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An Infinitely Old Universe with Planck Fields Before and After the Big Bang

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Abstract: The Robertson–Walker minimum length (RWML) theory considers stochastically perturbed spacetime to describe an expanding universe governed by geometry and diffusion. We explore the possibility of static, torsionless universe eras with conserved energy density. We find that the RWML theory provides asymptotically static equations of state under positive curvature both far in the past and far into the future, with a Big Bang singularity in between.

Keywords: asymptotically static universe; big bang; curvature; planck era; hierarchy problem

1. Introduction

'Does time begin with the Big Bang?' remains one of several large cosmological questions. In search of the answer, some research considers bouncing cosmological models to show that a growing scale factor allowing entropy to dissipate cannot be cyclic in time [1,2]. Another line of research considers torsion which induces gravitational repulsion thus potentially preventing the cosmological singularity [3]. In the quantum realm, various theories have been considered in attempting to define the scalar field action for gravitational fields. The quintessence theory considers a canonical kinetic term in the Lagrangian with Λ as the scalar field potential along with additional dark energy terms [4]. Similarly, other theories have been proposed which directly insert a kinetic term into the energy–momentum tensor [5]. Here we consider the addition of stochastic perturbations to spacetime under a Robertson–Walker (RW) metric.

In general relativity, spacetime is a dynamic medium. The Robertson–Walker minimum length (RWML) theory extends this notion to include diffusion within a geometric framework quantified by the RW metric. The introduction of stochastic perturbations in the very fabric of spacetime under a conserved proper time functional results in extended evolution equations and Friedmann equations where diffusion and geometry both play a role [6–8]. The action defined entirely by the resulting RWML Ricci scalar is emergent rather than formulated by choice, as is the ϕ^4 theory of the Planck fields.

Most early universe theories begin with an assumed action. A theory with phantom energy originating from the action of a general scalar tensor theory in the presence of non-minimal coupling [9] results in an asymptotically static universe where phantom energy, stiff matter, and dust matter all play a role. In comparison, the RWML theory is defined by the standard species with the stochastic spacetime fields introducing the diffusive species and the resulting action emerging entirely from the Ricci scalar.

Unlike the theoretical framework where the metric is subjected to weak gravitational perturbations, the RWML subjects spacetime to perturbations while the metric remains unperturbed in Cartesian coordinates. Spontaneous symmetry breaking resulting from a transition to spherical coordinates suggests emergent gravity but retains an otherwise unperturbed metric.

Theories based on Moffat stochastic gravity arguments [10] where the gravitational constant has a stochastic element corresponding to metric fluctuations differ from the RWML theory where uncertainty is fundamental to spacetime itself and independent of the

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geometry. The resulting RWML universe is thus subjected to the individual effects of geometry and diffusion but also to their coupling. It is this coupling of diffusion and geometry which gives rise to curvature, traditionally something associated only with geometry.

We subject the RWML theory to the assumption of a torsionless universe with a conserved energy density and we consider possible outcomes. We find that under a positive curvature, the RWML provides an asymptotically static universe equation of state infinitely far in the past and infinitely far in the future, with a singularity in between. Diffusion dominates before the Big Bang, while vacuum dominates after the Big Bang. Figure 1 shows the universe equation of state parameter and acceleration across X, the ratio of relative magnitudes of diffusion and geometry parametrized by the diffusion parameter D and Hubble parameter H, respectively.

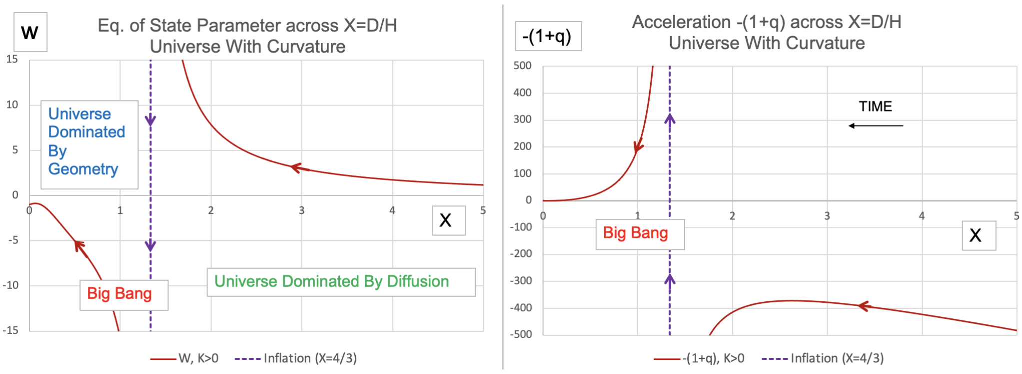

Figure 1. Universe with curvature: equation of state (**left**) and acceleration (**right**). In both plots, the x-axis is the ratio of diffusion parameter D to the Hubble parameter H: $\mathrm{X} \equiv \frac{D}{H}$. The equation of state plot shows the ratio of energy density to pressure, both provided by the extended RWML Friedmann equations. For the acceleration plot, y-axis is given by $-(1+\mathrm{q}) \equiv \frac{\dot{H}}{H^2}$. The arrows on the plots correspond to the arrow of time.

Uncertainty or unpredictability is not by default contrary to a deterministic universe [11]. In the case of stochastic spacetime, order is maintained via smooth two-point functions. Indeed, the stochastic fields giving spacetime uncertainty at a very small scale cannot only be quantized but have been shown to retain Lorentz invariance [6]. We consider the RWML-extended Ricci scalar to find it rich in information about the dynamics of these Planck scale fields and to directly provide us with the corresponding Lagrangian.

The notion of evolution without evolution has been considered based on system energy conservation with subsystems which are allowed to evolve [5]. Loosely, we discuss two different sets of subsystems. First, we have the subsystem of geometry and its contribution to the dynamics of the universe. Opposite geometry, we have diffusion. The two play a balancing role providing us overall with a constant energy density through time. Energy density conserved across time gives the Lorentzian universe a classical feature.

Similarly, we have various species, each evolving according to its species-specific equation of evolution. While further research is required to ascertain the path of evolution the new diffusive species takes as the universe experiences a violent implosion followed by the violent explosion corresponding to the Big Bang, the asymptotically static states of the universe infinitely far in the past and infinitely far in the future provide a clear understanding of the dominant species corresponding to each static era and are in agreement with the overall universe equation of state.

The paper follows the development of the RWML theory application to universe evolution under the assumption of a torsionless universe with a conserved energy density. We begin with a brief review of the RWML theory, as introduced in [6–8]. First, we consider universe evolution under the assumption of a flat universe to discover a tension between species-derived equation of state vs. equation of state deriving from the extended Friedmann equations. The introduction of nonzero curvature resolves the tension. We discover that curvature is both the interaction term between geometry and diffusion and a deciding factor in determining the equation of state for the new diffusive species. Ricci scalar alone defines the Planck fields' action, and we find Planck fields in a ϕ^4 potential. Spontaneous symmetry breaking both introduces a possible relationship to the Schwarzschild metric and gives one of the fields a nonzero vacuum value. We finish with a consideration of the coupling constants for Planck fields by briefly considering the U(1)×SU(2) gauge invariance of the GWS theory and sketching out the scales of interaction.

Unless stated otherwise, we work in natural units, in spatially local coordinates ($r \approx 0$), with a mostly positive RW metric, and under Itô calculus.

2. Review of RWML Theory

The RWML theory begins with the notion that each point in spacetime, x, also carries some very small uncertainty generated by a stochastic field unique to x, $\delta\xi_x$. Such small random spacetime perturbations give rise to the concept of minimum length at Planck scale and should not be confused for the lack of precision in the meter stick but rather should be understood as fundamental to the spacetime and thus to our universe. Ultimately, we would like to understand how stochastic fields $\delta\xi_x$ affect universe dynamics.

Despite the introduction of stochastic perturbations in spacetime, the RW metric in Cartesian coordinates remains unperturbed (further discussed in Section 9). While we work mostly in the RW geometry, we first review several concepts under the simpler Minkowski metric to then promote the discussion to the RW metric.

In General Relativity, a proper time functional must remain invariant under a stochastic translation in addition to the regular translation. The addition of such a translation provides for new terms in the Christoffel connection and thus can be readily applied to many problems in cosmology (Appendix A summarizes the derivation of the extended Christoffel connection).

While [6] explains many of the details about uncertainty in spacetime, here, we introduce the concepts relevant to their application in an evolving universe. We allow spacetime to take on an element of uncertainty $\delta\xi_x$ over some segment δx, which we assume to be at Planck scale (we begin by working with a Minkowski metric):

$$\delta\tilde{x} \equiv \delta x + \delta\xi_x \tag{1}$$

$$\delta\xi^\mu_x \sim \mathcal{N}(0, \sigma_\xi\sqrt{\delta t}), \quad \langle\delta\xi^\mu_x\rangle = 0 \tag{2}$$

We quantize the uncertainty in spacetime via two-point functions, which we separate out across space and time:

$$\langle\delta\xi^0\delta\xi^0\rangle = -\sum_i \sigma^2_\epsilon \delta x^i, \quad \langle\delta\xi^i\delta\xi^j\rangle = g^{ij}\sigma^2_\xi\delta t \tag{3}$$

Here, the parameters σ_ϵ and σ_ξ (both of dimension $L^{\frac{1}{2}}$) are measures of the magnitude of uncertainty in time and space, respectively. These can be simplified further under the assumption of a torsionless universe [7], something we can expect in the case of a static universe. With stress energy tensor T^{0i} elements proportional to σ^2_ϵ, we thus approximate σ_ϵ to zero corresponding to a torsionless universe, giving us

$$\langle\delta\xi^0\delta\xi^0\rangle \approx 0 \tag{4}$$

We can also greatly simplify the two-point functions for the kinetic stochastic terms:

$$\langle\partial_\alpha\delta\xi^0\partial_\beta\delta\xi^0\rangle \approx 0, \quad \langle\partial_\alpha\delta\xi^i\partial_\beta\delta\xi^j\rangle = 2\delta_{\alpha\beta}g^{ij}c_\xi\sigma^2_\xi\delta t \tag{5}$$

where now, we also have a diffusive parameter corresponding to the kinetic stochastic fields, c_ξ, of dimension L^{-2}.

As explained in [6] (and shown in Figure 3 of [6]) , the resulting spacetime follows a random walk. However, the kinetic spacetime fields have a measure of uncertainty which is bounded across time. In other words, the uncertainty in spacetime grows in time, whereas the uncertainty in the kinetic spacetime fields remains proportional to the segment of time δt. Consider a particle on a world line parametrized by λ, with an uncertainty in space $\xi^\lambda = \int \delta\xi^\lambda$ accumulated over some time t. It has the following two-point functions:

$$\langle(\xi^\lambda)^2\rangle = \bar{\sigma}_\xi^2 \delta t, \quad \bar{\sigma}_\xi^2 \approx \sigma_\xi^2 \frac{t}{\delta t}, \quad \langle\partial_\alpha \xi^\lambda \partial_\beta \xi^\lambda\rangle = \delta_{\alpha\beta} c_\xi \sigma_\xi^2 \delta t \tag{6}$$

where we have approximated the overall measure of uncertainty in space, $\bar{\sigma}_\xi^2$ as some multiple of the instantaneous uncertainty in space over each segment σ_ξ^2. We can think of space as a lattice where each point on the lattice is allowed to 'randomly walk around', albeit taking extremely small steps. As the number of steps grows, the uncertainty of each point on the lattice grows, and with it the lattice itself expands. In a sense, the 'room' where the random walk takes place for each point on the lattice expands with time. But the same is not true for the kinetic space fields. Regardless of the passage of time, t, kinetic fields always exhibit the same amount of uncertainty, provided that $\sigma_\xi^2 \delta t$ remains the same.

(You might also notice the factor of two in the two-point function for the kinetic fields, Equation (5), has 'disappeared' in Equation (6). This is purely cosmetic, since $\epsilon^\alpha \partial_\alpha \delta\xi_x^\mu \equiv \delta\xi_{x+\epsilon}^\mu - \delta\xi_x^\mu$, whereas $\epsilon^\alpha \partial_\alpha \xi_x^\mu \equiv \xi_{x+\epsilon}^\mu - \xi_x^\mu = \delta\xi_x^\mu$. Hence, it is a matter of choice, do we work with ξ or with $\delta\xi$? Our choice is $\delta\xi$.)

The resulting simulated phase space diagram in Figure 3a of [6] shows the time evolution of uncertainty across space vs. momentum clearly: the uncertainty in the momentum remains bounded. Finally, the corresponding energy density function for such a space has the following functional form,

$$\rho \sim \int \mathcal{D}\xi_x^\alpha \int \mathcal{D}\partial_\mu \xi_x^\beta \prod_x \frac{1}{\sqrt{\bar{\sigma}_\xi^2 t}} \frac{1}{\sqrt{2c_\xi \sigma_\xi^2 \delta t}} e^{-\frac{(\xi_x^\alpha)^2}{4\bar{\sigma}_\xi^2 t}} e^{-\frac{(\partial_\mu \delta\xi_x^\beta)^2}{8c_\xi \sigma_\xi{}^2 \delta t}} \tag{7}$$

Appendix B explains the relationship between the diffusion equation and the functional form of Equation (7).

As long as our analysis depends only on the kinetic stochastic fields and as long as we are not considering the evolution of a system over time, the relationship between the uncertainty over t vs. over the segment δt shown in Equation (6) becomes irrelevant. This was mostly the case in [6,7]. However, here, we are looking at the evolution of the universe, with the distinct possibility of a universe with a nonzero curvature. The uncertainty in space over t does play a role in this case, and it is important that we understand how this uncertainty relates to that over a single time segment δt.

The elements of the Christoffel connection for the RWML have already been derived for us in [7,8] by requiring the conservation of the proper time functional under a stochastic translation. After performing a non-trivial metric transformation to the local ($r \approx 0$) RW geometry (see Appendix A) , we find that the resulting elements of the extended Christoffel connection include new terms in addition to the standard terms. We simply quote these here [7,8]:

$$\tilde{\Gamma}_{tt}^t = 3D \tag{8}$$

$$\tilde{\Gamma}_{xx}^t = \tilde{\Gamma}_{yy}^t = \tilde{\Gamma}_{zz}^t = a^2(H + 3D + \frac{3}{2}\frac{k}{a^2}\frac{\bar{\sigma}_\xi^2}{a^2}) \tag{9}$$

$$\tilde{\Gamma}_{tt}^x = \tilde{\Gamma}_{tt}^y = \tilde{\Gamma}_{tt}^z = -D_\epsilon \tag{10}$$

$$\tilde{\Gamma}^{x}_{tx} = \tilde{\Gamma}^{y}_{ty} = \tilde{\Gamma}^{z}_{tz} = H \tag{11}$$

$$\tilde{\Gamma}^{x}_{xx} = \tilde{\Gamma}^{x}_{yy} = \tilde{\Gamma}^{x}_{zz} = a^2(-D_\epsilon + \frac{\bar{\sigma}^2_\epsilon}{2}(1-q)H^2) \tag{12}$$

where a is the scale factor, and where σ_ξ has a scale factor dependence $\sigma_\xi \sim \frac{1}{a^2}$. Above, we also have the diffusion parameters $D \equiv \frac{c_\xi \sigma^2_\xi}{a^2}, D_\epsilon \equiv c_\epsilon \sigma^2_\epsilon$, and the Hubble parameter $H \equiv \frac{\dot{a}}{a}$. Note that the terms above include the curvature parameter, k, and the standard deceleration parameter defined as $-q \equiv 1 + \frac{\dot{H}}{H^2}$.

Also note that D has the same power dependence on the scale factor as stiff matter [12], $D \sim a^{-6}$. The scale factor dependence for D was derived in [7,8] by considering how a stochastic field ought to transform between the Minkowski and a local ($r \approx 0$) RW metric (see Appendix A). While diffusion has a different equation of state from stiff matter, the similarity of the power dependence on the scale factor warrants further research and understanding of the relationship between stiff matter and D .

From [7,8], we can also quote the Ricci scalar:

$$R = g^{\mu\nu} R_{\mu\nu} = -R_{tt} + \frac{3}{a^2} R_{xx} =$$
$$6(1-q)H^2 + 6\frac{k}{a^2} - 27HD + 27D^2 \tag{13}$$

The expanded RWML connection allows us to formulate the equation of energy–momentum conservation for a cosmic medium described by the energy momentum tensor $T_{\mu\nu}$, assuming an isotropic universe:

$$\nabla_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu} + \tilde{\Gamma}^{\alpha}_{\alpha\beta} T^{\beta\nu} + \tilde{\Gamma}^{\nu}_{\alpha\beta} T^{\alpha\beta} = 0,$$
$$T_{\mu\nu} = \rho(1, wa^2, wa^2, wa^2) \tag{14}$$

The standard $T_{\mu\nu}$ describes any one of the several cosmic species with the corresponding equation of state parameter w_i for vacuum, matter, and radiation: $w_V = -1, w_M = 0, w_R = \frac{1}{3}$.

The pressure-specific conservation equation gives us:

$$D_\epsilon = \frac{2w}{1+4w}(1-q)H^2\sigma^2_\epsilon \tag{15}$$

while the species-specific energy density evolution, with i referencing the particular species ($i = R/M/D/V$ for radiation, matter, diffusion due to Planck fields, and vacuum, respectively) is given by,

$$\frac{\dot{\rho}_i}{\rho_i} = -3\{(1+w_i)H + (2+3w_i)D + \frac{3}{2}w_i \frac{k}{a^2}\frac{\bar{\sigma}^2_\xi}{a^2}\} \tag{16}$$

The species-specific energy density evolution contains the standard model term corresponding to H as well as a new diffusion term, D, along with a new curvature term, k.

In a flat, non-stochastic, but expanding universe, with $k = 0, D = 0, H > 0$, only a universe dominated by Lorentz Invariant Vacuum Energy (LIVE) [13] can be static, allowing for a constant energy density during the expansion, $\dot{\rho} = 0$. Alternatively, in a flat stochastic universe without expansion, with $k = 0, H = 0, D > 0$ (corresponding to a Minkowski space), Equation (16) suggests that a new diffusive species with $w_D \equiv -\frac{2}{3}$ allows for a static universe. As we will see, a nonzero k becomes quite important in defining the equation of state for diffusion and in enabling a possible pre-Big-Bang static universe.

For an expanding, diffusive universe, with $D, H > 0$, regardless of curvature, none of the species (vacuum, diffusion, matter, or radiation) alone can provide for a static universe during expansion. However, as shown in the following, a LIVE-dominated asymptotically static universe is possible [8] going far into the future, and a diffusion-dominated static universe is possible going far back in time .

Requiring the Lorentzian universe to also exhibit a classical feature by setting $\dot{\rho} = 0$, the universe equation of state derived from the species' evolution equation, w_S, becomes,

$$\Omega_i \equiv \frac{\rho_i}{\rho}, \quad \dot{\rho} = 0 \quad \Rightarrow \quad \sum_i \dot{\Omega}_i = 0$$
$$\Rightarrow \quad w_S \equiv -\frac{H + 2D}{H + 3D + \frac{3}{2}\frac{k}{a^2}\frac{\bar{\sigma}^2}{a^2}} \tag{17}$$

where we have defined Ω_i as the fraction of the total energy density for species i. The first and second Friedmann equations in the RWML theory are also given by:

$$\frac{8\pi G}{3}\rho = H^2 - \frac{3}{2}HD + \frac{9}{2}D^2 + \frac{k}{a^2} + \frac{3}{4}\frac{k}{a^2}\frac{\bar{\sigma}^2}{a^2}(3D - 5H) \tag{18}$$

$$-8\pi G p = (1 - 2q)H^2 - \frac{15}{2}HD + \frac{9}{2}D^2 + \frac{k}{a^2} + \frac{3}{4}\frac{k}{a^2}\frac{\bar{\sigma}^2}{a^2}(3D - 5H) \tag{19}$$

These are the extended Friedmann equations.

Choosing to work with universe energy density and pressure in relation to H^2, i.e., $\frac{\rho}{H^2}$ and $\frac{p}{H^2}$, allows us to express the extended Friedmann equations as a function of X, the ratio between the diffusion parameter and the Hubble parameter:

$$X \equiv \frac{D}{H} \tag{20}$$

We can think of X as a relative measure of the effect spacetime uncertainty and geometry have on the universe. As X represents the relative magnitude of universe diffusion to its expansion at some point in time t, it is natural to consider the universe behavior as we allow X to go to infinity and zero. For $X \to 0$, the assumption of a conserved energy density together with a scale factor which grows with time constrains the possible scenarios for values of H and D and defines the arrow of time relative to X.

3. The Arrow of Time

The scale factor $a(t)$ grows with time in the expanding post-Big-Bang era. It is common practice to normalize the scale factor to 1 for present-day value, and to take $a(0) = 0$ at the Big Bang, also corresponding to the beginning of time, $t = 0$. We instead must take a more general approach as the RWML suggests the existence of time prior to the Big Bang.

With the assumption of a conserved energy density, ρ must remain finite regardless of X. This means that during the static eras, neither D nor H can go to infinity (unless we want to introduce some form of a fine-tuning problem between D and H!). Let us first consider $X \to 0$: if $H \to \infty$ is not allowed, then it must be that $D \to 0$. Given that $D \sim a^{-6}$, this is only possible if $a \to \infty$. As the scale factor is a growing function of time in the post-Big-Bang era, we can direct the arrow of time in the $X \to 0$ direction.

Now let us see what happens as $X \to \infty$. We must consider the possibility that a changes its functional form at the Big Bang. $D \to \infty$ as $X \to \infty$ is contrary to a finite energy density. Instead, D must approach some constant value, while $H \to 0$. One possibility consistent with a number of past theories employing or deriving non-traditional scale factors [9,14] is to allow the scale factor to approach some constant positive value infinitely far ago. Such a positive asymptotic value for the scale factor has been considered by past asymptotic universe theories, including de Sitter solutions with a negative gravitational constant [15] and fluctuating spacetime geometry under Moffat stochastic gravity [10], to mention a few.

While the functional form for a in the pre-Big-Bang universe remains somewhat elusive, we leave these considerations for future research, particularly as a proper treatment of the scale factor just before the Big Bang might very well require a better understanding of how torsion plays a role as the universe approaches the Big Bang era forwards in time. For now, we simply extend the same relationship between the arrow of time and X found in the post-Big-Bang era to the pre-Big-Bang era: we relate the forward arrow of time to a decreasing X.

4. Equation of State Tension: Extended Friedmann Equations vs. Species' Evolution

Given the equations of evolution of each species, we derived the equation of state parameter under a conserved energy density, w_S (Equation (17)). Alternatively, the extended Friedmann equations (Equations (18) and (19)) provide us with the universe equation of state parameter $w \equiv \frac{p}{\rho}$, devoid of the condition of a constant universe energy density, $\dot{\rho} = 0$.

We can use the extended Friedmann equation for energy density, Equation (18), to readily obtain $\dot{\rho}$ and thus solve for the deceleration parameter q under the assumption of a conserved universe energy density. Together with Equations (18) and (19), we can map out the universe evolution as a function of X, and consider its asymptotic behavior as X goes to zero and infinity. However, this evolution must be asymptotically consistent with the evolution of individual species. We thus consider under which circumstances an asymptotically static universe is possible by requiring $w = w_S$.

We find that for $k = 0, D = 0$, both $w_S = -1$ and $w = -1$ are in agreement with the vacuum (or more appropriately, LIVE [8]) as a dominant species. As the vacuum evolution is independent of geometry, i.e., $\dot{\rho}_V$ does not depend on H, we see that $X \to 0$ allows for a possible static universe as long as vacuum is the dominant species.

However, for $k = 0, H = 0$, we find that in a flat universe, $w = -\frac{1}{3}$ and $w_S = -\frac{2}{3}$ are clearly in tension with each other. The diffusion species' equation of state parameter in a flat universe, $w_D = -\frac{2}{3}$, equals w_S but not w. This tension motivates our considerations of a nonzero curvature parameter.

A positive curvature universe resolves this tension and provides us with an asymptotically static vacuum-dominated universe forwards in time and a static diffusion-dominated universe far back in time. Curvature is thus a key element providing for an asymptotically static universe. Additionally, as we shall also see, it is key in establishing the equation of state for the new species corresponding to the Lorentz invariant Planck fields [6].

Before we delve into the derivations of a flat and curved universe dynamics, we first summarize the requirements and assumptions of an asymptotically static universe. We then consider a universe with flat geometry in more detail to formally discover the tension between w and w_S followed by the introduction of a positive curvature to resolve the w_S vs. w tension and provide a new value for the diffusion equation of state, $w_D = -\frac{1}{3}$.

5. The Requirements for Asymptotic Eras Corresponding to $X = 0, \infty$

We enforce two assumptions consistent with a static universe at all times:

1. We require the energy density to be conserved at all times. This assumption allows us to obtain an estimate for q by setting the derivative of the first extended Friedmann equation, Equation (18), to zero, $\dot{\rho} = 0$.

2. We assume the universe is torsionless. As $T^{0i} \sim \sigma_\epsilon^2$ [7,8], we can set $\sigma_\epsilon \approx 0$, making $\nabla_\mu T^{\mu i} = 0$ automatically satisfied. An interesting consequence is that $\sigma_\epsilon \approx 0$ also makes Equation (15) for c_ϵ irrelevant. It is important to understand the significance of a torsionless universe, in addition to what it means when it comes to its evolution. $\delta\xi^t$ and kinetic stochastic fields in time, $\partial\delta\xi^t$, have two-point functions proportional to σ_ϵ^2. Hence, a torsionless universe effectively removes the effect of uncertainty in time, and we are left with Planck fields and their two-point functions only in space.

In addition to the above requirements, which we apply to the universe at all times, as we consider the limits $X \to 0, \infty$, we also require the following for an asymptotically static universe:

3. In the asymptotic limit, pressure must also be constant, and the universe can be described by the equation of state $p = w\rho$. We can thus use the extended Friedmann Equations (18) and (19) to obtain $w = \frac{p}{\rho}$.

4. The relationship $w_S = \sum_i w_i \Omega_i$, where $\Omega_i \equiv \frac{\rho_i}{\rho}$ and where the sum is performed over species, must hold. Note that we are thus using a definition of $\Omega = \sum_i \Omega_i = 1$ which is different from the traditional definition of $\Omega_c \equiv \frac{\rho}{\rho_c}$. Also note that under this definition, we must also have $\sum_i \dot{\Omega}_i = 0$, thus providing us with a species' specific equation of state parameter which we refer to as w_S. We thus have a test of the theory: w_S must asymptotically match the value for w, i.e., $w_S = w$.

Finally, it is important to realize that the last condition diverges significantly from the traditional approach which provides the traditional acceleration equation. In the traditional approach, we obtain $\dot{\rho}$ from the first Friedmann equation and we set it equal to the value we obtain for the species energy density evolution by requiring $\nabla_\mu T^{\mu 0} = 0$. The traditional approach generally postulates the dominant species, and the appropriate equation of state follows. Instead, we keep things very simple. In a stable universe, the above conditions must hold, and we require that an appropriate theory provides a match between the asymptotic value for the equation of state resulting from the extended Friedmann equations, w, and the extended species equation for w_S.

6. Universe Evolution for $k = 0$ and w vs. w_S Tension

We now consider a flat universe where $k = 0$, and w_S is given by:

$$\frac{\dot{\rho}_i}{\rho_i} = -3H\{(1 + w_i) + (2 + 3w_i)X)\} \Rightarrow w_S \equiv -\frac{1 + 2X}{1 + 3X} \tag{21}$$

$$X \to 0 \quad \Rightarrow \quad w_S \to -1 + X \to -1 \tag{22}$$

$$X \to \infty \quad \Rightarrow \quad w_S \to -\frac{2}{3} \tag{23}$$

Similarly, the first and second Friedmann equations in the RWML theory are now given by:

$$\frac{8\pi G\rho}{3H^2} = 1 - \frac{3}{2}X + \frac{9}{2}X^2 \tag{24}$$

$$-\frac{8\pi G p}{H^2} = 1 - 2q - \frac{15}{2}X + \frac{9}{2}X^2 \tag{25}$$

$$\Rightarrow \quad w = -\frac{1 - 2q - \frac{15}{2}X + \frac{9}{2}X^2}{3(1 - \frac{3}{2}X + \frac{9}{2}X^2)} \tag{26}$$

We use Equation (24) to set $\dot{\rho} = 0$ to obtain $\dot{H}(D - \frac{4}{3}H) = 6\dot{D}(D - \frac{1}{6}H)$. As $D \sim \frac{1}{a^6}$, $\dot{D}$ is readily obtained, $\dot{D} = -6HD$. Together with $\dot{H} = -(1 + q)H^2$ and $X \equiv \frac{D}{H}$, we find that $-(1 + q)$ has a singularity at $X = \frac{4}{3}$:

$$-(1 + q) = (-36X)\frac{(X - \frac{1}{6})}{(X - \frac{4}{3})} \tag{27}$$

$$X \to 0 \quad \Rightarrow \quad -(1 + q) \to -\frac{9}{2}X \tag{28}$$

$$X \to \infty \quad \Rightarrow \quad -(1 + q) \to -36X \tag{29}$$

By substituting the value for acceleration from Equation (27) into Equation (26) for w, after a bit of algebra, we obtain the equation of state parameter and its limits:

$$w = -\frac{-2(-36X)(X-\frac{1}{6}) + (3 - \frac{15}{2}X + \frac{9}{2}X^2)(X-\frac{4}{3})}{3(1-\frac{3}{2}X+\frac{9}{2}X^2)(X-\frac{4}{3})} \tag{30}$$

$$X \to 0 \quad \Rightarrow \quad w \to -1 + 4X \to -1 \tag{31}$$

$$X \to \infty \quad \Rightarrow \quad w \to -\frac{1}{3} \tag{32}$$

A comparison of Equations (22) and (31) in the case of $X \to 0$ and Equations (23) and (32) in the case of $X \to \infty$ shows that in both cases, the asymptotic values for equation of state parameter w differ from what we have derived using evolution Equation (21) for individual species. We thus have an equation of state tension in a flat universe. Furthermore, Equations (27) and (30) suggest that the universe experiences a violent implosion at $X \to \frac{4}{3}|_+$ as $-(1+q)$ becomes infinitely large in magnitude and negative, followed by a violent explosion as $-(1+q)$ flips to being infinitely large in magnitude and positive for $X \to \frac{4}{3}|_-$. Figures 2 and 3 provide a zoomed-in look at the equation of state for $X \to 0, \infty$ to clearly show the tension.

Requiring invariance of Planck fields to D gives us $w_D = -\frac{2}{3}$. Using Equation (21), we can summarize the evolution of each species in Table 1.

Table 1. Evolution of species energy density with $k = 0$.

Species	w_i	$\dot{\Omega}_i$	$X \to 0$	$X \to \infty$
Radiation	$\frac{1}{3}$	$(-4H - 9D)\Omega_R$	$-4H\Omega_R$	$-9D\Omega_R$
Matter	0	$(-3H - 6D)\Omega_M$	$-3H\Omega_M$	$-6D\Omega_M$
Diffusion	$-\frac{2}{3}$	$-H\Omega_D$	$-H\Omega_D$	0
Vacuum	-1	$3D\Omega_V$	0	$3D\Omega_V$

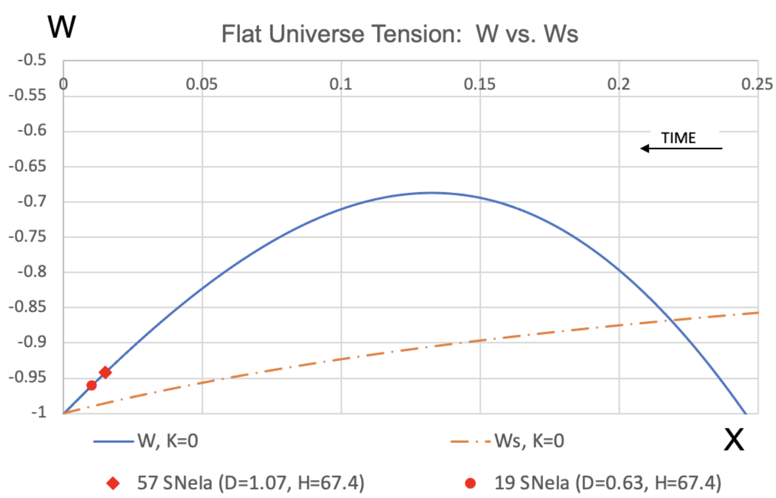

Figure 2. Flat universe equation of state–zoom in view after the Big Bang.

Thus, a flat universe does not provide a satisfactory static universe under $X \to 0, \infty$. Instead, we must allow for a universe with nonzero curvature to obtain an asymptotically static universe infinitely long ago and far into the future.

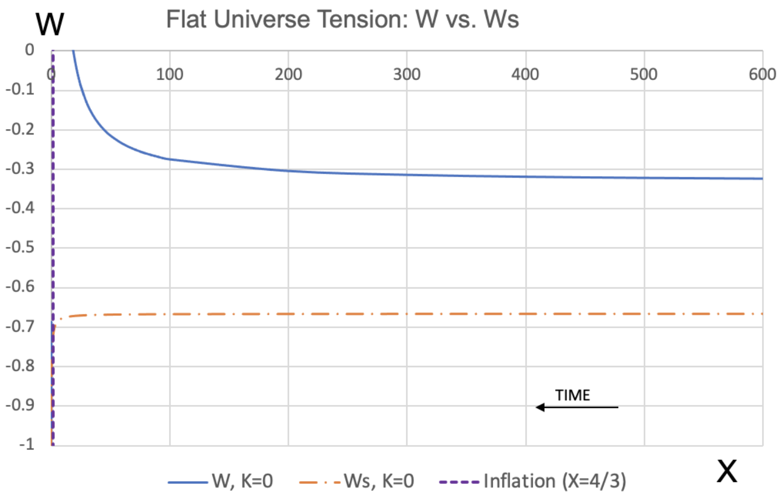

Figure 3. Flat universe equation of state–zoom in view before the Big Bang.

7. Curvature Formulation and Equation of State Tension Resolution

To resolve the problem of the equation of state tension, we venture an ansatz, where α, β are some parameters to be derived by requiring w_S to equal w,

$$\frac{k}{a^2}\frac{\bar{\sigma}_\xi^2}{Ha^2} \equiv 2\alpha_1 X, \quad \alpha_2 \equiv \frac{a^2}{H\bar{\sigma}_\xi^2} \tag{33}$$

We then have,

$$w_S = -\frac{1+2X}{1+3(1+\alpha_1)X} \tag{34}$$

The first and second Friedmann equations in the RWML theory are now given by:

$$\frac{8\pi G\rho}{3H^2} = 1 - (\frac{3}{2} - 2\alpha_1\alpha_2 + \frac{15}{2}\alpha_1)X + \frac{9}{2}(1+\alpha_1)X^2 \tag{35}$$

$$-\frac{8\pi G p}{H^2} = 1 - 2q - (\frac{15}{2}(1+\alpha_1) - 2\alpha_1\alpha_2)X + \frac{9}{2}(1+\alpha_1)X^2 \tag{36}$$

Applying $\dot{\rho} = 0$ to Equation (35), we obtain q:

$$-(1+q) = (-54X)\frac{(1+\alpha_1)X - \frac{1}{9}(\frac{3}{2} - 2\alpha_1\alpha_2 + \frac{15}{2}\alpha_1)}{(\frac{3}{2} - 2\alpha_1\alpha_2 + \frac{15}{2}\alpha_1)X - 2} \tag{37}$$

We can now solve for the two parameters. By considering the asymptotic behavior infinitely far in the past, we can first solve for α_1 to see that $\alpha_1 = 1$. The above relationships simplify, and we solve for α_2 by looking into the future. We thus have

$$X \to \infty \quad \Rightarrow \quad w \to -\frac{1}{3} \quad \Rightarrow \alpha_1 = 1, \quad w = w_S \tag{38}$$

$$X \to 0 \quad \Rightarrow \quad w_S \to -1 + 4X \quad \Rightarrow \alpha_2 = \frac{15}{4}, \quad w = w_S \tag{39}$$

$$\frac{k}{a^2} = \frac{15}{2}DH, \quad \frac{H\bar{\sigma}^2}{a^2} = \frac{4}{15} \tag{40}$$

Note that the curvature parameter is defined by the interaction term DH.

The first and second Friedmann equations are now asymptotically in agreement with the species' equations of evolution and are given by:

$$\frac{8\pi G\rho}{3H^2} = 1 - \frac{3}{2}X + 9X^2 \tag{41}$$

$$-\frac{8\pi Gp}{H^2} = 1 - 2q - \frac{15}{2}X + 9X^2 \tag{42}$$

and the species' evolution equations now become

$$\frac{\dot{\rho}_i}{\rho_i} = -3H\{(1+w_i) + 2(1+3w_i)X\} \quad \Rightarrow \quad w_S \equiv -\frac{1+2X}{1+6X} \tag{43}$$

Note that the equation of state for the diffusive species—which must be invariant to D—is now given by $w_D = -\frac{1}{3}$. Diffusion in the RWML theory has the same SF dependence as curvature in the ΛCDM model.

The acceleration equation with its asymptotic behavior becomes

$$-(1+q) = (-72X)\frac{(X-\frac{1}{12})}{(X-\frac{4}{3})} \tag{44}$$

$$X \to 0 \quad \Rightarrow \quad -(1+q) \to -\frac{9}{2}X \tag{45}$$

$$X \to \infty \quad \Rightarrow \quad -(1+q) \to -72X \tag{46}$$

The evolution of species for $X \to 0, \infty$ is summarized in Table 2.

Table 2. Evolution of species energy density with $\frac{k}{a^2} = \frac{15}{2}HD$.

Species	w_i	$\dot{\Omega}_i$	$X \to 0$	$X \to \infty$
Radiation	$\frac{1}{3}$	$(-4H-12D)\Omega_R$	$-4H\Omega_R$	$-12D\Omega_R$
Matter	0	$(-3H-6D)\Omega_M$	$-3H\Omega_M$	$-6D\Omega_M$
Diffusion	$-\frac{1}{3}$	$-2H\Omega_D$	$-2H\Omega_D$	0
Vacuum	-1	$12D\Omega_V$	0	$12D\Omega_V$

We can take a rather simplistic approach to the scale factor to get some sense of the Hubble parameter values in the pre- and post-Big Bang eras. We find that in this overly simplistic approach, we still obtain the correct scaling between σ_ξ^2 and δt:

$$\dot{H} = -(1+q)H^2 \tag{47}$$

$$a \sim t^\beta \quad \Rightarrow \quad H \sim \frac{\beta}{t}, \quad -(1+q) \sim -\frac{1}{\beta}, \quad a \sim t^{\frac{1}{(1+q)}} \tag{48}$$

$$\Rightarrow \quad \bar{\sigma}_\xi^2 \sim \frac{1}{H}, \quad \bar{\sigma}_\xi^2 \sim \sigma_\xi^2 \frac{t}{\delta t} \quad \Rightarrow \quad \sigma_\xi^2 \sim \delta t \tag{49}$$

We can also rewrite the first and second Friedmann equations in terms of H, D, and k to clearly see the effects of geometry, diffusion, and the interaction term between the two:

$$\frac{8\pi G\rho}{3} = H^2 - \frac{1}{5}\frac{k}{a^2} + 9D^2 \tag{50}$$

$$-8\pi Gp = (1-2q)H^2 - \frac{k}{a^2} + 9D^2 \tag{51}$$

Figures 1 and 4–6 show the resulting universe equation of state and acceleration across X.

Finally, with the help of Table 2, we can now consider the possible mix of species which corresponds to the universe equation of state of $-\frac{1}{3}$ in the far past and $-1+4X \rightarrow -1$ in the far future.

For $X \rightarrow 0$, with $X^2 \approx 0$, the universe is vacuum-dominated with some small amount of matter and diffusion (we treat radiation as an insignificant contribution). In this case, we can see that for $\Omega_M = 2X, \Omega_D = 3X$, and $\Omega_V = 1-5X$, we have a constant total universe energy density and a universe equation of state converging to -1 as X goes to 0:

$$\Omega = \sum_i \Omega_i = \Omega_V + \Omega_D + \Omega_M = (1-5X) + 3X + 2X = 1 \tag{52}$$

$$\dot{\Omega} = \sum_i \dot{\Omega}_i = \dot{\Omega}_V + \dot{\Omega}_D + \dot{\Omega}_M = 12D - 6HX - 6HX + O(X^2) \approx 0 \tag{53}$$

$$w_S = \sum_i w_i \Omega_i = -\Omega_V - \frac{1}{3}\Omega_D = -(1-5X) - X = -1 + 4X \tag{54}$$

For $X \rightarrow \infty$, we can readily see that a universe dominated by diffusion alone is static. The diffusion equation of state matches the universe equation of state in this limit. The universe is entirely defined by Lorentz invariant Planck fields infinitely far in the past.

Appendix C explains the redshift data analysis used as a test of the RWML theory. Using the CMB estimate for the Hubble parameter and distance ladder methodology, a fit of the data was performed to obtain the 57 SNeIa and 19 SNeIa estimates for X [7] shown in the plots. The CMB Hubble estimate puts an upper bound on the value for X at ∼0.06. The two redshift estimates give $X_{19SNeIa} = 0.0093$ and $X_{54SNeIa} = 0.0159$, hence X is in the range of ∼0.01. The estimates are within the acceptable range. Appendix D also discusses the CMB sound horizon estimate and provides a rough estimate for the value of X based on the CMB sound horizon value of 147.5 Mpc.

Finally, it is important to note that in the present day, the universe curvature is very small and approaching zero, as it is linear in X: $\frac{k}{a^2 H^2} = \frac{15}{2} X$.

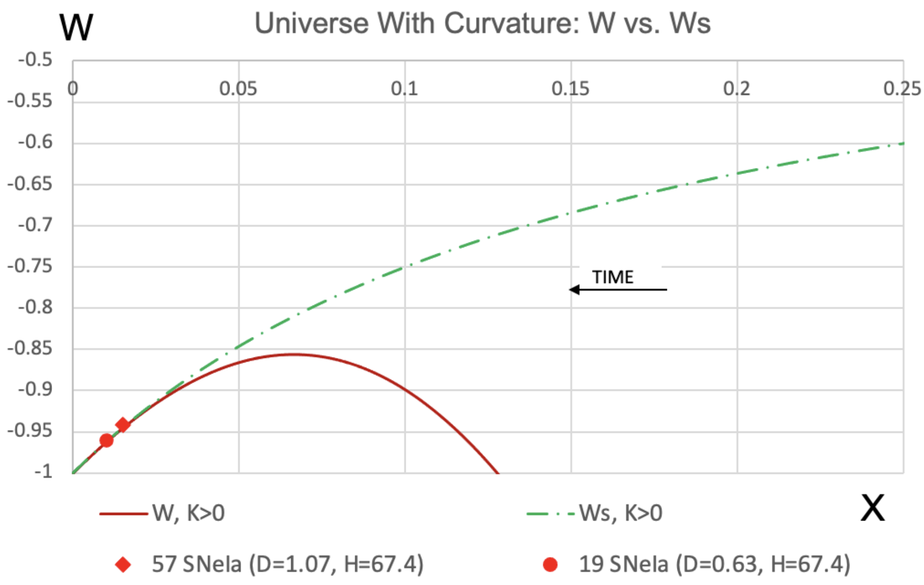

Figure 4. Universe with curvature equation of state–zoom in view after the Big Bang.

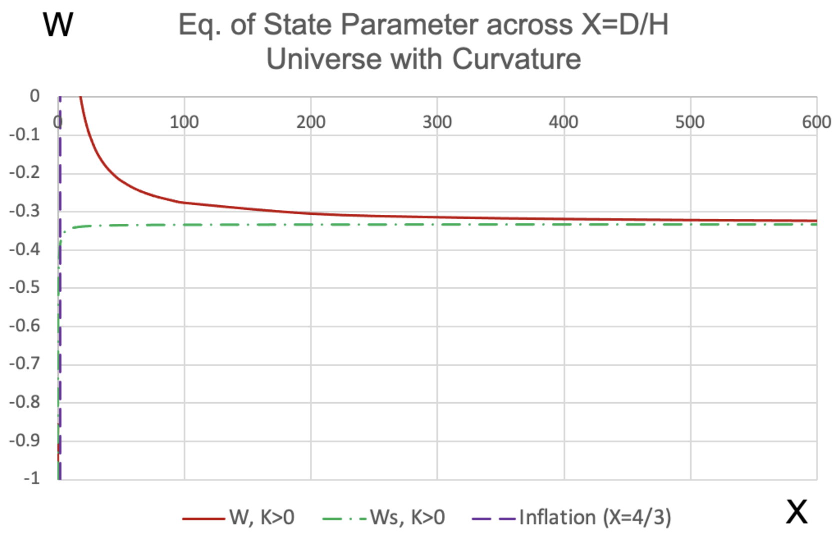

Figure 5. Universe with curvature equation of state–zoom in view before the Big Bang.

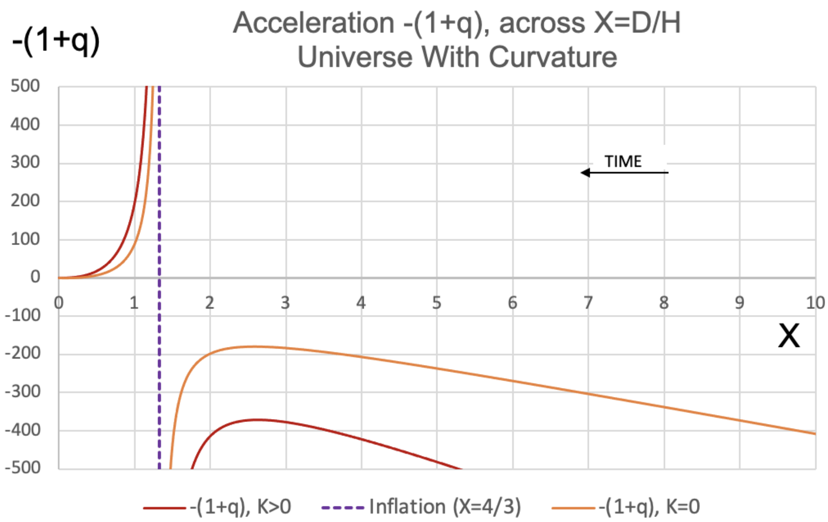

Figure 6. Universe acceleration with positive and zero curvature.

8. $X \longrightarrow 0$ Static Universe

Let us consider the problem of today's universe in more detail. Using the extended Friedmann equations, and applying similar steps used to derive scale factors for a flat Friedmann–Lemaitre universe in [13], we can derive the differential equation for H_t in terms of D_t to obtain,

$$\frac{dH_t}{dt} = \frac{3}{4}\frac{dD_t}{dt} \tag{55}$$

For $X \rightarrow 0$, we have $D \rightarrow 0$. We can integrate the differential equation from today's time t to infinitely far in the future to obtain:

$$H_t = \frac{3}{4}D_t + \sqrt{\frac{\Lambda}{3}}, \quad \Lambda \equiv 8\pi G\rho \tag{56}$$

where we have used the limit for ρ in the case of $X \rightarrow 0$ and notation similar to that in [13] for the total energy density of the universe. While we cannot apply the $a(0) = 0$ boundary value for the scale factor usually employed in the (New) Standard Model for the beginning of time, we can at the least normalize the scale factor to 1 corresponding to the estimate of $H_0 \approx$ 67.4 km s^{-1} Mpc^{-1}, and write the resulting scale factors as

$$H_0 = \frac{3}{4}c\sigma_0^2 + \sqrt{\frac{\Lambda}{3}} \quad \Rightarrow \quad a_t^6 = 1 + \sqrt{\frac{3H_0^2}{\Lambda}}(e^{6\sqrt{\frac{\Lambda}{3}}(t-t_{H_0})} - 1) \tag{57}$$

If we estimate Hubble time $t_{H_0} \approx 14 \times 10^9$ years corresponding to $\Omega_V \approx 0.7$, and if we assume that this Hubble estimate occurs late enough in time to approximate $\Omega_V \approx 1 - 5X \Rightarrow X \approx 0.06$, we can then roughly relate the Hubble time to a change in X from $X = \frac{4}{3}$ to $X \approx 0.06$. This gives us a rough relationship between X and time: for every $\Delta X = 0.01$, we have $\Delta t \approx 10^8$ years.

9. Ricci Scalar as the Planck Field Action

For simplicity, from here on, we take $a = 1$, and we consider the Planck fields in the pre-Big-Bang era. We begin with the Einstein–Hilbert action, with a choice of the sign which should become clear as we work through the derivation for the action of the Planck fields:

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R \tag{58}$$

where R is the Ricci scalar now given by

$$R = 6((1-q)H^2 - \frac{9}{2}HD + 9D^2) \tag{59}$$

With $-(1+q)H^2 \sim -72DH$, we have

$$R = 6(2H^2 - \frac{153}{2}HD + 9D^2) \approx 6(2H^2 + 9D^2) \tag{60}$$

where all omitted terms are DH terms, considered insignificant relative to X^2 terms as $X \to \infty$.

We construct Planck fields normalized to D as follows:

$$\phi^i_\mu \equiv \frac{1}{\sqrt{24\delta t}}\partial_\alpha \delta\xi^i, \quad \phi^i_\mu \phi^j_\nu = g^{ij}\delta_{\mu\nu}\frac{D}{12}, \quad D = (\phi^i_\alpha)^2 \tag{61}$$

Note that the dimension of the Planck fields normalized to the value for the diffusion parameter is $[\phi] = L^{-\frac{1}{2}}$.

For slowly rolling fields we take $\partial_\mu \phi^\alpha_\nu \approx 0$, leaving us with $\nabla_\mu \phi^\alpha_\nu \approx \Gamma^\alpha_{\mu\beta}\phi^\beta_\nu - \Gamma^\beta_{\mu\nu}\phi^\alpha_\beta$. $\partial_0 D = -6HD$ further confirms that we can treat the HD terms as insignificant in static eras. We thus obtain

$$\nabla_0\phi^i_0 \approx (H - 3D)\phi^i_0 \tag{62}$$

$$\nabla_k\phi^i_0 = \nabla_0\phi^i_k \approx -H(\phi^i_0 - \phi^i_k) \tag{63}$$

$$\nabla_k\phi^i_j \approx -\delta_{kj}(H + 6D)\phi^i_0 \tag{64}$$

and

$$(\nabla\phi)^2 \equiv (\nabla^\mu\phi^i_\alpha)(\nabla_\mu\phi^i_\alpha)^2 = -(\nabla_0\phi^i_\alpha)^2 + (\nabla_k\phi^i_\alpha)^2 \tag{65}$$

$$= -(\nabla_0\phi^i_0)^2 - (\nabla_0\phi^i_j)^2 + (\nabla_k\phi^i_0)^2 + (\nabla_k\phi^i_j)^2 \tag{66}$$

$$= \frac{3D}{4}(H^2 + 2DH - 3D^2) \tag{67}$$

$$\Rightarrow \quad H^2 = \frac{4}{3c_\xi\sigma_\xi^2}(\nabla\phi)^2 - 2DH + 3D^2 \approx \frac{4}{3c_\xi\sigma_\xi^2}(\nabla\phi)^2 + 3D^2 \tag{68}$$

Putting all this together, we have

$$R = 6(\frac{8}{3c_{\xi}\sigma_{\xi}^2}(\nabla\phi)^2 + 15(\phi^2)^2), \quad \phi^2 \equiv (\phi_{\alpha}^i)^2 \tag{69}$$

$$S = \frac{1}{\pi G}\int d^4x\sqrt{-g}(-\frac{1}{c_{\xi}\sigma_{\xi}^2}(\nabla\phi)^2 - V(\phi)), \quad V(\phi) \equiv \frac{45}{8}(\phi^2)^2 \tag{70}$$

We can renormalize the fields one more time to give the action a 'look' we are more accustomed to seeing:

$$\hat{\phi}_{\alpha}^i \equiv \sqrt{\frac{2}{\pi G c_{\xi}\sigma_{\xi}^2}}\phi_{\alpha}^i, \quad \lambda \equiv \frac{45}{8}\pi G(c_{\xi}\sigma_{\xi}^2)^2 \tag{71}$$

$$\Rightarrow \quad S = \int d^4x\sqrt{-g}(-\frac{1}{2}(\nabla\hat{\phi})^2 - \frac{\lambda}{4}(\hat{\phi}^2)^2) \tag{72}$$

This is the action for a $\hat{\phi}^4$ potential. Note that $[\hat{\phi}] = L^{-1}, [\lambda] = L^0$.

Throughout the paper we have discussed the relationship of the effects of geometry vs. diffusion on the universe. The action for Planck fields reinterprets this relationship to that of the universe kinetic vs. potential energy, respectively. In the pre-Big-Bang era, potential energy dominates and provides a compact asymptotically static universe infinitely long ago. In the post-Big-Bang era, kinetic energy dominates and provides an expanding asymptotically static universe infinitely far into the future.

The next step then is to consider the possible perturbed matter equations ultimately explaining the seeds of galaxy formation. For this, we must relax the assumption of a torsionless universe and possibly also the assumption of a conserved energy density. We leave this discussion for future research and papers. However, we can consider the effects of spontaneous symmetry breaking to see that it suggests emergent gravity.

10. Spontaneous Symmetry Breaking

In dealing with spherical potentials, we must derive the corresponding spherical ML spacetime. In this case, with the choice of $\theta = \frac{\pi}{2}$ and with the above two-point function simplifications, we have [6],

$$\delta\tilde{r} = \delta r + \frac{\sigma_{\xi}^2\delta t}{2r} + \delta\xi_{r,\phi}^r, \quad \langle\delta\xi_{r,\phi}^r\rangle = 0, \quad \langle(\delta\xi_{r,\phi}^r)^2\rangle = \sigma_{\xi}^2\delta t \tag{73}$$

$$\delta\tilde{\phi} = \delta\phi + \delta\xi_{r,\phi}^{\phi}, \quad \langle\delta\xi_{r,\phi}^{\phi}\rangle = 0, \quad r^2\langle(\delta\xi_{r,\phi}^{\phi})^2\rangle = \sigma_{\xi}^2\delta t, \tag{74}$$

where

$$\delta\xi^r = cos\phi\ \delta\xi^x + sin\phi\ \delta\xi^y \tag{75}$$

$$r\delta\xi^{\phi} = -sin\phi\ \delta\xi^x + cos\phi\ \delta\xi^y \tag{76}$$

Note that in spherical coordinates, the ML radius obtains a singularity at $r = 0$ proportional to the diffusive parameter σ_{ξ}^2. A similar problem is encountered in the case of the Schwarzschild metric. From the point of view of practical applications, tests of gravity are inherently limited by the limitation of the Schwarzschild radius. A higher accuracy requires a higher mass of the probe which in turn requires a smaller volume for the mass to inhabit, forcing us to 'cross' the Schwarzschild radius at some point [16]. However, this singularity is of a non-physical nature and resolved by the choice of frame of reference corresponding to the observer sitting inside or outside the sphere defined by the Schwarzschild radius. Hence, we do not concern ourselves with the singularity per se at that point and leave it for further discussion in the future.

Let us now consider how this new term impacts the geometry:

$$dr \to dr + \delta\xi^r + \frac{\sigma_\xi^2}{2r} dt \tag{77}$$

$$\Rightarrow \quad dr^2 \to dr^2 + \frac{\sigma_\xi^2}{2r}(drdt + dtdr) + \sigma_\xi^2 dt \tag{78}$$

$$ds^2 = -dt^2 + dr^2 + \frac{\sigma_\xi^2}{2r}(drdt + dtdr) + O(r^2), \quad r^2 d\Omega^2 = O(r^2) \tag{79}$$

$$\to \quad -(dt - \frac{1}{2}\sigma_\xi^2)^2 + dr^2 + \frac{\sigma_\xi^2}{2r}(drdt + dtdr) + O(r^2) + O(\sigma_\xi^4) \tag{80}$$

We can simplify the expression by performing a change of variables for dt, and then we can rename dt' to show that the σ_ξ^2 term is in fact of no physical consequence:

$$dt' \equiv dt - \frac{1}{2}\sigma_\xi^2 \quad \Rightarrow ds^2 = -(dt')^2 + dr^2 + \frac{\sigma_\xi^2}{2r}(drdt' + dt'dr) + O(r^2) + O(\sigma_\xi^4) \tag{81}$$

$$dt \equiv dt' \quad \Rightarrow ds^2 = -dt^2 + dr^2 + \frac{\sigma_\xi^2}{2r}(drdt + dtdr) + O(r^2) + O(\sigma_\xi^4) \tag{82}$$

However, the same is not true for the $\frac{\sigma_\xi^2}{2r}$ term!

In Minkowski space, for a particle on a null path, we have $dr = \pm dt$. Clearly, this is not the case here. For some small perturbation ϵ ($|\epsilon| \ll 0$), we can instead try $dr = \pm(1+\epsilon)dt$. With $r \approx 0$, we require $dr > 0$ as r must be greater than or equal to zero. Hence, for $dt > 0$, only $dr = (1+\epsilon)dt$ is possible, and for $dt < 0$, only $dr = -(1+\epsilon)dt$ is allowed. We then have:

$$dr = (1+\epsilon)dt \quad \Rightarrow \quad ds^2 \to dt^2(2\epsilon + 2\frac{\sigma_\xi^2}{2r}) = 0 \quad \Rightarrow \quad \epsilon = -\frac{\sigma_\xi^2}{2r} \tag{83}$$

$$dr = -(1+\epsilon)dt \quad \Rightarrow \quad ds^2 \to dt^2(2\epsilon - 2\frac{\sigma_\xi^2}{2r}) = 0 \quad \Rightarrow \quad \epsilon = +\frac{\sigma_\xi^2}{2r} \tag{84}$$

This provides us with the following values for $\frac{dt}{dr}$:

$$dt > 0 \quad \Rightarrow \quad \frac{dt}{dr} = \frac{1}{1 - \frac{\sigma_\xi^2}{2r}} \tag{85}$$

$$dt < 0 \quad \Rightarrow \quad \frac{dt}{dr} = -\frac{1}{1 + \frac{\sigma_\xi^2}{2r}} \tag{86}$$

Going backwards in time ($dt < 0$) puts the particle outside the light cone and thus outside the physical spacetime. Thus, going back in time is not a physical possibility, and only a change in time into the future is possible and corresponds to the $\frac{dt}{dr}$ term of the Schwarzschild metric, provided that $\sigma_\xi^2 = 4G\delta m$, where δm is the mass of the Schwarzschild metric. In solving for the curvature parameter and $\frac{H\bar{\sigma}_\xi^2}{a^2} = \frac{4}{15}$, we have already established that $\sigma_\xi^2 \sim \delta t$. With δt at the scale of the Planck length, we can relate $\delta m \sim \frac{\delta t}{4G} \sim \frac{1}{4\sqrt{G}}$. We see that δm scales with the Planck mass. The resulting emergent gravity appears to be associated only with the forward moving arrow of time.

We can now turn our attention to the Planck fields. In Cartesian coordinates, we have $\langle\hat{\phi}\rangle = 0$:

$$\hat{\phi}_\mu^i \sim \partial_\mu \delta\xi^i \quad \Rightarrow \quad \langle\hat{\phi}_\mu^i\rangle = 0 \tag{87}$$

In spherical coordinates, $\hat{\phi}^r_r$ obtains a nonzero vacuum expectation value (see Equation (73)),

$$\hat{\phi}^r_r \sim \partial_r \delta\xi^r - \frac{\sigma_\xi^2 \delta t}{2r^2} \quad \Rightarrow \quad \langle \hat{\phi}^r_r \rangle \sim -\frac{\sigma_\xi^2 \delta t}{2r^2} \tag{88}$$

This spontaneous symmetry breaking must be accompanied by a new term in the Lagrangian corresponding to a sombrero hat-type potential:

$$S = \int d^4x \sqrt{-g}(-\frac{1}{2}(\nabla\hat{\phi})^2 + \mu^2\hat{\phi}^2 - \frac{\lambda}{4}(\hat{\phi}^2)^2) \tag{89}$$

$$\mu = 0 \quad \rightarrow \quad \mu = \sqrt{\lambda}\sqrt{\frac{\sigma_\xi^2 \delta t}{48\pi G c_\xi}}\frac{1}{r^2} \tag{90}$$

This is exactly the case of spontaneous symmetry breaking in a Linear Sigma Model [17]. The $\hat{\phi}^r_r$ field obtains a bare mass term $(2\mu^2)$, and the potential is minimized at $(\hat{\phi}^r_r)^2 = \frac{\mu^2}{\lambda}$ (see p. 349 of [17]).

11. U(1)×SU(2) and the Scale of Planck Fields' Couplings

We leave a more detailed discussion of the impact of Planck fields in the Glashow–Weinberg–Salam (GWS) theory for future research, but we show that the force carriers for Planck fields carry an extra factor of $2c_\xi\sigma_\xi^2\delta t$ in their self-interaction coupling constants.

First, consider that any function in stochastic spacetime can be written as a function in regular space plus a term containing $\delta\xi$:

$$f(\tilde{x}) = \alpha(x) + \beta_\mu(x)\delta\xi^\mu_x \tag{91}$$

$$\alpha(x) = f(x) + \frac{1}{2}\partial^2 f(x)\sigma^2\delta t, \quad \beta_\mu(x) = \partial_\mu f(x) \tag{92}$$

Hence, even a simple phase transformation in stochastic spacetime carries uncertainty.

Let us first consider a much simpler problem, a U(1) gauge transformation. With the help of the steps in [6] which show how to formulate the connection in stochastic spacetime, we can derive the covariant derivative in this case:

$$\psi(\tilde{x}) \rightarrow e^{\alpha(x)+\beta_\mu(x)\delta\xi^\mu_x}\psi(\tilde{x}) \tag{93}$$

$$U(x+\epsilon, x) = 1 + ie\epsilon^\mu A_\mu(x) + ig\epsilon^\mu R_\alpha(x)\partial_\mu\delta\xi^\alpha_x \tag{94}$$

$$T_{\delta\xi_{x+\epsilon}}U(x+\epsilon, x)\tilde{T}_{-\delta\xi_x} = 1 + \epsilon^\mu(-\tilde{\partial}_\mu - \partial_\mu) + ie\epsilon^\mu A_\mu(\tilde{x}) + ig\epsilon^\mu R_\alpha(\tilde{x})\partial_\mu\delta\xi^\alpha_x \tag{95}$$

$$\tilde{\partial}_\mu\delta\xi^\alpha_x = \partial_\mu\delta\xi^\alpha_x - \partial_\mu\delta\xi^\beta_x\partial_\beta\delta\xi^\alpha_x = \partial_\mu\delta\xi^\alpha_x - 2c_\xi\sigma_\xi^2\delta t g^{\beta\alpha}\delta_{\mu\beta} \tag{96}$$

$$\tilde{D}_\mu = \tilde{\partial}_\mu - ieA_\mu(\tilde{x}) - igR_\alpha(\tilde{x})\tilde{\partial}_\mu\delta\xi^\alpha_x - 2igc_\xi\sigma_\xi^2\delta t R_\alpha(\tilde{x})g^{\beta\alpha}\delta_{\mu\beta} \tag{97}$$

We see that the covariant derivative in stochastic spacetime carries a new force carrier for Planck fields, a vector boson R_α. But this vector boson does not appear alone. Instead, it is coupled to a kinetic stochastic field, $\partial\delta\xi$.

The above connection formulation assumes that $\partial_\mu\beta_\alpha\delta\xi^\alpha << \beta_\alpha\partial_\mu\delta\xi^\alpha$. This condition is satisfied as long as β is some smooth function of x. In this case, the new vector boson transforms according to $R_\alpha \rightarrow R_\alpha + \frac{1}{g}\beta_\alpha$.

With everything now expressed in terms of $\tilde{x}$, we can revert to the regular notation (remembering that we are working in the stochastic spacetime). We can continue on with this simple U(1) analysis to find

$$\langle|D^2|\rangle = \partial^2 + e^2A^2 + 4c_\xi\sigma_\xi^2\delta t g^2 R^2 + \ldots \tag{98}$$

$$\langle[D_\mu, D_\nu]\rangle = -ie\mathcal{F}_{\mu\nu} - ig\langle(\partial_\mu R_\alpha\partial_\nu\delta\xi^\alpha_x - \partial_\nu R_\alpha\partial_\mu\delta\xi^\alpha_x)\rangle = -ie\mathcal{F}_{\mu\nu} \tag{99}$$

The story is no different when we look at the GWS gauge transformation. The fact that the vector bosons of the Planck fields come coupled with the kinetic Planck fields guarantees the Planck bosons obtain a factor of $2c_\xi\sigma_\xi^2\delta t$ when we formulate $\Delta\mathcal{L} = \frac{1}{2}\begin{pmatrix}0 & v\end{pmatrix}|\tilde{D}^2|\begin{pmatrix}0\\v\end{pmatrix}$.

Strong, electromagnetic, and weak force coupling constants scale as $\alpha_s \sim 1$, $\alpha \sim \frac{1}{137}$, and $\alpha_W \sim 10^{-6}$, respectively. But gravity scales as $\alpha_g \sim 10^{-39}$. With $c_\xi\sigma_\xi^2 \sim D \sim 10^{-2}$, and δt scaling with the Planck length, we see that for $g^2 \sim e^2$, the coupling constant for the force carriers of Planck fields scales as 10^{-37}. The question remains as to the exact value for $\frac{g^2}{e^2}$; however, we see that Planck fields provide for couplings at a much smaller scale, in fact, a scale compatible with what we expect to see in the case of gravity.

Past models have formulated quantum field theory with fields as classical random variables [18] when considering inflation. We can draw on such research to consider how the above Planck fields might play a role in inflation and the Big Bang. We leave this as a topic for further research.

12. Conclusions

There are still many questions remaining in cosmology. Some of the favorite remaining puzzles, such as the Hubble Tension, keep appearing in the news. Not only are we often trying to understand the theory behind the various cosmological parameters of interest, but often, even our experimental observations have trouble obtaining scientific community consensus [19,20].

We found that the RWML theory suggested an infinitely old universe with Planck fields deriving from stochastic spacetime fluctuations dominating over geometry in the pre-Big Bang era, while geometry dominated over diffusion after the Big Bang. Under the assumption of a torsionless stress energy tensor, the theory suggested that a violent implosion preceded the Big Bang and was followed by the post-Big Bang era so successfully described by the ΛCDM theory. While the RWML theory clearly supported the Big Bang singularity, thus negating a repetitive cycle of expansion and compression proposed by the Big Bounce hypothesis, it might provide for the possibility of a less violent Big Bang muted by torsion.

Under the RWML theory, the Hubble parameter H measured the speed of metric expansion as usual. Diffusion was parametrized by the diffusive parameter D and contributed new terms to the total universe energy density and pressure. During the pre-Big Bang Planck era, D dominated over H. The present-day universe was, in contrast, dominated by its expansion, $H >> D$. The end conditions of a stable universe required an agreement between the universe equation of state and the sum of species' equation of state. Imposing such a condition provided an estimate of the curvature parameter. We found that the curvature parameter took on the same positive functional form during the pre-Big Bang era as during the post-Big Bang era. We also learned that the curvature, k, was a measure of the interaction between diffusion and geometry, $k \sim HD$.

We constructed the diffusion parameter D in terms of Planck fields, $D \sim \phi^2$, while the H^2 term in the Ricci scalar provided the corresponding kinetic term for Planck fields, $(\nabla\phi)^2$. With the resulting action defined entirely by the Ricci scalar, we found Planck fields in a ϕ^4 potential, similar to the linear sigma model of the quantum field theory (QFT) prior to spontaneous symmetry breaking.

The transition from Cartesian to spherical space broke the symmetry of stochastic spacetime by giving one of the Planck fields a nonzero expectation value. This spontaneous symmetry breaking motivated a sombrero hat potential and a positive bare mass term for one of the fields, with other fields remaining massless.

We briefly considered the impact of Planck fields in U(1)×SU(2) algebra. We found the Planck field couplings to be at a much smaller scale compared to the couplings of the standard model's weak interactions and consistent with the scale expected for gravitational interactions.

Finally, the RWML leaves us with the question of how torsion might play a role in possibly muting the effects of the violent implosion and consequent explosion at the Big Bang singularity. More importantly, how might the torsion and evolving energy density possibly provide an alternative model to inflationary perturbations allowing Planck fields to ultimately explain the seeds of galaxy formation? Here, we provided the groundwork allowing us to consider these next steps.

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Data Availability Statement: The data that support the findings of this study are openly available. See [7,21–27] for additional information and data sources.

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Appendix A. Additional Summary of RWML Derivations

We summarize here some of the derivations in [7]. Beginning with the requirement that the proper time functional must be conserved, including under a stochastic translation, we obtain the extended Christoffel connection. From [7],

$$\begin{aligned} I &= -\frac{1}{2}\int d\tau \frac{dx^\mu}{d\tau} g_{\mu\nu} \frac{dx^\nu}{d\tau}, \\ & \quad x^\mu \to x^\mu + \delta\overline{x}^\mu + \delta\xi^\mu, \quad \delta\xi^\mu \sim \mathcal{N}(0, \sigma\sqrt{\delta t}) \end{aligned} \tag{A1}$$

$$\begin{aligned} I \to -\frac{1}{2}\int d\tau \{ & \frac{d(x^\mu + \delta\overline{x}^\mu + \delta\xi^\mu)}{d\tau} \\ & \times [g_{\mu\nu} + (\partial_\rho g_{\mu\nu})(\delta\overline{x}^\rho + \delta\xi^\rho) \\ & + \frac{1}{2}(\partial_\alpha\partial_\beta g_{\mu\nu}) h_\rho^{\alpha\beta} \delta\overline{x}^\rho] \frac{d(x^\nu + \delta\overline{x}^\nu + \delta\xi^\nu)}{d\tau} \} \end{aligned} \tag{A2}$$

$$\langle \delta I \rangle = \int d\tau g_{\mu\rho} \delta\overline{x}^\rho \{ \frac{d^2x^\mu}{d\tau^2} + \tilde{\Gamma}^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} \} = 0 \tag{A3}$$

$$\tilde{\Gamma}^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} - \mathcal{H}^\mu_{\alpha\beta} \tag{A4}$$

It is straightforward, although algebraically a bit cumbersome, to obtain the extended Christoffel connection in Minkowski geometry (see A.2 in [7]). To be able to transform to the RW geometry requires a formulation of the stochastic variable transformation under a metric transformation.

We allow the stochastic variables to transform between $g_{i,j}$ and $g_{\hat{i},\hat{j}}$ most generally by setting

$$\frac{\partial x^i}{\partial x^{\hat{i}}} = a, \quad \frac{\partial \delta\xi^{\hat{i}}}{\partial \delta\xi^i} = \frac{1}{a^\gamma} \tag{A5}$$

where γ is to be defined. By requiring the resulting energy density to have a minimum with a positive Hubble parameter, corresponding to the present-day universe, we obtain the lowest integer value for γ, $\gamma = 4$. Setting γ to four, the connection elements provided in Equations (8)–(12) follow, and D obtains the SF dependence of $D \sim \frac{1}{a^6}$.

Appendix B. Diffusion and Probability Density

The solution to the diffusion equation in one spatial dimension $\frac{d\rho}{dt} = k\partial_x^2\rho$ is given by

$$\rho(x) = \frac{1}{\sqrt{4\pi kt}} e^{-\frac{x^2}{4kt}} \tag{A6}$$

Any function with the functional form in Equation (A6) must satisfy a corresponding diffusion equation.

For a stochastic variable $\xi \sim \mathcal{N}(0, \bar{\sigma}\sqrt{t})$ where $\langle \xi^2 \rangle = \bar{\sigma}^2 t$, the normalized probability density function takes on the form

$$\psi(\xi) = \frac{1}{\sqrt{4\pi\bar{\sigma}^2 t}} e^{-\frac{\xi^2}{4\bar{\sigma}^2 t}} \tag{A7}$$

$\psi(\xi)$ has the same functional form as in Equation (A6), hence it is diffusive in ξ over t. This can be generalized for any choice of ξ and t. If t grows, the width of the distribution grows with t.

Appendix C. Fitting to Redshift Data

The ΛCDM model with the commonly used value for deceleration of $q = -0.55$ provides estimates for the Hubble parameter based on late redshift data in tension with the CMB estimate (see Figure A1 for the ΛCDM estimate based on 19 SNe Ia galaxies' data). The RWML model was used in [7] to fit to the late-time redshift data by using the CMB Hubble estimate value and in order to obtain the late-time estimates for X. In [7], the author reports values for X in the vicinity of 0.01. Here, we perform the same analysis but with some modifications. We assume a slowly changing value for X in both fits, allowing us to estimate a single X corresponding to the small ($z_{helio} < 0.077$) and very small ($z_{helio} < 0.0177$) redshift values for 57 SNe Ia and 19 SNe Ia galaxies, respectively.

The analysis here differs from [7] in several ways. First, the analysis here uses the results for a universe with a positive curvature and consequently resolves the w vs. w_S tension, while [7] performed the redshift analysis assuming a zero curvature. Therefore, the extended Friedmann equations and evolution equations are different between [7] and here. Furthermore, [7] approximated the vacuum together with the diffusive species as a 'diffusive vacuum', whereas here, we correctly differentiate between all the species.

With $\frac{dt}{d\lambda} \sim \frac{1}{a} e^{\frac{D}{H}}$ as the solution for Equation (17) from [7],

$$\frac{d^2 t}{d\lambda^2} = -(H + 6D)(\frac{dt}{d\lambda})^2 \tag{A8}$$

we obtain the redshift for a very slowly varying Hubble parameter ($\dot{H} \approx 0$),

$$(1+z) = \frac{a_2}{a_1} e^{\frac{D_1}{H_1} - \frac{D_2}{H_2}}, \quad t_1 < t_2 \tag{A9}$$

where t_1 is the time of signal emission, and t_2 is the time of its observation. We assume $H_1, H_2 \approx H_0$, the Hubble parameter CMB estimate of H_0 = 67.4 km s^{-1} Mpc^{-1} corresponding to $\Omega_V \approx 0.7$. Further assuming we are in the era of small X, we can apply $\Omega_V = 1 - 5X$, to obtain $X \approx 0.06$, consistent with what we see on Figure 4. Choosing t_2 to correspond to today's time, and thus normalizing X, a to $X_2 \approx 0$, $a_2 = 1$, the redshift in Equation (A9) reduces to

$$(1+z) = \frac{1}{a} e^{X} \tag{A10}$$

where we have relabeled the parameters associated with the time of emission, $a \equiv a_1$, $X \equiv X_1$.

With $\dot{H} \approx 0$, we can approximate $\int_{t_1}^{t_2} dt' D_{t'} \approx -\frac{1}{6H_0}(D_2 - D_1) = \frac{1}{6}X$ to obtain,

$$\Omega_M = \Omega_{M,0}(1+z)^3\, e^{-4X}, \quad \Omega_{M,0} = 2X \tag{A11}$$

$$\Omega_D = \Omega_{D,0}(1+z)^2\, e^{-2X}, \quad \Omega_{D,0} = 3X \tag{A12}$$

$$\Omega_V = \Omega_{V,0}\, e^{2X}, \quad \Omega_{V,0} = 1 - 5X \tag{A13}$$

With the above, we can write

$$H(z)^2(1-\frac{3}{2}X+9X^2) \approx \\ H_0^2(\Omega_{M,0}e^{-4X}(1+z)^3+\Omega_{D,0}e^{-2X}(1+z)^2+\Omega_{V,0}e^{2X}) \tag{A14}$$

From Equation (A104) in [7], the luminosity distance under the RWML is given by $d_L = (1+z)\frac{e^{-X}}{1+6X}\int_0^z \frac{dz'}{H(z')}$ [7]. After a bit of algebra, assuming a small value for z, we obtain

$$d_L = \frac{1}{H_0}\frac{e^{-X}}{(1+6X)}\sqrt{\frac{1-\frac{3}{2}X+9X^2}{\Omega_{M,0}e^{-4X}+\Omega_{D,0}e^{-2X}+\Omega_{V,0}e^{2X}}} \\ \cdot(z(1+z)-z^2\frac{3\Omega_{M,0}e^{-4X}+2\Omega_{D,0}e^{-2X}}{4(\Omega_{M,0}e^{-4X}+\Omega_{D,0}e^{-2X}+\Omega_{V,0}e^{2X})}) \\ +O(z^3) \tag{A15}$$

We fit the above luminosity distance model to available redshift data from Riess et al. (2022) [7,21–26] and PantheonPlusSHOES Github [27].

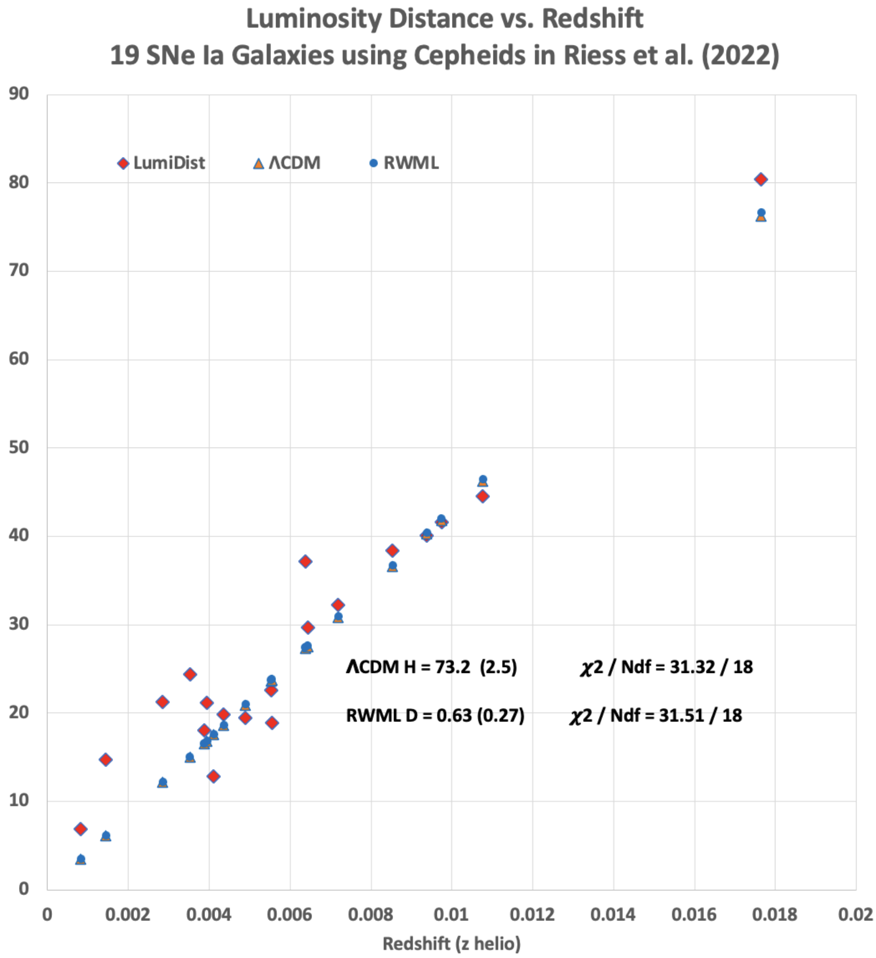

Figure A1. Nineteen SNe Ia galaxies with distances calibrated using Cepheids data from Table 1 in Riess et al. (2022) [7,21–26] corresponding to redshift data $z_{helio} < 0.0177$, along with z_{helio} data from PantheonPlusSHOES Github [27]. The RWML model fit was performed using H_0 = 67.4 km s^{-1} Mpc^{-1}. The ΛCDM model used $q = -0.55$.

The resulting estimates for X were used in the various Figures above. The RWML model with a curvature provided a slightly higher value for X corresponding to the 57 SNe Ia Hubble Flow H galaxies than what was reported in [7] and a slightly lower value for the case of 19 SNe Ia galaxies using Cepheids but remained in the 0.01 range.

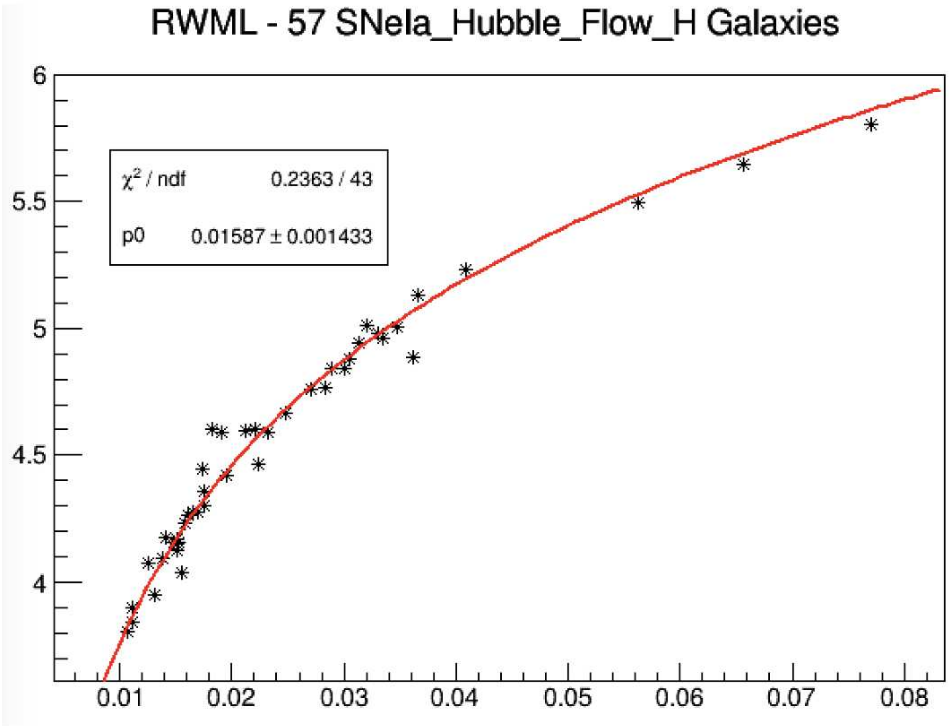

Figure A2. Natural log of luminosity distance fitted with an implementation of the RWML model for a positive curvature in ROOT with user-defined functions using $z_{helio} < 0.077$ for 57 SNe Ia Hubble flow galaxies shown in Table 2 of Riess et al. (2022) [7,21–26]. The RWML model fit was performed using H_0 = 67.4 km s^{-1} Mpc^{-1}.

Appendix D. Cosmic Distance Scale Ladder and CMB Considerations

In considering cosmic microwave background (CMB), we quote from [28], 'It is important to stress that the CMB estimates of H_0 are extrapolations, and therefore are cosmological model-dependent'.

CMB data provide us with means of estimating cosmological parameters in some cases to a very high degree of precision. However, to fit to CMB data appropriately in the case of the RWML model described here, we must first better understand the evolution of Planck fields in the vicinity of the Big Bang.

However, we can go ahead and approximate the sound horizon, r_d, assuming a large redshift ($\sim$1100), in order to obtain an estimate for X in the vicinity of the sound horizon values provided by a variety of different models. $r_d = 147.5$ Mcp [28], with the expectation that the result ought to be somewhat higher from the linear expectation of $X \approx 0.05$. Hence, we are simply testing for a reasonable answer under the RWML, knowing full well that the asymptotic approximation is no longer applicable as X becomes a bit larger.

The sound horizon can be formulated from Equations (2) and (8) in [28] but now applied to the RWML under the assumption of a small X ($\Omega_M = 2X, \Omega_D = 3X, \Omega_V = 1 - 5X$):

$$H(z) = \frac{H_0}{\sqrt{1 - \frac{3}{2}X + 9X^2}} (\Omega_M (1+z)^3 e^{-4X} + \Omega_D (1+z)^2 e^{-2X} + \Omega_V z^{12X} e^{2X})^{\frac{1}{2}} \quad \text{(A16)}$$

where the LIVE density function redshift dependency follows from Equation (2) in [28] and the linear approximation for the universe under a small X.

We can now perform the integration for large redshift $z >> 1$, using Equation (8), to obtain the sound horizon:

$$r_d = \frac{ce^{2X}\sqrt{1-\frac{3}{2}X+9X^2}}{H_0\sqrt{3\Omega_M}}\left(\frac{2}{z^{\frac{1}{2}}} - \frac{1}{3}\frac{\Omega_D}{\Omega_M}\frac{e^{2X}}{z^{\frac{3}{2}}} - \frac{1}{4(1-6X)}\frac{\Omega_V}{\Omega_M}\frac{e^{6X}}{z^{2(1-6X)}}\right) \quad \text{(A17)}$$

where c is the speed of light. Matching r_d to the value of 147.5 corresponding to the predictions from the other models, we obtain $X \approx 0.11$.

To properly perform this fit along with the full spectral analysis requires a formulation of the RWML theory to incorporate the effects of torsion and possibly relax the requirement of conserved energy density and thus is left for a future paper.

References

1. Riordon, J. The Universe Began with a Bang, Not a Bounce, New Studies Find. SciAm. 2023. Available online: https://www.scientificamerican.com/article/the-universe-began-with-a-bang-not-a-bounce-new-studies-find/ (accessed on 5 August 2024).
2. Kinney, W.; Stein, N. Cyclic cosmology and geodesic completeness. *J. Cosmol. Astropart. Phys.* **2022**, *2022*, JCAP06(2022)011. [CrossRef]
3. Poplawski, N. Cosmology With Torsion: An Alternative To Cosmic Inflation. *Phys. Lett. B* **2010**, *694*, 3. [CrossRef]
4. Gao, C.; Kunz, M.; Liddle, A.; Parkinson, D. Unified dark energy and dark matter from a scalar field different from quintessence. *Phys. Rev. D* **2010**, *81*, 043520. [CrossRef]
5. Vedral, V. Classical Evolution without Evolution. *Universe* **2023**, *9*, 394. [CrossRef]
6. Pilipović, D. The Algebra and Calculus of Stochastically Perturbed Spacetime with Classical and Quantum Applications. *Symmetry* **2024**, *16*, 36. [CrossRef]
7. Pilipović, D. Late-time dark energy and Hubble tension. *Open Astron.* **2023**, *32*, 20220221. [CrossRef]
8. Pilipović, D. Minimum length universe in a Robertson Walker geometry. *arXiv* **2022**, arXiv: 2210.08968 .
9. El-Nabulsi, R.A. Asymptotically Static Universe Dominated by Phantom Energy. *Z. Naturforschung A* **2015**, *70*, 2. [CrossRef]
10. El-Nabulsi, R.A.; Anukool, W. Oscillating gravity, non-singularity and mass quantization from Moffat stochastic gravity arguments. *Commun. Theor. Phys.* **2022**, *74*, 105405. [CrossRef]
11. Marletto, C.; Vedral, V. Unpredictability is perfectly possible in a deterministic universe. *arXiv* **2022**, arXiv: 2210.09050.
12. Grøn, Ø. Can Stiff Matter Solve the Hubble Tension? *Axioms* **2024**, *13*, 526. [CrossRef]
13. Grøn, Ø. A new standard model of the universe. *Eur. J. Phys.* **2002**, *23*, 135. [CrossRef]
14. Kouwn, S.; Lee, J.; Lee, T.H.; Moon, T.; Oh, P. Asymptotically static universe. *Phys. Rev. D* **2012**, *85*, 063519. [CrossRef]
15. Atale, O. Asymptotically de-Sitter solution with negative gravitational constant from El-Nabulsi's non-standard Lagrangian. *Indian J. Phys.* **2023**, *98*, 1893–1900. [CrossRef]
16. Vedral, V. Are There Any Real Problems With Quantum Gravity? *arXiv* **2022**, arXiv: 2204.01718.
17. Peskin, M.; Schroeder, D. *An Introduction to Quantum Field Theory*; Addison-Wesley: Reading, PA, USA, 1995.
18. Prokopec, T.; Tsamis, N.C.; Woodard, R.P. Stochastic Inflationary Scalar Electrodynamics. *Ann. Phys.* **2008**, *323*, 1324–1360. [CrossRef]
19. Freedman, W.L. Measurements of the Hubble Constant: Tensions in Perspective. *Astrophys. J.* **2021**, *919*, 16. [CrossRef]
20. Kruesi, L. The Webb Telescope Further Deepens the Biggest Controversy in Cosmology. *Quanta Magazine*, 13 August 2024.
21. Galbany, L.; Jaeger, T.; Riess, A.G.; Müller-Bravo, T.E.; Dhawan, S.; Phan, K.; Stritzinger, M.D.; Karamehmetoglu, E.; Leibundgut, B.; Burns, C.; et al. An updated measurement of the Hubble constant from near-infrared observations of Type Ia supernovae. *arXiv* **2022**, arXiv: 2209.02546. [CrossRef]
22. Riess, A.G.; Strolger, L.G.; Casertano, S.; Ferguson, H.C.; Mobasher, B.; Gold, B.; Challis, P.J.; Filippenko, A.V.; Jha, S.; Li, W.; et al. New Hubble Space Telescope discoveries of Type Ia supernovae at z >=1: Narrowing constraints on the early behavior of dark energy. *Astrophys. J.* **2007**, *659*, 98–121. [CrossRef]
23. Kamionkowski, M.; Riess, A.G. The Hubble Tension and Early Dark Energy. *Annu. Rev. Nucl. Part. Sci.* **2023**, *73*, 153–180. [CrossRef]
24. Altavilla, G.; Fiorentino, G.; Marconi, M.; Musella, I.; Cappellaro, E.; Barbon, R.; Benetti, S.; Pastorello, A.; Riello, M.; Turatto, M.; et al. Cepheid calibration of Type Ia supernovae and the Hubble constant. *Mon. Not. R. Astron. Soc.* **2004**, *349*, 1344–1352. [CrossRef]
25. Brout D.; Scolnic, D.; Popovic, B.; Riess, A.G.; Carr, A.; Zuntz, J.; Kessler, R.; Davis, T.M.; Hinton, S.; Jones, D.; et al. Pantheon + Analysis: Cosmological Contstraints. *Astrophys. J.* **2022**, *938*, 110. [CrossRef]
26. Braglia, M.; Riess, A.G. Early-Time Modified Gravity and the Hubble Tension. In *Di Valentino, E. Brout Hubble Constant Tension*; Springer: Singapore, 2024.

27. PantheonPlusSH0ES Github. Available online: https://github.com/PantheonPlusSH0ES (accessed on 20 January 2023).
28. Cuesta, A.J.; Verde, L.; Riess, A.; Jimenez, R. Calibrating the cosmic distance scale ladder: The role of the sound-horizon scale and the local expansion rate as distance anchors. *Mon. Not. R. Astron. Soc.* **2015**, *448*, 3463–3471. [CrossRef]