PARALLEL SESSION ON

THE UNIFICATION OF WEAK AND EM INTERACTIONS

Organiser: B W Lee, FNAL

PROBLEMS IN GAUGE FIELD THEORIES

S Weinberg

Harvard University, U.S.A.

(Notes prepared by G G Ross)

The Chairman has asked me to "emphasize problems in gauge theories of strong, weak and electromagnetic interactions that we must face in the coming years rather than dwell on past and present successes". I would like to comply by discussing three problems that seem to me at present to stand in the way of further progress:

(1) The η problem

(2) The problem of gauge hierarchies

(3) The renormalisation of gravitation.

(1) The η problem(1,2,3). This consists of two parts: Why is the η mass not near the π mass? And why is $\eta \to 3\pi$ not suppressed by powers of m_π^2/m_η^2 These problems have been around for a long time but they are particularly troublesome in a picture of the weak, electromagnetic and strong interactions which has become increasingly popular recently. The main features of this "standard" picture are as follows:

(a) The quark fields carry two indices, a "color" index which presumably runs over three hues and a "charge etc." index which runs over the values $\mathcal{P}, \mathcal{N}, \lambda, \mathcal{P}'$ etc.

(b) The weak and electromagnetic interactions are based on a gauge group G_W, which acts on the "charge etc." indices.

(c) The strong interactions are based on a non-chiral semisimple gauge group G_S (presumably SU(3)) which acts on the color indices.

(d) There may be a set of color-neutral weakly coupled scalar fields whose vacuum expectation values break G_W. There are no strongly interacting scalar fields.

In this picture the strong intereactions are described to zeroth order in e by an effective field theory consisting only of massive quarks and massless G_S gauge fields, with the quark mass matrix m arising from the large vacuum expectation values of the weakly coupled scalars (if any). This immediately leads to several of the attractive features of the standard picture;

(1) To order α parity is conserved. So too is charge conjugation and any quantum numbers (such as charge, strangeness, charm, baryon number) expressible in terms of the number of quarks (summed over color) of each type. If some of the quark masses are zero or equal or approximately zero or equal then the strong interactions obey other exact or approximate unitary global conservation laws. For instance the usual picture of an exact zeroth order isospin conservation superimposed on an approximate SU(2) x SU(2) which is itself part of a less exact SU(3) x SU(3) can be achieved if we suppose that the zeroth order p, n and λ quark masses are small, with the p and n in zeroth order equal and somewhat smaller than the λ mass.

(2) The approximate unitary global symmetries of the strong intereactions can be broken spontaneously by the strong interactions, giving rise to Goldstone bosons of small mass.

(3) The strong interaction theory is asymptotically free, provided only that we do not include too many types of fermion.

(4) The G_S gauge symmetry of the strong interactions may be dynamically broken or it may be unbroken. In the latter case it is necessary to arrange that infrared effects make the gluons as well as the quarks unobservable[3,4].

When we add the effects of the weak and electromagnetic interactions and discard all terms which are suppressed by powers of m_W^2, we find that the remaining "order α" effects appear only as corrections to the quark mass matrix, together with the familiar one photon exchange term. This has further desirable consequences.

(5) The order α corrections to isospin conservation consist only of one photon exchange plus $\Delta I = 1$ terms. Thus ordinary one photon exchange calculations should work for the $\Delta I = 2$ corrections to isospin conservation, (such as the $\pi^+-\pi^0$ mass difference and the quadratic term in the Σ mass difference), but not for the $\Delta I = 1$ corrections (such as the n-p mass difference and $\eta \to 3\pi$ decay).

(6) Not only are the order α corrections to the natural zeroth order symmetries of the quark mass matrix finite - if the theory is asymptotically free these corrections are even calculable (including all effects of the strong interactions) using ordinary one loop perturbation theory.

These consequences of the standard picture are very pleasing. However, the η problem stands out as a warning not to become too comfortable.

The η mass problem arises because in addition to the usual isovector axial vector current

$$\vec{J}^\mu = -i \sum_{\text{colors}} \bar{\psi} \gamma_5 \gamma^\mu \vec{\tau} \psi$$

there is also an isoscalar axial vector current

$$J_0^\mu = -i \sum_{\text{color}} (\bar{\psi}_{\mathcal{P}} \gamma_5 \gamma^\mu \psi_{\mathcal{P}} + \bar{\psi}_{\mathcal{N}} \gamma_5 \gamma^\mu \psi_{\mathcal{N}})$$

$$= -\frac{2i}{3} \sum_{\text{color}} (\bar{\psi} \gamma_5 \gamma^\mu (1+\frac{\sqrt{3}}{2} \lambda_8) \psi)$$

Apart from anomalies the only mechanism which breaks the conservation of either $\vec{J}^\mu$ or $J_0^{\ \mu}$ is the common $\mathcal{P},\mathcal{N}$ quark mass. Hence if both chiral SU(2) x SU(2) and chiral U(1) are strongly spontaneously broken, we would expect an isoscalar pseudoscalar Goldstone boson with a mass comparable to that of the isovector Goldstone boson. The problem then is why is $m_\eta^{\ 2}$ so much larger than $m_\pi^{\ 2}$? More quantitatively, if we further assume ordinary SU(3) is not spontaneously broken we can actually show that

$$m_\eta < \sqrt{3}\, m_\pi \ .$$

To prove this, we note first that if there were only $\mathcal{P}$, $\mathcal{N}$ and λ quarks, there would be just two isotopic singlet Goldstone bosons (corresponding to the two isosinglet channels λ_8 and 1). Under our assumption that SU(3) is not spontaneously broken we may calculate their masses by familiar current algebra techniques. The result is that one mass eigenvalue is less than $\sqrt{3}\, m_\pi$ while the other is greater than the Gell-Mann Okubo value $\sqrt{\frac{4}{3}}\, m_K$. When we add charmed quarks we increase the number of unitary singlet Goldstone bosons with which these two could mix. However the lowest eigenvalue of a Hermitian matrix is less than the eigenvalues of any diagonal submatrix and the lower mass eigenvalue is pulled down even further than $\sqrt{3}\, m_\pi$.

The η decay problem also arises from the same source, the existence of $J_0^{\ \mu}$. If we use soft pion theorems to evaluate the resulting matrix element for $\eta \to 3\pi$ we get a result proportional to a zero momentum transfer matrix element of the operator

$$\sum_{\text{color}} \bar{\psi} \gamma_5 \psi$$

In the standard picture this operator is proportional to $\partial_\mu J_o^\mu$ and therefore vanishes between equal four-momentum states.

I can think of just three possible ways out of the η problem.

(a) The "standard" picture may be wrong. In particular SU(3) may be spontaneously broken. However then we wonder how to save the successes of SU(3), including the Gell-Mann Okubo formula for π, η and K. Alternatively we might add strongly or semistrongly interacting scalar [5] fields. However then we wonder how to save the successes of the standard picture, in particular conservation of parity to order α.

(b) The approximate conservation of J_o^μ may be violated by anomalies. There certainly are expected anomalies of the Adler-Bell-Jackiw type which gave

$$\partial_\mu J_o^\mu = g_o^2 \; \varepsilon_{\mu\nu\lambda\rho} \, F_\alpha^{\mu\nu} \, F_\alpha^{\lambda\rho}$$

where $F_\alpha^{\mu\nu}$ is the covariant cure of the gluon fields. However the right hand side is itself a divergence and we can construct a current

$$J_{o,c}^{\mu} \equiv J_o^{\mu} - g_o^2 \; \varepsilon^{\mu\nu\lambda\rho} \, A_{\alpha\nu} \, F_{\alpha\lambda\rho}$$

which is conserved; presumably $J_{o,c}^\mu$ can be used in place of J_o^μ to rederive the undesired results above, so this is no way out.

Langacker and Pagels[2] have proposed that non-perturbative anomalies of the Baker-Johnson type occur here and they do not allow us to construct a conserved current. However I then wonder why field theory can be trusted at all - why every current conservation law is not full of anomalies not seen in perturbation theory. Also, even within the context of their approximations, they have not shown that non-perturbative anomalies do occur, but only that Goldstone bosons do not.

A third alternative is that the current is conserved and that this implies the nucleon is massless which just means the theory does not describe the real world.

(c) The possible solution I would like to suggest is that the standard model is right but the resulting low mass Goldstone boson ($m < \sqrt{3}\, m_\pi$) behaves like quarks and gluons, and does not appear in collisions of color-neutral particles. The observed η is the other unitary singlet Goldstone boson with a mass given (in the limit that $J_{o,c}^\mu$ conservation is much more strongly spontaneously broken than $\vec{J}_\mu$ conservation) by the Gell-Mann Okubo result, roughly $\sqrt{\frac{4}{3}}\, m_K$.

This last suggestion sounds terribly *ad hoc*, but while it cannot yet be justified it can at least be restated in a more plausible way. This is done by generalizing the idea of trapping so that physical singularities corresponding to trapped particles such as quarks and gluons do not appear in the matrix elements of strong gauge invariant operators, whether or not these are color-neutral. We note $J_{o,c}^\mu$ which is conserved and contains the unwanted pole is not strong gauge invariant, so this pole can correspond to a trapped particle.

2. The Problem of Gauge Symmetry Breaking

If there are no unobserved leptons or weak interactions then the gauge theory of weak and electromagnetic interactions must be based on the group SU(2) x U(1).

However this group is not simple. In consequence there are free dimensionless parameters which cannot be determined on a purely a priori basis.

One possibility is to introduce new leptons and thereby to change the gauge group to a simple group. One theory of this type is the SU(2) model of Georgi and Glashow.

Another possibility is to add unobserved weak interactions so that SU(2) x U(1) is promoted to a larger simple group. If the group is simple there is only one gauge coupling constant so the unobserved weak interactions can only be unobserved because the corresponding gauge bosons are superheavy,[(6)] i.e. much heavier than the $W^{\pm}$ and Z^{o}. Thus the new gauge symmetries must be subject to a superstrong spontaneous symmetry breakdown. The problem then is how is it possible for gauge symmetries to suffer very different degrees of symmetry breakdown to different subgroups.

The embedding of the observed gauge groups in a larger group determines the weak mixing angle uniquely. Georgi, Quinn, and I[(7)] have shown that it is possible to determine the mixing angle just by counting the particles in any irreducible representation of the simple group. For any group we can normalise the generators T_α so that in any given irreducible representation D we have

$$\mathrm{Tr}\,(T_\alpha T_\beta) = N_D\, \delta_{\alpha\beta}.$$

The simplicity of the group then requires that all the gauge coupling constants are equal.

If the observed weak and electromagnetic interactions are described by an SU(2) x U(1) subgroup of the simple group, the charge generator must be of the form

$$Q = T_3 - T_o/\tan\theta$$

with T_3 and T_o normalised as above (Note that the coefficient of T_o is $1/\tan\theta$ because the operator $Y \equiv Q - T_3$ has coupling g' and so we may construct an operator with the same coupling constant g as T_3 by dividing by $g'/g = \tan\theta$).

To calculate θ, we note that

$$\mathrm{Tr}(Q^2)/\mathrm{Tr}(T_3{}^2) = 1 + 1/\tan^2\theta = 1/\sin^2\theta$$

For instance if the simple group has an irreducible representation[(6)] consisting of a T_3 singlet μ^+ and a T_3 doublet ν, e^- we have

$$\mathrm{Tr}\, Q^2 = 2 \text{ and } \mathrm{Tr}T_3{}^2 = \tfrac{1}{2} \quad \text{so } \sin^2\theta = \tfrac{1}{4}\ .$$

It is not even necessary to specify the simple group; all we have to do is count.

There seem to be just three possibilities to explain how the superheavy gauge particles get their mass.

(a) The most obvious possibility is that the simple gauge group G is broken down to the observed gauge group G_o in zeroth order perturbation theory by the vacuum expectation value of some scalar field ϕ_i, and that G_o is further broken by higher-order corrections which are small because the gauge coupling is small. This is the same mechanism that has been invented to explain approximate global symmetries such as isotopic spin conservation. However, a gauge symmetry is necessarily a symmetry of the whole Lagrangian and it is difficult to see how it could be broken at all if it is not broken in zeroth order perturbation theory. More formally, suppose the vacuum expectation value of a scalar field multiplet ϕ_i is given in the tree approximation by a minimum λ_i of the polynomial $P(\phi)$ in the Lagrangian, which leaves some gauge generators θ_α unbroken, in the sense that

$$(\theta_\alpha)_{ij}\ \lambda_j = 0.$$

With radiative corrections $\lambda_i \to \lambda_i(\varepsilon)$, given by the minimum of the "potential"

$$V_\varepsilon(\phi) = P(\phi) + \varepsilon V_1(\phi) + \varepsilon^2 V_2(\phi) + \ldots$$

where V_N is the N loop correction and ε is a small parameter. The question is, whether or not $\lambda(\varepsilon)$ is also invariant under θ? To answer this, consider the eigenvalues of the matrix

$$M^2_{ij}(\phi, \varepsilon) = \frac{\partial^2 V_\varepsilon(\phi)}{\partial\phi_i \partial\phi_j}$$

According to Goldstone's theorem, for each broken symmetry there is an eigenvector with eigenvalue zero

$$M^2(\phi, \varepsilon)\Theta_\alpha \lambda(\varepsilon) = 0$$

But if no symmetry is broken for $\varepsilon = 0$ we expect M^2 to have no zero eigenvalue at $\varepsilon = 0$ and $\phi = \lambda(0)$, and then continuity leads us to expect that it has no zero eigenvalues for a finite range of ε and λ around this point. It is therefore not possible for Θ_α to be broken at any minimum $\lambda(\varepsilon)$ which goes over smoothly to $\lambda(0)$ as $\varepsilon \to 0$.

(b) It is possible for the unperturbed scalar mass matrix to develop zero eigenvalues unrelated to broken symmetries in some special circumstances. This is the case, for instance, in the theories discussed by Coleman and E Weinberg[8]. If $P(\phi) \sim \phi^4$ then there is a unique zeroth order solution at $\phi = 0$, but adding a one-loop correction $V_1(\phi)$ can shift this solution to a symmetry breaking point, because for ϕ sufficiently small $V_1(\phi)$ dominates over $P(\phi)$. Theories of this kind are not natural in the sense that the scalar masses only vanish because of some arbitrary choice of parameters in the Lagrangian, not as in the case of "accidental" symmetry because of unavoidable constraints. We can always arrange for gauge hierarchies to occur if we are willing to make the proper adjustment of parameters so this possibility does not seem to be what we are looking for.

(c) There remains the possibility that at least part of the symmetry breaking is dynamical, i.e. does not involve the scalar fields at all. The point I want to make here is that dynamical symmetry breaking does appear to offer at least one thoroughly natural mechanism for the appearance of gauge hierarchies. Suppose the Lagrangian does contain elementary scalar fields ϕ_i. The vacuum expectation value $<\phi_i>$ breaks the gauge group G to a subgroup G_0 and it is natural to expect that all gauge fields will be massless (G_0) or superheavy; all fermions will be massless or superheavy, and all scalars will be Goldstone bosons.

Then at mass scales m << M (M superheavy) such theories look like a G_0 gauge theory with massless vector and fermion fields and no scalars[9]. Such a theory may perhaps exhibit dynamical symmetry breaking breaking[10]. However, the only dimensional parameter is the renormalisation point μ of the gauge couplings $g_R(\mu)$. Hence m_R must be determined by

$$g_R^{\,2}(m_R) = A \quad \text{(A a pure number} \simeq 1?)$$

If the theory is asymptotically free and $g_R^{\,2}(M^2) < A$ then because the evolution of the effective coupling $g_R^{\,2}(m)$ is logarithmic in m we have

$$m_R^{\,2} << M^2 \quad \text{(exponentially!)}$$

Theories of weak, electromagnetic and strong interactions based on simple groups have been suggested by several authors[11]. We consider the SU(5) version of Georgi and Glashow which is spontaneously broken to $G_0 \equiv SU(3)_{Strong} \times (SU(2) \times u(1))_{weak\ and\ em.}$

For $M >> m >> M_{W,Z}$ we may use the renormalization group equations

$$\frac{1}{m}\frac{d}{dm}\, g_i(m) = \beta_i(g(m))$$

to compute the effective couplings

$$g_i^{-2}(m) \sim \text{const} - 2b_i \ln m \quad \{b_i \text{ known constants}\}$$

The integration constants can be fixed by noting that at $m \sim M$, $g_i(M)$ (assuming they are also small) are essentially equal to g, the coupling constant of G. We thus have[7]

$$g_i^{-2}(\mu) \sim g^{-2} + 2b_i \ln \frac{M}{\mu}$$

We can use our knowledge of e and the b_i to relate the strong coupling g_3 at μ = 10 GeV to the mixing angle θ and the superheavy mass M, as shown in the following table:

$\frac{g_3^2\ (10\ \text{GeV})}{4\pi}$	M(GeV)	$\sin^2\theta$
0.5	2.10^{17}	.17
0.2	2.10^{16}	.19
0.1	5.10^{14}	.21
0.05	2.10^{11}	.25

It may thus be possible then for theories of this kind to explain in a natural way the existence of two mass scales. One question remains. Why is the quark mass m_q very much less than $m_{W,Z}$?

From the Goldberger-Treiman relation we expect

$$m_q \simeq FG/g.$$

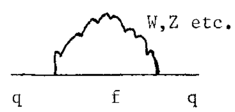
And from the Higgs phenomenon we expect[10]

$$m_W \simeq F$$

where F is the coupling of the Goldstone boson to the current and G is the coupling of the Goldstone boson to the quark. So we need G <<g. But composite particles always have G ~ 1. One possible solution is that SU(2) x U(1) is broken with a Goldstone boson, a fermion antifermion bound state $\bar{f}f$ with

$$m_f \simeq m_W/g \simeq 300\ \text{GeV}.$$

In this picture the quark mass arises via diagrams of the type



A heuristic argument suggests that $m_Q = 0(\frac{\alpha}{\pi} m_f) \simeq 300$ MeV. One must of course ask what is the effect of "intermediate fermions" on models and on phenomenology?

3. Renormalization of Gravitation

Recently several authors[12] have used dimensional regularization to study the infinities in the one-loop approximation to general relativity. Their result is that the infinities cannot be removed by a renormalization of the parameters of the theory.

Deser and I have independently suggested a modification of general relativity, based on the action

$$I = \int d^4x\sqrt{g}\left[G^{-1}R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right]$$

where α and β are dimensionless constants, presumably of order unity. This appears to be a renormalizable theory, at least according to the simple power-counting arguments which show that Einstein's theory is non-renormalizable. Furthermore, in this respect it appears to be unique. It also presents a nice analogy with gauge theories; even the coupling G^{-1} of the "superrenormalizable" term R is not too different in magnitude from the corresponding coupling M^2 in the gauge theories discussed in the last section. Of course, for distances large compared with $G^{-\frac{1}{2}}$, this theory behaves just like general relativity.

The big outstanding question is whether this theory makes sense as quantum theory, with unitarity, positive energies, etc. Deser will discuss these matters further in his talk at this Conference, so I will not go into them here.

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A REVIEW OF ASYMPTOTIC FREEDOM

D Gross

Princeton University, U.S.A

1. The Road to Asymptotic Freedom

a. Deep Inelastic Scattering

Asymptotically Free Gauge Theories (AFGT) of the strong interactions were discovered in an attempt to understand Bjorken scaling within the framework of renormalizable quantum field theory [1,2,3]. The resurgence of quantum field theory as a description of the strong interactions of hadrons is mainly due to the fact that local currents are naturally contained in such a framework. Other approaches to hadronic physics, such as the dual resonance model, have severe, if not insuperable, problems in incorporating local currents.

Deep inelastic scattering probes the structure of hadrons at short distances. One might expect renormalizable theories to yield scaling in this region, since the only dimensional parameters appearing in such theories are masses which can be neglected (Weinberg's theorem) at large (Euclidean) momenta. However this does not occur because even in the absence of masses the theory contains a scale parameter. In order to define the theory one must define the (dimensionless) coupling constants and the scale of all fields and operators. In doing so one introduces a "renormalization scale parameter", μ, say, to specify the point in momentum space where the vertex characterizing the interaction is defined to equal the coupling constant. Consequently naive dimensional analysis is wrong and even if one were to obtain simple scaling laws one might expect the dimensions of fields and operators to depend on the interaction (anomalous dimensions).

Deep inelastic scattering measures the Fourier transform of the product of electromagnetic currents. In the scaling region one probes this product near the light cone where one can employ the Wilson operator product

$$J_\mu(x)J_\nu(0) \underset{x\to 0}{\sim} \sum_N C_N(x^2, g) x^{\mu_1} \ldots x^{\mu_N} \theta^N_{\mu\nu\mu_1 \ldots \mu_N}(0)$$

The q^2 dependence of the standard structure functions, νW_2 and W_1 is determined by the x^2 singularities of the Wilson (c-number) coefficients C_N of the dominant operator (twist = dimension-spin = 2) in the expansion. The operators of spin N + 2 are projected out by the N^{th} moments of the structure functions $F(q^2, x = q^2/2\nu)$

$$\int_0^1 dx\, x^N F(q^2, x) \underset{q^2\to\infty}{\sim} \tilde{C}_N(q^2, q) < H|\theta^N|H>.$$

If there was a simple scaling law one might expect that

$$\tilde{C}_N(q^2, g) \underset{q^2\to\infty}{\sim} (\frac{1}{q}2)^{\frac{1}{2}\gamma_N},$$

where γ_N is the "anomalous dimension" of the operator 0^N.

The experimental indications are that:

1) Bjorken scaling is quite well satisfield. This means that $F(q^2, x) \underset{q^2\to\infty}{\to} F(x)$ and that $\gamma_N = 0$.

2) The parton model works extremely well, particularly in relating electron and neutrino scattering. The relations that follow from this model, or equivalently the light cone models, are equivalent to assuming that the Wilson coefficients $\tilde{C}_N(q^2, g)$ are identical, for large q^2, to their values in a free field theory $\tilde{C}_N(q^2, o)$ - i.e. that at short distances one sees non-interacting quarks in the hadron.

b. The Renormalization Group

The primary tools in analysing the short distance behaviour of quantum field theories are the renormalization group equations. These state that a change in μ, the only scale parameter once masses are neglected, is equivalent to a change in the coupling constants and in the scale of all operators. Thus $\tilde{C}_N(q^2/\mu^2, g)$ satisfied the equation:[4,5,6]

$$(\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_N(g)) \tilde{C}_N(q^2/\mu^2, g) = 0.$$

The solution of this equation is written in terms of an "effective coupling constant" $\bar{g}(t = \frac{1}{2} \ln q^2/\mu^2)$, which satisfies

$$\frac{d\bar{g}}{dt} = \beta(\bar{g}) = \tfrac{1}{2}b_0\bar{g}^3 + b_1\bar{g}^5 + \ldots, \quad \bar{g}(0) = g,$$

and represents the coupling measured at momentum q^2 in terms of the 'physical coupling constant' measured at μ^2.

$$\tilde{C}_N(q^2/\mu^2, g) = \tilde{C}_N(1, \bar{g}(t)) \exp \int_0^t \gamma_N(\bar{g}(t'))dt'$$

The short distance behaviour of the theory is then determined by the effective coupling at infinite q^2. We can distinguish two cases.

A. $\bar{g}(t) \underset{t\to\infty}{\to} g_\infty \neq 0.$

For example if $\beta(g)$ has the form illustrated in Fig. 1.

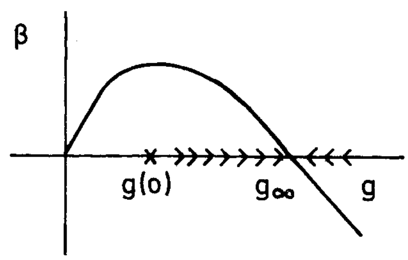

Figure 1

then $\bar{g}(t) \to g_\infty$, where $\beta(g_\infty) = 0$ and $\beta'(g_\infty) > 0$. In that case $\tilde{C}_N(q^2/\mu^2, g) \to \tilde{C}_N(1, g_\infty)\ (\frac{\mu}{q})^{\gamma_N(g_\infty)}$ $\exp \int_0^t [\gamma_N(\bar{g}(t')) - \gamma_N(g_\infty)]\, dt'$. We would expect $g_\infty \sim O(1)$, $\gamma_N(g_\infty) \sim 1$, and thus that there would be large power deviations from Bjorken scaling. In addition we would not expect the parton model to be valid, since $C_N(1,g)$ should be quite different from $C_N(1,0)$. The only exception would be if the ultra-violet stable fixed point g were to be very small, however this would appear unlikely.

B. Asymptotic Freedom

If $\beta(g)$ is negative for small g, as in Fig. 2,

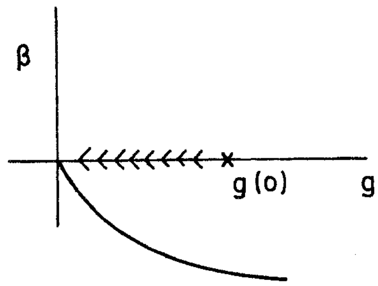

Figure 2

then $\bar{g}^2(t) \underset{t\to\infty}{\to} 0$ (like $\frac{1}{t}$). Such a theory is asymptotically free, since the interaction "turns off" for sufficiently high momenta. In this case (since $\gamma_N(g_\infty = 0) = 0$)

$$\tilde{C}_N(q^2/\mu^2, g) \underset{q^2\to\infty}{\to} \tilde{C}_N(1,0) \exp \int_0^t \gamma_N[\bar{g}(t')]\, dt.$$

Asymptotic Freedom thus guarantees the validity of the parton model for deep inelastic scattering and the absence of power violations of Bjorken scaling. Logarithmic violations will however occur since $\gamma_N(\bar{g}(t)) = \gamma_N \bar{g}^2(t) + .. \underset{t\to\infty}{\to} \frac{\gamma_N}{t} + \ldots$.

An even more powerful argument can be made that Bjorken scaling alone (at least in theories involving only scalars and fermions) implies asymptotic freedom[6,7]. In other words $\gamma_N(g_\infty)$ can't all vanish unless $g_\infty = 0$. (This means that exact Bjorken scaling cannot occur in quantum field theory, at the very least there must be logarithmic violations).

C. The Price of Asymptotic Freedom

Asymptotic Freedom is a rare phenomena in quantum field theory. It has been shown that no theory involving only scalar, Yukawa or Abelian gauge interactions can be asymptotically free[8]. Asymptotic Freedom requires the presence of non-abelian gauge interations. For quantum electrodynamics the physics is clear - at short distances there is less screening due to the polarization of the vacuum so that the effective charge increases. A similar physical explanation of the increasing couplings of Yakawa and scalar theories at short distances is still lacking.

II The Structure of Asymptotically Free Gauge Theories of the Strong Interactions

The preceeding arguments suggest the necessity of asymptotically free theories of the strong interactions. The discovery that non-abelian gauge theories are asymptotically free[1,2,3], and indeed that they alone possess this feature makes a compelling case for constructing Yang-Mills theories of the strong interactions[1,2].

a. Models

Yang Mills theories based on a semi-simple gauge group are always asymptotically free. One can easily incorporate a reasonable number of fermions without destroying the asymptotic freedom. To specify such a model, one must choose a gauge group G, as well as the representation R to which the fermions are assigned. If one demands that at short distances one "sees" (with leptonic probes) free quarks then the gauge group G must commute with SU(3), so that the gauge gluons be SU(3) singlets.

Thus one is forced to consider "coloured quark" models, invariant under SU(3) x G; where the quark field ψ is given by:

$$\psi = \underbrace{\begin{pmatrix} p_1 n_1 \lambda_1 \\ p_2 n_2 \lambda_2 \\ p_N n_N \lambda_N \end{pmatrix}}_{SU(3)\times SU(3)} \Bigg\} \text{Strong Gauge Group G}$$

G mixing the rows of ψ, which can be labelled by their colours; and ordinary SU(3) transformation mixing the columns of ψ. [One could imagine that G and SU(3) are merely sub-groups of a unified gauge theory of the weak, electromagnetic and strong interactions, all of equal strength at superhigh energies, which undergoes symmetry breaking so as to yield a SU(3) $\otimes$ G structure at present energies. There has been some speculation along these lines [9].]

If one demands that there be three colours (red-white-blue) as suggested by the successes of the naive quark model, then one must choose G to be SU(3). In that case one has nine quarks, the model is asymptotically free and one is guaranteed to see at short distances non interacting quarks.

The effective expansion parameter $\bar{\alpha}(Q^2) = \frac{1}{4\pi^2}\bar{g}^2(Q^2)$ will behave asymptotically like:

$$\bar{\alpha} \rightarrow \frac{4}{9\ \ell n\ q^2/\mu^2}$$

The parameter μ^2 indicates the value of Q^2 where the strong intereactions become strong. If one chooses this to be of the order of hadronic masses, say $\mu = 1$ GeV, then $\alpha(Q^2 = 10) = 0.2$; and the strong interactions are relatively weak at present energies. The $\frac{4}{9}$ appearing in the above formula, which is determined by the Casimir operators of the group, is therefore crucial. If indeed one had obtained $\frac{9}{4}$ then one would have had to go to values of $Q^2 \geqslant 1000$ GeV^2 before $\bar{\alpha}$ became small - and asymptotic freedom would have been irrelevant at present energies. The dependence of $\bar{\alpha}$ on μ is mild (logarithmic) unless one changes μ by an order of magnitude. If one adds 3 more charmed quarks the effect is small $(\frac{4}{9} \rightarrow \frac{12}{25})$.

b. Two Loop Calculation of β

Recently a two loop calculation of β has been performed for non-abelian gauge theories[10]. Together with the original one-loop calculation[1,2] one has: $(\alpha = q^2/4\pi^2)$

$$\beta(\alpha) \equiv \frac{d\bar{\alpha}}{d\ell n Q^2} = -\alpha^2\left(\frac{11}{12}\ C_2(G) - \frac{1}{3}\,T(R)\right)$$

$$-\alpha^3\left(\frac{17}{24}\,C_2^2(G) - \frac{1}{4}C_2(R)T(R) - \frac{5}{12}\,C_2(G)T(R)\right) + O(\alpha^4)$$

$$= -\frac{9}{4}\ \alpha^2 - 4\alpha^3 + O(\alpha^4)$$

where $C_2(G)$ is the quadratic Casimir operator of the Group G ($C_2 = N$) for SU(N)) and T(R) the value of the trace of the square of a matrix in the fermion representation ($T = \frac{1}{2}$ for one triplet in SU(3)). The numerical values given above are for the red-white and blue model. These terms are independent of how one defines the coupling constant g, and, if one uses dimensional regularization both are gauge independent [1,11]. The net effect on $\bar{\alpha}(Q^2)$ is to decrease its asymptotic value slightly:

$$\bar{\alpha}(Q^2) \underset{Q^2\to\infty}{\rightarrow} \frac{4}{9\,\ell n Q^2/\mu^2} - \frac{256}{729}\ \frac{\ell n\ell n Q^2/\mu^2}{(\ell n Q^2/\mu^2)^2}$$

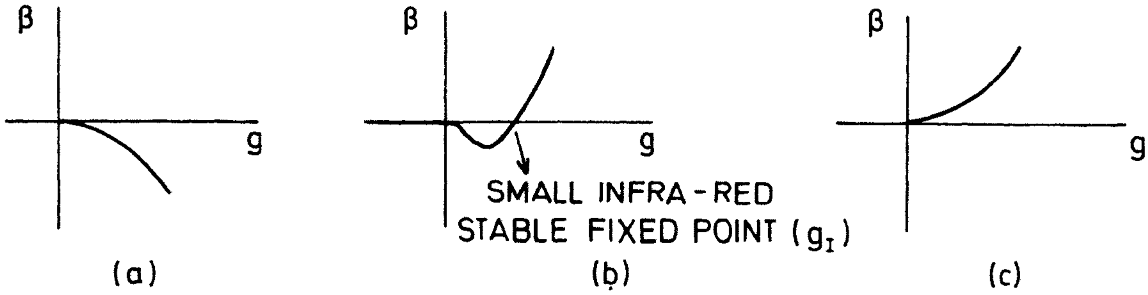

Figure 3

Having calculated $\beta(g)$ to this order one can investigate the effect of increasing the number of fermions. In the standard models ($G = SU(3)$ and 3 or 4 types of quarks) β is illustrated in Fig. 3A. Of course β might have a zero, presumably at $\alpha = \frac{g^2}{4\pi^2} \stackrel{\sim}{\sim} O(1)$, but this can't be ascertained to finite order in perturbation theory. If one includes a critical number of fermion triplets, N_c, ($N_c = 16$ if $G = SU(3)$) then β develops a small ($\alpha_I = 0.01$) zero, which is infra-red stable as illustrated in Fig. 3B. Since this zero is very small one can (presumably) trust perturbation theory (the corrections to α_I are themselves of order α_I^2). Finally if one has more fermions than N_c (more than 16 triplets if $G = SU(3)$) then the theory is no longer asymptotically free (as in Fig. 3C).

The existence of this calculable infra-red stable fixed point has revived an old idea,[12] namely that there exists a finite ultra-violet cut off Λ in the theory (say $\Lambda \sim 10^{18}$ GeV where gravity becomes important) and that the effective coupling $\bar{g}(\Lambda)$ is of order unity. For momenta of order 1-100 GeV one is then in the infra-red-domain, relative to Λ, and the effective coupling is essentially g_I. Thus one would have small anomalous dimensions, and the deep inelastic structure functions would behave much as they do in an asymptotically free gauge theory[13]. For smaller momenta, whereupon masses become important the effective strength of interaction will increase since the quarks (whose masses say are of order $\sim$1 GeV) will decouple and the dynamics will revert to that of an asymptotically free gauge theory. The main objections to this idea are that it requires a finite cut-off and a very large number of quark types for which there is no evidence (these additional 12 or so quarks must have roughly the same masses as the p, n, λ quarks if the infra-red stable fixed point is to control the dynamics for $q^2 \stackrel{\sim}{\sim} 10\text{-}100\ \text{GeV}^2$).

c. Scalar Mesons

In general it is very difficult to incorporate scalar mesons into a gauge theory without destroying asymptotic freedom. With a random choice of couplings (including quantic scalar and Yukawa couplings) zero coupling is not ultraviolet stable unless the number of scalar mesons is very small and the number of fermions is close to the critical value N_C[1]. Thus one cannot merely add Higgs scalars to break the gauge symmetry spontaneously. However[1] some interesting exceptions to this general picture have been discovered recently - namely the existence of theories with "unstable lines of ultra-violet attraction". In such theories, for a particular choice of coupling constants the theory is asymptotically free - but an infinitesimal change in the initial values of the couplings will cause the effective couplings to increase for large momenta. The prime example of such theories are those possessing a super-symmetry relating bosons to fermions. There exist super-symmetric gauge theories, in which all the couplings are related to each other by the super-symmetry which forces one to be on an asymptotically free line in coupling constant space [14]. It has also been pointed out recently that standard (non super-symmetric) theories, such as the Georgi-Glashow model, possess such lines of asymptotic freedom[15]. Here there appears to be no symmetry that forces one to be on the asymptotically free line - and one must arbitrarily adjust the relative values of the coupling constants of the model. This might be an advantage since it reduces the number of free parameters and increases the predictive power of the model.

An interesting question arises in this connection. Is there a hidden symmetry in such models (say the Georgi-Glashow model) that leads to the existence of such a line? In general can one probe for new symmetries of gauge theories by exploring the asymptotically free submanifolds of coupling constant space?

d. Low Energy Structure

Asymptotically free theories, for which the coupling turns off at short distances will inevitably develop large couplings at large distances. The actual behaviour of the effective coupling will be determined by the form of $\beta(g)$. If β vanishes for $g = g_o \neq 0$ then $\bar{g}(Q^2) \to g_o$ as $Q^2 \to 0$, and one expects g_o to be of order unity. On the other hand if β remains forever negative then $\bar{g}(Q^2) \to \infty$ when $Q^2 \to 0$ (if $\int^{\infty} \frac{dx}{\beta(x)} = \infty$) or when $Q^2 \to Q_o^2$ (if $\int^{\infty} \frac{dx}{\beta(x)} < \infty$). The low energy structure of an asymptotically free theory is thus a strong coupling problem for which perturbation theory is useless. The difficulties are compounded in a gauge theory due to the infra-red singularities. On the other hand the increasing coupling at large distances might provide a dynamical mechanism for:

A. Dynamical Symmetry Breaking

The increasing coupling at large distances in asymptotically free theories with attractive interactions might produce bound state scalar mesons which could serve as Goldstone bosons or Higgs bosons to generate spontaneous symmetry breaking and produce masses. In fact a theory involving no mass parameters and only one coupling constant which produces masses in this fashion would end up involving no adjustable parameters (except the overall scale of lengths). In such a theory it has been shown that the produced masses, m, must depend on the coupling constant g, for small g and fixed renormalization scale like $\exp\left(\frac{1}{b_o g^2}\right)$ where $\beta(g) = b_o g^3 + \ldots$[16]. In asymptotically free theories this implies $m \underset{g\to o}{\sim} \exp\left(-\frac{1}{g^2}\right)$ like the binding energy of a Cooper pair, whereas for non asymptotically free theories (massless QED) $m \underset{g\to o}{\sim} \exp\left(+\frac{1}{g^2}\right)$.

Examples of such theories have been constructed in two dimensions [16]. These are $(\bar{\psi}\psi)^2$ theories, which are renormalizable and asymptotically free. They exhibit dynamical symmetry breaking ($\langle\psi\bar{\psi}\rangle \neq 0$), bound state Goldstone or Higg's bosons and models with no adjustable parameters. Unfortunately they are not gauge theories and the methods used to solve the models are hard to extend to four dimensions.

B. Confinement

A more exciting possibility for theories of the strong interactions is that the strong (perhaps infinitely strong) coupling and infra-red singularities of asymptotically free gauge theories provide the forces necessary to confine the coloured quarks and gluons within the colour singlet hadronic bound states, and eliminate all coloured asymptotic states[17]. Thus one would have a theory which for small distances one would see weakly coupled quarks inside the hadrons but these would be forever confined due to the large couplings at large distances.

In this connection it has been noted that in the N (of SU(N)) limit of non-abelian gauge theories the topological structure of the Feynman graphs is that of the dual resonance model, and that the quarks might be confined at the end of strings - thus "deriving" the dual resonance models[18]. Another approach taken is to consider a lattice gauge theory, in which confinement is automatic[19]. However the relation to the continumn theory is unclear at present.

This is clearly the most important problem which must be solved, if asymptotically free gauge theories are to explain the strong interactions.

III Applications

A. Deep Inelastic Scattering

Deep inelastic scattering is the most reliable way of probing the short distance structure of hadrons and could provide sensitive tests of asymptotic freedom. The generic prediction of an asymptotically free gauge theory is that the moments, M_N, of a structure function (νW_2, W_1, νW_3 etc ...) should, for large q^2, behave as:

$$M_N(q^2) = \int_0^1 dx\, x^N F(x, q^2) = \int_1 \frac{d\omega}{\omega^{N+2}} F(\omega, q^2)$$

$$\sim \Sigma\, C_i^N (1, \bar{g}^2(q^2))(\ell n q^2)^{-A_N^i} + O(\frac{M^2}{q^2}),$$

where the sum runs over the twist-two operators in the Wilson expansion, and $\bar{g}^2(q^2)$ is the effective coupling constant which is vanishing for large q^2.

a. Where is Asymptopia?

When does asymptopia set in? One can at present only give necessary conditions for the onset of asymptopia. One clearly requires that q^2 be larger than the 'relevant' mass parameters (say hadronic masses). The precocious scaling, for some particular choice of scaling variable, in the region where $\frac{q^2}{M^2} \sim 0.2 - 1.0$ is a dynamical mystery which is certainly not explained by asymptotic freedom. To be convinced that scaling really breaks down one must be in a region where q^2/M^2 corrections are small. In addition, in asymptotically free theories, one must be in a region where the effective couplings are small. The asymptotic form of $\bar{\alpha} = \frac{\bar{g}^2(q^2)}{4\pi^2}$ is determined by the gauge group. For the nine quark model we have: $\bar{\alpha} \to \frac{4}{9\ell n\, q^2/\mu^2}$. The value of μ is determined by the value of q^2 where the strong interactions become strong. Setting this equal to 1 GeV we have:

$$\alpha(5\ \text{BeV}^2) \sim \frac{1}{4} \qquad \frac{M^2}{Q^2} \sim .2$$

(μ = 1, RWB_{quarks})

$$\alpha(50\ \text{BeV}^2) \sim .1 \qquad \frac{M^2}{Q^2} \sim .02.$$

Thus the predictions for M_N, being asymptotic expansions in α and M^2/q^2 should be good to about 10% at $Q^2 = 50\ \text{BeV}^2$.

b. Sum Rules

All the standard parton or light-cone model sum rules for the moments of the structure functions are true asymptotic theorems. In addition asymptotically free theories allow one to calculate the corrections to these sum rules (which are of order α, α^2 etc). These can be calculated by calculating the Wilson coefficients in perturbation theory.

Thus, for example, the baryon-number sum rule becomes:[20]

$$\int_0^1 dx\, [F_3^{\nu p} + F_3^{\nu n}] \sim -b\, [1 + \tfrac{5}{8}C_2(R)\alpha(Q^2) + O(\alpha^2)]$$

and the ratio of longitudinal to transverse moments is:[20]

$$\frac{\int_0^1 dx\, x^N F_L(x, q^2)}{\int_0^1 dx\, x^N F_T(X, q^2)} \underset{q^2\to\infty}{\to} \alpha(Q^2) C_N (1 + O(\alpha)),$$

where the C_N's can be calculated for the non-singlet (I = 1) structure functions. These corrections could be used to test the theory.

In addition there is a new sum rule, for the moment which is determined by the energy momentum tensor. One form of this sum rule is:[21,22]

$$\int_0^1 dx \{6 [F_2^{ep} + F_2^{en}] - [F_2^{\nu p} + F_2^{\nu n}] \}$$

$$\underset{q^2\to\infty}{\to} \frac{4}{3} r + O\ (\alpha^{\cdot 6}) \to O(\alpha)$$

where r essentially measures the fraction of momentum carried by the quarks and $r = \frac{T(R)}{2\ C_2(R) + T(R)}$ = (36% in the RWB model).

c. Determination of A_N^i

The coefficients A_N^i have been determined by calculating the anomalous dimensions of the twist two operators in second order perturbation theory. For the non-singlet structure functions one has: [1,21,22]

$$A_N^{N.S} = G\left[1 - \frac{2}{(N+2)(N+3)} + 4 \sum_{K=2}^{N} \frac{1}{K}\right]$$

where G is determined by the group and the fermion representation: $G = \frac{3C_2(R)}{11\ C_2(G) - 4T(R)}$ $\left[= \frac{4}{27}\right.$ in the RWB model$\left.\right]$. It is of importance to note that for large N: $A_N \to 4G\ell nN$ which implies large deviations from scaling near threshold; and for $N \sim -2$ (corresponding to $J \sim 0$) $A_N^{N.S} \sim -\frac{2G}{N+2}$. The singlet A_N's have also been calculated,[21,22] here it suffices to note that:

$$A_N^{SING.} \underset{N\to\infty}{\approx} A_N^{N.S} + O(\frac{1}{N^2 \ell nN})\ A_0^{SING} = 0$$

and $A_N^{SING.}$ has a pole at $N = -1 (J=1)$.

The original calculation of the singlet A_N's utilized the fact that in the axial gauge ($n.A = 0$, $n^2 = 0$) there are no Fadeev-Popov ghosts and thus there is no mixing of manifestly gauge invariant operators with ghost operators[21]. This is not the case in other gauges. The structure of the Wilson expansion for non-abelian gauge theories in arbitrary gauges, and in particular the determination of which operators are multiplicatively renormalizable and which contribute to S-matrix elements is still unresolved[23].

d. Deviations from Scaling

The ideal test of asymptotically free gauge theories is the measurement of the q^2-dependence of the moments M_N, which are determined by the coefficients A_N^i. This test is however impractical at present. This is evident if we note that since

$$\int_1^\infty \frac{d\omega}{\omega^2} F(\omega, q^2) \underset{q^2\to\infty}{\to} \text{const.}$$ whereas all higher moments decrease with q^2, $F(\omega, q^2)$ on q^2 will increase with q^2 for $\omega > \omega_c(q^2)$ and decrease for $\omega < \omega_c(q^2)$. The increase (decrease) is greatest for large (small) ω. The value of ω_t can be estimated to be proportional to $[\alpha(q^2)]^{-\text{const.}}$ (in the RWB model $\omega_t \sim \left(\frac{1}{\alpha(Q^2)}\right)^{1.5}$ and $\omega_t(50\ \text{GeV}^2) \sim 20 \pm 5$). To determine the q^2 dependence of the moments it is clearly necessary to measure F out to values of $\omega > \omega_t$. However ω is severely limited for large q^2. Indeed

$\omega < \frac{2\nu_{MAX}}{q^2} < \frac{2E_B}{q^2}$, where E_B is the energy of the lepton beam. At NAL this means that $\omega_{MAX}(50) \approx 6$, whereas $\omega_t(50) \approx 20$. Thus the moments cannot be fruitfully measured at present energies.

The moment relations can be used to derive an "asymptotic extrapolation formula", using the measured structure function $F(\omega, Q'^2)$ at one asymptotic value of Q'^2 to extrapolate to another $F(\omega, Q^2)$. One derives that [24]

$$F(\omega,Q^2) = \int_1^\omega \frac{d\omega'}{\omega'} F(\frac{\omega}{\omega'}, Q'^2)\, T(\frac{\ell n Q^2}{\ell n Q'^2}, \omega')$$

where T is the inverse Mellin transform of $(\frac{\ell n q'^2}{\ell n q^2})^{A_N}$.

$$T(\frac{\ell n q^2}{\ell n q'^2}, \omega) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} ds\, \omega^s \left(\frac{\ell n q'^2}{\ell n q^2}\right)^{A_N}$$

This expression contains all the information in the moment relations, for q^2, $q'^2 \to \infty$, if there is only one A_N^i. This is the case for the non-singlet structure functions, or for values of ω close to one where large N dominates.

The advantage of this extrapolation formula is that it requires only knowledge of $F(\omega', Q'^2)$ for $1 \le \omega' \le \omega$ in order to extrapolate to $F(\omega, Q^2)$. Furthermore it assumes a simple form near $\omega \approx 1$ (threshold) since only the large N-behaviour of $A_N (A_N \sim > 4G\ell nN)$ will be relevant.

$$T(\omega \approx 1) = \left(\frac{\ell n q^2}{\ell n q'^2}\right)^{.69G} \frac{(\ell n\omega)^{p-1}}{\omega\Gamma(p)}\,(1+O(\ell n\omega));$$

$$p = 4G\ell n \frac{\ell n q^2/\mu^2}{\ell n q'^2/\mu^2}$$

In particular it illustrates how the violations of scaling grow large near threshold. If one assumes that $F(\omega, Q'^2) \approx (\omega-1)^d$ as $\omega \to 1$ then one derives: [24]

$$R(\omega;q^2,q'^2) \equiv \frac{F(\omega,q^2)}{F(\omega,q'^2)} = \left(\frac{\ell n q^2}{\ell n q'^2}\right)^{.69G} \frac{\Gamma(1+d)}{\Gamma(1+d+p)}\,(\ell n\omega)^p$$

In the RWB model, with $\mu=1$ and extrapolating from $Q'^2 = 5$(SLAC) to $Q^2 = 50$ (NAL) and assuming $d \approx 3$ one has: $R(\omega, 50; 5) = .54\,(\ell n\omega)^{.53}$. These predictions are only logarithmically dependent on μ (which should be taken as a free parameter) and change very slightly if one adds more quarks (say charmed quarks).

The threshold dependence of $R(\omega;q^2,q'^2)$ offers the best hope at present of testing asymptotically free gauge theories. The $(\ell n\omega)^{4G\ell n\left(\frac{\ell n q^2}{\ell n q'^2}\right)}$ behaviour is characteristic of these theories, arising from the logarithmic behaviour of the moments and the ℓnN blow-up of the A_N's. Other models of scaling deviations give radically different behaviour. If one assumes parton form factors, the violation of scaling is ω independent and $R(\omega;q^2,q'^2) = \frac{(M^2+q'^2)}{(M^2+q^2)}$. In non gauge theories, which possess a finite fixed point and anomalous dimensions the A_N approach a finite limit as $N\to\infty$ [6]. Thus:

$$R(\omega;q^2,q'^2) \underset{\omega\to 1}{\longrightarrow} \left(\frac{q'^2}{q^2}\right)^{\gamma_F}$$

where γ_F is the anomalous dimension of the underlying fields. These would easily be distinguished from asymptotically free theories by experiments in the vicinity of threshold $\omega = 1.5$ to 3 for $q^2 = 20\text{-}50\ GeV^2$.

e. Scaling for Large and Small

In inverting the moments to deduce the behaviour of $F(\omega,q^2)$ one must assume uniformity in N of the corrections at a given q^2. This assumption might well be incorrect for $\omega \gtrsim 1$ (which stresses large N) or $\omega >> 1$ (which stresses small N). If one however assumes uniformity for all N, then one can use the extrapolation formula in to these regions with interesting conclusions.

Near threshold, the structure function behaves like $(\ell n\omega)^{4G\ell n\ell nq^2}$. If one extrapolates this to the resonance region $\omega \gtrsim 1 + \frac{M^2}{q^2}$, and uses Bloom-Gilman duality in the sense that one demands that the resonance contribution is a fixed part of $F(\omega,q^2)$ for increasing q^2 one can deduce an extrapolation formula for hadronic form factors. Thus one is lead to expect that these will, in addition to the dipole form factor, contain terms[25,26] like $(\frac{1}{q^2})^{4G\ell n\ell nq^2}$.

In the Regge region, $\omega \to \infty$ q^2 fixed and large, the moment relations indicate the existence of essential singularities in the angular momentum, due to the poles in A_N^i. In the singlet channel these occur at J = 1, leading to a behaviour[27] $F \underset{\omega\to\infty}{\sim} C^{\sqrt{\ell n\omega\ell nq^2}}$

However both of the above extrapolations are of doubtful validity; since the corrections to the lowest order moment relations are large (for fixed q^2) when $N \to \infty$ or $N \to -1$. The moments $M_N(q^2)$ behave like:

$$M_N(q^2) = C_N(0)\, \frac{g^{-2}(Q^2)^{A_N}}{q^2} \left[1+K_1^N\alpha + . + K_i^N\alpha^i + \ldots\right]$$

where $\alpha = \frac{g^{-2}(Q^2)}{4\pi^2} \to \frac{1}{\ell nQ^2}$. The K_i^N arise from higher order terms in the anomalous dimensions and the Wilson coefficients and can be calculated in perturbation theory.

Near threshold the effective N that contributes to F is of order $\frac{\ell n\alpha}{\ell n\omega}$. However one can show that $K_i^N \underset{N\to\infty}{\sim} (\ell nN)^{3i}$; and thus the corrections will be important unless Δ = const. $\alpha(Q^2)\quad(\ell n\frac{4G\ell n\alpha}{\ell n\omega})^3 \ll 1$. Since near threshold $\ell n\omega \gtrsim \frac{M^2}{q^2}$ this excludes the resonance region [28]. The dominant corrections to K_i^N arise from the large N anomalous dimensions, $\gamma_N(g^2)=\sum_{i=1}^{\infty}(g^2)^i\gamma_N^i$,

$\gamma_N^i \underset{N\to\infty}{\sim} (\ell nN)^{2i-1}$, which can be calculated in terms of the graphs of the type shown in Fig.4.

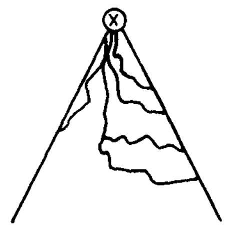

Fig. 4

The leading logarithms can be easily summed and used to extrapolate towards the resonance region. These corrections are small (10%) in the RWB model for Q^2:10 to 50 if $\omega \geq 1.3$.

For large ω the effective value of N is of order $N+1 \gtrsim \sqrt{\frac{\ell n\alpha}{\ell n\omega}}$. Here one can show that the corrections K_N^i have themselves poles at N = -1, in fact $K_N^i \underset{N\to -1}{\sim} (\frac{1}{N+1})^{3i}$. Therefore the corrections will become important unless $\Delta \sim \alpha(\frac{\ell n\omega}{9G\ell n\ 1/\alpha})^{\frac{3}{2}} << 1$ [28]. This excludes the Regge region. The leading pole terms, for the non-singlet structure functions which have poles at N = -2, can be summed. One finds that the sum converts the pole in $\gamma_N(g^2)$ into a square root branch point at $N = -2 + 0(g^2)$

B. Other Applications

a. e^+e^-

Asymptotically free theories predict that

$$R = \frac{\sigma_{e^+e^- \to had}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \to \sum Q_i^2\ (1+0(g^{-2})),$$

and that the limit be approached from above [29]. One expects scaling to set in at higher values of q^2 than in the time-like region, and at present $q^2 \gtrsim 25\text{GeV}^2$ there is no serious difficulty with reconciling the lack of scaling in e^+e^- experiments with the predictions of the theory for the space like region. The situation would be easier to understand if charmed thresholds were being opened for $q^2 \sim 10\text{-}25\ \text{GeV}^2$. This would hardly affect deep inelastic scattering and one would predict that $R \to 10/3$. One certainly expects this limit (or $R = 2$ if there are no charmed quarks) to be approached for $q^2 \gtrsim 80\ \text{GeV}^2$. It should also be noted that the validity of the standard parton model predictions for $e^+e^- \to$ hadron + x have not been established in asymptotically free gauge theories.

b. $\Delta I = \frac{1}{2}$ Rule

A recent calculation has suggested that the $\Delta I = \frac{1}{2}$ rule could be explained in a gauge theory of the weak interactions if the strong interactions are asymptotically free, as a consequence of the logarithmic enhancement of the $\Delta I = \frac{1}{2}$ operators in the short distance expansion of weak currents [30].

c. Hadronic Reaction

There have been attempts to apply renormalization group techniques to analyse elastic form factors, large angle scattering, inclusive reactions at large P_T etc. Although some results have emerged, the severe infra-red singularities of gauge theories have thus far prevented one from discussing asymptotically free gauge theories [31].

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GAUGE THEORIES AND WEAK INTERACTIONS

M K Gaillard

FNAL and Orsay

Gauge theories of both strong and weak interactions allow us to calculate measurable quantities with a degree of confidence which was previously lacking. I wish to report on some recent progress along these lines. The topics I will discuss include:

1) Conventional radiative corrections to μ- and β- decay.

2) Induced neutral currents in K-decay and constraints on charmed quark masses.

3) Electromagnetic and weak mass shifts.

4) Non-leptonic interactions

I shall focus my attention primarily on the Weinberg-Salam model[1] as modified by the GIM mechanism[2] to include charm. Where strong interactions play a role, I will consider the model with three coloured quartets of fractionally charged quarks: $(p_i, n_i, \lambda_i, p_i')$, i = red, white, blue. The strong interactions may be mediated by a colour singlet gluon or by an octet of colour gauge gluons. In the second case the asymptotic freedom[3] of the theory allows us to valuate perturbatively the effects of strong interactions on certain hadronic matrix elements. In either case it is assumed that weak and electromagnetic currents are singlets with respect to the gauge group of strong interactions.

1. Radiative corrections to β-decay and μ-decay.

Renormalization of the weak couplings in the Weinberg Salam-model has been studied by several authors[4]. I shall follow the treatment of Sirlin who assumed that (a) Schwinger terms are c-numbers, (b) weak and electromagnetic currents are colour singlets, and (c) operators in the Wilson expansion have canonical dimensions up to logarithms. Then the Ward identities which follow from the once integrated current algebra reduce the radiative corrections to nucleon β-decay to expressions involving the matrix elements:

$$\langle p'|T^*(J^\mu(x)\ J_\mu'(o)|p\rangle \ .$$

Under the above assumptions, the short distance expansion of the T^* product of current operators leads to a logarithmic divergence which cancels when all contributions (W, Z, γ, physical and unphysical Higgs exchange) are taken into account. The only remaining logarithmic divergence appears in a term proportional to the matrix element of the charged weak current:

$$\langle p'\ |J_\mu{}^W(x)|p\rangle$$

which is the zeroth order matrix element. Since this result is a consequence of the current algebra, and since leptonic and hadronic currents satisfy the same algebra, the divergent part of the radiative corrections to any leptonic or semi-leptonic weak process may be removed by a universal renormalization of the W-coupling constant.

Under the further assumptions of (a) asymptotic freedom and (b) an early approach to asymptopia, the finite parts of the radiative corrections can also be calculated. It is anticipated[5] that the results will be similar to those obtained[6] using slightly less sophisticated methods. To evaluate the finite parts, Sirlin first studied the β-decay of the free neutron-like quark. He classified the corrections into three categories:

(a) those which are identical for μ- and β- decay and therefore do not affect the ratio of coupling constants.

(b) those of order $\alpha m_e^2/m_w^2$ or $\alpha m_q^2/m_w^2$ which are negligible.

(c) the remaining corrections reduce to those previously calculated in a local V-A theory, but with the cut-off replaced by m_z.

Corrections of type (c) to the Fermi coupling in the decay of the physical neutron are evaluated using the methods of current algebra, as previously discussed by Abers et.al.[7] The contribution from the vector coupling is model independent up to a term controlled by short distance behaviour. The contribution from the axial coupling depends as well on the mean quark charge and on a non-asymptotic contribution which is estimated to be small.[7] Assuming the short distance behaviour of free field theory and fractionally charged coloured quarks, the value of the Cabibbo angle can be extracted from the ft value in ^{14}O decay. The result depends on the Z mass, related to the Weinberg angle by

$$M_z = 38 \text{ GeV}/\sin\theta_w \cos\theta_w .$$

For $\sin^2\theta_w = 0.35$, one finds $M_z \simeq 80$ GeV and

$$\sin\theta_c \simeq 0.223$$

in excellent agreement with the values obtained from Ke_3[8] and the data on baryon decay[9].

Other recent calculations include the deviation from μ-e universality in the leptonic decay of the intermediate boson[10] and the branching ratio[11] for $E^o \to \nu\gamma$ in theories where a heavy lepton is required.

2. <u>The suppression of neutral currents in K-decay.</u>

The GIM mechanism plays a very delicate rôle in the rare decay modes of the K-meson, and experimental data provides constraints on the quark masses. This is seen in the comparison of the amplitudes for $K_L \to \mu\mu$ and $K_L \to \gamma\gamma$, which to lowest order in the semi-weak and electromagnetic couplings ($e^4 \simeq g^4$) are of order of magnitude:

$$A(K_L \to \mu\mu) \sim G_F\alpha(\Delta m^2/m_w^2\sin^2\theta_w) \ln (m_w^2/m^2)$$
$$A(K_L \to \gamma\gamma) \sim G_F\alpha \ (\Delta m^2/m^2)$$

where $\Delta m^2 = m_{p'}^2 - m_p^2$ and $m^2 = m_p^2$ or $m_{p'}^2$ are quark masses. The observed strong suppression of the leptonic mode and the non suppression of the photonic mode imply

$$m_p^2 << m_p'^2 << m_w^2\sin^2\theta_w \sim (38 \text{ GeV})^2$$

In explicit calculations[12] the free quark model was used to obtain an effective coupling for $\lambda\bar{n} \to \gamma\gamma$ or $\ell\bar{\ell}$. The hadronic part of the effective interaction turns out to be a V-A current operator whose matrix elements are known. The contribution of order g^4

to $K_L \to \mu\mu$ vanishes identically in this model, due to a fortuitous cancellation, so that this decay is probably dominated by the two photon intermediate state as indicated by experiment.

A similar calculation of the K_L, K_S mass difference, which is of order

$$\frac{\Delta m_K}{m_K} \sim G_F\alpha \quad \Delta m^2/(38 \text{ GeV})^2$$

together with the observed rate for $K_s \to \gamma\gamma$ give the more explicit quark mass constraints:

$$m_{p'} \simeq 1.5 \text{ GeV} \quad , \quad m_p \lesssim m_K$$

These values are then used as input to determine other K-decay amplitudes; of those calculated the most immediately accessible to experiment is $K^+ \to \pi^+ ee$, of order:

$$A(K^+ \to \pi^+ ee) \sim G_F\alpha \ \ln(m_{p'}/m_K)$$

The predicted branching ratio is

$$\Gamma(K^+ \to \pi^+ ee) \simeq 0.5 \times 10^{-6}.$$

It is gratifying that a result near this value(13) ($\Gamma_{exp} \simeq 0.3 \times 10^{-6}$) has been reported at this conference. It should be emphasized that this result is not a confirmation of the GIM mechanism and/or gauge theories since in any model a similar order of magnitude is expected. However a very different result would have been embarrassing to the gauge model since the calculation is now more reliable than a simple order of magnitude estimate.

The real test of the particular model studied here lies in the experimental search for charmed particles. The masses of the charmed hadrons cannot be directly inferred from the constraints on quark masses, but it is reasonable to assume that they lie in the range of one to ten GeV. Their life times are expected to be in the range

$$10^{-13} \lesssim \tau \lesssim \quad 5 \times 10^{-12}$$

depending on the leptonic branching ratio

$$0.01 \lesssim B_\ell \lesssim 0.5$$

However it is also likely that the lowest lying charmed baryon states will decay strongly into an uncharmed baryon and a charmed meson.

3. Are weak and electromagnetic mass shifts calculable?

Aside from the renormalizability of the theory, it may be hoped that mass differences among members of an isospin multiplet are calculable, i.e. do not require infinite renormalization. Several authors have studied this question(14) and the following features appear to emerge from their work:

(a) In the chiral symmetry limit ($m_q \to 0$) mass shifts are finite in the Weinberg-Salam model.

(b) For $m_q \neq 0$, the leading logarithmic divergence (order α) is determined by the short distance behaviour of current products. It is proportional to the matrix element of a quark-antiquark operator, so there is no contribution with $\Delta I = 2$ and therefore no contribution to the pion mass difference.

(c) Fayyazudin and Ryazuddin have further pointed out that since V±A currents give no logarithmic divergence of order α, there exists a class of models in which mass differences are finite to this order.

However in all of these discussions, calculability is established only up to terms $O(\alpha m_q^2/m_W^2)$. Infinity times a small number is still infinity; if counterterms (or a renormalization of the Higgs couplings) must be introduced at any order, mass differences remain uncalculable.

Closely related problems are the study of spectral function sum rules, chiral symmetry breaking and the pseudoscalar system. A number of papers presented to this conference[15] study these questions using various features of strong and weak gauge theories.

4. Non-leptonic weak interactions.

The apparatus of the models for strong and weak interactions outlined in the introduction has also been applied to the study of non-leptonic weak interactions. In this case the asymptotic freedom of the theory involving colour gauge gluons plays a crucial role. The effective local non-leptonic interaction

$$\mathcal{H}_{eff} \sim \int d^4x\, D_F(x, m_W^2)\; T(J_\mu(x), J^{\mu'}(o)),$$

where D_F is the propagator function for the intermediate boson, is studied by means of the Wilson operator product expansion:

$$T(J_\mu(x),\ J^\mu\ (o)) = \Sigma\ F^i(x^2) O_i(o)$$

where the O^i are local operators and $F^i(x^2)$ are functions of the space-time separation of the currents. In free field theory these functions must be such that when the operator expansion is inserted in $\mathcal{H}_{eff}$ one obtains a local current-current interaction (to order m_h^2/m_W^2)

$$\mathcal{H}_{eff} \rightarrow :J_\mu(o)\ J^\mu\ (o):$$

In the presence of strong interactions the coefficient functions may be renormalized in such a way that the effective hamiltonian does not reduce to the primary current current coupling:

$$\mathcal{H}_{eff} = C^i O^i(o) \neq :J_\mu(o)\ J^\mu\ (o):$$

In asymptotically free theories the coefficient functions F^i can be calculated perturbatively in the limit $x^2 \to 0$, which determines the leading behaviour (to order m_h^2/m_W^2).

For the effective hamiltonian responsible for non-leptonic decays of strange particles, the operators O^i in the Wilson expansion must have the properties (up to corrections of order m_q^2/m_W^2):

(a) singlet with respect to colour SU_3.

(b) 1st component of a U-spin vector ($\mathcal{H}_{eff} \sim \bar{\lambda}n + \bar{n}\lambda$)

(c) 3rd component of a "P-spin" vector ($\mathcal{H}_{eff} \sim \bar{p}p - \bar{p}'p'$)

(d) left handed helicity

The operators of lowest dimension (i.e. those which determine the short distance behaviour) which have the above properties are V-A current-current products. However the effective local interaction differs from the free field case in that the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ components are renormalized differently. Explicitly one finds[16]

$$C_{\frac{1}{2}}/C_{\frac{3}{2}} \sim \left\{1 + \frac{g^2}{4\pi}\frac{25}{6\pi}\ln\frac{M_W}{\mu}\right\}^{0.72} \simeq 5$$

where the values $m_W \simeq 100$ GeV, $\mu \simeq 1$ GeV (the onset of scaling), and $\frac{g^2}{4\pi} \simeq 1$ (the gluon coupling at 1 GeV) have been used. In effect, there is an enhancement of the $\Delta I = \frac{1}{2}$ operator by a factor of about 2.5 and a slight suppression (~ 0.6) of the $\Delta I = \frac{3}{2}$ operator.

This is not sufficient in itself to explain the observed $\Delta I = \frac{1}{2}$ rule. However there are many arguments in the literature tending to support the possibility of an additional suppression of the $\Delta I = \frac{3}{2}$ contribution in the matrix elements.

The question of parity violation in nuclear physics is being studied with the same techniques by the authors of refs. 16. In the Cabibbo theory of charged currents, the effective $\Delta S = 0$ coupling is of the form:

$$\mathcal{H}_{eff} = \cos^2\theta_c \; J_1{}^+ J_1{}^- + \sin^2\theta_c \; J_{\frac{1}{2}}{}^+ \; J_{\frac{1}{2}}{}^-$$

The parity violating pion exchange potential, which must have $\Delta I = 1$, derived from this coupling has a suppression factor $\sin^2\theta_c \simeq .04$. Although nuclear effects are not entirely understood, there seems to be a consensus that the observed pion exchange contribution is much larger than can be accounted for in this theory. In the Weinberg-Salam model, there is an additional $\Delta I = 1$ component which is induced by Z-exchange

$$\mathcal{H}_{eff} = J_1{}^o(V - A) \; J_o{}^o(V) \; \sin^2\theta_w .$$

This term arises from the interference of the V - A isovector neutral current with the isoscalar component of the electromagnetic current and is suppressed only by $\sin^2\theta_w \simeq .4$.

The isospin properties of the $\Delta S = 0$ couplings in a variety of gauge models have been studied by Bailen et. al.[17] What is of interest to us here is the effect of strong interactions on the structure of the effective local interaction. In the Wilson expansion of a colour singlet operator, two types of 4-quark operators may appear:

$$O_1 = (\bar{q}q)(\bar{q}q) \; , \quad O_2 = (\bar{q}\,\vec{\lambda}\,q)\;(\bar{q}\,\vec{\lambda}\,q)$$

where λ are the matrices of colour SU(3). Operators of type O_2 may be reduced to those of type O_1 by a Fierz rearrangement of spin indices. Under this transformation a V-A current current coupling remains a V-A current current coupling, but a (V-A)x(V+A) coupling gives scalar pseudoscalar couplings: SP + PS + ... In the asymptotically free theory, it turns out that there is a considerable enhancement of the $\Delta I = 1$ SP + PS term. Evaluation of the matrix element is not straightforward and depends explicitly on the p, n and λ quark masses. However in the factorization approximation

$$\langle \pi N \, | \mathcal{H}_{eff} | \, N \rangle \sim \langle N | S | N \rangle \, \langle \pi | P | o \rangle$$

the sign, and, for sufficiently small quark mass, the order of magnitude of the pion exchange potential appear to be in agreement with that derived from polarization experiments in nuclear γ-decay[18].

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ABNORMAL NUCLEAR STATES

T D Lee

Columbia University, New York

(Notes compiled by H. Danskin and unchecked by the author).

Recently, it has been suggested[1,2] that if there exists a strongly interacting 0^+ meson, then when the nucleon density becomes sufficiently high over an extended volume, there exists the possibility of abnormal nucleon states, hitherto unobserved. This report contains a summary of the quasi-classical arguments that led to such a suggestion, and a few comments on the theoretical basis in quantum field theory; for more details, see ref. 3.

For definiteness, consider a simple renormalizable theory of a spin-0 Hermitian field ϕ and a spin-$\frac{1}{2}$ nucleon field ψ. The Lagrangian density **L** is

$$\mathbf{L} = -\tfrac{1}{2}\, (\partial_\mu\phi)^2 - U(\phi) - \psi^+\gamma_4 \left[\gamma_\mu\partial_\mu + (m_N + g\phi)\right]\psi$$

$$+ \text{ counter terms} \qquad (1)$$

where $U(\phi) = \frac{1}{2}\, a\, \phi^2 + \frac{1}{3!}\, b\phi^3 + \frac{1}{4!}\, c\, \phi^4$, (2)

the parameters a, b, c, m_N and g are the renormalized constants, and we assume that $c > o$ and $a \geq b^2/3c > o$. The vacuum state is defined as the lowest eigenstate of the system with baryon number 0, and we assume

$$< \text{Vac} \,|\, \phi(x) \,|\, \text{Vac} > \;=\; 0. \qquad (3)$$

Let $|\, n >$ be the <u>lowest energy eigenstate</u> of the total Hamiltonian H that satisfies the constraint

$$\Omega^{-1} \int d^3 r \, < n | \psi^+\psi | \, n > \;\equiv\; n, \qquad (4)$$

where n denotes the average nucleon density, and Ω is the (finite) volume of the box in which the theory is quantised. A useful concept is the energy density

$\varepsilon(n)$, defined as

$$\varepsilon(n) = \lim_{\Omega\to\infty} \Omega^{-1} < n|H \mid n> . \qquad (5)$$

Now, if the nucleon density n is sufficiently high, the system may exist in an "abnormal nuclear state", in which the effective nucleon mass becomes zero, or nearly zero (instead of m_N). The simplest way to see why such an abnormal state may develop is to examine the quasi-classical solution.

Assume that the nucleons form a Fermi gas and ϕ is a classical field; then the lowest energy state $|n>$ is one in which ϕ is a constant and the Fermi gas is completely degenerate. The corresponding energy density $\varepsilon(n)$ is

$$\varepsilon(n) = U(\phi) + (4\pi^3)^{-1} K \int_F d^3\underline{p} \left[\underline{p}^2 + m^2_{eff}\right]^{\frac{1}{2}} \qquad (6)$$

where m_{eff} ($\equiv m_N + g\phi$) is the effective nucleon mass, the subscript F indicates that the integral extends only over the Fermi sea, and K = 1(2) for a neutron (nuclear) medium.

In Eq.(6), ϕ is determined by the minimum of ε. As n increases, the Fermi-sea contribution to the energy becomes increasingly more important and one finds

$$\lim_{n\to\infty} \phi = -(m/g) \quad \text{and} \quad \lim_{n\to\infty} m_{eff} = 0; \qquad (7)$$

thus, the state becomes abnormal, defined by $m_{eff} \cong 0$. As illustrated in Fig. 1, depending on the paramters in the theory, the transition from the low density "normal" solution to the high density "abnormal" solution may or may not be a continuous one. The transition is discontinuous only if $b^2 > \frac{8}{3}ac$, b and g have the same sign, and the point $\phi = -m_N/g$ is to the left of $\phi = -c^{-1}\left[b - (b^2 - 2ac)^{\frac{1}{2}}\right]$, one of the points of inflection of $U(\phi)$.

A particularly interesting case is the σ- model[(4)], since its quasi-classical solution gives a discontinuous transition from the normal to the abnormal state as n increases. The critical density n_c is

$$n_c \simeq 11.6 \left(\frac{m_\sigma}{m_N}\right)^2 \left(\frac{g^2}{4\pi}\right)^{-1} n_o \qquad (8)$$

where $n_o^{-1} = \frac{4\pi}{3}(1.2 \text{ fm})^3$.

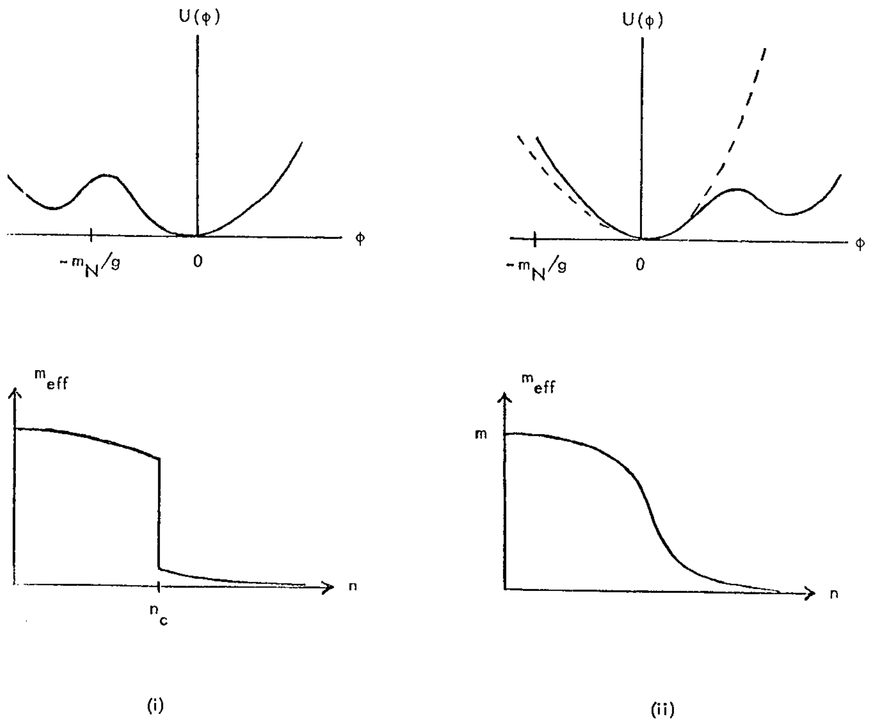

Fig. 1 Schematic drawings of transitions between the normal state ($m_{eff} \cong m_N$) and the abnormal state ($m_{eff} \cong 0$). In case (ii), $U(\phi)$ can have either only one minimum (dashed curve), or two minima (solid curve).

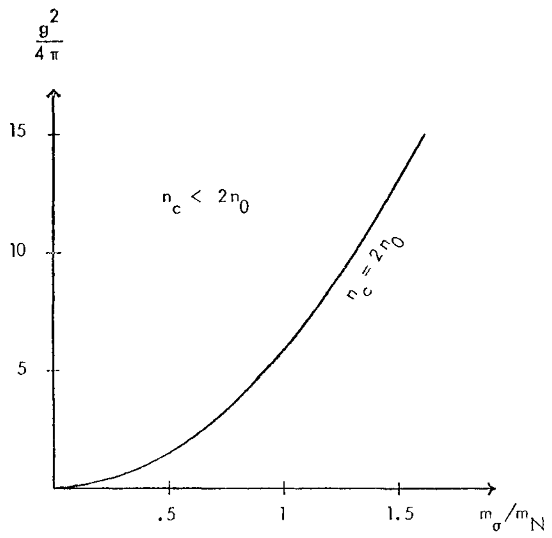

Fig. 2 The solid curve denotes $n_c = 2n_0$.

Although there is at present no reliable data on either m_σ or g^2, the range given in Fig. 2 is not incompatible with most of the available discussions on 0^+ mesons in the literature(5). Since it is possible that, through heavy ion collisions, we may increase the nucleon density from n_0 to around $2n_0$, this suggests that experimental investigations of abnormal nuclear states may become feasible.

Questions naturally arise as to effects not included in the above quasi-classical solution, especially those related to quantum fluctuations because of multi-loop diagrams. At present, there exists no reliable technique capable of handling a relativistic local field theory with strong coupling (except in 2 dimensions), but from the quasi-classical solution, the abnormal nuclear state is expected to exist even for a weakly coupled meson field, provided the nucleon density is sufficiently high. Thus, it seems worthwhile to develop systematically the high density but weak coupling expansion of a Fermion medium interacting with a 0^+ meson field. This problem is not completely trivial, since the usual perturbation series is applicable only if the coupling is weak and the density is relatively low. A suitable rearrangement of this series is therefore necessary in order to derive a high density expansion; the details are given elsewhere(3). The result is that, to order $O(g^2)$, the energy density ε is

$$\varepsilon = U(\bar{\phi}) + (4\pi^3)^{-1} K \int_F d^3\underline{p} \; [\underline{p}^{\,2} + m^2_{eff}]^{\frac{1}{2}} + (32\pi^4)^{-1} g^2 k_F{}^4 \qquad (9)$$

where U is given by Eq.(2), $m_{eff} = m_N + g\bar{\phi}$, and $\bar{\phi}$, defined by $\bar{\phi} \equiv \lim_{\Omega\to\infty} \Omega^{-1} \int <n| \; \phi(x) \; |n> d^3\mathbf{x}$, is determined by the minimum of ε. By comparing Eq.(9) with the quasi-classical solution (Eq.(6)), we see that $\bar{\phi}$ assumes the role of the classical field; the only difference is the last term $(32\pi^4)^{-1} g^2 k_F{}^4$ in Eq.(9), which is $\bar{\phi}$ -independent. Thus, the quasi-classical solution fo m_{eff} and the related phenomenon of phase transition remain correct in a quantum-mechanical description, provided that g^2 is sufficiently small.

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MAGNETIC MONOPOLES IN UNIFIED THEORIES

G 't Hooft

CERN, Geneva

In any system described by interacting field equations with particular symmetries, one can expect solutions of a very remarkable type, even at the classical level. A well known example is the Schwartzschild solution in General Relativity.

Hedgehogs

In pure gauge theories with unbroken symmetry, there are spherically symmetric solutions[1] which have arbitrary energy. In pure chirally symmetric scalar field theories, one might consider solutions with unconventional boundary conditions[2], but their energy is infinite except when they occur in rotating pairs.

In gauge theories with Higgs mechanism there are spherically symmetric solutions with definite energy[3,4], but many of them will be unstable[4]. We report a stable solution[3] in all gauge models where the electromagnetic group U(1) is a subgroup of a semi-simple local gauge group like $[SU(2)]^{n_2} \otimes [SU(3)]^{n_3} \otimes \ldots$ etc. It will present itself to the observer as a finite mass magnetic monopole[5]. The magnetic charge g satisfies Dirac's quantization condition[6]

$$eg = 2\pi n \quad ,$$

where n is an integer. Indeed, n may be set equal to one if we replace e by the minimal charge allowed by the group structure (which for instance is ½e in the Georgi-Glashow model[7]), and we see that Schwinger's condition[8] is violated.

The calculation has been performed in more detail in the Georgi-Glashow model[3,7]. We put for the Higgs field

$$Q_a(\bar{x},t) = x_a\, Q(|x|) , \tag{2}$$

and for the gauge vector field

$$\begin{aligned} W_i^{\,a}(\bar{x},t) &= \varepsilon_{iab}\, x_b\, W(|x|) , \quad i = 1,2,3 \\ W_4^{\,a}(\bar{x},t) &= 0 \end{aligned} \tag{3}$$

The boundary conditions are

$$\begin{aligned} &Q \xrightarrow[|x| \to \infty]{} F/|x| \\ &W \xrightarrow[|x| \to \infty]{} a/|x|^n \quad , \quad (n > 1) \\ &Q,W \text{ finite as } |x| \to 0. \end{aligned} \tag{4}$$

The magnetic charge then follows from the field equations, and the mass of the particle turns out to be

$$M_{monopole} = \frac{4\pi}{e^2} M_W\, C \quad , \tag{5}$$

where M_W is an intermediate vector boson mass; e is the charge of the electron and C depends on the parameter(s) of the theory but it is close to one.

It has been proposed[2] to call classical solutions with the spherical topology "hedgehogs".

Vortices

Some theories might also allow for solutions with the topology of the dual string: a well known example is the Abelian Higgs model with the quantized magnetic flux lines[9]. We do not expect that analogous, stable, solutions exist in the current models for weak and electromagnetic interactions, but attempts have been made recently to construct mathematically more complicated vortex solutions[10].

In an attempt to understand qualitatively the strong interactions that lead to quark confinement, one can construct models with a non-renormalizable "effective" Lagrangian[11], in which not the magnetic but the electric field lines are squeezed into quantized vortex lines. These "strings" will bind automatically all quarks and other coloured objects in triality-zero states only.

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LEPTON NUMBER AS THE FOURTH COLOUR

J C Pati

University of Maryland, U.S.A

I wish to talk about the idea that there exists only one variety of fermionic matter composed of baryonic quarks and leptons with no sharp distinction between them at the deepest level. The fact that only the baryons exhibit strong interactions at present energies and not the leptons is to be attributed to be a consequence of spontaneous symmetry-breaking; in this case universality of all interactions (weak, electromagnetic and strong) with respect to baryons and leptons must manifest itself at an appropriately high energy. Initial suggestion along this line was made [1] by Salam and myself in the last High Energy Conference (1972) held at Batavia; it has subsequently been developed in three recent notes [2),3),4)] by us.

The need for treating baryons and leptons in such a unified manner appear to be compelling by our desire to understand: (a) the emergence of baryon and lepton numbers, (b) baryon-lepton universality in the weak and electromagnetic interactions with $(1 + \gamma_5)/2$ projection for baryons and leptons $(\nu_e, e^-, \mu^-, \nu_\mu)$ rather than for antileptons $(\bar{\nu}_e, e^+, \mu^+, \bar{\nu}_\mu)$ and (c) quantization of electric charge (which demands no abelian contribution to electromagnetic current), etc. as necessary consequences of the theory.

The idea of such unification may be realized in a variety of ways. However there appear to be two general consequences: (i) there must exist gauge particles (X's) carrying baryonic and leptonic quantum numbers coupled to quark-lepton currents. The mass of these particles define a new scale of energy in particle physics; at such energies leptonic and semileptonic interactions would become as strong as hadronic. (ii) There is the logically independent but very likely possibility (especially if quarks carry integer charges) that baryonic quarks may transform themselves into leptons with a violation of baryonic and leptonic quantum numbers (such that fermion number F = B + L is still conserved). This would have profound effects on quark-searches, proton-stability and perhaps on cosmology.

The basic scheme, which we propose, consists of two 16-folds of chiral fermionic multiplets Ψ_L and Ψ_R :

$$\Psi_{L,R} = \begin{bmatrix} p_a & p_b & p_c & p_d = \nu_e \\ n_a & n_b & n_c & n_d = e^- \\ \lambda_a & \lambda_b & \lambda_c & \lambda_d = \mu^- \\ \chi_a & \chi_b & \chi_c & \chi_d = \nu_\mu \end{bmatrix}_{L,R}$$

transforming as (4,1,4) and (4,1,4), respectively, under the fundamental symmetry structure $G = SU(4)_L \times SU(4)_R \times SU(4')_{L+R}$. The colours (a,b,c) denote baryonic quarks, while the fourth colour "d" denotes lepton number. Universal weak, electromagnetic and strong interactions of all matter are generated by gauging a non-abelian, anomaly-free renormalizable subgroup $\mathcal{G}$ of G :

$$\mathcal{G} = SU(2)_L^{I+II} \times SU(2)_R^{I+II} \times SU(4')_{L+R}$$

for which Ψ_L and Ψ_R transform as (2+2,1,4) and (1,2+2,4), respectively. The groups $SU(2)^{I}_{L,R}$ and $SU(2)^{II}_{L,R}$ act on the indices $(p,n)_{L,R}$ and $(\chi,\lambda)_{L,R}$, respectively, while $SU(2)^{I+II}_{L,R}$ are their diagonal sums. The $SU(4')_{L+R}$ act on the colour indices (a,b,c,d).

<u>The gauge particles:</u>

$$W_L = (\underset{\sim}{3},1,1) \ , \ W_R = (1,\underset{\sim}{3},1)$$

$$V = (1,1,\underset{\sim}{15}) = \left[\begin{array}{c|c} V(\underset{\sim}{8}) - \dfrac{S^0 \times \mathbb{1}}{\sqrt{12}} & X \\ \hline \bar{X} & \sqrt{\dfrac{3}{4}}\, S^0 \end{array}\right]$$

<u>Coupling constants:</u>

$$g^2_{L,R}/4\pi \simeq \alpha \ , \ f^2/4\pi \simeq (1 - 10) \ .$$

(The possibility that all three (bare) coupling constants $g_L^{(0)}$, $g_R^{(0)}$ and $f^{(0)}$ are related can, of course, be entertained in such a scheme by imbedding $\mathcal{G}$ in a bigger group [5]). In the above $V(\underset{\sim}{8})$ denotes the SU(3')-colour-octet of gauge mesons consisting of (V_ρ, V_{K*}, V_8) which are coupled to the (a,b,c)-quarks; X is an exotic (B = +1, L = -1) SU(3')-triplet; S^0 is a SU(3') singlet,

The complete Fermi-Lagrangian of the scheme is given by:

$$-\mathrm{Tr}\ \left\{ \bar{\Psi}_L (\gamma_\mu \nabla_\mu)_L\ \psi_L + L \leftrightarrow R \right\}$$

where

$$\nabla_\mu \psi = \partial_\mu \psi + igW\psi_\mu - if\psi V_\mu \ .$$

Note the left-right symmetric nature of this Lagrangian (if $g_L^{(0)} = g_R^{(0)}$).

The restrictions on gauge-meson masses, which must be satisfied from experimental considerations are the following:

Process	Exchange	Limit
$K_L \to \bar{\mu} e$	X-triplet	$(f^2/m_X^2) \lesssim 10^{-9}\ (\mathrm{BeV})^{-2}$
$\nu + p \to \nu + h$)) $\nu + e \to \nu + e$)	S^0	$(f^2/m^2_{S^0}) \lesssim 10^{-5}\ (\mathrm{BeV})^{-2}$
(V + A)-interaction	W_R	$(g_R^2/m^2_{W_R}) \lesssim \frac{1}{10} G_{\mathrm{Fermi}}$
(V - A)-interaction	W_L	$(g_L^2/m^2_{W_L}) \simeq G_{\mathrm{Fermi}}$
$(q^{a,b,c} - q^{a,b,c}$ interaction)	$V(\underset{\sim}{8})$	$f^2/m^2(V(8)) \sim (1\text{-}10)/m_\rho^2$

($V(\underset{\sim}{8})$-gluons may be left massless for the fractionally charge-quark model (see below), if the associated infra-red problem can be resolved satisfactorily).

I list below some of the major consequences of the scheme:

(1) Quarks may carry either integer or fractional charges; however remarkably enough the group structure uniquely fixes the charges of the leptons to be (0,-1,-1,0). This explains why proton quark and e^- (not e^+) should carry same helicity in low-energy weak interaction.

(2) If $g_L^{(0)} = g_R^{(0)}$, there is the attractive possibility that parity violation is entirely due to spontaneous symmetry-breaking, which leads to $m_{W_R} \neq m_{W_L}$. Thus parity non-conservation and dominance of (V-A) interactions may entirely be a low-energy phenomenon to disappear at high energies.

(3) The left-right symmetric nature of the Fermi-Lagrangian provides a desirable basis for CP non-conservation[6].

(4) The gauge group is non-abelian, which is desirable for possibly realizing asymptotic freedom for the complete theory. In the SU(3') sector, asymptotic freedom is retained either if V(8) receive mass by dynamical symmetry-breaking or if they are left massless (a possibility which is appropriate in the fractionally charged quark scheme[7]).

(5) The scheme very definitely calls for V+A interactions in nature. Improvements in the longitudinal polarization of $e^{\pm}$ in $\beta^{\pm}$-decays and other related experiments would be desirable to improve the presently available 10% limit of (V+A) amplitudes compared to (V-A) amplitudes.

(6) The scheme leads to neutral neutrino-current-leptonic and semileptonic processes via exchanges of an effective Z^0 (this is identical to the Z^0 in the simple $SU(2)_L \times U(1)$ scheme for pure leptonic currents) as well as S^0. If S^0 is superheavy, the neutral-current processes may be described by one angle $\sin^2\theta = g_R^2/(g_L^2 + g_R^2)$, for which our preferred value is $\approx \frac{1}{2}$ corresponding to the left-right symmetric Fermi-Lagrangian for which $g_L^{(0)}=g_R^{(0)}$. If, however, S^0 is light enough to give $f^2/m_{S^0}^2 \approx G_F$, it can be as potent as Z^0 to contribute to neutral-current processes. It is worth noting that S^0 is coupled to pure vector current[8] of quarks and leptons and that the quark part is pure isoscalar and colour singlet. S^0 contribution is a new ingredient in our scheme.

(7) If quarks carry fractional charges, it turns out in our scheme that baryon-number conservation is a consequence of electric charge and fermion-number conservation. If on the other hand, they carry integer charges, it appears most likely that the mass matrix (arising through spontaneous symmetry-breaking) would induce a mixing of $W_L^{\pm}$ of $W_R^{\pm}$ with X's; which leads to a breakdown of baryon and lepton-number conservation. The quarks in this case are unstable. They decay to $(\ell+\pi)$ or $(\ell+\ell+\bar{\ell})$ with lifetimes as short as 10^{-10} sec. (if $m_q \approx$ 5-10 BeV); on the other hand, a proton having the quantum numbers of a 3-quark system can only decay by a conversion of each of the three quarks to leptons like triple β-decay (provided quark and diquark are heavier than proton). This accounts for the extraordinary stability of the proton ($\tau_p \geq 10^{37}$ sec). The allowed decay modes of the proton (consistent with fermion-number conservation in the scheme) are:

$$P \rightarrow \nu + \nu + \nu + \pi^+$$
$$\rightarrow 4\ell + \bar{\ell}\text{ , etc.}$$

Note that no 2- or 3-body decays of the proton are allowed.

In such a picture there is the intriguing possibility that quarks carrying integer charges have already been produced but are missed due to their instability. We urge a search for integer charge quarks decaying into highly energetic leptons with lifetimes of order 10^{-10} sec. or longer (phenomenologically considerably shorter quark lifetimes may also be allowed without conflicting with proton lifetime). Note that the leptons thus produced should carry helicity. We also strongly urge a search for proton decaying into multiparticle system (4 or higher).

As a possibility, one may ask if the prompt leptons produced in p+p collision, as reported in this Conference are due to $(q\bar{q})$ production with the quarks decaying rapidly into leptons. This could explain the similarity between lepton and pion distributions. The leptons thus produced can be distinguished from lepton-pair production via vector-meson decays.

Lack of time does not permit me to talk about variations of our basic scheme, which would be relevant[9)] for enhanced lepton-hadron interactions at present energies ($e^-e^+ \to$ hadrons, $e^{\pm}p \to e^{\pm}+X$, $p + p \to \ell + X$, etc.) and the question of spontaneous symmetry-breaking and generation of masses in the scheme.

In summary, it seems a new era in physics has begun in which one may increasingly see the unity of baryons and leptons.

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2 J C Pati and Abdus Salam, Phys. Rev. D8, 1240 (1973)

3 J C Pati and Abdus Salam, Phys. Rev. Letters 31, 661 (1973).

4 J C Pati and Abdus Salam, Phys. Rev. D, 1 July 1974.

5 This has been considered by us together with Mohapatra and independently by Fritsz and Minkowski for our basic scheme.

6 R N Mohapatra and J C Pati, University of Maryland preprint (1974). Ebel and Frenkel, University of Wisconsin preprint (1974). These two papers are somewhat similar for the hadronic sector; however certain exact consequences such as $\eta_{+-} = \eta_{00}$, $\eta_{+-0} = \eta_{000}$ are not noted in the second paper.

7 This possibility has been advocated by Gross and Wilczek, Fritsz and Gell-Mann and Weinberg as a possible mechanism to confine quarks and $V(\underset{\sim}{8})$ gluons.

8 This could enhance the pure vectorial character of neutral currents, which could tend to make ν and $\bar{\nu}$ cross-sections equal.

9 See Ref. 4 and J C Pati and Abdus Salam, Phys. Rev. Letters 32, 1083 (1974) for a discussion of these possibilities.