

Review

A Review of the Exceptional Supersymmetric Standard Model

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Received: 29 January 2020; Accepted: 16 March 2020; Published: 4 April 2020



Abstract: Local supersymmetry (SUSY) provides an attractive framework for the incorporation of gravity and unification of gauge interactions within Grand Unified Theories (GUTs). Its breakdown can lead to a variety of models with softly broken SUSY at low energies. In this review article, we focus on the SUSY extension of the Standard Model (SM) with an extra $U(1)_N$ gauge symmetry originating from a string-inspired E_6 GUTs. Only in this $U(1)$ extension of the minimal supersymmetric standard model (MSSM) can the right-handed neutrinos be superheavy, providing a mechanism for the baryon asymmetry generation. The particle content of this exceptional supersymmetric standard model (E_6 SSM) includes three 27 representations of the E_6 group, to ensure anomaly cancellation. In addition it also contains a pair of $SU(2)_W$ doublets as required for the unification of gauge couplings. Thus, E_6 SSM involves exotic matter beyond the MSSM. We consider symmetries that permit suppressing flavor changing processes and rapid proton decay, as well as gauge coupling unification, the gauge symmetry breaking and the spectrum of Higgs bosons in this model. The possible Large Hadron Collider (LHC) signatures caused by the presence of exotic states are also discussed.

Keywords: Grand Unified Theories; supersymmetry; Higgs bosons

1. Introduction

Symmetries play a key role in modern high energy physics. Indeed, it was realized a long time ago that light hadron resonances form representations of the $SU(3)$ group, which is associated with light quark flavors, while the physics of strong interactions is described by the colored $SU(3)_C$ gauge symmetry. It was also established that weak and electromagnetic forces represent electroweak (EW) interactions based on the $SU(2)_W \times U(1)_Y$ gauge group. Within the standard model (SM) of elementary particles, which describes rather precisely almost all experimental data measured in earth based experiments, $SU(2)_W \times U(1)_Y$ is spontaneously broken to the abelian $U(1)_{em}$ gauge group associated with electromagnetism by means of the Higgs mechanism. The latter predicts the existence of the scalar (i.e., Higgs particle) which was recently discovered at the LHC. Thus, the Lagrangian of the SM is invariant under the transformations of the Pointcaré group and $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge symmetry. The transformations of the Pointcaré group involve time and space translations as well as a set of transformations of the Lorentz group that includes Lorentz boosts along three axes and rotations about them.

At very high energies, the SM can be embedded into GUTs [1] based on the $SU(5)$ or $SO(10)$ gauge groups. In the case of $SU(5)$ GUTs, each SM family of quarks and leptons fills in a complete one antifundamental and one antisymmetric second-rank tensor representations of $SU(5)$, i.e., $\bar{5} + 10$. Within $SO(10)$ GUTs, each family of SM fermions may belong to a single 16-dimensional spinor

representation of $SO(10)$. Such models predict the existence of right-handed neutrinos, which may be used for the see-saw mechanism [2] and leptogenesis [3].

SUSY GUTs permit to place fermions and bosons of the SM within one supermultiplet. To combine Pointcaré and gauge (internal) symmetries to attain the unification of gauge interactions with gravity, one needs to overcome the Coleman–Mandula theorem. According to it, in the most general case, quantum field theory has a symmetry that is a product of the gauge and Pointcaré groups [4]. This theorem can be overcome within graded Lie algebras. The structure of these algebras can be presented as

$$[\hat{B}, \hat{B}] = \hat{B}, \quad [\hat{B}, \hat{F}] = \hat{F}, \quad \{\hat{F}, \hat{F}\} = \hat{B}, \quad (1)$$

where \hat{B} and \hat{F} are bosonic and fermionic generators. Supersymmetries are Graded Lie algebras that involve the Pointcaré algebra. The simplest $N = 1$ supersymmetry includes a set of generators of the Pointcaré group (bosonic generators) and a single Weyl spinor operator Q_α as well as its complex conjugate $Q_\alpha^\dagger = \bar{Q}_\alpha$ (fermionic generators). SUSY algebra implies that each supermultiplet has the same number of bosonic and fermionic degrees of freedom.

In $N = 1$, SUSY GUTs with the E_6 gauge group the fundamental representation of E_6 , which decomposes under $SO(10) \times U(1)_\psi$ subgroup, as

$$27 \rightarrow \left(16, \frac{1}{\sqrt{24}} \right) \oplus \left(10, -\frac{2}{\sqrt{24}} \right) \oplus \left(1, \frac{4}{\sqrt{24}} \right), \quad (2)$$

contains one family of the SM fermions and Higgs doublet. The Higgs bosons are components of $\left(10, -\frac{2}{\sqrt{24}} \right)$. The SM gauge bosons are assigned to the adjoint representation of E_6 , i.e., a 78-plet. In $N = 2$, SUSY GUTs with the E_8 gauge symmetry all SM particles are components of 248 representation of E_8 . This representation decomposes under the E_6 subgroup of E_8 as follows:

$$248 \rightarrow 78 \oplus 3 \times 27 \oplus 3 \times \bar{27} \oplus 8 \times 1. \quad (3)$$

The local SUSY (supergravity) leads to a partial unification of gauge interactions with gravity [5–7]. Because supergravity (SUGRA) is a non-renormalizable theory, it should be regarded as a low energy limit of some renormalizable or finite theory. The best candidate for such theory is a ten-dimensional superstring theory with $E_8 \times E'_8$ gauge symmetry [8]. Compactification of the extra dimensions in this theory gives rise to the breakdown of E_8 to E_6 or its subgroups associated with the observable sector [9]. The remaining E'_8 gauge group plays the role of a hidden sector in which the breakdown of local SUSY is induced. As a consequence, a set of soft SUSY breaking terms [10–13], which is characterized by the gravitino mass ($m_{3/2}$), is generated. The non-perturbative effects in the hidden sector which cause the breakdown of local SUSY may also induce a large mass hierarchy between $m_{3/2}$ and the Planck scale M_P [14].

When $m_{3/2} \ll M_P$, the breakdown of the E_6 gauge group near the GUT scale M_X can result in a variety of SUSY models at low energies including MSSM and its extensions with an extra $U(1)'$ gauge symmetry, which is a linear combination of $U(1)_\chi$ and $U(1)_\psi$, i.e.,

$$U(1)' = U(1)_\chi \cos \theta_{E_6} + U(1)_\psi \sin \theta_{E_6}. \quad (4)$$

Here, $U(1)_\psi$ and $U(1)_\chi$ can originate from the breakdown of E_6 , i.e., $E_6 \rightarrow SO(10) \times U(1)_\psi$, $SO(10) \rightarrow SU(5) \times U(1)_\chi$, whereas the SM gauge group is a subgroup of $SU(5)$, i.e., $SU(5) \supset SU(3)_C \times SU(2)_W \times U(1)_Y$. In the simplest case, $U(1)_\chi \times U(1)_\psi$ is broken down to its discrete subgroup $Z_2^M = (-1)^{3(B-L)}$, which is the so-called matter parity. If in this case the low energy matter content involves three families of the SM fermions and their scalar superpartners as well as two $SU(2)_W$ doublets of the Higgs bosons (H_1 and H_2) and their fermionic partners (Higgsinos), then this model corresponds to the simplest SUSY extension of the SM—the MSSM. Matter parity conservation

implies that the lightest SUSY particle (LSP) is stable. Therefore, it can play the role of dark matter. To generate the masses of all quarks and charged leptons, the MSSM superpotential has to include the following sum of the products of chiral superfields

$$W_{\text{MSSM}} = y_{ab}^U Q_a u_b^c H_2 + y_{ab}^D Q_a d_b^c H_1 + y_{ab}^L L_a e_b^c H_1 + \mu H_1 H_2, \quad (5)$$

where a and b are family indices that run from 1 to 3. In Equation (5), the left-handed quark and lepton doublets are denoted by Q_a and L_a ; the right-handed charged leptons and up- and down-type quarks are denoted by e_a^c , u_a^c and d_a^c , respectively; and the Yukawa couplings y_{ab}^U , y_{ab}^D and y_{ab}^L are dimensionless 3×3 matrices. The analysis of the renormalization group (RG) flow of gauge couplings indicates that they converge to a common value near the scale $M_X \simeq 2 \times 10^{16}$ GeV in the framework of the MSSM [15–18]. This allows one to embed the MSSM into SUSY GUTs.

The MSSM superpotential in Equation (5) contains only one bilinear term $\mu H_1 H_2$. This term can be present before SUSY is broken. Therefore, the parameter μ is expected to be either zero or of the order of M_X . If $\mu \sim M_X$, then the EW symmetry breaking (EWSB) does not take place. In contrast, when μ vanishes near the GUT scale, M_X it is not induced below M_X due to the non-renormalization theorems [19,20]. In this case, near the physical vacuum $\langle H_d \rangle = 0$ and the down-type quarks as well as the charged leptons remain massless. To ensure the correct EWSB, μ should be of the order of the SUSY breaking scale M_S .

In the framework of the simplest extension of the MSSM, the next-to-MSSM (NMSSM), the Z_3 symmetry ($\Phi_i \rightarrow e^{2\pi i/3} \Phi_i$) forbids the bilinear term $\mu(H_1 H_2)$. The superpotential of the NMSSM is given by [21]

$$W_{\text{NMSSM}} = \lambda S(H_1 H_2) + \frac{\kappa}{3} S^3 + W_{\text{MSSM}}(\mu = 0), \quad (6)$$

where S is an extra singlet superfield. It acquires a vacuum expectation value (VEV), i.e., $\langle S \rangle = s/\sqrt{2}$, inducing an effective μ parameter ($\mu = \lambda s/\sqrt{2} \sim M_S$). The non-zero value of the coupling κ in Equation (6) explicitly breaks an additional global $U(1)$ symmetry, which is a common way to avoid the appearance of the axion in the particle spectrum. However, the VEVs of the Higgs fields break the exact Z_3 symmetry, leading to the formation of domain walls in the early universe [22]. Such domain structure of vacuum creates unacceptably large anisotropies in the microwave background radiation [23]. Because of this, the NMSSM superpotential should contain additional operators that violate the Z_3 symmetry and prevent the appearance of domain walls [24,25].

In the $U(1)'$ extensions of the MSSM inspired by E_6 the extra $U(1)'$ gauge symmetry in Equation (4) forbids an elementary μ term if $\theta_{E_6} \neq 0$ or π . Nevertheless, these extensions of the SM allow the interaction $\lambda S(H_d H_u)$ in the superpotential while the S^3 term is forbidden by the $U(1)'$ gauge symmetry. Near the scale M_S , superfield S develops a non-zero VEV that breaks $U(1)'$ inducing an effective μ term of the required size. There are no problems associated with the appearance of domain walls in such models because there is no discrete Z_3 symmetry. Different aspects of the phenomenology of the $U(1)'$ extensions of the MSSM inspired by E_6 have been extensively studied in the past [26–36]. Previously, the implications of these SUSY extensions of the SM have been studied for the muon anomalous magnetic moment [37,38], the EWSB [39–45], the electric dipole moment of the electron [46] and of the tau lepton [47], neutrino physics [48,49], lepton flavor violating processes such as $\mu \rightarrow e\gamma$ [50], fermion mass hierarchy and mixing [51], CP-violation in the Higgs sector [52], collider signatures associated with the exotic quarks and squarks [53], leptogenesis [54–57], EW baryogenesis [58,59], the Z' mass limits [60] and neutralino sector [44,46,47,50,61–68]. The Higgs sector in these SUSY models was explored in [45,68–71].

In this review article, we consider a specific E_6 inspired SUSY realization of the above $U(1)'$ type model associated with $\theta_{E_6} = \arctan \sqrt{15}$. This choice of Abelian $U(1)'$ corresponds to $U(1)_N$ gauge symmetry. Thus, such an exceptional supersymmetric standard model (E_6 SSM) [69,70] is based on $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N$ gauge symmetry. Right-handed neutrinos in this model do not participate in the gauge interactions. Therefore, only in such a $U(1)'$ extension of the MSSM

inspired by E_6 GUTs the right-handed neutrinos can be superheavy. This allows using a see-saw mechanism to generate the mass hierarchy in the lepton sector. Moreover, the decays of the heavy Majorana right-handed neutrinos may generate the lepton and baryon asymmetries within this SUSY model [54–56].

The layout of this paper is as follows. In Section 2, we specify the $U(1)_N$ extensions of the MSSM and discuss global symmetries that prevent non-diagonal flavor transitions as well as rapid proton decay in these SUSY models. The two-loop RG flow of the SM gauge couplings in the framework of the E_6 SSM is examined in Section 3. The Higgs sector dynamics and the emerging spectrum are discussed in Sections 4 and 5, respectively. In Section 6, the possible LHC signatures of the E_6 SSM are considered. Section 7 is reserved for our conclusions.

2. The $U(1)_N$ Extensions of the MSSM

The E_6 SSM implies that near the scale M_X the gauge symmetry of the E_6 GUTs is broken down to $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N \times Z_2^M$ [69,70]. In the $U(1)'$ extensions of the MSSM inspired by E_6 , the anomalies are canceled automatically if the low-energy particle spectrum involves complete representations of E_6 . Consequently, in the E_6 SSM, the particle spectrum is extended by a number of exotics so that it contains three complete 27-dimensional representations of E_6 (27_i with $i = 1, 2, 3$). These 27_i supermultiplets decompose under the $SU(5) \times U(1)_N$ subgroup of E_6 as follows:

$$27_i \rightarrow \left(10, \frac{1}{\sqrt{40}}\right)_i + \left(5^*, \frac{2}{\sqrt{40}}\right)_i + \left(5^*, -\frac{3}{\sqrt{40}}\right)_i + \left(5, -\frac{2}{\sqrt{40}}\right)_i + \left(1, \frac{5}{\sqrt{40}}\right)_i + (1, 0)_i. \quad (7)$$

The first and second quantities in the brackets represent the $SU(5)$ representation and extra $U(1)_N$ charge, respectively. An ordinary SM family is assigned to $\left(10, \frac{1}{\sqrt{40}}\right)_i + \left(5^*, \frac{2}{\sqrt{40}}\right)_i$. Right-handed neutrinos N_i^c are associated with the last term in Equation (7), $(1, 0)_i$. The next-to-last term, $\left(1, \frac{5}{\sqrt{40}}\right)_i$, represents new SM-singlet fields S_i . These fields have non-zero $U(1)_N$ charges. Therefore, they survive down to the EW scale. The pair of $SU(2)_W$ -doublets (H_i^d and H_i^u) which belong to $\left(5^*, -\frac{3}{\sqrt{40}}\right)_i$ and $\left(5, -\frac{2}{\sqrt{40}}\right)_i$ have the quantum numbers of Higgs doublets. These multiplets form either Higgs or Inert Higgs $SU(2)_W$ multiplets, i.e., Higgs-like doublets that do not develop VEVs. Other components of the corresponding $SU(5)$ multiplets form color triplets of exotic quarks \bar{D}_i and D_i . These quarks carry electric charges $\left(\pm \frac{1}{3}\right)$. They also have $B-L$ charges $\left(\pm \frac{2}{3}\right)$. Therefore, in phenomenologically viable $U(1)_N$ extensions of the MSSM, they can be either diquarks or leptoquarks.

In addition to the complete 27_i multiplets, the splitting of $27'_i$ and $\bar{27}'_i$ within the E_6 GUTs can give rise to a set of M_i and \bar{M}_i supermultiplets that have opposite quantum numbers. In the simplest case, the E_6 SSM spectrum is supplemented by $SU(2)_W$ doublet L_4 and anti-doublet \bar{L}_4 from the extra $27'$ and $\bar{27}'$, where L_4 supermultiplet has the quantum numbers of left-handed leptons. This allows preserving gauge coupling unification. Thus, the E_6 SSM involves Z' and exotic matter beyond the MSSM. Extra matter fill in three $5 + 5^*$ representations of $SU(5)$ plus three SM singlet superfields S_i . Such SUSY extensions of the SM can originate from the orbifold GUTs [57,72].

Over the last fifteen years, several variants of the E_6 SSM have been proposed [69,70,72–82]. The $U(1)_N$ extensions of the MSSM have been investigated in the context of non-standard neutrino models [49], EWSB [43–45], Z - Z' mixing [61], dark matter [83], the RG flow of couplings [44,84] and the neutralino sector [44,61,62]. In the vicinity of the quasi-fixed point the theoretical upper bound on the mass of the lightest Higgs boson and the RG flow of the Yukawa couplings were studied in [85,86]. This quasi-fixed point arises as a result of the intersection of the quasi-fixed and invariant lines [87,88]. Detailed studies of the E_6 SSM have established that extra exotic matter and Z' predicted by this model may give rise to distinctive LHC signatures [69,70,74,77,89–93], as well

as may lead to non-standard Higgs decays for sufficiently light exotics [81,86,94–100]. The particle spectrum and associated phenomenological implications within the constrained version of the E_6 SSM (cE_6 SSM) and its modifications were explored in [91,101–106] while the degree of fine tuning was examined in [107,108]. The threshold corrections and their impact in the cE_6 SSM were analyzed in [109]. The renormalization of the VEVs in the E_6 SSM was considered in [110,111].

The superpotential of the $U(1)_N$ extensions of the MSSM contains the renormalizable part that comes from the $27 \times 27 \times 27$ decomposition of the E_6 fundamental representation. It can be written as

$$\begin{aligned} W_{E_6} &= W_0 + W_1 + W_2, \\ W_0 &= \lambda_{ijk} S_i (H_j^d H_k^u) + \kappa_{ijk} S_i (D_j \bar{D}_k) + h_{ijk}^N N_i^c (H_j^u L_k) + h_{ijk}^U u_i^c (H_j^u Q_k) + \\ &+ h_{ijk}^D d_i^c (H_j^d Q_k) + h_{ijk}^E e_i^c (H_j^d L_k), \\ W_1 &= g_{ijk}^Q D_i (Q_j Q_k) + g_{ijk}^q \bar{D}_i d_j^c u_k^c, \\ W_2 &= g_{ijk}^N N_i^c D_j d_k^c + g_{ijk}^E e_i^c D_j u_k^c + g_{ijk}^D (Q_i L_j) \bar{D}_k, \end{aligned} \quad (8)$$

where $i, j, k = 1, 2, 3$. In Equation (8), the summation over repeated family indexes is implied. The part of the superpotential in Equation (8) possesses a global $U(1)$ symmetry which is a linear superposition of $U(1)_Y$ and $U(1)_\chi$. This $U(1)$ symmetry is associated with $B-L$ number conservation. On the other hand, the baryon and lepton numbers are violated if all terms in W_1 and W_2 are simultaneously present in W_{E_6} . In other words, the baryon and lepton numbers of the exotic quarks cannot be defined so that the Lagrangian is invariant separately under $U(1)_B$ and $U(1)_L$ global symmetries. Thus, as in any other SUSY extension of the SM, the gauge symmetry of the models under consideration does not forbid lepton and baryon number violating operators. Because of this, all these models in general suffer from problems related with rapid proton decay.

Moreover, exotic states in the $U(1)_N$ extensions of the MSSM give rise to new Yukawa interactions that may induce unacceptably large flavor changing processes. Indeed, in the most general case three families of H_i^u and H_i^d may couple to ordinary quarks and charged leptons of different generations leading to the phenomenologically unwanted non-diagonal flavor transitions even at the tree level. Such flavor changing interactions contribute to the amplitude of $K^0 - \bar{K}^0$ oscillations. They also result in new channels of muon decay such as $\mu \rightarrow e^- e^+ e^-$. To suppress flavor changing neutral currents (FCNCs), one can postulate Z_2^H symmetry. If all matter supermultiplets except one pair of H_i^u and H_i^d (say $H_d \equiv H_3^d$ and $H_u \equiv H_3^u$) as well as one SM-type singlet superfield ($S \equiv S_3$) are odd under this symmetry, then only H_u interacts with up-type quarks and only H_d couples to the down-type quarks and charged leptons [69,70]. The couplings of all other exotic states to the ordinary quark and lepton supermultiplets are forbidden that eliminates any problems related with the non-diagonal flavor transitions at the tree level. In this original E_6 SSM model, the scalar components of the supermultiplets H_u , H_d and S compose the Higgs sector. In particular, the third family SM-singlet superfield S_3 gets a VEV, $\langle S_3 \rangle = s/\sqrt{2}$, breaking $U(1)_N$ gauge symmetry. This VEV is responsible for the effective μ term and D-fermion masses. The first and second families of Higgs-like doublets and SM-singlets do not get VEVs. Because of this, they are called “inert”. At the same time, the modified version of the E_6 SSM, in which three SM-singlet superfields S_i are taken to be even under the Z_2^H symmetry, was also recently considered [82]. In this case, all superfields S_i develop VEVs. They couple to H_u , H_d as well as other exotic bosons and fermions.

Although the Z_2^H symmetry forbids not only flavor changing processes but also the most dangerous baryon and lepton number violating operators, it cannot be an exact symmetry. Indeed, this symmetry forbids all terms in W_1 and W_2 that permit the lightest exotic quarks to decay. The Lagrangian of the corresponding model is invariant under $U(1)_D$ symmetry transformations, i.e.,

$$D \rightarrow e^{i\alpha} D, \quad \bar{D} \rightarrow e^{-i\alpha} \bar{D}. \quad (9)$$

The $U(1)_D$ as well as $U(1)_L$ and $U(1)_B$ global symmetries should be broken by the non-renormalizable operators. These operators are suppressed by inverse power of M_X . All dimension five operators that break $U(1)_D$ symmetry are forbidden by E_6 . The dimension six operators result in the lifetime of the lightest exotic quarks, which is of order of

$$\tau_D > M_X^4 / \mu_D^5, \quad (10)$$

where μ_D is the mass of this exotic quark. For $\mu_D \simeq \text{TeV}$, the lifetime $\tau_D > 10^{49} \text{ GeV}^{-1} \sim 10^{17} \text{ years}$ so that it is much larger than the age of the Universe. During early epochs of the Big Bang, so long-lived exotic quarks had to be copiously produced. Stable quarks which survive annihilation should be confined in nuclei. Thus, nuclear isotopes with stable exotic quarks should be present in terrestrial matter. The concentration of such remnant particles is expected to be 10^{-10} per nucleon if the mass of the lightest exotic quarks varies from 1 GeV to 10 TeV [112,113]. At the same time, different experiments indicate that the relative concentrations of such nuclear isotopes have to be less than 10^{-15} per nucleon or even smaller [114–116]. Therefore, the extensions of the SM with so long-lived exotic quarks are basically ruled out. This means that the discrete Z_2^H symmetry can only be an approximate one.

To prevent rapid proton decay within the $U(1)_N$ extensions of the MSSM, one can impose either Z_2^L or Z_2^B discrete symmetry. The Z_2^L symmetry implies that all superfields except lepton ones (including L_4 and \bar{L}_4) are even and all Yukawa interactions in W_2 are forbidden. Then, the baryon number is conserved only when exotic quarks are diquarks (Model I). In Model I, the most general renormalizable superpotential can be presented in the following form:

$$\begin{aligned} W_{E_6\text{SSM I}} &= W_0 + W_1 + \frac{1}{2} M_{ij} N_i^c N_j^c + W'_0, \\ W'_0 &= \mu_L L_4 \bar{L}_4 + \tilde{h}_{ij}^L e_i^c (H_j^d L_4) + h_{ij}^L N_i^c (H_j^u L_4). \end{aligned} \quad (11)$$

The terms in W'_0 are caused by the splitting of $27'$ and $\bar{27}'$ representations of E_6 . In the case of Z_2^B symmetry, the supermultiplets of ordinary leptons, exotic quarks D_i and \bar{D}_i , as well as L_4 , \bar{L}_4 are all odd whereas the others remain even. As a consequence, all terms in W_1 are ruled out and the baryon number conservation requires the exotic quarks to carry lepton ($L_D = 1$ and $L_{\bar{D}} = -1$) and baryon ($B_D = 1/3$ and $B_{\bar{D}} = -1/3$) numbers simultaneously (Model II). Thus, in Model II, D_i and \bar{D}_i are leptoquarks. The most general renormalizable superpotential in Model II is given by

$$W_{E_6\text{SSM II}} = W_0 + W_2 + \frac{1}{2} M_{ij} N_i^c N_j^c + W'_0 + g_{ik}^L (Q_i L_4) \bar{D}_k, \quad (12)$$

The last term in Equation (12) appears because of the splitting of $27'$. In the superpotentials in Equations (11) and (12), the $SU(2)_W$ doublet L_4 is redefined in such a way that W'_0 contains only one bilinear term. The mass parameter μ_L should not be too large. Otherwise, the gauge coupling unification gets spoiled. Within SUGRA models, the appropriate term $\mu_L L_4 \bar{L}_4$ in the superpotentials in Equations (11) and (12) can be induced if the Kähler potential contains an extra term $(Z L_4 \bar{L}_4 + h.c.)$ [117,118]. This is the same mechanism which is used in the MSSM to solve the μ problem. Within the $U(1)_N$ extensions of the MSSM, the bilinear term involving H_d and H_u are forbidden by the $U(1)_N$ gauge symmetry and the mechanism mentioned above cannot be used.

The superpotentials of Models I and II also include bilinear terms, $\frac{1}{2} M_{ij} N_i^c N_j^c$, responsible for the right-handed neutrino masses. The corresponding mass parameters M_{ij} are expected to be at intermediate mass scales. They can be induced through the non-renormalizable interactions of the form

$$\delta W = \frac{\kappa_{ij}}{M_{Pl}} (\bar{27}_H 27_i) (\bar{27}_H 27_j) \quad \Rightarrow \quad M_{ij} = \frac{2\kappa_{ij}}{M_{Pl}} < \bar{N}_H^c >^2, \quad (13)$$

where N_H^c and \bar{N}_H^c are components of some extra 27_H and $\bar{27}_H$ representations which develop VEVs along the D -flat direction $\langle N_H^c \rangle = \langle \bar{N}_H^c \rangle$. These VEVs can also break $U(1)_\psi \times U(1)_\chi$ down to $U(1)_N \times Z_2^M$ symmetry [72]. If such breakdown takes place somewhere around the GUT scale M_X , a reasonable pattern for the left-handed neutrino masses can be obtained.

The superpotentials in Equations (11) and (12) involve many new Yukawa couplings in comparison to the SM. In general, the exact Z_2^L and Z_2^B discrete symmetries do not guarantee the absence of FCNCs in the $U(1)_N$ extensions of the MSSM. At the same time, it is worth noting that the most of the Yukawa couplings in the SM as well as in the MSSM are rather small. Therefore, it seems natural to assume some hierarchical structure of the Yukawa interactions that may permit to suppress non-diagonal flavor transitions. In addition, it is reasonable to use the approximate Z_2^H symmetry to eliminate problems related with flavor changing processes. The appropriate suppression of the non-diagonal flavor interactions can be achieved if all Z_2^H symmetry violating couplings are less than 10^{-4} . In the limit when all Yukawa couplings that explicitly break the Z_2^H symmetry are negligibly small, the superpotential of the E₆SSM reduces to

$$\begin{aligned} W_{E_6\text{SSM}} = & \lambda S(H_u H_d) + \lambda_{\alpha\beta} S(H_\alpha^d H_\beta^u) + \kappa_{ij} S(D_i \bar{D}_j) + \tilde{f}_{\alpha\beta} S_\alpha(H_\beta^d H_u) + f_{\alpha\beta} S_\alpha(H_d H_\beta^u) \\ & + \mu_L L_4 \bar{L}_4 + \frac{1}{2} M_{ij} N_i^c N_j^c + W_{L_4} + W_{\text{MSSM}}(\mu = 0), \end{aligned} \quad (14)$$

where

$$W_{L_4} = \tilde{h}_{i3}^L e_i^c (H_d L_4) + h_{i3}^L N_i^c (H_u L_4), \quad (15)$$

$\alpha, \beta = 1, 2$ and $i, j = 1, 2, 3$. If some of the couplings λ , $\lambda_{\alpha\beta}$ or κ_{ij} are rather large at the GUT scale M_X , they can lead to negative values of m_S^2 at low energies. This triggers the breakdown of $U(1)_N$ gauge symmetry giving rise to the large VEV of the singlet superfield S that generate sufficiently large masses of the exotic particles and Z' boson. On the other hand, the generation of the VEVs of H_u and H_d , that break the $SU(2)_W \times U(1)_Y$ gauge symmetry, can be caused by the large value of the top-quark Yukawa coupling.

Since in the $U(1)_N$ extensions of the MSSM the Z_2^M symmetry and R -parity are conserved, the lightest R -parity odd state, i.e., the lightest SUSY particle (LSP), must be stable. Using the approach discussed in [119–121], it was shown that in the E₆SSM the masses of the LSP and next-to-lightest SUSY particle (NLSP) are smaller than 60–65 GeV [94]. The LSP and NLSP (\tilde{H}_1^0 and \tilde{H}_2^0) are predominantly linear combinations of the fermion components of the two SM singlet superfields S_α . The couplings of \tilde{H}_1^0 to the SM particles are quite small. However, if LSP had a mass close to half the Z mass, it could account for some of the observed dark matter density. In these scenarios, LSP annihilate mainly through an s -channel Z -boson [94]. The SM-like Higgs state decays mostly into either \tilde{H}_1^0 or \tilde{H}_2^0 in this case. All other branching ratios would be strongly suppressed. Nowadays, such scenario are ruled out by the LHC experiments. If fermion components of the SM singlet superfields S_α are considerably lighter than M_Z , then the cross section of the annihilation of $\tilde{H}_1^0 \tilde{H}_1^0 \rightarrow \text{SM}$ particles becomes too small, leading to cold dark matter density which is much larger than its measured value.

Nevertheless, in the E₆SSM with approximate Z_2^H symmetry, one of the lightest R -parity odd state can account for some of the observed dark matter density. To prevent the decays of this state into the LSP and NLSP, an additional Z_2^S symmetry needs to be postulated [78]. In the corresponding variant of the E₆SSM, couplings $\tilde{f}_{\alpha\beta}$ and $f_{\alpha\beta}$ vanish. As a result, the fermion components of the SM singlet superfields S_α remain massless and decouple. If Z' boson is sufficiently heavy, the presence of these massless states does not affect Big Bang Nucleosynthesis (BBN) [78]. Since $\tilde{f}_{\alpha\beta} = f_{\alpha\beta} = 0$, the branching ratios of the SM-like Higgs decays into \tilde{H}_1^0 and \tilde{H}_2^0 vanish.

Instead of Z_2^H , Z_2^L and Z_2^B , one can impose a single discrete \tilde{Z}_2^H symmetry which forbids tree-level flavor-changing transitions as well as the most dangerous operators that lead to rapid proton decay. In this case H_u , H_d , S , L_4 and \bar{L}_4 are even under the \tilde{Z}_2^H symmetry while all other supermultiplets are

odd [72]. Neglecting non-renormalizable interactions, the superpotential of this variant of the E_6 SSM is given by Equation (14) with

$$W_{L_4} = g_{ik}^L (Q_i L_4) \bar{D}_k + \tilde{h}_{i\alpha}^L e_i^c (H_\alpha^d L_4) + h_{i\alpha}^L N_i^c (H_\alpha^u L_4), \quad (16)$$

where $\alpha = 1, 2$ and $i, k = 1, 2, 3$. Since the low-energy effective Lagrangian of these SUSY models is invariant under both Z_2^M and \tilde{Z}_2^H symmetries and $\tilde{Z}_2^H = Z_2^M \times Z_2^E$, the Z_2^E symmetry associated with exotic states is also conserved. The transformation properties of different components of 27_i supermultiplets under the $Z_2^H, Z_2^L, Z_2^B, Z_2^S, \tilde{Z}_2^H, Z_2^M$ and Z_2^E symmetries are summarized in Table 1. The Z_2^E symmetry conservation ensures that the lightest exotic state, which is odd under this symmetry, is stable. The simplest phenomenologically viable scenarios imply that $f_{\alpha\beta} \sim \tilde{f}_{\alpha\beta} < 10^{-6}$. As a consequence, two lightest exotic states (\tilde{H}_1^0 and \tilde{H}_2^0), which are formed by the fermion components of the superfields S_α , are substantially lighter than 1 eV. They compose hot dark matter in the Universe [72]. However, these states give only a very small contribution to the dark matter density. The presence of extremely light neutral fermions might also have some implications for the neutrino physics [122]. The invariance of the Lagrangian under the Z_2^M ensures that the lightest R -parity odd state with $Z_2^E = +1$, which is most commonly the lightest ordinary neutralino in this case, is also stable and can account for some of the observed dark matter density [106].

Table 1. Transformation properties of matter supermultiplets under $Z_2^H, Z_2^L, Z_2^B, Z_2^S, \tilde{Z}_2^H, Z_2^M$ and Z_2^E discrete symmetries in the E_6 SSM. The signs + and – correspond to the states which are even and odd under different Z_2 symmetries.

Q_i, u_i^c, d_i^c	L_i, e_i^c, N_i^c	\bar{D}_i, D_i	H_α^d, H_α^u	S_α	H_d, H_u, S	L_4, \bar{L}_4
Z_2^H	–	–	–	–	+	–
Z_2^L	+	–	+	+	+	–
Z_2^B	+	–	–	+	+	–
Z_2^S	+	+	+	+	+	+
\tilde{Z}_2^H	–	–	–	–	+	+
Z_2^M	–	–	+	+	+	–
Z_2^E	+	+	–	–	+	–

3. Gauge Coupling Unification

In this section, we consider the RG flow of the gauge couplings within the E_6 SSM between M_Z and the GUT scale M_X . The evolution of the corresponding couplings is affected by a kinetic term mixing. In the Lagrangian of any extension of the SM, which involves an additional $U(1)'$ factor, there can arise a kinetic term consistent with all symmetries [123]. This term mixes the gauge fields of the $U(1)'$ and $U(1)_Y$. The E_6 SSM is not an exception. In the basis in which the couplings of the gauge bosons to matter fields have the canonical form, for example a covariant derivative, D_μ which acts on the left-handed quark field is given by

$$D_\mu = \partial_\mu - ig_3 A_\mu^a T^a - ig_2 W_\mu^b \tau^b - ig_Y Q_i^Y B_\mu^Y - ig_N Q_i^N B_\mu^N, \quad (17)$$

and the mixing between the $U(1)$ field strengths can be written as

$$\mathcal{L}_{mix} = -\frac{\sin \chi}{2} F_{\mu\nu}^Y F_{\mu\nu}^N. \quad (18)$$

Here, $A_\mu^a, W_\mu^b, B_\mu^Y$ and B_μ^N represent $SU(3)_C, SU(2)_W, U(1)_Y$ and $U(1)_N$ gauge fields; $G_{\mu\nu}^a, W_{\mu\nu}^b, F_{\mu\nu}^Y$ and $F_{\mu\nu}^N$ are field strengths for the corresponding gauge interactions; and g_3, g_2, g_Y and g_N are the $SU(3)_C, SU(2)_W, U(1)_Y$ and $U(1)_N$ gauge couplings in this basis. Since $U(1)_Y$ and $U(1)_N$ factors

come from the breakdown of the simple gauge group E_6 , the parameter $\sin \chi$ is expected to vanish at tree-level. However, the non-zero value of this parameter is induced by loop corrections because

$$\text{Tr} \left(Q^Y Q^N \right) = \sum_i \left(Q_i^Y Q_i^N \right) \neq 0. \quad (19)$$

In Equation (19), trace is restricted to the states which are lighter than M_X . The contribution of the complete E_6 supermultiplets to this trace cancels. The non-zero value of the trace in Equation (19) is induced by L_4 and \bar{L}_4 supermultiplets which survive to low energies.

For non-zero values of the parameter $\sin \chi$, the mixing in the gauge sector in Equation (18) can be eliminated by means of a non-unitary transformation [39,124–127]:

$$B_\mu^Y = B_{1\mu} - B_{2\mu} \tan \chi, \quad B_\mu^N = B_{2\mu} / \cos \chi. \quad (20)$$

In the basis $(B_{1\mu}, B_{2\mu})$, the covariant derivative in Equation (17) becomes

$$D_\mu = \partial_\mu - ig_3 A_\mu^a T^a - ig_2 W_\mu^b \tau^b - ig_1 Q_i^Y B_{1\mu} - i(g'_1 Q_i^N + g_{11} Q_i^Y) B_{2\mu}, \quad (21)$$

where the redefined gauge couplings are

$$g_1 = g_Y, \quad g'_1 = g_N / \cos \chi, \quad g_{11} = -g_Y \tan \chi. \quad (22)$$

In this basis, the mixing effect is concealed in the interaction parameterized by a new off-diagonal gauge coupling g_{11} . The gauge coupling constant g'_1 differs from the original one. In the new basis, the covariant derivative in Equation (21) can be rewritten in a compact form

$$D_\mu = \partial_\mu - ig_3 A_\mu^a T^a - ig_2 W_\mu^b \tau^b - iQ^T G B_\mu, \quad (23)$$

where $Q^T = (Q_i^Y, Q_i^N)$, $B_\mu^T = (B_{1\mu}, B_{2\mu})$ and G is a 2×2 matrix of new gauge couplings in Equation (22)

$$G = \begin{pmatrix} g_1 & g_{11} \\ 0 & g'_1 \end{pmatrix}. \quad (24)$$

Now, all physical phenomena can be examined using the Lagrangian with the modified structure of the gauge interactions in Equations (21)–(23). In this approximation, the gauge kinetic mixing changes effectively the $U(1)_N$ charges of all fields to

$$\tilde{Q}_i \equiv Q_i^N + Q_i^Y \delta, \quad (25)$$

where $\delta = g_{11}/g'_1$, whereas the $U(1)_Y$ charges remain the same. The effective charges \tilde{Q}_i are scale dependent. The particle spectrum in the basis $B_\mu^T = (B_{1\mu}, B_{2\mu})$ depends on the effective $U(1)_N$ charges \tilde{Q}_i .

The running of four diagonal gauge couplings, i.e., $g_3(t)$, $g_2(t)$, $g_1(t)$ and $g'_1(t)$, and one off-diagonal gauge coupling $g_{11}(t)$ is described by a system of RG equations (RGEs), which can be written as:

$$\frac{dG}{dt} = G \times B, \quad \frac{dg_2}{dt} = \frac{\beta_2 g_2^3}{32\pi^2}, \quad \frac{dg_3}{dt} = \frac{\beta_3 g_3^3}{32\pi^2}, \quad (26)$$

where $t = 2 \ln (q/M_Z)$, q is a renormalization scale, G is the 2×2 matrix in Equation (24) while B is a 2×2 matrix given by

$$B = \frac{1}{32\pi^2} \begin{pmatrix} \beta_1 g_1^2 & 2g_1 g'_1 \beta_{11} + 2g_1 g_{11} \beta_1 \\ 0 & g'_1 \beta'_1 + 2g'_1 g_{11} \beta_{11} + g_{11}^2 \beta_1 \end{pmatrix}. \quad (27)$$

In Equations (26) and (27), β_i and β_{11} are beta functions. Here, the RG flow of the gauge couplings is explored in the two-loop approximation. In this approximation,

$$\beta'_1 = b'_1 + \frac{\tilde{b}'_1}{4\pi}, \quad \beta_i = b_i + \frac{\tilde{b}_i}{4\pi}, \quad (28)$$

where b_i and b'_1 are one-loop beta functions of the diagonal gauge couplings while \tilde{b}_i and \tilde{b}'_1 correspond to the two-loop contributions to these functions.

It seems to be rather natural to expect that just after the breakdown of the E_6 symmetry near the GUT scale M_X the mixing parameter $\sin \chi$ vanishes, while the $SU(3)_C$, $SU(2)_W$, $U(1)_Y$ and $U(1)_N$ gauge interactions are characterized by a unique E_6 gauge coupling g_0 , i.e.,

$$g_3(M_X) = g_2(M_X) = g_1(M_X) = g'_1(M_X) = g_0, \quad g_{11}(M_X) = 0. \quad (29)$$

The previous analysis performed in [84] revealed that $g_{11}(t)$ being set to zero at the GUT scale remains substantially smaller than the diagonal gauge couplings at any scale below M_X . Therefore, the two-loop corrections to the off-diagonal beta function β_{11} can be neglected. In the one-loop approximation, the beta function of the off-diagonal gauge coupling is given by $\beta_{11} = -\frac{\sqrt{6}}{5}$.

To simplify our analysis here, we further assume that the interactions of matter supermultiplets in the E_6 SSM are described by the superpotential in Equation (14), in which all interactions in W_{L_4} can be ignored, $\tilde{f}_{\alpha\beta} \simeq f_{\alpha\beta} \rightarrow 0$, $\lambda_{\alpha\beta} = \lambda_{\alpha}\delta_{\alpha\beta}$ and $\kappa_{ij} = \kappa_i\delta_{ij}$. The part of the superpotential in Equation (14) associated with $W_{\text{MSSM}}(\mu = 0)$ reduces to

$$W_{\text{MSSM}}(\mu = 0) = h_t Q_3 u_3^c H_u + h_b Q_3 d_3^c H_d + h_\tau L_3 e_3^c H_d, \quad (30)$$

because only third-generation fermions have Yukawa couplings to H_d and H_u which can be of the order of unity.

In the one-loop approximation the beta functions of the diagonal gauge couplings are given by

$$b_1 = \frac{3}{5} + 3N_g, \quad b'_1 = \frac{2}{5} + 3N_g, \quad b_2 = -5 + 3N_g, \quad b_3 = -9 + 3N_g, \quad (31)$$

where parameter N_g is the number of complete E_6 fundamental representations at low energies ($E \ll M_X$). In the E_6 SSM, $N_g = 3$, which is the critical value for b_3 . Indeed, for $N_g = 3$, the one-loop beta function of the strong interactions is equal to zero and the $SU(3)_C$ gauge coupling remains constant everywhere from the EW scale to M_X . Thus, any reliable analysis of the RG flow of the gauge couplings within the E_6 SSM requires the inclusion of two-loop corrections to β'_1 and β_i . Using the results of the calculation of two-loop beta functions in a general models with softly broken SUSY [128], one obtains

$$\begin{aligned} \tilde{b}_1 &= 8N_g\alpha_3 + \left(\frac{9}{5} + 3N_g\right)\alpha_2 + \left(\frac{9}{25} + 3N_g\right)\alpha_1 + \left(\frac{6}{25} + N_g\right)\alpha'_1 \\ &\quad - \frac{26}{5}y_t - \frac{14}{5}y_b - \frac{18}{5}y_\tau - \frac{6}{5}\Sigma_\lambda - \frac{4}{5}\Sigma_\kappa, \\ \tilde{b}'_1 &= 8N_g\alpha_3 + \left(\frac{6}{5} + 3N_g\right)\alpha_2 + \left(\frac{6}{25} + N_g\right)\alpha_1 + \left(\frac{4}{25} + 3N_g\right)\alpha'_1 \\ &\quad - \frac{9}{5}y_t - \frac{21}{5}y_b - \frac{7}{5}y_\tau - \frac{19}{5}\Sigma_\lambda - \frac{57}{10}\Sigma_\kappa, \\ \tilde{b}_2 &= 8N_g\alpha_3 + \left(-17 + 21N_g\right)\alpha_2 + \left(\frac{3}{5} + N_g\right)\alpha_1 + \left(\frac{2}{5} + N_g\right)\alpha'_1 \\ &\quad - 6y_t - 6y_b - 2y_\tau - 2\Sigma_\lambda, \\ \tilde{b}_3 &= \alpha_3 \left(-54 + 34N_g\right) + 3N_g\alpha_2 + N_g\alpha_1 + N_g\alpha'_1 - 4y_t - 4y_b - 2\Sigma_\kappa, \\ \Sigma_\lambda &= y_{\lambda_1} + y_{\lambda_2} + y_\lambda, \quad \Sigma_\kappa = y_{\kappa_1} + y_{\kappa_2} + y_{\kappa_3}, \end{aligned} \quad (32)$$

where $\alpha_i = \frac{g_i^2}{4\pi}$, $\alpha'_i = \frac{g'_i}{4\pi}$, $y_t = \frac{h_t^2}{4\pi}$, $y_b = \frac{h_b^2}{4\pi}$, $y_\tau = \frac{h_\tau^2}{4\pi}$, $y_\lambda = \frac{\lambda^2}{4\pi}$, $y_{\lambda_\alpha} = \frac{\lambda_\alpha^2}{4\pi}$ and $y_{\kappa_i} = \frac{\kappa_i^2}{4\pi}$.

For the analysis of the RG flow of the SM gauge couplings, it is convenient to use an approximate solution of the two-loop RGEs (see [129]). At high energies, this solution is given by

$$\frac{1}{\alpha_i(t)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} t - \frac{C_i}{12\pi} - \Theta_i(t) + \frac{b_i - b_i^{SM}}{2\pi} \ln \frac{T_i}{M_Z}, \quad (33)$$

where b_i^{SM} are the one-loop beta functions in the SM, the $\overline{MS} \rightarrow \overline{DR}$ conversion factor with $C_1 = 0$, $C_2 = 2$, $C_3 = 3$ correspond to the third term on the right-hand side of Equation (33) [130,131], while

$$\Theta_i(t) = \frac{1}{8\pi^2} \int_0^t \tilde{b}_i d\tau, \quad T_i = \prod_{k=1}^N \left(m_k \right) \frac{\Delta b_i^k}{b_i - b_i^{SM}} \quad (34)$$

In Equation (34), m_k and Δb_i^k are masses and one-loop contributions to b_i due to sparticles, heavy Higgs bosons and exotic states appearing in the E₆SSM. Since the two-loop corrections to the RG flow of the SM gauge couplings $\Theta_i(t)$ are substantially smaller than the leading terms, the solutions of the one-loop RGEs for the gauge and Yukawa couplings are normally used for the calculation of $\Theta_i(t)$. The threshold corrections associated with the last terms in Equation (33) are even smaller than $\Theta_i(t)$. Because of this, only leading one-loop threshold effects are taken into account in Equations (33) and (34).

Relying on the approximate solution of the two-loop RGEs, one can find the relationships between $\alpha_i(M_X)$ and the values of these couplings at low energies. Then, one can estimate the scale M_X where $\alpha_1(M_X) = \alpha_2(M_X) = \alpha_0$ as well as the value of the overall gauge coupling α_0 . Substituting M_X and $\alpha_3(M_X) = \alpha_0$ into the approximate solution for $\alpha_3(t)$ one can find the value of $\alpha_3(M_Z)$, for which the unification of the SM gauge couplings takes place (see [132]):

$$\frac{1}{\alpha_3(M_Z)} = \frac{1}{b_1 - b_2} \left[\frac{b_1 - b_3}{\alpha_2(M_Z)} - \frac{b_2 - b_3}{\alpha_1(M_Z)} \right] - \frac{1}{28\pi} + \Theta_s + \frac{19}{28\pi} \ln \frac{T_S}{M_Z}, \quad (35)$$

$$\Theta_s = \left(\frac{b_2 - b_3}{b_1 - b_2} \Theta_1 - \frac{b_1 - b_3}{b_1 - b_2} \Theta_2 + \Theta_3 \right), \quad \Theta_i = \Theta_i(M_X).$$

In Equation (35), the threshold scale T_S can be expressed in terms of T_1 , T_2 and T_3

$$T_S = \frac{T_2^{172/19}}{T_1^{55/19} T_3^{98/19}}. \quad (36)$$

In the E₆SSM, the effective threshold scales T_1 , T_2 and T_3 are given by

$$\begin{aligned} T_1 &= \mu^{4/55} m_A^{1/55} \mu_L^{4/55} m_L^{2/55} \left(\prod_{i=1,2,3} m_{\tilde{Q}_i}^{1/165} m_{\tilde{d}_i}^{2/165} m_{\tilde{u}_i}^{8/165} m_{\tilde{L}_i}^{1/55} m_{\tilde{e}_i}^{2/55} m_{\tilde{D}_i}^{4/165} \mu_{D_i}^{8/165} \right) \times \\ &\quad \times \left(\prod_{\alpha=1,2} m_{H_\alpha}^{2/55} \mu_{H_\alpha}^{4/55} \right), \\ T_2 &= M_{\tilde{W}}^{8/43} \mu^{4/43} m_A^{1/43} \mu_L^{4/43} m_L^{2/43} \left(\prod_{i=1,2,3} m_{\tilde{Q}_i}^{3/43} m_{\tilde{L}_i}^{1/43} \right) \left(\prod_{\alpha=1,2} m_{H_\alpha}^{2/43} \mu_{H_\alpha}^{4/43} \right), \\ T_3 &= M_{\tilde{g}}^{2/7} \left(\prod_{i=1,2,3} m_{\tilde{Q}_i}^{1/21} m_{\tilde{u}_i}^{1/42} m_{\tilde{d}_i}^{1/42} m_{\tilde{D}_i}^{1/21} \mu_{D_i}^{2/21} \right), \end{aligned} \quad (37)$$

where $M_{\tilde{g}}$ and $M_{\tilde{W}}$ are the masses of gluinos and winos; m_A and μ are the masses of heavy Higgs bosons and effective μ -term; μ_{D_i} and $m_{\tilde{D}_i}$ are the masses of exotic quarks and their superpartners; μ_{H_α} and m_{H_α} are the masses of the fermion and scalar components of H_α^u and H_α^d ; m_L and μ_L are

the masses of the scalar and fermion components of L_4 and \bar{L}_4 ; $m_{\tilde{Q}_i}$ and $m_{\tilde{L}_i}$ are the masses of the left-handed squarks and sleptons; and $m_{\tilde{u}_i}$, $m_{\tilde{d}_i}$ and $m_{\tilde{e}_i}$ are the masses of the right-handed squarks and sleptons.

It is worth noting here that in the limit when the two-loop and threshold corrections are neglected, i.e., $\Theta_s = 0$ and $T_S = M_Z$, Equation (33) leads to the same prediction for $\alpha_3(M_Z)$ in the MSSM and E_6 SSM. Indeed, since extra matter in the E_6 SSM form complete $SU(5)$ representations these multiplets contribute equally to b_i . Due to this the differences of the coefficients $b_i - b_j$ and the form of Equation (33) remain the same in the MSSM and E_6 SSM. However, the inclusion of the two-loop and threshold corrections may spoil the gauge coupling unification within the E_6 SSM.

In general, the effective threshold scales T_1 , T_2 and T_3 are quite different. Nevertheless, from Equation (35), it follows that the unification of the SM gauge couplings is determined by a single combined threshold scale T_S . Therefore, without loss of generality, one can set three effective threshold scales be equal to each other. Then, from Equation (36), it follows that $T_1 = T_2 = T_3 = T_S$. The results of our numerical analysis of the gauge coupling unification within the E_6 SSM are presented in Figure 1 where the evolution of the SM gauge couplings computed in the two-loop approximation is shown. In particular, we use the two-loop SM beta functions to evaluate the running of $\alpha_i(t)$ between M_Z and T_S . Then, we apply the two-loop RG equations of the E_6 SSM to calculate the evolution of $\alpha_i(t)$ from T_S to M_X which is around $2-3 \times 10^{16}$ GeV. At low energies, the values of g_{11} and g'_1 are chosen so that the conditions in Equation (29) are fulfilled. For the computation of the RG flow of Yukawa couplings, a set of one-loop RG equations is used. The corresponding set of the one-loop RGEs are specified in [69].

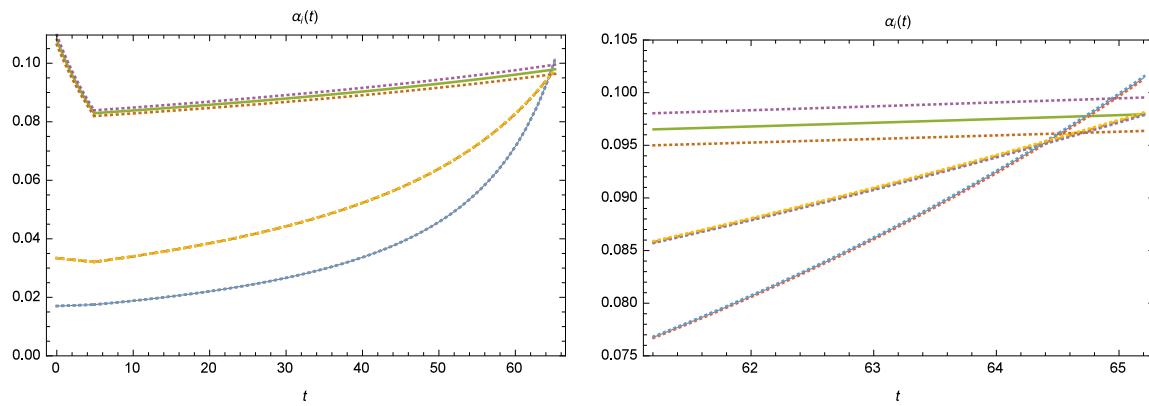


Figure 1. (Left) Two-loop RG flow of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings in the E_6 SSM as a function of $t = 2 \ln(q/M_Z)$ for $T_S = 2$ TeV in the case when q varies from M_Z to M_X . (Right) Evolution of these couplings in the vicinity of M_X . Thick, dashed and solid lines represent the evolution of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings, respectively. Here, we fix $\kappa_1(T_S) = \kappa_2(T_S) = \kappa_3(T_S) = \lambda_1(T_S) = \lambda_2(T_S) = \lambda_3(T_S) = g'_1(T_S)$, $\sin^2 \theta_W = 0.231$, $\alpha(M_Z) = 1/127.9$, $\alpha_3(M_Z) = 0.118$ and $\tan \beta = 10$. The dotted lines correspond to the uncertainty in $\alpha_i(t)$ induced by the variation of $\alpha_3(M_Z)$ from 0.116 to 0.120.

In Figure 1, we fix $T_1 = T_2 = T_3 = T_S = 2$ TeV and $\tan \beta = 10$. Although to simplify our analysis we also set $\kappa_i(T_S) = \lambda_\alpha(T_S) = \lambda(T_S) = g'_1(T_S)$, the RG flow of $\alpha_i(t)$ depends rather weakly on the values of the Yukawa and extra $U(1)_N$ gauge couplings. Dotted lines in Figure 1 show the modifications of the RG flow of the SM gauge couplings caused by the variations of $\alpha_3(M_Z)$ from 0.116 to 0.120. The corresponding interval of variations of $\alpha_3(t)$ is always substantially wider than the ones for $\alpha_1(t)$ and $\alpha_2(t)$. The dependence of $\alpha_1(t)$ and $\alpha_2(t)$ on $\alpha_3(M_Z)$ is expected to be relatively weak because $\alpha_3(t)$ arises only in the two-loop contributions to β_1 and β_2 . It is worth pointing out that at high energies the uncertainty in $\alpha_3(t)$ induced by the variations of $\alpha_3(M_Z)$ is considerably bigger in the E_6 SSM than in the MSSM. This happens because $\alpha_3(t)$ grows with increasing renormalization scale q in the E_6 SSM while in the MSSM it decreases at high energies. Thus, in the E_6 SSM, the uncertainty

in $\alpha_3(M_X)$ is almost equal to the low energy uncertainty in $\alpha_3(M_Z)$, whereas within the MSSM the interval of variations of $\alpha_3(M_X)$ shrinks drastically. As a consequence, it is much easier to achieve the unification of gauge couplings within the E₆SSM as compared with the MSSM where in the two-loop approximation the exact gauge coupling unification requires $\alpha_3(M_Z) > 0.123$, well above the experimentally measured central value [129,132–140].

The results of the numerical analysis shown in Figure 1 indicate that for $T_S = 2$ TeV almost exact gauge coupling unification can be achieved in the E₆SSM if $\alpha_3(M_Z) \approx 0.116$. The value of $\alpha_3(M_Z)$, at which the exact unification of the SM gauge couplings takes place, becomes lower (greater) when the combined threshold scale T_S increases (decreases). In the E₆SSM, T_S can be considerably lower than 1 TeV even if the SUSY breaking scale is much larger than a few TeV. To demonstrate this let us assume that all scalars except the SM-like Higgs boson are almost degenerate around $m_A \approx M_S$ which is much larger than the masses of all fermions. Then, combining Equations (36) and (37), one finds

$$T_S = \frac{M_{\tilde{W}}^{32/19} M_S^{3/19}}{M_{\tilde{g}}^{28/19}} \left(\frac{\mu \mu_L \mu_{\tilde{H}_1} \mu_{\tilde{H}_2}}{\mu_{D_1} \mu_{D_2} \mu_{D_3}} \right)^{12/19}. \quad (38)$$

If $M_S \approx M_{\tilde{g}} \approx 10$ TeV and $\mu \approx \mu_L \approx \mu_{\tilde{H}_1} \approx \mu_{\tilde{H}_2} \approx 1$ TeV while $M_{\tilde{W}}$ and the masses of the exotic quarks μ_{D_i} are of the order of a few TeV, the effective threshold scale tends to be much smaller than 1 TeV. For $T_S = 400$ GeV, almost exact unification of the SM gauge couplings in the E₆SSM can be obtained if $\alpha_3(M_Z) \approx 0.118$ [72]. Thus, in this SUSY model the gauge coupling unification can be attained for the values of $\alpha_3(M_Z)$ which are in agreement with current data.

As mentioned above, the inclusion of the two-loop corrections to the diagonal beta functions could spoil the gauge coupling unification entirely within the E₆SSM. Since at any intermediate scale $\alpha_i(t)$ are considerably larger in the E₆SSM as compared to the ones in the MSSM, the corresponding two-loop corrections affect the running of the SM gauge couplings in the E₆SSM much more strongly than in the MSSM. The analysis of the RG flow of the couplings $\alpha_i(t)$ performed in [84] indicated that $\Theta_i(M_X)$ are a few times smaller in the MSSM than in the E₆SSM. On the other hand, the absolute value of Θ_s is more than three times larger in the MSSM as compared with the E₆SSM. The cancellation of different two-loop contribution to Θ_s in the case of the model under consideration is caused by the structure of \tilde{b}_i . As a consequence, the prediction for $\alpha_3(M_Z)$ obtained using Equation (35) is substantially lower in the E₆SSM than in the MSSM.

4. Gauge Symmetry Breaking and Higgs Sector

In the E₆SSM, the VEV of the SM singlet field S and the VEVs of the two Higgs doublets H_u and H_d can give rise to the breakdown of the $SU(2)_W \times U(1)_Y \times U(1)_N$ gauge symmetry in the simplest case. The interactions between these fields are determined by the structure of the gauge group and by the superpotential in Equation (14). The resulting Higgs effective potential is the sum of four pieces:

$$V = V_F + V_D + V_{\text{soft}} + \Delta V, \quad (39)$$

$$V_F = \lambda^2 |S|^2 (|H_d|^2 + |H_u|^2) + \lambda^2 |(H_d H_u)|^2, \quad (40)$$

$$\begin{aligned} V_D = & \sum_{a=1}^3 \frac{g_2^2}{8} (H_d^\dagger \sigma_a H_d + H_u^\dagger \sigma_a H_u)^2 + \frac{g'^2}{8} (|H_d|^2 - |H_u|^2)^2 + \\ & + \frac{g_1'^2}{2} (\tilde{Q}_{H_d} |H_d|^2 + \tilde{Q}_{H_u} |H_u|^2 + \tilde{Q}_S |S|^2)^2, \end{aligned} \quad (41)$$

$$V_{\text{soft}} = m_S^2 |S|^2 + m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 + \left[\lambda A_\lambda S (H_u H_d) + \text{h.c.} \right], \quad (42)$$

where σ_a ($a = 1, 2, 3$) denotes the three Pauli matrices, $g' = \sqrt{3/5} g_1$, $H_d^T = (H_d^0, H_d^-)$, $H_u^T = (H_u^+, H_u^0)$, $(H_d H_u) = H_u^+ H_d^- - H_u^0 H_d^0$, and \tilde{Q}_{H_d} , \tilde{Q}_{H_u} , and \tilde{Q}_S are the effective $U(1)_N$ charges of H_d , H_u and

S , respectively. At tree-level the potential of the Higgs sector in Equation (39) is determined by the sum of the first three terms. V_F and V_D correspond to the F - and D -term contributions. They do not violate SUSY. V_F is exactly the same as the contribution of the F -terms in the NMSSM without the self-interaction of the SM singlet superfield S . However, V_D contain new terms which are proportional to $g_1'^2$. These terms are not present in the MSSM or NMSSM. They represent D -term contributions due to the additional $U(1)_N$ factor. The low-energy values of g_2 and g' are well known. Assuming gauge coupling unification one can compute the low-energy value of the extra $U(1)_N$ coupling g'_1 . In this case, the effective $U(1)_N$ charges of H_d , H_u and S may be calculated as well.

The terms in the Higgs potential in Equation (39), which break global SUSY, are collected in V_{soft} . The set of the soft SUSY breaking terms involves the soft masses $m_{H_d}^2$, $m_{H_u}^2$, m_S^2 as well as trilinear coupling A_λ . V_{soft} coincides with the corresponding part of the NMSSM scalar potential in the limit when the NMSSM parameters κ and A_κ vanish. Because the only possible complex phase of (λA_λ) in the tree-level Higgs potential in Equation (39) may easily be absorbed by a suitable redefinition of H_d , H_u and S , CP-invariance is preserved in the sector responsible for the breakdown of the $SU(2)_W \times U(1)_Y \times U(1)_N$ symmetry at tree-level.

The last term ΔV in Equation (39) represents the contribution of loop corrections to the effective potential of the Higgs sector. In the one-loop approximation the contributions of different states to ΔV are determined by their masses, i.e.,

$$\Delta V = \frac{1}{64\pi^2} Str |M|^4 \left[\log \frac{|M|^2}{Q^2} - \frac{3}{2} \right]. \quad (43)$$

In Equation (43), M is the mass matrix for the bosons and fermions within the SUSY model under consideration. The supertrace operator in Equation (43) counts positively (negatively) the number of degrees of freedom associated with the different bosonic (fermionic) fields, while Q is the renormalization scale. The inclusion of loop corrections draws into the consideration many other soft SUSY breaking parameters which determine masses of different sparticles. Some of these parameters may be complex giving rise to potential sources of CP-violation.

At the physical minimum of the Higgs potential in Equation (39), the Higgs fields develop VEVs

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = \frac{s}{\sqrt{2}}. \quad (44)$$

The equations for the extrema of the effective potential of the E_6 SSM Higgs sector in the directions in Equation (44) in field space read:

$$\frac{\partial V}{\partial s} = m_S^2 s - \frac{\lambda A_\lambda}{\sqrt{2}} v_1 v_2 + \frac{\lambda^2}{2} (v_1^2 + v_2^2) s + \frac{g_1'^2}{2} D' \tilde{Q}_{SS} + \frac{\partial \Delta V}{\partial s} = 0, \quad (45)$$

$$\frac{\partial V}{\partial v_1} = m_{H_d}^2 v_1 - \frac{\lambda A_\lambda}{\sqrt{2}} s v_2 + \frac{\lambda^2}{2} (v_2^2 + s^2) v_1 + \frac{\bar{g}^2}{8} (v_1^2 - v_2^2) v_1 + \frac{g_1'^2}{2} D' \tilde{Q}_{H_d} v_1 + \frac{\partial \Delta V}{\partial v_1} = 0, \quad (46)$$

$$\frac{\partial V}{\partial v_2} = m_{H_u}^2 v_2 - \frac{\lambda A_\lambda}{\sqrt{2}} s v_1 + \frac{\lambda^2}{2} (v_1^2 + s^2) v_2 + \frac{\bar{g}^2}{8} (v_2^2 - v_1^2) v_2 + \frac{g_1'^2}{2} D' \tilde{Q}_{H_u} v_2 + \frac{\partial \Delta V}{\partial v_2} = 0, \quad (47)$$

where $D' = \tilde{Q}_{H_d} v_1^2 + \tilde{Q}_{H_u} v_2^2 + \tilde{Q}_{SS} s^2$ and $\bar{g} = \sqrt{g_2^2 + g'^2}$. Instead of specifying v_1 and v_2 , it is more convenient to use $\tan \beta = v_2/v_1$ and $v = \sqrt{v_1^2 + v_2^2} \approx 246$ GeV.

The Higgs sector of the E_6 SSM includes ten degrees of freedom. Four of them are massless Goldstone modes which are swallowed by the W^\pm , Z and Z' vector bosons. The mass M_W of the charged W^\pm bosons is induced via the interaction of these vector bosons with the neutral components of H_u and H_d . This results in $M_W = \frac{g_2}{2} v$. Meanwhile, the mechanism of the neutral gauge boson mass

generation differs substantially. Letting the Z and Z' states be the SM-like Z boson and the gauge boson associated with the $U(1)_N$ factor, the Z - Z' mass squared matrix is given by

$$M_{ZZ'}^2 = \begin{pmatrix} \frac{\tilde{g}^2}{4}v^2 & \frac{\tilde{g}g'_1}{2}v^2 \left(\tilde{Q}_{H_d} \cos^2 \beta - \tilde{Q}_{H_u} \sin^2 \beta \right) \\ \frac{\tilde{g}g'_1}{2}v^2 \left(\tilde{Q}_{H_d} \cos^2 \beta - \tilde{Q}_{H_u} \sin^2 \beta \right) & g'^2_1 D' \end{pmatrix}. \quad (48)$$

To ensure that the extra $U(1)_N$ vector boson is sufficiently heavy, the SM singlet fields S must acquire large VEV, $s \gg 1$ TeV. In this case, the mass of the lightest neutral vector boson Z_1 is very close to $M_Z = \tilde{g}v/2$, while the mass of Z_2 is set by s , i.e., $M_{Z'} \approx g'_1 \tilde{Q}_S s$.

To explore the spectrum of the Higgs bosons within the E_6 SSM, we express the soft masses $m_{H_d}^2$, $m_{H_u}^2$, m_S^2 in terms of other parameters using the minimization conditions in Equation (45)–(47). The charged components of H_u and H_d are not mixed with the neutral Higgs fields because of the electric charge conservation. They form a separate sector, whose spectrum is described by a 2×2 mass matrix. The determinant of this matrix vanishes leading to the appearance of charged Goldstone states

$$G^- = H_d^- \cos \beta - H_u^{+*} \sin \beta \quad (49)$$

which are absorbed by the W^\pm vector bosons. Their orthogonal linear combinations

$$H^+ = H_d^{-*} \sin \beta + H_u^+ \cos \beta \quad (50)$$

gain mass

$$m_{H^\pm}^2 = \frac{\sqrt{2}\lambda A_\lambda}{\sin 2\beta} s - \frac{\lambda^2}{2}v^2 + \frac{g^2}{2}v^2 + \Delta_\pm. \quad (51)$$

where Δ_\pm denotes the loop corrections to $m_{H^\pm}^2$.

The imaginary parts of the SM singlet field S and the neutral components of H_u and H_d do not mix with the real parts of these fields if CP-invariance is preserved. In this case, the imaginary parts of the neutral components of H_u and H_d as well as the imaginary part of the SM singlet field S form CP-odd Higgs sector. They compose two neutral Goldstone states

$$\begin{aligned} G &= \sqrt{2}(\text{Im } H_d^0 \cos \beta - \text{Im } H_u^0 \sin \beta), \\ G' &= \sqrt{2}\text{Im } S \cos \gamma - \sqrt{2}(\text{Im } H_u^0 \cos \beta + \text{Im } H_d^0 \sin \beta) \sin \gamma, \end{aligned} \quad (52)$$

which are swallowed by the Z and Z' vector bosons, and one physical state

$$A = \sqrt{2}\text{Im } S \sin \gamma + \sqrt{2}(\text{Im } H_u^0 \cos \beta + \text{Im } H_d^0 \sin \beta) \cos \gamma, \quad (53)$$

where $\tan \gamma = \frac{v}{2s} \sin 2\beta$. Two massless pseudoscalars G_0 and G' decouple from the rest of the spectrum. The physical CP-odd Higgs state A acquires mass

$$m_A^2 = \frac{\sqrt{2}\lambda A_\lambda}{\sin 2\gamma} v + \Delta_A. \quad (54)$$

In Equation (54), Δ_A denotes loop corrections. Since in the E_6 SSM the VEV of the SM singlet field S must be much larger than v , the value of γ is always small and A is mostly the superposition of the imaginary parts of the neutral components of the Higgs doublets. In the limit $s \gg v$, the masses of the charged and CP-odd Higgs states are approximately equal to each other.

The CP-even Higgs sector includes $Re H_d^0$, $Re H_u^0$ and $Re S$. In the field space basis (h, H, N) , where

$$\begin{aligned} Re H_d^0 &= (h \cos \beta - H \sin \beta + v_1) / \sqrt{2}, \\ Re H_u^0 &= (h \sin \beta + H \cos \beta + v_2) / \sqrt{2}, \\ Re S &= (s + N) / \sqrt{2}, \end{aligned} \quad (55)$$

the mass matrix of the Higgs scalars in the E_6 SSM takes the form [141–143]:

$$M^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 \\ M_{21}^2 & M_{22}^2 & M_{23}^2 \\ M_{31}^2 & M_{32}^2 & M_{33}^2 \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 V}{\partial v^2} & \frac{1}{v} \frac{\partial^2 V}{\partial v \partial \beta} & \frac{\partial^2 V}{\partial v \partial s} \\ \frac{1}{v} \frac{\partial^2 V}{\partial v \partial \beta} & \frac{1}{v^2} \frac{\partial^2 V}{\partial \beta^2} & \frac{1}{v} \frac{\partial^2 V}{\partial s \partial \beta} \\ \frac{\partial^2 V}{\partial v \partial s} & \frac{1}{v} \frac{\partial^2 V}{\partial s \partial \beta} & \frac{\partial^2 V}{\partial s^2} \end{pmatrix}. \quad (56)$$

Calculating second derivatives of the effective potential of the Higgs sector in Equations (39)–(42) and substituting $m_{H_d}^2$, $m_{H_u}^2$ and m_S^2 from the minimization conditions in Equations (45)–(47), one finds:

$$M_{11}^2 = \frac{\lambda^2}{2} v^2 \sin^2 2\beta + \frac{\tilde{g}^2}{4} v^2 \cos^2 2\beta + g_1'^2 v^2 (\tilde{Q}_{H_d} \cos^2 \beta + \tilde{Q}_{H_u} \sin^2 \beta)^2 + \Delta_{11}, \quad (57)$$

$$\begin{aligned} M_{12}^2 = M_{21}^2 &= \left(\frac{\lambda^2}{4} - \frac{\tilde{g}^2}{8} \right) v^2 \sin 4\beta + \frac{g_1'^2}{2} v^2 (\tilde{Q}_{H_u} - \tilde{Q}_{H_d}) \times \\ &\times (\tilde{Q}_{H_d} \cos^2 \beta + \tilde{Q}_{H_u} \sin^2 \beta) \sin 2\beta + \Delta_{12}, \end{aligned} \quad (58)$$

$$M_{22}^2 = \frac{\sqrt{2}\lambda A_\lambda}{\sin 2\beta} s + \left(\frac{\tilde{g}^2}{4} - \frac{\lambda^2}{2} \right) v^2 \sin^2 2\beta + \frac{g_1'^2}{4} (\tilde{Q}_{H_u} - \tilde{Q}_{H_d})^2 v^2 \sin^2 2\beta + \Delta_{22}, \quad (59)$$

$$M_{23}^2 = M_{32}^2 = -\frac{\lambda A_\lambda}{\sqrt{2}} v \cos 2\beta + \frac{g_1'^2}{2} (\tilde{Q}_{H_u} - \tilde{Q}_{H_d}) \tilde{Q}_S v s \sin 2\beta + \Delta_{23}, \quad (60)$$

$$M_{13}^2 = M_{31}^2 = -\frac{\lambda A_\lambda}{\sqrt{2}} v \sin 2\beta + \lambda^2 v s + g_1'^2 (\tilde{Q}_{H_d} \cos^2 \beta + \tilde{Q}_{H_u} \sin^2 \beta) \tilde{Q}_S v s + \Delta_{13}, \quad (61)$$

$$M_{33}^2 = \frac{\lambda A_\lambda}{2\sqrt{2}s} v^2 \sin 2\beta + g_1'^2 \tilde{Q}_S^2 s^2 + \Delta_{33}. \quad (62)$$

In Equations (57)–(62), Δ_{ij} denotes the loop corrections.

If the VEV of the SM singlet field S and all SUSY breaking parameters are much larger than the mass of the Z boson, then the mass matrix in Equations (56)–(62) has a hierarchical structure. In the field basis (h, H, N) , all off-diagonal elements of this matrix are relatively small $\sim M_S M_Z$. Therefore, the masses of the heaviest Higgs scalars are closely approximated by the diagonal entries of the mass matrix of the CP-even Higgs sector M_{22}^2 and M_{33}^2 . These entries are expected to be of the order of the SUSY breaking scale M_S^2 . Two heaviest CP-even Higgs bosons are predominantly formed by the components of the field basis H and N . Because the minimal eigenvalue of the mass matrix in Equations (56)–(62) does not exceed its smallest diagonal element, the mass of the lightest Higgs scalar m_{h_1} , which is predominantly h , remains always relatively small irrespective of the SUSY breaking scale, i.e., $m_{h_1}^2 < M_{11}^2$. In the interactions with other SM particles, the lightest Higgs scalar manifests itself as a SM-like Higgs boson if $M_S \gg M_Z$.

As follows from Equations (51), (54) and (56)–(62), at the tree level, the spectrum of the Higgs bosons depends on four variables only:

$$\lambda, \quad s, \quad \tan \beta, \quad A_\lambda. \quad (63)$$

5. Higgs Spectrum

The qualitative pattern of the Higgs spectrum in the E_6 SSM is determined by the Yukawa coupling λ . Let us start our analysis here from the MSSM limit of the E_6 SSM when $\lambda \ll g'_1$. In the case when λ goes to zero, s has to be sufficiently large so that $\mu = \lambda s / \sqrt{2}$ is held fixed in order to give an acceptable chargino mass and EWSB. The diagonal entry M_{33}^2 , which is set by $M_{Z'}^2$, is considerably larger than other elements of the mass matrix in Equations (56)–(62) in this scenario. From the first minimization conditions in Equation (45), one can see that such solution can be obtained for $m_S^2 < 0$ when the absolute value of m_S^2 is very large. If $\mu \ll M_{Z'}$ and $m_A^2 \ll M_{Z'}$, the CP-even Higgs mass matrix in Equations (56)–(62) can be reduced to the block diagonal form M'^2 using unitary transformation [144,145]

$$M'^2 \approx \begin{pmatrix} M_{11}^2 - \frac{M_{13}^4}{M_{33}^2} & M_{12}^2 - \frac{M_{13}^2 M_{32}^2}{M_{33}^2} & 0 \\ M_{21}^2 - \frac{M_{23}^2 M_{31}^2}{M_{33}^2} & M_{22}^2 - \frac{M_{23}^4}{M_{33}^2} & 0 \\ 0 & 0 & M_{33}^2 + \frac{M_{13}^4}{M_{33}^2} + \frac{M_{23}^4}{M_{33}^2} \end{pmatrix}. \quad (64)$$

In the limit when λ is small, the top-left 2×2 submatrix of the matrix in Equation (64) reproduces the mass matrix of the Higgs scalars in the MSSM. Such hierarchical structure of the mass matrix of the Higgs scalars in Equation (64) implies that the heaviest CP-even Higgs state associated with N and the Z' boson are almost degenerate. In other words, the singlet dominated CP-even state is always rather heavy and decouples from the rest of the spectrum. This makes the spectrum of the Higgs bosons indistinguishable from the one in the MSSM. The mass of the heaviest CP-even Higgs is determined by the VEV s . It does not change much if the other parameters λ , $\tan \beta$ and A_λ (m_A) vary. The masses of the second lightest Higgs scalar, which is predominantly H , the Higgs pseudoscalar and the charged Higgs states grow when m_A rises providing the degeneracy of the corresponding states at m_A when m_A is much larger than M_Z but is less than $M_{Z'}$. In this case, the expression for the SM-like Higgs mass $m_{h_1}^2$ is essentially the same as in the MSSM.

When $\lambda \geq g'_1$, the qualitative pattern of the Higgs spectrum is quite similar to the one that arises in the NMSSM with the approximate PQ symmetry [144–148]. In the NMSSM and E_6 SSM, the growth of the Yukawa coupling λ at low energies entails the increase of its value at the GUT scale M_X leading to the appearance of the Landau pole [87,88]. This spoils the applicability of perturbation theory at high energies. The requirement of validity of perturbation theory up to the scale M_X restricts the interval of variations of $\lambda(M_t)$ setting an upper limit on $\lambda(M_t)$ for each fixed value of $\tan \beta$ in these models. In the E_6 SSM, the restrictions on the low energy values of λ are weaker than in the NMSSM (see Figure 2, left). The presence of exotic matter change the evolution of the SM gauge couplings. Indeed, at the intermediate scales the values of these couplings rise when the number of extra $5 + \bar{5}$ -plets increases. In the RGEs that describe the evolution of the Yukawa couplings within the NMSSM and E_6 SSM, the gauge couplings occur on the right-hand side of these equations with negative sign. As a consequence, the growth of the SM gauge couplings reduces the values of the Yukawa couplings at the intermediate scales preventing the appearance of the Landau pole in the RG flow of these couplings. Therefore, within the E_6 SSM, the values of $\lambda(M_t)$ can be larger than in the NMSSM. The upper bound on the low energy values of λ grows with increasing $\tan \beta$ since the top-quark Yukawa coupling decreases. At large $\tan \beta$, this bound approaches the saturation limit. In the NMSSM and E_6 SSM the maximal possible values of $\lambda(M_t)$ are 0.71 and 0.84, respectively, whereas the low energy value of $g'_1 \approx g_1$ vary from 0.46 to 0.48.

If $\lambda \geq g'_1$, then M_{22}^2 tends to be the largest diagonal entry of the mass matrix in Equations (56)–(62), i.e., $M_{22}^2 \gg M_{33}^2 \gg M_{11}^2$. Relying on this mass hierarchy, the approximate analytical expressions for the masses of the CP-even Higgs bosons can be obtained. The perturbation theory method yields [141–145]

$$m_{h_3}^2 \approx M_{22}^2 + \frac{M_{23}^4}{M_{22}^2}, \quad m_{h_2}^2 \approx M_{33}^2 - \frac{M_{23}^4}{M_{22}^2} + \frac{M_{13}^4}{M_{33}^2}, \quad m_{h_1}^2 \approx M_{11}^2 - \frac{M_{13}^4}{M_{33}^2}. \quad (65)$$

All terms in Equation (65) suppressed by inverse powers of m_A^2 or $M_{Z'}^2$, which are of the order of M_Z^4/m_A^2 , $M_Z^4/M_{Z'}^2$ or even smaller, are neglected. At tree-level, the Higgs masses in the E₆SSM are given by

$$\begin{aligned} m_{h_3}^2 &\approx m_{H^\pm}^2 \approx m_A^2 \approx \frac{4\mu^2 x}{\sin^2 2\beta}, \quad m_{h_2}^2 \approx M_{Z'}^2, \\ m_{h_1}^2 &\approx \frac{\lambda^2}{2} v^2 \sin^2 2\beta + M_Z^2 \cos^2 2\beta + g_1'^2 v^2 \left(\tilde{Q}_{H_d} \cos^2 \beta + \tilde{Q}_{H_u} \sin^2 \beta \right)^2 \\ &\quad - \frac{\lambda^4 v^2}{g_1'^2 \tilde{Q}_S^2} \left(1 - x + \frac{g_1'^2}{\lambda^2} \left(\tilde{Q}_{H_d} \cos^2 \beta + \tilde{Q}_{H_u} \sin^2 \beta \right) \tilde{Q}_S \right)^2, \end{aligned} \quad (66)$$

where $x = \frac{A_\lambda}{2\mu} \sin 2\beta$ and $\mu = \frac{\lambda}{\sqrt{2}} s$. As evident from the explicit expression for $m_{h_1}^2$ given above at $\lambda^2 \gg g_1'^2$, the last term in this expression dominates and the mass squared of the lightest Higgs scalar tends to be negative if x is not close to unity. A negative eigenvalue of the mass matrix in Equations (57)–(62) implies that the vacuum configuration in Equation (44) ceases to be a minimum and turns into a saddle point. This means that there is a direction in field space in the vicinity of such point along which the energy density decreases leading to the instability of the vacuum configuration in Equation (44). Thus, large deviations of x from unity pulls the mass squared of the lightest Higgs scalar below zero destabilizing the vacuum. The requirement of stability of the physical vacuum therefore constrains the variable x around unity limiting the interval of variations of m_A from below and above. As a consequence, the masses of the charged, CP-odd and heaviest CP-even Higgs bosons are almost degenerate around m_A and are confined in the vicinity of $\mu \tan \beta$. They are considerably larger than the masses of the Z' and lightest CP-even Higgs boson. Together with the experimental lower limit on the Z' boson mass it maintains the mass hierarchy in the Higgs spectrum [69].

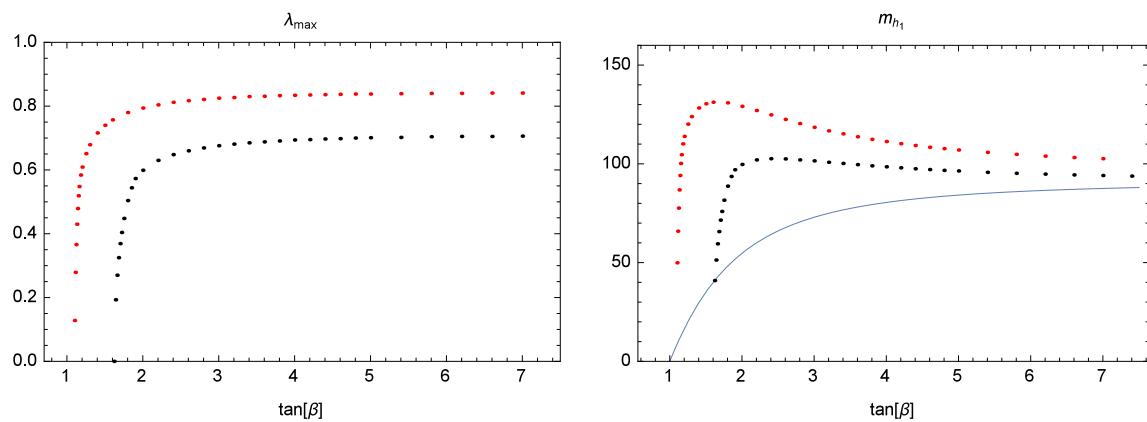


Figure 2. (Left) Upper limit on λ versus $\tan \beta$ in the NMSSM (lower dotted line) and E₆SSM (upper dotted line). (Right) Tree-level upper bound on the mass of the lightest Higgs scalar as a function of $\tan \beta$ in the MSSM (solid line), NMSSM (lower dotted line) and E₆SSM (upper dotted line).

From the explicit analytic expression for $m_{h_1}^2$, it is apparent that, at some value of x (or m_A), the mass of the lightest Higgs scalar mass attains its maximum value. It corresponds to the value of x for which the fourth term in the expression for $m_{h_1}^2$ vanishes. In this case, the mass squared of the lightest Higgs boson coincides with the theoretical upper bound on $m_{h_1}^2$ given by M_{11}^2 . The sum of the

first and second terms in the expression for M_{11}^2 are similar to the tree-level upper bound on $m_{h_1}^2$ in the NMSSM [149,150]. The third term in Equation (57) is a contribution which comes from the additional $U(1)_N$ D -term in the potential of the Higgs sector in Equations (39)–(42). At tree-level the upper bound on the mass of the lightest Higgs scalar in the E_6 SSM depends on λ and $\tan\beta$ only. Using the obtained theoretical restrictions on the low energy values of λ as a function of $\tan\beta$, one can calculate for each $\tan\beta$ the maximum possible value of m_{h_1} .

The tree-level upper bound on the mass of the lightest Higgs scalar in the E_6 SSM is presented in Figure 2 (see Figure 2, right) and compared to the corresponding bounds in the MSSM and NMSSM. At moderate values of $\tan\beta$ the theoretical restriction on lightest Higgs boson mass in the E_6 SSM and NMSSM exceeds the corresponding limit in the MSSM. This happens because of the contribution to M_{11}^2 induced by the first term in the right-hand side of Equation (57). It comes from the additional F -term in the Higgs scalar potentials of the E_6 SSM and NMSSM. For the values of $\tan\beta \sim 1\text{--}3$, this contribution to the upper bound on m_{h_1} in the E_6 SSM and NMSSM dominates. Its size is determined by the Yukawa coupling λ . Since in the E_6 SSM λ is allowed to be larger than in the NMSSM, the tree-level theoretical restriction on m_{h_1} in the E_6 SSM is also larger at moderate values of $\tan\beta$ as compared with the one in the NMSSM. In the framework of the E_6 SSM, the upper bound on m_{h_1} attains its maximum value of 130 GeV at $\tan\beta = 1.5\text{--}1.8$. Thus, large tree-level theoretical restriction on the mass of the lightest Higgs scalar means that in this model the contribution of loop corrections to $m_{h_1}^2$ is not needed to be as big as in the MSSM and NMSSM in order to get the SM-like Higgs boson with mass around 125 GeV.

With increasing $\tan\beta$, the contribution to M_{11}^2 associated with the first term on the right-hand side of Equation (57) decreases rapidly. For $\tan\beta > 10$, it becomes negligibly small. The second term on the right-hand side of Equation (57) grows when $\tan\beta$ increases. At $\tan\beta > 4$, it exceeds $\frac{\lambda^2}{2}v^2 \sin^2 2\beta$ and gives the dominant contribution to the tree-level theoretical restriction on m_{h_1} . Therefore, with increasing $\tan\beta$, the upper bound on the mass of the lightest CP-even Higgs state in the NMSSM diminishes and approaches the corresponding limit in the MSSM. In the case of the E_6 SSM, the third term on the right-hand side of Equation (57), that comes from the extra $U(1)_N$ D -term contribution to the Higgs scalar potential in Equations (39)–(42), gives the second largest contribution to M_{11}^2 at very large values of $\tan\beta$. Because of the contribution of this term the tree-level theoretical restriction on the mass of the lightest Higgs scalar in the E_6 SSM, which also diminishes when $\tan\beta$ rises, is about 6–7 GeV larger for $\tan\beta > 10$ than the corresponding upper bounds in the MSSM and NMSSM. As a consequence, at large $\tan\beta$, the presence of the 125-GeV Higgs boson in the particle spectrum of the E_6 SSM does not require as large contribution of loop corrections to $m_{h_1}^2$ as in the MSSM and NMSSM.

The inclusion of loop corrections substantially increases the mass of the lightest Higgs scalar in SUSY models. The dominant contribution comes from the loop diagrams involving the top quark and its superpartners because of the large top-quark Yukawa coupling h_t . Within the MSSM leading one-loop and two-loop corrections increase the theoretical upper bound on the lightest Higgs boson mass, which at the tree-level does not exceed M_Z [151,152], from M_Z to 130 GeV (see [153] and references therein). In the leading approximation, the two-loop upper bound on the lightest Higgs scalar mass in the E_6 SSM may be presented in the following form [69]

$$\begin{aligned} m_{h_1}^2 &\leq \left[\frac{\lambda^2}{2}v^2 \sin^2 2\beta + M_Z^2 \cos^2 2\beta + g_1'^2 v^2 \left(\tilde{Q}_{H_d} \cos^2 \beta + \tilde{Q}_{H_u} \sin^2 \beta \right)^2 \right] \left(1 - \frac{3h_t^2}{8\pi^2} l \right) \\ &\quad + \frac{3m_t^4}{2\pi^2 v^2} \left\{ \frac{1}{2}U_t + l + \frac{1}{16\pi^2} \left(\frac{3}{2}h_t^2 - 8g_3^2 \right) (U_t + l)l \right\}, \\ m_t(M_t) &= \frac{h_t(M_t)}{\sqrt{2}} v \sin \beta, \quad U_t = 2 \frac{X_t^2}{M_S^2} \left(1 - \frac{1}{12} \frac{X_t^2}{M_S^2} \right), \quad l = \ln \left[\frac{M_S^2}{m_t^2} \right], \end{aligned} \quad (67)$$

where X_t is a stop mixing parameter. In Equation (67), M_S is defined as $M_S^2 = m_Q^2 = m_U^2$, where m_Q^2 and m_U^2 are soft scalar masses of superpartners of the right-handed and left-handed components of

the t -quark, respectively. Using the relationship between the running ($m_t(Q)$) and t -quark pole (M_t) masses [154,155]

$$m_t(M_t) = M_t \left[1 - 1.333 \frac{\alpha_s(M_t)}{\pi} - 9.125 \left(\frac{\alpha_s(M_t)}{\pi} \right)^2 \right] \quad (68)$$

one can compute $m_t(M_t)$ for the world average mass $M_t = 173.1 \pm 0.9$ GeV [156]. Equation (67) is just a simple generalization of the approximate expressions for the theoretical restriction on the mass of the lightest Higgs scalar obtained in the MSSM [157] and NMSSM [158]. In the case $g'_1 = 0$ and $\lambda = 0$, Equation (67) coincides with the theoretical bound on the mass of the lightest CP-even Higgs state in the MSSM. The analytic approximation of the two-loop effects given above slightly underestimates the full two-loop corrections. In the MSSM, the approximate expression in Equation (67) results in m_{h_1} , which is typically a few GeV lower than the lightest Higgs mass which is computed using the Suspect [159] and FeynHiggs [160–163] packages. It was shown that in the two-loop approximation the lightest Higgs boson mass in the E_6 SSM does not exceed 150 GeV [69].

Although the inclusion of loop corrections changes considerably the lightest Higgs boson mass in the E_6 SSM, it does not change the qualitative pattern of the spectrum of the Higgs states for $\lambda \ll g'_1$ and $\lambda > g'_1$. The mass of the SM singlet dominated CP-even state is always set by $M_{Z'}$, whereas another Higgs scalar, CP-odd and charged Higgs states have masses close to m_A . In the phenomenologically viable scenarios, the masses of all Higgs particles except the lightest Higgs state are much larger than M_Z . Moreover, when $\lambda > g'_1$, the charged, CP-odd and heaviest CP-even Higgs states lie beyond the multi-TeV range and therefore cannot be detected at the LHC experiments.

6. LHC Signatures

We now turn to the LHC signatures of the E_6 SSM, which permit distinguishing this SUSY model from the MSSM or NMSSM. As discussed above, in the simplest phenomenologically viable scenarios, the lightest exotic fermion \tilde{H}_1^0 should have mass $m_{\tilde{H}_1^0} \ll 1$ eV. At the same time, the next-to-lightest exotic fermion \tilde{H}_2^0 may be considerably heavier. Let us assume that all sparticles and exotic states except \tilde{H}_1^0 and \tilde{H}_2^0 are rather heavy and can be integrated out. In particular, the parameters are chosen so that all fermion components of the supermultiplets H_α^u and H_α^d are heavier than 100 GeV, whereas $s \approx 12$ TeV. In this limit, the part of the Lagrangian involving the couplings of \tilde{H}_1^0 and \tilde{H}_2^0 to the SM-like Higgs state and the Z boson takes the following form:

$$\mathcal{L}_{Zh} = \sum_{\alpha, \beta} \frac{M_Z}{2v} Z_\mu \left(\tilde{H}_\alpha^{0T} \gamma_\mu \gamma_5 \tilde{H}_\beta^0 \right) R_{Z\alpha\beta} + \sum_{\alpha, \beta} X_{\alpha\beta}^h \left(\tilde{H}_\alpha^{0T} \tilde{H}_\beta^0 \right) h, \quad (69)$$

where $\alpha, \beta = 1, 2$. Although \tilde{H}_1^0 and \tilde{H}_2^0 are substantially lighter than 100 GeV, the interactions of these exotic states with the SM particles including Z boson tend to be rather weak because they are predominantly the fermion components of the superfields S_α . Therefore, any possible signal that \tilde{H}_1^0 and \tilde{H}_2^0 could give rise to at former and present experiments might be extremely suppressed and such states could remain undetected.

The couplings of the SM-like Higgs state h_1 to \tilde{H}_1^0 and \tilde{H}_2^0 are determined by the masses of these lightest exotic states [94]. Since \tilde{H}_1^0 is very light, it does not affect Higgs phenomenology. The absolute value of the coupling of h_1 to the second lightest exotic particle $|X_{22}^h| \simeq |m_{\tilde{H}_2^0}|/v$ [94]. This coupling gives rise to the decays of h_1 into \tilde{H}_2^0 pairs. The corresponding partial decay width is given by

$$\Gamma(h_1 \rightarrow \tilde{H}_2^0 \tilde{H}_2^0) = \frac{|X_{22}^h|^2 m_{h_1}}{4\pi} \left(1 - 4 \frac{|m_{\tilde{H}_2^0}|^2}{m_{h_1}^2} \right)^{3/2}. \quad (70)$$

The nonstandard Higgs boson decays were studied in the context of different extensions of the SM (see, for example, [98,164–180]). The partial decay width in Equation (70) depends rather strongly

on $m_{\tilde{H}_2^0}$. To avoid the suppression of the branching ratios for the lightest Higgs boson decays into SM particles, we restrict our consideration here to the GeV-scale masses of \tilde{H}_2^0 .

To compare the partial width that correspond to the exotic decays of h_1 in Equation (70) with other decay rates a set of benchmark points (see Table 2) is specified. The masses of the heavy Higgs bosons given in Table 2 are computed in the leading one-loop approximation. In the case of the mass of the lightest Higgs scalar, the leading two-loop corrections are taken into account. In all benchmark scenarios, the structure of the spectrum of the Higgs states is rather hierarchical. Since all heavy Higgs particles have masses, which are substantially larger than the lightest Higgs boson mass, the partial widths of the decays of h_1 into the SM particles are almost the same as in the SM. Therefore, in our analysis, we use the results presented in [181] where the corresponding decay rates were estimated within the SM for different values of the mass of the Higgs boson. When $m_{h_1} \simeq 125$ GeV, the SM-like Higgs boson decays mostly into $b\bar{b}$. The corresponding branching ratio is about 60% while the branching ratios associated with the decays of h_1 into WW and ZZ are about 20% and 2%, respectively [181]. The total decay width of such Higgs boson is about 4 MeV.

Table 2. Benchmark scenarios for $m_{h_1} \approx 125$ GeV; the partial width of the decays $h_1 \rightarrow \tilde{H}_2^0 \tilde{H}_2^0$, the branching ratios of the lightest Higgs scalar, the masses and couplings of \tilde{H}_1^0 and \tilde{H}_2^0 are calculated for $s = 12$ TeV, $\lambda = 0.6$, $\tan \beta = 1.5$, $m_{H^\pm} \simeq m_A \simeq m_{h_3} \simeq 9497$ GeV, $m_{h_2} \simeq M_{Z'} \simeq 4450$ GeV, $m_Q = m_U = M_S = 4000$ GeV and $X_t = \sqrt{6}M_S$.

	i	ii	iii	iv
λ_{22}	−0.03	−0.012	−0.06	0
λ_{21}	0	0	0	0.02
λ_{12}	0	0	0	0.02
λ_{11}	0.03	0.012	0.06	0
f_{22}	−0.1	−0.1	−0.1	0.6
f_{21}	−0.1	−0.1	−0.1	0.00245
f_{12}	0.00001	0.00001	0.00001	0.00245
f_{11}	0.1	0.1	0.1	0.00001
\tilde{f}_{22}	0.1	0.1	0.1	0.6
\tilde{f}_{21}	0.1	0.1	0.1	0.002
\tilde{f}_{12}	0.000011	0.000011	0.000011	0.002
\tilde{f}_{11}	0.1	0.1	0.1	0.00001
$ m_{\tilde{H}_1^0} /\text{GeV}$	2.7×10^{-11}	6.5×10^{-11}	1.4×10^{-11}	0.31×10^{-9}
$ m_{\tilde{H}_2^0} /\text{GeV}$	1.09	2.67	0.55	0.319
$ \tilde{R}_{Z11} $	0.0036	0.0212	0.00090	1.5×10^{-7}
$ \tilde{R}_{Z12} $	0.0046	0.0271	0.00116	1.7×10^{-4}
$ \tilde{R}_{Z22} $	0.0018	0.0103	0.00045	0.106
$X_{22}^{h_1}$	0.0044	0.0106	0.0022	0.00094
$\text{Br}(h_1 \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)$	4.7%	21.9%	1.23%	0.22%
$\text{Br}(h_1 \rightarrow b\bar{b})$	56.6%	46.4%	58.7%	59.3%
$\Gamma(h_1 \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)/\text{MeV}$	0.194	1.106	0.049	0.0088

The benchmark Scenarios (i)–(iv) presented in Table 2 demonstrate that the branching ratio of the exotic decays of h_1 changes from 0.2% to 20% when $m_{\tilde{H}_2^0}$ varies from 0.3 GeV to 2.7 GeV [98]. For smaller (larger) values of $m_{\tilde{H}_2^0}$ the branching ratio of these decays is even smaller (larger). On the other hand, the couplings of the exotic states \tilde{H}_1^0 and \tilde{H}_2^0 to the Z boson are so small that these fermions could not be observed before. In particular, the contribution of \tilde{H}_1^0 and \tilde{H}_2^0 to the Z-boson width is very small. After being produced \tilde{H}_2^0 sequentially decay into \tilde{H}_1^0 and fermion–antifermion pair via virtual Z. Thus, the exotic decays of h_1 result in two fermion–antifermion pairs and missing energy in the final state. Nevertheless, since $|\tilde{R}_{Z12}|$ is quite small, \tilde{H}_2^0 tends to live longer than 10^{-8} s and typically decays outside the detectors. As a consequence, the decay channel $h_1 \rightarrow \tilde{H}_2^0 \tilde{H}_2^0$ normally gives rise to an invisible branching ratio of the lightest Higgs scalar. The benchmark Scenarios (i),

(iii) and (iv) lead to such invisible decays of h_1 . In the case of benchmark Scenario (ii), $|R_{Z12}|$ is larger so that $\tau_{\tilde{H}_2^0} \sim 10^{-11}$ s and some of the decay products of \tilde{H}_2^0 could be observed at the LHC.

Because R_{Z12} is relatively small, \tilde{H}_2^0 may decay during or after Big Bang Nucleosynthesis (BBN) destroying the agreement between the observed and predicted abundances of light elements. To preserve the success of the BBN, \tilde{H}_2^0 must decay before BBN, i.e., the lifetime of the second lightest exotic fermion $\tau_{\tilde{H}_2^0}$ should not be longer than 1 s. This requirement sets lower bound on $|R_{Z12}|$. Indeed, for $m_{\tilde{H}_2^0} = 1$ GeV, the absolute value of the coupling R_{Z12} has to be larger than 1×10^{-6} [182]. The constraint on $|R_{Z12}|$ becomes more stringent when $m_{\tilde{H}_2^0}$ decreases because $\tau_{\tilde{H}_2^0} \sim 1/(|R_{Z12}|^2 m_{\tilde{H}_2^0}^5)$. The results of our analysis indicate that it is somewhat problematic to ensure that $\tau_{\tilde{H}_2^0} \leq 1$ s if $m_{\tilde{H}_2^0} \leq 100$ MeV.

The presence of a Z' boson and exotic matter which compose three $5 + 5^*$ representations of $SU(5)$ is another very peculiar feature of the E_6 SSM. LHC signatures associated with these states are determined by the structure of the particle spectrum which varies substantially depending on the choice of the parameters. At tree-level, the masses of the Z' boson and fermion components of $5 + 5^*$ supermultiplets are set by the VEV of the SM singlet superfield S , which is a free parameter in these models. Therefore, the masses of these states cannot be predicted. The lower experimental limits on the Z' mass, that comes from the direct searches ($pp \rightarrow Z' \rightarrow l^+l^-$) conducted at the LHC experiments, are already very stringent and vary around $3.8 - 3.9$ TeV [183,184]. This means that the scenarios with $s < 10 - 10.5$ TeV have been excluded. Possible decay channels of the Z' gauge bosons in E_6 inspired SUSY models were studied in [60,67].

Assuming that $f_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$ are very small the masses of the fermion components of extra $5 + 5^*$ supermultiplets of matter are given by

$$\mu_{D_i} = \frac{\kappa_i}{\sqrt{2}} s, \quad \mu_{H_\alpha} = \frac{\lambda_\alpha}{\sqrt{2}} s, \quad (71)$$

where μ_{D_i} are the masses of exotic quarks with electric charges $\pm 1/3$ and μ_{H_α} are the masses of the $SU(2)_W$ doublets of the Inert Higgsino states. Here, we set $\kappa_{ij} = \kappa_i \delta_{ij}$ and $\lambda_{\alpha\beta} = \lambda_\alpha \delta_{\alpha\beta}$. Requiring the validity of perturbation theory up to the scale M_X one can obtain upper bounds on the low-energy values of κ_i and λ_α . Nevertheless, the low-energy values of these couplings are allowed to be as large as $g_1'(q) \approx g_1(q) \approx 0.46 - 0.48$. On the other hand, since the exotic fermions must be sufficiently heavy to avoid any conflict with direct particle searches at former and present accelerators, couplings λ_α and κ_i have to be large enough. Although nowadays there are clear indications that Z' boson and sparticles have to be rather heavy some of the exotic fermions can be relatively light in the E_6 SSM. This may happen if the matrices of the Yukawa couplings of the exotic particles κ_{ij} and λ_α have hierarchical structure which is similar to the one in the quark and lepton sectors of the SM. Then, Z' boson can be much heavier than 10 TeV and the only manifestation of the E_6 SSM can be the presence of light exotic fermions in the particle spectrum.

If the relatively light exotic quarks of the nature described above do exist, they might be accessed through direct pair hadroproduction. The lifetime and decay modes of the lightest exotic quarks are determined by the Z_2^H symmetry violating couplings. Since in order to suppress FCNCs the Yukawa couplings of exotic states to the SM fermions of the first two generations have to be rather small, here we assume that exotic particles couple most strongly with the third family fermions and bosons. Then, because the lightest exotic quarks are R -parity odd states, they decay either via

$$\overline{D} \rightarrow t + b + E_T^{\text{miss}} + X, \quad (72)$$

if exotic quarks \overline{D}_i are diquarks or via

$$D \rightarrow t + \tau + E_T^{\text{miss}} + X, \quad D \rightarrow b + \nu_\tau + E_T^{\text{miss}} + X, \quad (73)$$

if D-fermions are leptoquarks. Thus, the pair production of light D-fermions at the LHC should result in some enhancement of the cross sections of either $pp \rightarrow t\bar{t}b\bar{b} + E_T^{\text{miss}} + X$ if exotic quark states are diquarks or $pp \rightarrow t\bar{t}\tau^+\tau^- + E_T^{\text{miss}} + X$ and $pp \rightarrow b\bar{b} + E_T^{\text{miss}} + X$ when D-fermions are leptoquarks.

In general, exotic squarks tend to be considerably heavier than the D-fermions because the masses of these scalars are determined by the soft SUSY breaking terms. Nevertheless, the exotic squark associated with the heavy exotic quark can be relatively light. This happens when the large mass of the heaviest D-fermion gives rise to the large mixing in the corresponding exotic squark sector. Such mixing can lead to the large mass splitting between the appropriate mass eigenstates. Because of this, the lightest exotic squark may be much lighter than all other scalars. Moreover, in principle, it can be even lighter than the lightest D-fermion. If this is a case, then in the variants of the E₆SSM with approximate Z_2^H symmetry the lightest exotic squark decays into either

$$\tilde{D} \rightarrow t + b + +X, \quad (74)$$

if it is a scalar diquark or

$$D \rightarrow t + \tau + X, \quad D \rightarrow b + \nu_\tau + X, \quad (75)$$

if this state is a scalar leptoquark. In the limit, when the couplings of this squark to the quarks and leptons of the first two generations are rather small, the lightest exotic squarks may only be pair produced at the LHC. Therefore, the presence of relatively light \tilde{D} is expected to lead to some enhancement of the cross sections of either $pp \rightarrow t\bar{t}\tau^+\tau^- + X$ and $pp \rightarrow b\bar{b} + E_T^{\text{miss}} + X$ if exotic squarks are leptoquarks or $pp \rightarrow t\bar{t}b\bar{b} + X$ when they are diquarks. On the other hand, in the variants of the E₆SSM with exact \tilde{Z}_2^H symmetry the Z_2^E symmetry conservation implies that the final state in the decay of \tilde{D} should always contain the lightest exotic fermion \tilde{H}_1^0 [72]. Because the lightest exotic squark is R-parity even $SU(3)_C$ triplet of scalar fields while \tilde{H}_1^0 is R-parity odd fermion the final state in the decay of \tilde{D} should also involve the lightest ordinary neutralino to ensure that R-parity is conserved. As a consequence, in such models, the decay patterns of the lightest exotic squarks and their LHC signatures are rather similar to the ones which appear in the case of the lightest exotic quarks. The presence of relatively light exotic squark and D-fermion may significantly modify the LHC signatures associated with the gluinos [92].

A few experiments at LEP, HERA, Tevatron and LHC have searched for the states which decay into either a pair of quarks or lepton and quark. Most searches focus on leptoquarks or diquarks which have integer-spin so that they can be either scalars or vectors. Such particles may be coupled directly to either quark and lepton or a pair of quarks. The most stringent constraints on the masses of scalar leptoquarks and scalar diquarks come from the non-observation of these exotic states at the LHC experiments. ATLAS and CMS collaborations ruled out first and second generation scalar leptoquarks (i.e., states which mainly interact with the first and second generation fermions, respectively) that have masses below 1230–1560 GeV depending on the branching ratios of their decays [185–187]. The experimental limits on the masses of the third-generation scalar leptoquarks are somewhat weaker. ATLAS and CMS collaborations excluded such exotic objects if they have masses below 800–1000 GeV [188–190]. The experimental lower limits on the masses of dijet resonances including diquarks tend to be considerably higher [191].

However, the LHC lower bounds mentioned above are not always directly applicable in the case of the E₆SSM. For instance, it is expected that scalar diquarks are mostly produced singly at the LHC and decay into final state that contains two quarks. At the same time, within the E₆SSM the couplings of all exotic scalars to the fermions of the first and second generation have to be quite small to avoid processes with non-diagonal flavor transitions. Therefore, in this SUSY model, diquarks can only be pair produced. It is also worth pointing out that the lightest exotic quarks in the E₆SSM give rise to collider signatures which are very different from the commonly established ones that have been thoroughly studied. Indeed, it is commonly assumed that scalar leptoquarks or diquarks

decay without missing energy. On the other hand, in the E₆SSM, exotic quarks are fermions and therefore R -parity odd states. Thus, R -parity conservation necessarily results in the missing energy and transverse momentum in the final state. Because of this, the pair production of the lightest exotic diquarks and the pair production of gluinos at the LHC may give rise to the enhancement of the same cross section of $pp \rightarrow t\bar{t}b\bar{b} + E_T^{\text{miss}} + X$.

The $SU(2)_W$ doublets of the Inert Higgsino states may also be heavy or light depending on free parameters. When at least one coupling λ_α is of the order of unity it may give rise to a large mixing in the Inert Higgs sector which can result in relatively light Inert Higgs bosons. Since these bosons have very small couplings to the fermions of the first and second generation such states can be produced in pairs at the LHC via off-shell W and Z bosons. As a consequence, the production cross section of the corresponding scalars is relatively small even when these particles have masses below the TeV scale. After being produced they sequentially decay into the third-generation fermions that should lead to some enlargement of the cross sections of $pp \rightarrow Q\bar{Q}\tau^+\tau^-$ and $pp \rightarrow Q\bar{Q}Q'\bar{Q}'$, where Q and Q' are heavy quarks of the third-generation.

From Equation (71), it follows that the lightest Inert Higgsinos can be relatively light if the corresponding Yukawa coupling λ_α is sufficiently small. If all sparticles and other exotic states are quite heavy, the corresponding fermionic states can be produced at the LHC in pairs via weak interactions. As a consequence, their production cross section is considerably smaller than the production cross section of the exotic quarks (see Figure 3). The Inert Higgsino states decay predominantly into the lightest exotic fermions (\tilde{H}_1^0 or \tilde{H}_2^0) as well as an on-shell Z or W boson. Thus, when pair produced Inert Higgsinos decay, they should lead to some enhancements in the rates of $pp \rightarrow ZZ + E_T^{\text{miss}} + X$, $pp \rightarrow WZ + E_T^{\text{miss}} + X$ and $pp \rightarrow WW + E_T^{\text{miss}} + X$. Similar enhancement of these cross sections could be caused by the pair production of ordinary chargino and neutralino in the MSSM if the mass of the LSP is negligibly small. Using the corresponding results of the analysis of ATLAS and CMS collaborations [192–194], one can conclude that the mass of the $SU(2)_W$ doublets of the Inert Higgsino states has to be larger than 650 GeV.

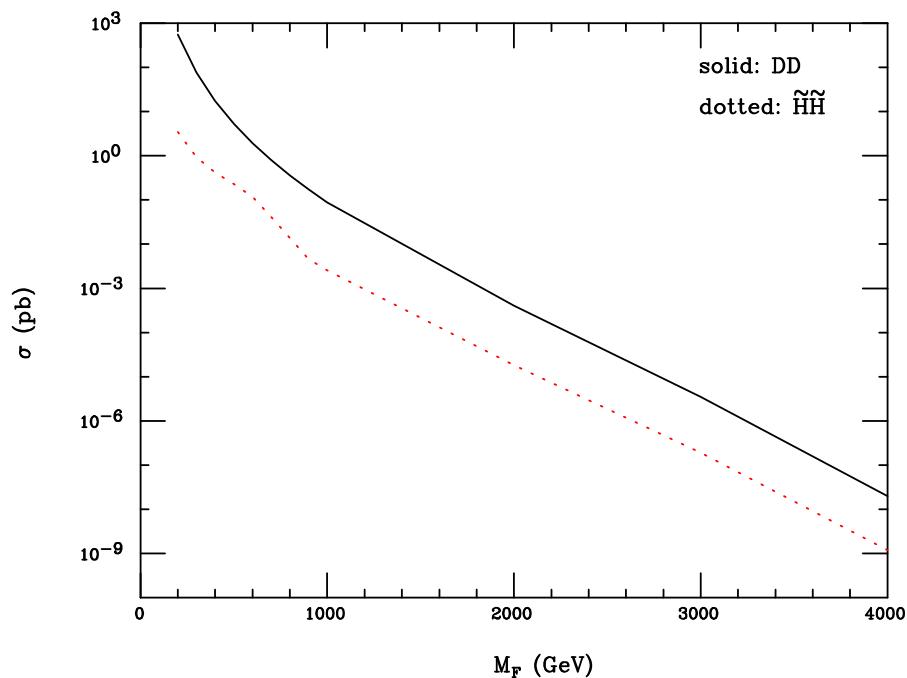


Figure 3. The cross section of pair production of D -fermions (via QCD interactions) and the cross section of pair production of Inert Higgsinos \tilde{H} which are components of $SU(2)_W$ doublets (via EW interactions) versus their (common) mass, denoted by M_F .

7. Conclusions

The breakdown of an extended gauge symmetry in the string-inspired E_6 GUTs may result in a variety of extensions of the SM with softly broken SUSY at low energies including MSSM, NMSSM, $U(1)$ extensions of the MSSM, etc. Among $U(1)$ extensions of the MSSM inspired by E_6 GUTs there is a model based on the SM gauge group together with an additional $U(1)_N$ gauge symmetry. Only in this $U(1)_N$ extension of the MSSM the right-handed neutrinos do not participate in gauge interactions, which allows them to be used for a high scale see-saw mechanism. In this Exceptional Supersymmetric Standard Model (E_6 SSM), the lepton asymmetry, which may be induced by the heavy right-handed neutrino decays, can be partially converted into baryon asymmetry via sphaleron processes [195,196]. The $U(1)_N$ symmetry forbids the bilinear term $\mu H_d H_u$ in the superpotential, but permits the term $\lambda S(H_u H_d)$, where S is a SM singlet superfield that carries $U(1)_N$ charge. When S develops VEV breaking $U(1)_N$ gauge symmetry, it also gives rise to an effective μ term. Thus, within the E_6 SSM, the μ problem can be solved without the accompanying problems of singlet tadpoles or domain walls which appear in the NMSSM.

In this review article, we discuss the particle content, the global symmetries which allow suppressing FCNCs and rapid proton decay and the RG flow of gauge couplings in the E_6 SSM. The low energy matter content of this SUSY model includes three copies of 27_i representations of E_6 so that anomalies get canceled generation by generation. In addition, an extra pair of $SU(2)_W$ doublets L_4 and \bar{L}_4 should survive to low energies to ensure high energy gauge coupling unification. As a consequence, the E_6 SSM involves extra matter beyond the MSSM contained in three supermultiplets of exotic charge 1/3 quarks (D_i and \bar{D}_i), two pairs of $SU(2)_W$ doublets of Inert Higgs states, three SM singlet superfields which carry $U(1)_N$ charges, L_4 , \bar{L}_4 and Z' vector superfield. As in the simplest SUSY extensions of the SM the gauge symmetry of the E_6 SSM does not permit to suppress baryon and lepton number violating interactions which can lead to rapid proton decay. Moreover, in general relatively, light exotic states induce unacceptably large flavor changing processes. To suppress the corresponding baryon and lepton number violating operators, one can impose either Z_2^L or Z_2^B discrete symmetry, which implies that the exotic D -fermions are either diquarks (Model I) or leptoquarks (Model II). To avoid the appearance of the FCNCs at the tree level, one can postulate an approximate Z_2^H symmetry, under which all supermultiplets of matter except a pair of Higgs doublets (H_d and H_u) and one SM singlet superfield S are odd. Instead of Z_2^H , Z_2^L and Z_2^B , one can use a single discrete \tilde{Z}_2^H symmetry, which forbids operators giving rise to rapid proton decay and tree-level flavor-changing transitions. The Higgs supermultiplets H_u , H_d and S as well as L_4 and \bar{L}_4 are even under the \tilde{Z}_2^H symmetry, whereas all other matter fields are odd. In this case, the exotic D -fermions are leptoquarks.

The results of the analysis of the two-loop RG flow within the E_6 SSM are presented taking into account kinetic term mixing between $U(1)_Y$ and $U(1)_N$ factors. If there is no mixing between $U(1)_Y$ and $U(1)_N$ near the GUT scale M_X , then the off-diagonal gauge coupling, which describes such mixing, remains negligibly small at any intermediate scale between M_X and TeV scale. In this limit, the gauge coupling of the extra $U(1)_N$ is always close to the $U(1)_Y$ gauge coupling. On the other hand, at high energies, the values of the gauge couplings are considerably larger in the E_6 SSM than in the MSSM due to the presence of the supermultiplets of exotic matter in the $U(1)_N$ extensions of the MSSM. Our analysis reveals that the unification of gauge couplings in the E_6 SSM can be achieved for phenomenologically acceptable values of $\alpha_3(M_Z)$, consistent with the central measured value of this coupling.

Because of the larger gauge couplings, the theoretical restrictions on the low energy values of the Yukawa couplings coming from the requirement of the validity of perturbation theory up to the scale M_X get relaxed in the E_6 SSM as compared with the MSSM and NMSSM. As a consequence, for moderate $\tan \beta$ the tree-level upper bound on the SM-like Higgs boson mass can be considerably bigger in the E_6 SSM than in the MSSM and NMSSM. In this SUSY model, it can be even larger than 115–125 GeV so that the contribution of loop corrections to the mass of the lightest Higgs scalar is not needed to be as large as in the MSSM and NMSSM in order to obtain 125 GeV Higgs boson.

In this article, the gauge symmetry breaking and the spectrum of the Higgs bosons within the E_6 SSM are reviewed as well. In the $U(1)_N$ extensions of the MSSM the SM singlet Higgs field, S is required to acquire a very large VEV $\langle S \rangle = s/\sqrt{2}$, where $s > 10$ TeV, to ensure that the Z' boson and exotic fermions gain sufficiently large masses. In particular, the results of the analysis of the LHC data imply that the $U(1)_N$ gauge boson must be heavier than 3.8–3.9 TeV. When CP-invariance is preserved, the E_6 SSM spectrum of the Higgs states involves two charged, three CP-even and one CP-odd bosons. The SM singlet dominated CP-even state and the Z' gauge boson are almost degenerate. The masses of the charged and another CP-even Higgs bosons are set by the mass of Higgs pseudoscalar m_A . All these states tend to be substantially heavier than the lightest Higgs scalar that manifests itself in the interactions with other SM particles as a SM-like Higgs boson. In the part of the E_6 SSM parameter space, where the mass of the lightest Higgs state can be larger than 100–110 GeV at the tree-level, all other Higgs bosons lie beyond the multi-TeV range and therefore cannot be discovered at the LHC.

We also consider possible manifestations of the E_6 SSM that may be observed at the LHC. The simplest phenomenologically viable scenarios imply that LSP and NLSP are the lightest exotic states (\tilde{H}_1^0 and \tilde{H}_2^0), which are formed by the fermion components of the SM singlet superfields S_α . One of these fermions \tilde{H}_1^0 should be much lighter than 1 eV composing hot dark matter in the Universe. Such states give only a very minor contribution to the density of the dark matter. The NLSP \tilde{H}_2^0 can have mass of the order of 1 GeV giving rise to exotic decays of the 125 GeV Higgs boson. Since \tilde{H}_2^0 tends to live longer than 10^{-8} s it decays outside the detectors. Therefore, the decay channel $h_1 \rightarrow \tilde{H}_2^0 \tilde{H}_2^0$ results in an invisible branching ratio of the SM-like Higgs state. The corresponding branching ratio can be as large as 20%.

Other possible manifestations of the E_6 SSM, which can permit distinguishing this model from the MSSM or NMSSM, are associated with the presence of the Z' gauge boson and exotic supermultiplets of matter that compose three $5 + 5^*$ representations of $SU(5)$. The most spectacular LHC signals can come from the exotic color states and Z' . The Z' boson production at the LHC should lead to unmistakable signal, i.e., $pp \rightarrow Z' \rightarrow l^+l^-$. Assuming that the Z_2^H symmetry is mainly broken by the operators which involve third-generation fermions, the pair production of the lightest exotic quarks with masses in a few TeV range can give rise to some enhancement of the cross section of either $pp \rightarrow t\bar{t}\tau^+\tau^- + E_T^{\text{miss}} + X$ and $pp \rightarrow b\bar{b} + E_T^{\text{miss}} + X$ if exotic quarks are leptoquarks or $pp \rightarrow t\bar{t}b\bar{b} + E_T^{\text{miss}} + X$ if exotic quarks are diquarks. Because of the large mass splitting in the exotic squark sector, which can be caused by the heavy D -fermion, one of the exotic squarks can be relatively light. If this is the case, then the pair production of the superpartners of D -fermions may result in some enlargement of the cross sections of either $pp \rightarrow t\bar{t}b\bar{b} + X$ when exotic squarks are diquarks or $pp \rightarrow t\bar{t}\tau^+\tau^- + X$ and $pp \rightarrow b\bar{b} + E_T^{\text{miss}} + X$ if these squarks are leptoquarks. As compared with the exotic quarks and squarks, the production of Inert Higgs bosons and Inert Higgsinos is rather suppressed at the LHC. The discovery of Z' and new exotic states predicted by the E_6 SSM would point towards an underlying E_6 gauge structure at high energies and open a new era in elementary particle physics.

Author Contributions: Conceptualization, S.F.K., S.M. and R.N.; methodology, S.F.K. and S.M.; validation, S.F.K., S.M. and R.N.; formal analysis, S.F.K., S.M. and R.N.; investigation, S.F.K., S.M. and R.N.; resources, S.M.; writing—original draft preparation, R.N.; and writing—review and editing, S.F.K. and S.M. All authors have read and agreed to the published version of the manuscript.

Funding: The work of S.F.K. and S.M. is supported by the Science and Technology Facilities Council, Consolidated Grant number ST/L000296/1. S.F.K. also acknowledges the European Union’s Horizon 2020 Research and Innovation programme under Marie Skłodowska-Curie grant agreements Elusives ITN No. 674896 and InvisiblesPlus RISE No. 690575. S.M. further acknowledges partial financial support through the NExT Institute.

Acknowledgments: R.N. would like to thank P. Athron, E. Boos, M. Binjonaid, S. Demidov, M. Dubinin, D. Gorbunov, D. Harries, M. Libanov, D. Kazakov, M. Mühlleitner, V. Rubakov, M. Sher, A. W. Thomas, S. Troitsky, X. Tata and A.G. Williams for fruitful discussions.

Conflicts of Interest: The authors declare no conflict of interest.

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