Design and simulation of a helical and a rectilinear channel with vacuum cavities for 6-dimensional muon cooling

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Abstract. Reducing the 6D emittance is an essential step for achieving high luminosity in a Muon Collider. Given the short lifetime of a muon particle, ionization cooling is the only practical method that can be realized. Here, we explore two individual ionization cooling schemes. In the first, emittance reduction is achieved within a helical channel with wedge absorbers for cooling and tilted solenoids for emittance exchange. In the second scheme, cooling is achieved by using a rectilinear channel with tilted solenoids. The latter configuration is expected to solve for many of the engineering challenges of a conventional helical channel. We numerically examine the performance of both channels and review the conductor current densities and absorber length requirements for all of the simulated scenarios.

1. Introduction

A key technical challenge in the development of a Muon Collider [1] is that the phase space of the beam that comes from pion decay greatly exceeds the acceptance of downstream accelerator systems and therefore, a cooling channel is required. Given the short lifetime of a muon particle, ionization cooling is the only practical method that can be realized [2]. Over the past decade much progress has been made in the design and simulation of 6D cooling lattices based on emittance exchange. This is generally accomplished by using a wedge shaped absorber or a differential path length in a region with non-zero dispersion. Particles with higher energies pass through more material than particles with lower energy as a result of dispersion, eventually leading to reduction of both longitudinal and transverse emittance. In particular, ring-shaped coolers that use tilted solenoids to generate dispersion have been shown to provide an impressive two orders of magnitude reduction of the normalized 6D phase-space volume with a transmission above 50% [3]. This design later evolved into a helical channel referred to as a "Guggenheim" in order to avoid serious problems with injection and extraction of large emittance beams. It was found that the performance of a Guggenheim lattice is comparable to the original cooler ring [4].

Although a helical lattice can be advantageous in paper, still many questions remain. For instance, the beam can be intercepted by fringe fields from the nearby solenoids of neighboring helix turns or engineering constraints may arise at the last stages as the radius of curvature becomes less than 1.5 m. To further relax the aforementioned problems we design and simulate here a rectilinear FOFO snake channel which has been proposed recently [5] and show that it can offer the same performance as the Guggenheim [6]. In particular, we show that by applying a new tapering scheme [7] in which numerous parameters of the structure change from stage to stage based on the emittance reduction rate and transmission, we can achieve performances above 40% (including muon decays) and our results are consistent with the baseline cooling requirements for a Muon Collider. Our rf voltages are also relatively low, for instance the 325 MHz voltage does not exceed 20 MV/m in any case.

2. Alternative 6-dimensional cooling systems

For a Muon Collider, it is desirable to reduce the transverse and longitudinal emittance. Below we explore two individual 6D cooling schemes to achieve this:

2.1. Helical cooling channel

The idea for the 6D cooling channel described here originated from the conventional Reverse Focus Focus (RFOFO) cooling ring [3]. The conventional ring design employs a single cell for both transverse cooling and emittance exchange. The overall layout of the ring is shown in Fig. 1(a). The ring consists of 12 identical cells with two or four coils (yellow) in each cell with opposite polarity to provide transverse focusing. The relative amount of cooling can be adjusted by changing the opening angle and transverse location of the wedge. A series of rf cavities (dark red) are used to restore the momentum along the longitudinal axis. The dispersion necessary for emittance exchange is provided from the bend field generated by tilting the axes of the solenoids above and below the orbital midplane. The ring design evolved into a helical channel [see Fig. 1(b)] commonly referred to as a "Guggenheim" in order to avoid serious problems with injection of large beams. This lattice inherits most of the parameters of the ring with little or no change and the only difference is the change of elevation of the elements [4].

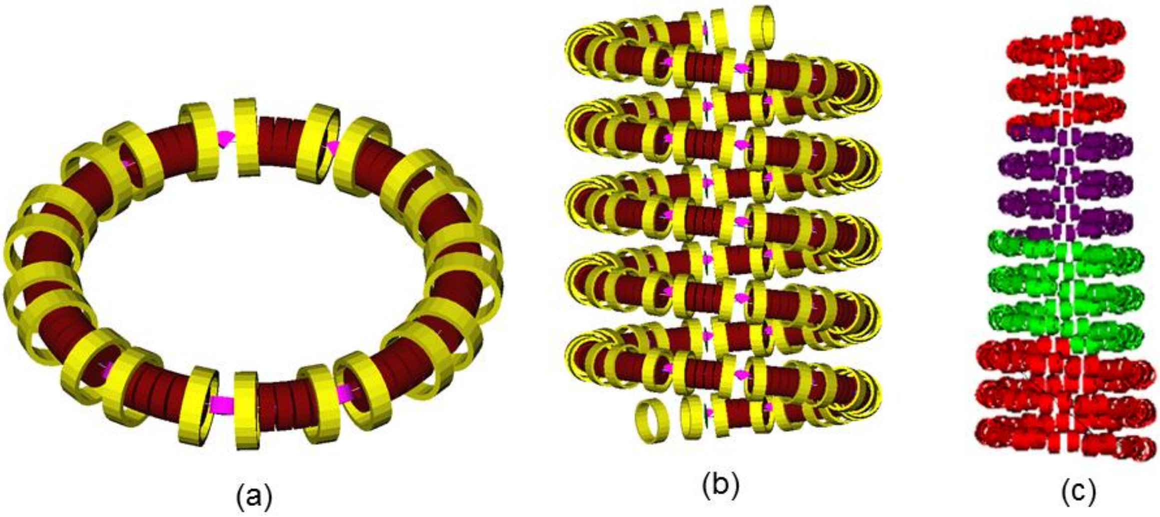

Figure 1. Progress of 6D cooling lattices: (a) RFOFO ring; (b) Helical channel (known as Guggenheim), and (c) tapered helical channel. The common feature of all schemes is that coils are tilted in order to generate the required dispersion.

The main disadvantage of the lattices shown in Fig. 1(a) and Fig. 1(b) is that they used a fixed set of parameters for the radius of curvature, cell length, rf frequency, amount of the absorbing material and magnetic field strength, consequently opposing any cooling beyond the equilibrium emittance. Note that the equilibrium emittance is the minimum emittance that can be achieved for a given absorber in a presumed focusing field and is reached when the cooling rate equals the heating rate. To further improve the performance of the cooling lattice, we propose here a tapered channel in which the aforementioned parameters change gradually with distance. Recent studies [4] indicated that such configuration would enhance the lattice performance substantially.

At the first stage of the tapered channel the focusing will be relatively weak to avoid excessive angular divergence that can arise from the large transverse emittance of the initial muon beam. However, the weak focusing implies that the beta function and thus the equilibrium emittance are also relatively large, so the transverse cooling weakens as the limit is approached. To avoid this, this stage is terminated and we couple into the next stage that has a lower beta. This is achieved by simultaneously scaling down the cell dimensions and raising the strength of the on-axis solenoidal field. As a result this will produce a piecewise constant multi-stage "ziggurat" shaped channel [see

Fig. 1(c)] where each stage will be a fixed-radius helical channel consisting of a series of identical cells similar to the one shown in Fig. 1(b). As we will demonstrate shortly, 17 tapered stages are enough to cool towards the baseline requirements of a Muon Collider.

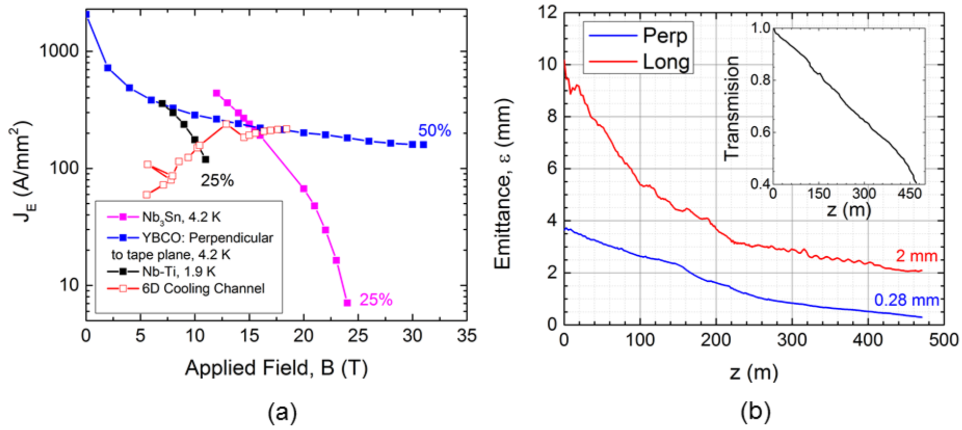

Figure 2. Characteristics of a helical channel: (a) magnet requirements, and (b) lattice performance.

Figure 2(a) shows the current densities vs. the maximum local fields in the coils used. For these estimates we have used published 'engineering' current densities, multiplied by factors to allow for the required support structure, the need for stabilizing copper, and the filling factor for a real conductor. Although with inclusion of reasonable safety factors, the needed fields are consistent with the critical limits of existing conductor technology, the last four stages are barely within the limits of YBCO. Simulations of the channel performance where done using the ICOOL code [8] with 100,000 particles. The transverse, longitudinal emittance and transmission including muon decays as a function of distance along stages 1 to 17 are shown in Fig. 2(b). The simulation produced a transverse emittance of 0.28 mm and a longitudinal emittance equal to 2 mm, which is close to the desired values for a Muon Collider.

2.2 Rectilinear cooling channel

A schematic of our proposed cooling system is shown in Fig. 3. Note that the coils (yellow) are not evenly spaced; those on either side of the absorber are closer in order to increase the focusing at the wedge absorber (magenta). In order to produce dispersion, they are tilted by 1.1-1.3^0. The absorber is wedge shaped so that higher momentum particles go through the thicker part. Each lattice cell [see Fig. 3(b)] contains the same number of rf cavities (dark red) which, depending on the stage, can be 4, 5, or 6. In contrast to the previous channel, the rf frequencies are either 325 MHz or 650 MHz. Those frequencies are chosen in order to match with the initial linac of Project X.

As before, the channel is tapered and 16 stages are designed and simulated with the ICOOL code. Each stage is designed to produce a smaller beta and thus has a stronger axial magnetic field compared to the previous one. The maximum field on the coil and the corresponding hoop stress are shown in Fig. 4(a). The transverse, longitudinal emittance and transmission including muon decays as a function of distance along stages 1 to 16 are shown in Fig. 4(b). The simulation produced a transverse emittance of 0.32 mm and a longitudinal emittance equal to 1.6 mm. While the longitudinal emittance is within the specifications for a Muon Collider it is likely that an additional stage needs to be added at the end of the channel to cool to 0.30 mm which is the current baseline parameter for the transverse emittance. The transmission without decays is above 60%. If space-charge is included in the simulation an additional 10% of losses is predicted [9]. The quoted emittance values are normalized rms.

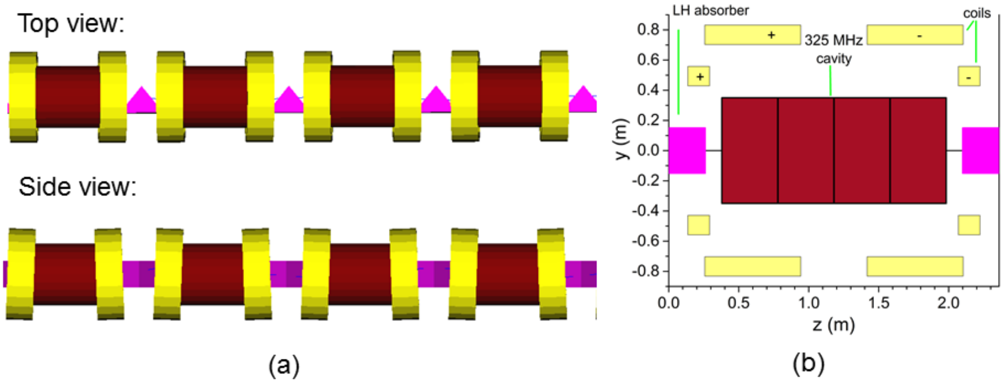

Figure 3. Visualization of a rectilinear channel: (a) in 3D view, (b) single cell.

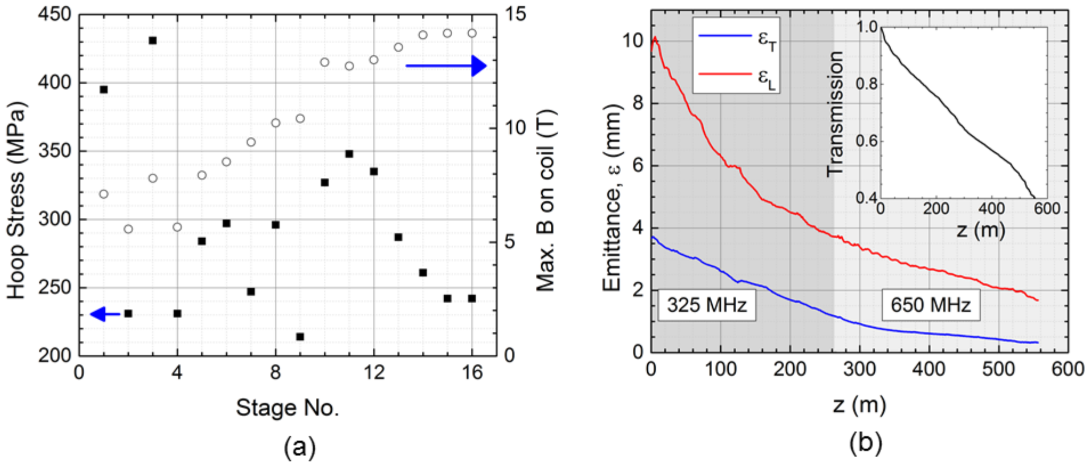

Figure 4. Evaluation of a rectilinear 6D cooling channel: (a) Hoop stress (left vertical axis) and maximum field on the coil (right vertical axis) for each stage, and (b) evolution of emittance and transmission along the channel.

3. Summary

Cooling large emittance muon beams is an essential step for a Muon Collider. A rectilinear FOFO snake channel may reduce the engineering challenges compared to conventional helical structures (like in a Guggenheim helix). As this study demonstrated, by implementing a tapering concept, the desired emittances for a Muon Collider can be almost obtained with a transmission above 60% without decays and 40% with muon decays. Furthermore, using a rectilinear channel the maximum field on the coil is 14 T which is within the published [10] critical limit for Nb_3Sn. This value is less by 3 T compared to a Guggenheim helix [see Fig. 2(a)].

References

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