

Type Ia Supernovae and Cosmology

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We review recent progress in supernova cosmology. We discuss inconsistencies recently unveiled by the SDSS-II collaboration, between distance measurements performed with the SALT2 and MLCS2k2 light curve fitters. Finally, we briefly review the main systematics affecting the measurements of the dark energy equation of state with type Ia supernovae.

1 Introduction

Comparing nearby and distant type Ia supernova luminosities allows one to study the relation between luminosity distance and redshift and to constrain the expansion history of the Universe. Twelve years ago, relying on the first distant supernova samples obtained in the 1990s, two independent teams reported that the expansion of the Universe seems to be accelerating^{1,2}. This acceleration implies either modifications of gravity on cosmological scales, or the existence of a fluid with negative pressure, called “Dark Energy”³.

This result was confirmed by complementary cosmological probes^{4,5}. With more observations than basic parameters, we have over-constrained the standard cosmological model, and found a consistent explanation, with Dark Energy accounting for over 75% of the total energy density of the Universe. To unveil the nature of Dark Energy one must measure its equation of state parameter w , defined as the ratio of its pressure p_X over its density ρ_X : $w = p_X/\rho_X$. SN Ia observations remain a key ingredient in this measurement. They give access to the history of the expansion in the redshift range $0.01 < z < 1$, when Dark Energy started to dominate over matter.

Given the size of the current SN Ia samples (~ 1000 nearby and distant SNe Ia), systematic uncertainties now dominate the error budgets. Recent studies seem to suggest that the uncertainty on cosmological parameters is much larger than previously assessed and that the uncertainties related to the empirical modeling of SNe Ia have been largely underestimated. In this short review, we will examine these claims. We will also discuss the main sources of uncertainties affecting the current results.

2 Constraining the Cosmic Expansion History with Type Ia Supernovae

Type Ia supernovae are rare (1 SN / galaxy / millennium), bright (10^{10} solar luminosities) and transient (~ 1 month) events. They are quite easily identified using spectroscopy. Spectra taken around maximum luminosity reveal broad absorption features from materials ejected at speeds of $\sim 20,000$ km/s. SN Ia spectra are characterized by their lack of hydrogen lines, and the presence of strong silicon features. These observations suggest that SNe Ia come from the thermonuclear explosion of white dwarfs having reached the Chandrasekhar mass by accreting

mass from a companion. However, the exact nature of the companion (evolved main sequence star, red giant, white dwarf) is not precisely known, nor are the ignition scenarios and the explosion mechanisms (detonation, deflagration, delayed detonation...) ⁶.

Type Ia supernovae display an impressive homogeneity. The dispersion of their maximum restframe luminosity in the Johnson B -band does not exceed 40%. More importantly, the SN Ia restframe luminosity is correlated with the shape of their light curves, and with their restframe colors. Taking into account these two relations — respectively known as the “brighter-slower” and “brighter-bluer” relations — allows one to reduce the dispersion of the standardized maximum luminosities to about 15%. Hence, SNe Ia permit to measure (relative) luminosity distances on cosmological scales with an remarkable precision of about 7%.

Several methods have been developed to derive standardized luminosity distances from multi-band SN Ia light curve measurements. They all rely on empirical models of the supernova light curves, build from a training sample of well measured SNe. Currently, two approaches are in wide use in the community. We discuss them briefly below.

2.1 The MLCS Distance Estimator

The Supernova Multicolor Light Curve Shape method (MLCS2k2⁹) allows one to directly derive a standardized luminosity distance from a supernova light curve, measured in several bands in the observer frame. The model is trained on a set of nearby supernovae *with known relative distances*. Light curve templates in a set of pre-defined filters are derived from this training sample, allowing to interpolate between the observations. Since the supernovae that enter the cosmological measurements span a large redshift range, observer frame magnitudes of supernovae at higher redshifts do not cover the same spectral region. To account for this effect, K-corrections are applied to the light curve points. MLCS2k2 uses tabulated K-corrections that depend on the supernova redshift, color and epoch, determined from a sample of about 100 nearby supernova spectra, covering a large range in phase.

MLCS2k2 parametrizes the diversity of SNe Ia with a single parameter, Δ . The prediction of the model, at a given epoch τ (relative to the maximum of luminosity) and in a given observer band f may be written as:

$$m_f(\tau) = \mu + A_f^{MW}(\tau) + K_{ff'}(\tau) + M_{f'}(\tau) + p_{f'}(\tau)\Delta + q_{f'}(\tau)\Delta^2 + A_{f'}^{Host}(\tau) \quad (1)$$

f' denote the corresponding restframe filters ($UBVRI$) for which the model is defined, $K_{ff'}$ is the K -correction, A_f^{MW} is the absorption by the Milky way and $A_{f'}^{Host}$ is the absorption by dust in the SN host galaxy. M , p and q are the model vectors, determined during the training set. Finally, μ is the “true” (standardized) distance modulus.

This parametrization encodes a series of strong assumptions. First, it assumes that SN Ia form a one dimensional family, whose diversity may be accurately parametrized using the single parameter Δ . It also assumes that the model accurately captures the full color diversity of SNe Ia and that any remaining color dispersion is entirely due to dust reddening (in the SN host galaxy). Finally, absorption by dust in the Milky-Way or in the host galaxy is parametrized with the Cardelli-Clayton-Mathis (CCM) law ¹³. In particular, we have explicitly, for example in the B band: $A_B = R_B \times E(B - V)$ with $R_B = 4.1$. In order to ensure that the estimates of A are positive, the authors apply a prior, which is determined using simulations.

From a multiband supernova light curve, the trained model gives an estimate of μ along with its uncertainty, as well as an estimate of the extinction by dust in the host galaxy. Then, the standardized distance moduli are compared with the cosmology predictions:

$$\mu_i = \mu(z, \vec{\theta}) \equiv 5 \log_{10} \left(\frac{d_L(z_i, \vec{\theta})}{10\text{pc}} \right) \quad (2)$$

where $d_L(z)$ is the Hubble parameter-free luminosity distance, and $\vec{\theta}$ is the vector of cosmological parameters. The MLCS method was used by the High-z Supernova Search in the discovery of cosmic acceleration. Since then it has been used in many projects^{11,12,10,21}.

2.2 Fitting Standardized Distances along with the Cosmology

An alternate approach consists in determining a subset of observables from the supernova light curves (or possibly spectra), and then try to combine these observables in order to minimize empirically the dispersion in the Hubble diagram.

In practice, three quantities are usually estimated from the supernova light curves (1) the restframe magnitude of the supernova in a reference band, usually B -Johnson (m_B^*) (2) a light curve shape parameter, such as the *stretch* parameter as defined in Perlmutter *et al*¹ (3) and a restframe color of the supernova, c . These quantities are estimated using an empirical light curve model, itself derived from a training set. Following the original approach proposed by Tripp⁷, they are combined to form a standardized distance modulus μ :

$$\mu = m_B^* + M_B + \alpha(s - 1) - \beta c \quad (3)$$

As shown above, μ is then compared to the predictions of the cosmological models under study, $\mu(z, \vec{\theta})$. In the equation above α accounts for the brighter-slower relation, β , for the brighter-bluer relation, and M_B is the restframe absolute magnitude of type Ia supernovae (fully degenerated with H_0). These three quantities are nuisance parameters. They are usually fit along with the cosmological parameters and then marginalized over.

Several light curve fitters have been developed by various collaborations in order to determine m_B^* , s and c from the supernova light curves. As an example, the Supernova Legacy Survey (SNLS)¹⁴ have developed SALT2¹⁵ and SiFTO¹⁶. These two models do not apply K-corrections to the data, but attempt to directly predict the observed supernova fluxes as a function of SN epoch from a restframe spectral sequence $F(\lambda, \tau)$, using the telescope passband functions $F(\lambda)$. The SALT2 model determines $F(\lambda, \tau)$ using a large training set incorporating over a hundred supernova observed simultaneously photometrically and spectroscopically. The model parametrization may be written as:

$$F(\lambda, \tau) = x_0 [M_0(\lambda, \tau) + x_1 M_1(\lambda, \tau)] \exp(c CL(\lambda)) \quad (4)$$

where x_0 , x_1 and c are respectively the restframe flux in the reference (B) band, x_1 a shape parameter, and c a color parameter. $M_0(\lambda, \tau)$ is the mean spectral sequence, while $M_1(\lambda, \tau)$ describes the variations that go with the shape parameter (analog to a first component in a PCA decomposition). Finally, $CL(\lambda)$ describes a color law. $M_0(\lambda, \tau)$, $M_1(\lambda, \tau)$ and $CL(\lambda)$ are all empirical functions (modeled as splines) determined during the training.

With this approach, the treatment of supernova diversity of foregrounds is totally agnostic. First, it is not assumed that the supernova-to-supernova color variations is due to dust extinction. Instead, it is accounted for empirically as can be seen in equation 3. Furthermore, they do not assume that CCM law applies for dust in distant galaxies. Instead, SALT2 re-determines empirically a color law $CL(\lambda)$ during the training set. Another advantage of this approach, is that the supernova luminosity information is not used in the training. Hence, the training set can be supplemented with several hundreds of high-quality distant supernova light curves, such as those published by the SNLS or SDSS-II.

Note that alternate methods have also been developed, following the same approach, but using different standardization observables. For example, CMAGIC^{17,18} utilizes a linear relationship found in the color-magnitude diagram of SNe Ia for an extended period of time after maximum. Other methods rely on correlations between SN Ia luminosities and spectral flux ratios or on the equivalent width of selected spectral features^{19,20}.

3 Are SN Ia Luminosity Distance Measurements Reliable ?

In an attempt to estimate the impact of the assumptions made by the various light curve fitters on the cosmology, Kessler *et al* ²¹ analyze the first year SDSS-II dataset (supplemented with nearby, SNLS, ESSENCE, and HST data) with two different light curve fitters: MLCS2k2 and SALT2. Combining supernova constraints with the WMAP-5 year results, and with the measurements of the BAO feature in the SDSS Luminous Red Galaxy sample ²², they obtain two estimates of w . Using MLCS2k2 they find: $w = -0.76 \pm 0.07(stat) \pm 0.11(sys)$, while using SALT2 they obtain: $w = -0.96 \pm 0.06(stat) \pm 0.12(sys)$ respectively. These two estimates are incompatible at the 2σ -level. This result has had a strong impact on the community and deserves some discussion.

The source of these discrepancies is analyzed in detail in Kessler *et al*. They show that the differences can be explained by two effects:

1. First, there seem to be an anomaly in the SN models in the region of rest frame U -band. This effect is easily tested by studying the difference between the SDSS-II SN distance moduli computed without including the restframe data corresponding to the observer frame passbands ($\mu_{no\ U}$), or including it ($\mu_{with\ U}$) as a function of the redshift: $(\mu_{no\ U} - \mu_{with\ U})(z)$ Both SALT2 and MLCS2k2 seem to exhibit this anomaly.

This effect has been studied on more recent SNLS data, with an updated SALT2 training and an updated photometric calibration ²³. It has been shown that it can be completely explained by the impact of an inaccurate calibration of the SNLS first year sample on the training of SALT2. For the MLCS2k2 fitter, this effect may also be explained by the inclusion of inaccurately calibrated U -band data. Ground-based U -band observations are indeed notoriously difficult to calibrate. Indeed, the atmospheric transmission is extremely variable around ~ 300 nm, and the atmospheric cutoff is also variable, making the effective passband ill-defined.

2. The second effect tracked by Kessler *et al*, is related to the treatment of supernova colors. As discussed above, MLCS2k2 assumes that the observed color variation is not intrinsic to the supernova, but due to dust. Furthermore, MLCS2k2 ensures that the fitted extinction is positive, using a prior. Hence, as noted by Kessler *et al*, since there high-redshift SNe tend to have bluer colors, the distant moduli estimated using both methods will differ: the method relying on SALT2 will assign them larger luminosities and distances, hence the differences in the estimates of w . Furthermore, the authors show that the shape of the prior on supernova colors has a very strong impact on w , that can be as large as $\Delta w \sim 0.2$, explaining the main part of the effect.

There is still ongoing controversy around these two explanations. More details may be found in ^{21,23}. To summarize the situation, we may point out that:

1. the U -band anomaly has disappeared from the more recent SALT2 trainings. This is mainly due to changes in the photometric calibration of the SNLS survey. There is evidence that currently available U -band photometry suffer from large photometric errors: the dispersion of rest-frame U -band data is much lower for high- z data than for nearby data. Furthermore, there is significant tension in the residuals from the cosmological fit between the nearby and distant data if U -band is included.
2. there is evidence that the β correction in equation 3 does not identify with the R_B parameter of the Cardelli law. Indeed, $R_B \sim 4.1$, while the cosmology fits have consistently yield $\beta \sim 2$ over the last decade. This low value suggests either a very unusual extinction law in the environment of SNe Ia (either in their host galaxy, or in dust shells around the SN), or an intrinsic color variation that dominates the effect of dust.

3. there is evidence from the SALT2 training (and also from the SiFTO training) that SNe Ia follow a color law that cannot be explained by the Cardelli-Clayton-Mathis law.

As a conclusion, the causes of the discrepancies observed by Kessler *et al* have been identified. They seem to be related to photometric calibration (a solvable issue, in principle) and potentially incorrect assumptions (that may always be relaxed).

4 Systematic Uncertainties

The most recent measurements of w with SNe Ia are now dominated by systematic uncertainties. For example, the SDSS-II collaboration reports systematics twice as large as the statistical uncertainties. Next generation large surveys such as DES^a will study thousands of SNe Ia, while LSST^b will derive cosmological constraints with more than 10^5 supernovae. In this section, we examine a few selected systematics that dominate the current budgets.

4.1 Photometric Calibration

Measuring SN Ia luminosities relies on precision photometry performed with CCD imagers. The photometric calibration of these imagers is a key ingredient of the cosmological measurements with SNe Ia. Indeed, this cosmological information comes from comparing the luminosity distances of distant supernovae, observed in the redder bands of the imagers (~ 800 nm), with the distances of nearby SNe, observed at bluer wavelengths (around 400 nm). Therefore, a good control of the flux intercalibration of the imager passbands is essential. Otherwise, a redshift-dependent bias may affect the distance estimates.

Calibration consists in two steps: first, the observations are standardized onto some magnitude system, using a catalog of standard stars^{24,25,26,27}. Then, it is necessary to convert from the standard system into fluxes. This is done using a fundamental flux reference, i.e. a source with known magnitudes and a well measured spectral energy distribution (SED). Modern supernova surveys have greatly benefited for the Hubble Space Telescope (HST) CALSPEC^c calibration program²⁸. CALSPEC tied the flux calibration of the HST on observations of pure hydrogen white dwarfs whose spectra were known theoretically. This calibration was then propagated to a larger sample of spectrophotometric standards.

Modern surveys such as SNLS or SDSS-II have put significant efforts in improving the calibration of their datasets^{29,30}. Interestingly enough, both surveys have independently devised very similar strategies, anchoring their flux calibration on red CALSPEC standards. The precision obtained with the current strategies is slightly less than 1% in the visible, and of about 2% in the near-infrared (z -band). Despite these efforts, photometric calibration is still the dominant contribution to the systematic uncertainty budgets.

It is likely that the precision of $\sim 0.1\%$ in all bands, required by future surveys such as LSST will not be attained by these methods. One has to keep in mind indeed, that the CALSPEC flux calibration ultimately relies on theoretical models of pure hydrogen white dwarfs. Hence, there is some unknown systematic uncertainty associated with it. Several cosmology groups have started R&D programs, to calibrate the imagers with dedicated illumination systems^{31,32,33,34}. This will allow to tie the imager flux calibration on laboratory standards instead of astrophysical objects.

^a<http://www.darkenergysurvey.org/>

^b<http://www.lsst.org>

^c<http://www.stsci.edu/hst/observatory/cdbs/calspec.html>

4.2 Modeling of Supernova Light Curves

The analysis of SN Ia light curves requires empirical models trained from real data. Aside from the controversies around the MLCS- or SALT-like approaches, there are additional sources of systematics, coming from the finite size of the training sample, and also from various assumptions made in the implementation of the model. Both may be quantified and propagated into the analysis (see²³ for details). For SNLS for example, there are always smaller (up to a factor 2) than the calibration uncertainties.

4.3 Gravitational Lensing

Gravitational lensing of SN light due to the presence of Dark Matter haloes along the line of sight is expected to affect the SN signal, causing increased dispersion in the Hubble diagram. The distortion is asymmetric, most SN being slightly demagnified, while some are strongly magnified. For current large surveys such as SNLS, the impact of lensing is negligible, however, it may become an issue for future surveys targeting $z > 1$ supernovae^{35,36}.

A 2σ detection of the lensing signal in the supernova data has been published by SNLS^{40,41}. With more data, it may become a powerful probe of mass of galaxies embedded in the dark matter haloes.

4.4 Evolution of SN Ia

Evolution of SNe Ia luminosities with redshift is fully degenerate with cosmology. Observationally, it can be tested either by comparing the properties of nearby and distant supernovae, or by studying the correlations of these properties with those of their host galaxies (which are known to evolve with redshift).

Modern surveys have undertaken large efforts to detect differences in the spectral properties (ejecta velocities, equivalent width. . .) of nearby and distant supernovae^{38,37,39}. No systematic differences between both population have been detected yet. Before, Astier *et al* had performed a similar test on the light curve properties of nearby and distant SNe and obtained negative results.

Another way to test for evolution is to study whether the standardization relations evolve with redshift. Kessler *et al*²¹ unveiled a strong decrease of β with redshift using SALT2 and MLCS2k2. These results have been put under scrutiny by the SNLS collaboration²³. The measurement of β is very sensitive to the estimate of the uncertainties affecting the color estimates. It was shown that with more sophisticated color-error modeling the effect became much smaller than that measured by Kessler *et al*, and it was concluded that there is no evidence for an evolution of α and β given the size of the current samples.

4.5 Host Galaxy Environment

Another test for evolution consists in comparing the SN Ia photometric properties, such (stretch and color) with the properties of their host galaxies (star formation rate, mass. . .). It is known that galaxy properties evolve with redshift. Hence, any unaccounted-for correlation between SNe luminosities and their host galaxies properties could translate into a redshift-dependent bias affecting the SN Ia distance estimates.

Several authors^{42,43,44,45,46} have shown that the width of SN Ia light curves is correlated with the host galaxy morphology. Sullivan *et al*⁴⁷ confirmed this result, using SNLS data and stellar evolutionary models fitted on the galaxy colors: SNe Ia have a tendency to exhibit a smaller light curve width when exploding in a passive environment. Hence, it was shown that there is a correlation between the SN Ia properties and their environment. However this effect was fully accounted for by the brighter-slower relation.

More recently, several authors^{48,47,49} published evidence that SN Ia residuals from the Hubble diagram are correlated with host galaxy mass. The correlation between the SN absolute luminosities and the galaxy properties are still weak (4σ at best), but the effect on the peak absolute magnitude is substantial, of the order of 0.08 mag. There is no physical explanation for this effect yet. It was shown however that splitting the sample by host galaxy mass, and fitting for a different peak absolute magnitude for each sub-sample corrects for the effect.

4.6 Combining the Systematic Uncertainties

Most identified systematics affecting the distances to supernovae (calibration, light curve fitter training, selection bias ...) are correlated and redshift-dependent. As a consequence, it is necessary to publish the full covariance matrix of the supernova luminosity distances, so that SN data can be used without loss of information. Up to very recently, this information was not available, with the consequence that most subsequent analyses just ignored them⁴. This situation has changed recently, as authors started to provide that information^{50,21}.

5 Conclusion

Supernova cosmology has been a very active field over the last decade. Large surveys such as SNLS and SDSS-II have started rolling out very high quality distant SN Ia samples of a few hundred supernovae. The precision of the measurements is now limited by the quality of the low- z datasets, and by the intercalibration of the high- z and low- z data.

There has been a controversy recently on the size of the systematic errors associated with the empirical modeling of SNe Ia, as the SDSS-II collaboration published two incompatible estimates of w from the same dataset. A careful analysis of these results show that these discrepancies mostly come from photometric calibration problems in the U -band, and from incorrect assumptions in the treatment of supernova colors.

The uncertainties affecting the cosmological parameter estimates is now dominated by systematic uncertainties. However, a careful analysis shows that all identified systematics may be reduced as the size of the supernova samples grows. As an example, the quality of the empirical light curve models will be improved with larger training samples. For the first time, correlations between residuals in the Hubble diagrams and the environment of SNe Ia have been detected. However, it has been shown that taking into account the host galaxy properties in the cosmological fit corrects for the effect. Despite substantial progress and considerable efforts invested by the supernova surveys, photometric calibration remains the dominant contribution to the systematic error budgets and obtaining the precision required by the future surveys will be a considerable challenge.

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