD-PC1) interaction.

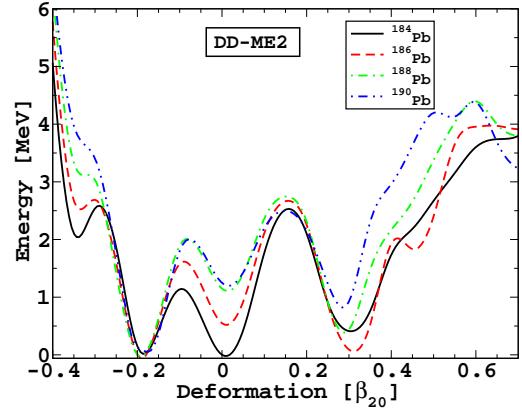


FIG. 1: The potential energy surface of  $^{184-190}Pb$ .

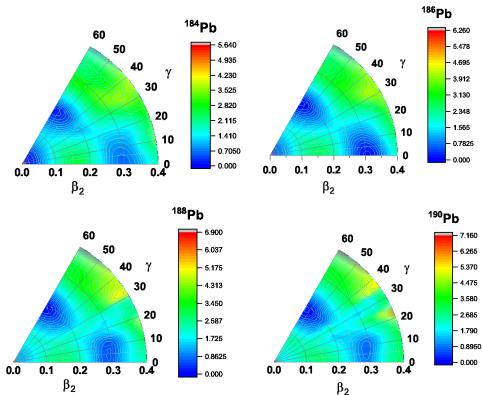


FIG. 2: The potential energy surfaces of  $^{184-190}Pb$  isotopes. The color scale shown at the right has the unit of MeV, and scaled such that the ground state has a zero MeV energy

### Results and Discussion

The energy surface as a function of quadrupole deformation parameters is obtained by solving the RHB equation with

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constraints on the axial and triaxial mass quadrupole moments. The method of quadratic constraints uses an unrestricted variation of the function

$$\langle \hat{H} \rangle + \sum_{\mu=0,2} C_{2\mu} (\langle \hat{Q}_{2\mu} \rangle - q_{2\mu})^2$$

where  $\langle \hat{H} \rangle$  is the total energy and  $\langle \hat{Q}_{2\mu} \rangle$  denotes the expectation value of the mass quadrupole operators

$$\hat{Q}_{20} = 2z^2 - x^2 - y^2$$

$$\hat{Q}_{22} = x^2 - y^2$$

$q_{2\mu}$  is the constrained value of the multipole moment and  $C_{2\mu}$  the corresponding stiffness constant. Moreover, the quadratic constraint adds an extra force term  $\sum_{\mu=0,2} \lambda_\mu \hat{Q}_{2\mu}$  to the system, where  $\lambda_\mu = 2C_{2\mu}(\langle \hat{Q}_{2\mu} \rangle - q_{2\mu})$ . Such a term is necessary to force the system to a point in deformation space different from the stationary point. We have done the axial calculations for  $^{178-220}Pb$  isotopes to study their ground state properties (binding energy, deformation,  $r_{ch}$ ,  $S_{2n}$ , etc), not presented here, among which, we found triplet states in  $^{184-190}Pb$ . We tried to understand the phenomena of triple state shape coexistence in these nuclei in terms of potential energy surface (PES). We did both axial and triaxial calculations for these nuclei. The Fig. 1 shows the PES as a function of quadrupole deformation using the DD-ME2 interaction for  $^{184-190}Pb$ . The three minimas at different  $\beta_2$ , corresponding to three different shapes, reflects the shape coexistence in these isotopes. These triplet states in all  $^{184-190}Pb$  isotopes are in accordance with the experimental findings [2] and with other theoretical models ([1] and the references therein). It is interesting to notice that the oblate minima becomes more pronounced with increasing neutron number from  $N=104-108$ . The same behaviour is shown with other interactions parameters (DD-PC1 and NL3\*) not shown here.

The results using the triaxial basis are displayed as contour plots in the (mass)  $\beta$ - $\gamma$  plane

TABLE I: Location of the ground state indicated by  $(\beta, \gamma^\circ)$  and difference in energy from the ground state for  $^{184-190}Pb$  isotopes using DD-ME2 interaction.

Nuclei	DD-ME2				
	Sph.	Pro.	Obl.	$E_0 - E_I$	$E_0 - E_{II}$
$^{184}Pb$	$(0,0^\circ)$	$(0.3,0^\circ)$	$(0.2,60^\circ)$	0.291	0.641
$^{186}Pb$	$(0,0^\circ)$	$(0.3,0^\circ)$	$(0.2,60^\circ)$	0.092	0.685
$^{188}Pb$	$(0,0^\circ)$	$(0.3,5^\circ)$	$(0.2,60^\circ)$	0.621	1.294
$^{190}Pb$	$(0,0^\circ)$	$(0.3,10^\circ)$	$(0.2,60^\circ)$	0.831	1.449

in the Fig. 2. Table I shows the exact location of triplet states in the  $^{184-190}Pb$  as can be analysed from Fig. 2. We also calculated the energy difference (in MeV) between the ground state and the first two excited states as shown in the Table I, which lies within 1.5 MeV energy range, clearly shows the presence of shape coexistence in these isotopes. We have shown only data obtained from DD-ME2 interaction, same has been observed with DD-PC1 interaction not presented here.

## Conclusion

Axial and triaxial calculations within RHB have been done to study the shape coexistence phenomena in the lead isotopes. Triplet states have been found in the  $^{184-190}Pb$  which are in accordance with the experimental and other theoretical observations. The energy difference (in MeV) between the first two excited states also gives the evidence for the same.

## References

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