Diffusion of heavy quarks in the early stage of high-energy nuclear collisions

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Abstract. We study the diffusion of heavy quarks in the early stage of high-energy nuclear collisions. The pre-equilibrium stage of relativistic heavy-ion collisions, commonly known as Glasma, evolves according to the classical Yang-Mills equations. Heavy quarks are coupled to the evolving Glasma fields via relativistic kinetic theory. We compute the momentum broadening, σ_p as well as the angular momentum fluctuations of heavy quarks in the early stage, which turn out to be anisotropic due to the anisotropy of the background gluon fields. We observe that $\sigma_p \propto t^2$ at very initial times. This non-Markovian diffusion of heavy quarks in the early stages is explained by the memory effect present in the gluon fields.

1 Introduction

Relativistic heavy-ion collisions (HICs) at the Large Hadron Collider (LHC) and the Relativistic Heavy Ion Collider (RHIC) offer unique opportunities to study hadronic matter under extreme conditions. Following the collision, a pre-equilibrium phase is formed, which transitions into a deconfined medium of quarks and gluons called the quark-gluon plasma (QGP), eventually leading to hadronization. The pre-equilibrium stage, the earliest phase of HICs, is characterized by high gluon occupation numbers and a highly non-linear regime referred to as the Glasma.

Heavy quarks (HQs) [1–15] are valuable probes of the early stages of HICs. Charm and beauty quarks, in particular, are produced with very short formation times that they can probe the entire evolution of the system, starting from the Glasma phase. Their formation time, estimated as $\tau_{\text{form}} \approx \frac{1}{2m}$,where m is the quark mass, is approximately 0.06 fm/c for charm quarks and 0.02 fm/c for beauty quarks. While the Glasma may only last for a fraction of a fm/c, the strong interactions between HQs and the dense gluon fields during this time can leave observable imprints on the final state. Therefore, studying HQs in the Glasma phase is

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essential for a comprehensive understanding of relativistic HICs. In this work, we focus on the propagation of HQs in the very early phase of HICs, namely the Glasma phase.

2 Framework

In high-energy nuclear collisions, we utilize the effective theory of the color glass condensate (CGC) [16–18]. The CGC operates within the high-energy limit of Quantum Chromodynamics. The CGC framework conceptualizes the collision of two colliding objects as the interaction of two thin sheets of colored glass. During this collision, the interaction generates two sets of effective color charges, one for each sheet, connected by dense gluon field filaments. These gluon fields have a high occupation number, indicating a classical nature. These fields extend throughout the interaction region of the colliding objects and are collectively referred to as the Glasma [17]. Within the CGC framework, the Glasma represents the initial condition for the system produced in high-energy nuclear collisions.

In our work, we utilize the McLerran-Venugopalan (MV) model, where the static color charge densities ρ^a on the colliding nuclei L and R are postulated to be random variables. These variables follow a normal distribution such that

$$\langle \rho^a_{L,R}(\vec{x}_\perp) \rangle = 0, \tag{1}$$

$$\langle \rho^a_{L,R}(\vec{x}_\perp) \rho^b_{L,R}(\vec{y}_\perp) \rangle = (g\mu_{L,R})^2 \delta^{ab} \delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp). \tag{2}$$

This study is constrained to SU(2) color group, hence $a, b = 1, 2, 3$. The parameter μ signifies the density of color charge carriers in the transverse plane, with $g\mu$ representing the color charge density. Importantly, $g^2\mu = O(Q_s)$, where Q_s stands for the saturation momentum scale.

The equations of motion governing the fields and their conjugate momenta, i.e., the classical Yang-Mills (CYM) equations, in the static box are given by

$$\frac{dA^a_i(x)}{dt} = E^a_i(x), \tag{3}$$

$$\frac{dE^a_i(x)}{dt} = \partial_j F^a_{ji}(x) + g f^{abc} A^b_j(x) F^c_{ji}(x). \tag{4}$$

The dynamics of the colored probes within the Glasma fields are governed by a set of equations known as the Wong equations [5–10, 19, 20]:

$$\frac{dx^i}{dt} = \frac{p^i}{E}, \tag{5}$$

$$\frac{dp^i}{dt} = g Q_a F^{i\nu}_a p_\nu, \tag{6}$$

$$\frac{dQ_a}{dt} = \frac{g}{E} f_{abc} A^i_b p^i Q_c, \tag{7}$$

where $i = x, y, z$ and $E = \sqrt{\vec{p}^2 + m^2}$ represent the relativistic energy of the HQ. Here, x^i, p^i, and Q_a denote the position, momentum, and effective color charge of the HQs, respectively.

3 Results

To determine Glasma parameters, we vary Q_s to obtain the QCD coupling g, see [7]. In Fig. 1, we show the evolution of transverse momentum broadening, $\sigma_p = \frac{1}{2}\langle (p_x(t) - p_{0x})^2 + (p_y(t) - p_{0y})^2 \rangle$, over proper time t for charm quarks in the Glasma fields for different values of

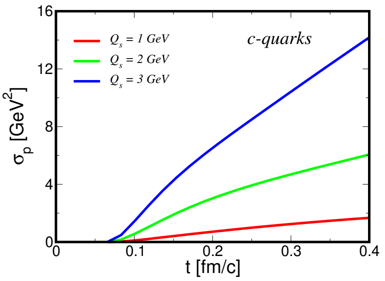

Figure 1. Transverse momentum broadening, σ_p w.r.t. proper time, t for charm quarks, considering initial $p_T = 0.5$ GeV. This calculation pertains to the evolving Glasma fields within a static box.

Q_s. Adjusting Q_s alters the energy density, thereby affecting the effective temperature of the Glasma medium where HQs diffuse. Consequently, a higher Q_s leads to increased diffusion of HQs in the Glasma fields.

An intriguing observation is that, in the very early time period, the relationship between σ_p and time is non-linear, in contrast to the standard Brownian motion (BM). This is followed by a linear growth of σ_p at a later stage. This discrepancy at initial times is attributed to the memory effects associated with the gluon fields within the bulk. These fields exert a correlated Lorentz force on the charm quarks, leading to the observed non-linearity of σ_p. As mentioned earlier, subsequent to the initial transient, σ_p exhibits a linear growth, resembling BM without drag, as observed in [6]. The transition between these two regimes occurs over a timescale of $t \approx \tau_{\text{mem}}$ [6]. Hence, Fig. 1 highlights how the diffusion of the charm quarks during the very early stage of the evolution deviates from standard BM due to the memory of the background gluon fields in the evolving Glasma bulk.

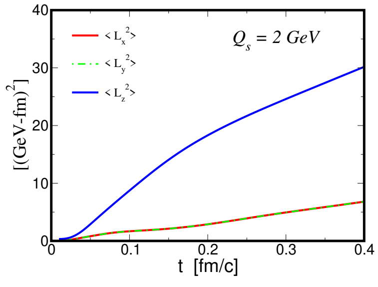

Figure 2. Evolution of the square of the components of orbital angular momentum with proper time for $m = 10$ GeV at $Q_s = 2$ GeV. The initial momentum is set with $p_T = 0.5$ GeV and $p_z = 0.5$.

Moreover, we study the fluctuations of angular momentum of HQs as well. To this end, we study the dynamics of the HQs in a classical color field within the non-relativistic limit, see [9, 20] for more a detailed framework. In Fig. 2, we present a plot depicting the evolution of squares of the components of angular momentum, namely $\langle L_x^2 \rangle$, $\langle L_y^2 \rangle$, and $\langle L_z^2 \rangle$ for $Q_s = 2$ GeV. Throughout the entire evolution, $\langle L_x \rangle = \langle L_y \rangle = \langle L_z \rangle = 0$, indicating that the quantities depicted in this figure represent the spreading of the components of $\vec{L}$.

Clearly, $\langle L_z^2 \rangle > \langle L_x^2 \rangle, \langle L_y^2 \rangle$; indicating that the interaction of HQs with the gluon fields leads to anisotropic fluctuations in orbital angular momentum. Futhermore, fluctuations of the spin angular momentum are much smaller in magnitude than orbital angular momentum fluctuations and remain isotropic, see [9]. Additionally, calculations show that spin and orbital angular momentum fluctuations are uncorrelated. Thus, we can approximate the fluctuations of total angular momentum, $\vec{J} = \vec{L} + \vec{S}$, as $\langle J_i^2 \rangle = \langle (L_i + S_i)^2 \rangle \approx \langle L_i^2 \rangle$.

4 Summary

We performed numerical simulations for the HQs in the background Glasma fields, focusing on linear momentum broadening and angular momentum fluctuations of HQs in the evolving Glasma fields. Our findings reveal that the diffusion of HQs in the pre-equlibrium phase of high-energy collisions is non-linear, influenced by strong, coherent gluon fields and significant memory effects. Additionally, we observed that the fluctuations of total angular momentum of HQs within the Glasma fields show pronounced anisotropy.

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